Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1 R	Signature	

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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9	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 36 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \quad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(a) $z - w$,

(1)

(b) zw.

(2)

Question Number	Scheme		Marks
1.	$z = 8 + 3\text{i}, \quad w = -2\text{i}$		
(a)	$z - w \left\{= (8 + 3\text{i}) - (-2\text{i})\right\} = 8 + 5\text{i}$	$8 + 5\text{i}$	B1
			[1]
(b)	$zw \left\{= (8 + 3\text{i})(-2\text{i})\right\} = 6 - 16\text{i}$	Either the real or imaginary part is correct.	M1
		$6 - 16\text{i}$	A1
			[2]
			3

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2. (i) $\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, where k is a constant

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix, find

(a) $\mathbf{B}$ in terms of k,

(2)

(b) the value of k for which $\mathbf{B}$ is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = (2 \ \ -1 \ \ 5)$$

and

$$\mathbf{E} = \mathbf{CD}$$

find $\mathbf{E}$.

(2)

Question Number	Scheme		Marks
2.	$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, $\mathbf{B} = \mathbf{A} + 3\mathbf{I}$		
(i)(a)	$\mathbf{B} = \mathbf{A} + 3\mathbf{I} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	For applying $\mathbf{A} + 3\mathbf{I}$. Can be implied by three out of four correct elements in candidate's final answer. Solution must come from addition.	M1
	$= \begin{pmatrix} 2k+4 & k \\ -3 & -2 \end{pmatrix}$	Correct answer.	A1
			[2]
(b)	$\mathbf{B}$ is singular $\Rightarrow$ $\det \mathbf{B} = 0$.		
	$-2(2k+4) - (-3k) = 0$	Applies "$ad - bc$" to $\mathbf{B}$ and equates to 0	M1
	$-4k - 8 + 3k = 0$		
	$k = -8$	$k = -8$	A1cao
			[2]
(ii)	$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$, $\mathbf{E} = \mathbf{C}\mathbf{D}$		
	$\mathbf{E} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}\begin{pmatrix} 2 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 10 \\ -6 & 3 & -15 \\ 8 & -4 & 20 \end{pmatrix}$	Candidate writes down a 3×3 matrix.	M1
		Correct answer.	A1
			[2] **6**

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3. $$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 2$ and $x = 2.5$

(2)

(b) Starting with the interval $[2, 2.5]$ use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $f(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

Question Number	Scheme		Marks
3.	$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$		
(a)	$f(2) = -1$ $f(2.5) = 3.40625$	Either any one of $f(2) = -1$ or $f(2.5) =$ awrt 3.4	M1
	Sign change (and $f(x)$ is continuous) therefore a root α exists between $x = 2$ and $x = 2.5$	both values correct, sign change and conclusion	A1
			[2]
(b)	$f(2.25) = 0.673828125 \left\{= \frac{345}{512}\right\}$ $\{\Rightarrow 2 \leqslant \alpha \leqslant 2.25\}$	$f(2.25) =$ awrt 0.7	B1
		Attempt to find $f(2.125)$	M1
	$f(2.125) = -0.2752685547...$ $\Rightarrow 2.125 \leqslant \alpha \leqslant 2.25$	$f(2.125) =$ awrt -0.3 with $2.125 \leqslant \alpha \leqslant 2.25$ or $2.125 < \alpha < 2.25$ or $[2.125, 2.25]$ or $(2.125, 2.25)$.	A1
			[3]
(c)	$f'(x) = 2x^3 - 3x^2 + 1\ \{+ 0\}$	At least two of the four terms differentiated correctly.	M1
		Correct derivative.	A1
	$f(-1.5) = 1.40625\ \left(= 1\tfrac{13}{32}\right)$ $\{f'(-1.5) = -12.5\}$	$f(-1.5) =$ awrt 1.41	B1
	$\beta_2 = -1.5 - \left(\frac{\text{"}1.40625\text{"}}{\text{"}-12.5\text{"}}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= -1.3875 \quad \left(= -1\tfrac{31}{80}\right)$	-1.3875 seen as answer to first iteration, award M1A1B1M1	
	$= -1.39$ (2 dp)	-1.39	A1 **cao**
			[5] **10**

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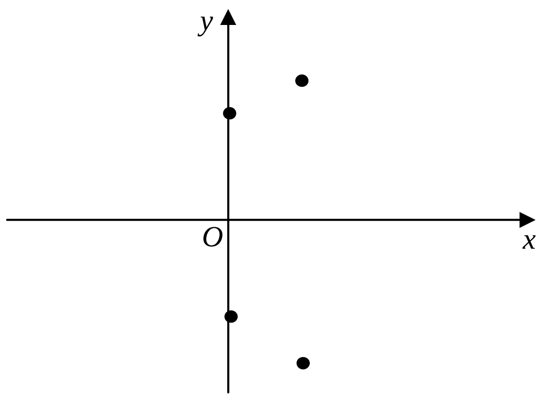
4. $$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of $f(x) = 0$

(4)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

Question Number	Scheme		Marks
4.	$\text{f}(x) = (4x^2 + 9)(x^2 - 2x + 5) = 0$		
(a)	$(4x^2 + 9) = 0 \Rightarrow x = \frac{3\text{i}}{2}, -\frac{3\text{i}}{2}$	An attempt to solve $(4x^2 + 9) = 0$ which involves i.	M1
		$\frac{3\text{i}}{2}, -\frac{3\text{i}}{2}$	A1
	$(x^2 - 2x + 5) = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$	Solves the 3TQ	M1
	$\Rightarrow x = \frac{2 \pm \sqrt{-16}}{2}$		
	$\Rightarrow x = 1 \pm 2\text{i}$	$1 \pm 2\text{i}$	A1 **[4]**
(b)		Any two of their roots plotted correctly on a single diagram, which have been found in part (a).	B1ft
		Both sets of their roots plotted correctly on a single diagram with symmetry about $y = 0$.	B1ft **[2]** **6**

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to x =...

Leave blank

5.

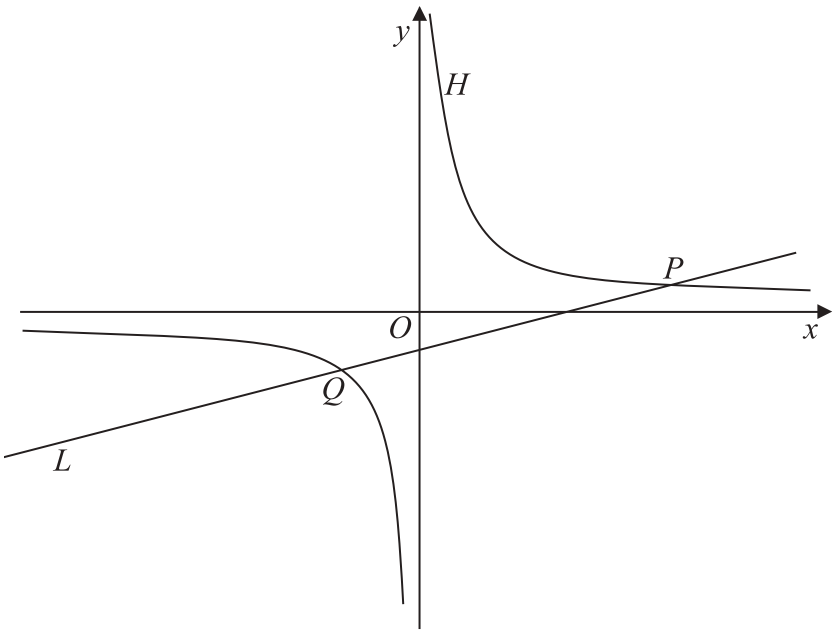


Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

Question Number	Scheme		Marks
	Ignore part labels and mark part (a) and part (b) together		
5.	$H:\ x = 3t,\ y = \dfrac{3}{t},\quad L: 6y = 4x - 15$		
(a)	$H = L \ \Rightarrow\ 6\left(\dfrac{3}{t}\right) = 4(3t) - 15$	An attempt to substitute $x = 3t$ and $y = \dfrac{3}{t}$ into L Correct equation in t.	M1 A1
	$\Rightarrow 18 = 12t^2 - 15t \ \Rightarrow\ 12t^2 - 15t - 18 = 0$		
	$\Rightarrow 4t^2 - 5t - 6 = 0$ *	Correct solution only, involving at least one intermediate step to given answer.	A1 **cso**
			[3]
(b)	$(t-2)(4t+3)\ \{= 0\}$	A valid attempt at solving the quadratic.	M1
	$\Rightarrow t = 2, -\frac{3}{4}$	Both $t = 2$ and $t = -\frac{3}{4}$	A1
	When $t = 2$, $x = 3(2) = 6,\ y = \dfrac{3}{2} \ \Rightarrow \left(6, \dfrac{3}{2}\right)$	An attempt to use one of their t-values to find one of either x or y.	M1
	When $t = -\dfrac{3}{4}$,	One set of coordinates correct or both x-values are correct.	A1
	$x = 3\left(-\dfrac{3}{4}\right) = -\dfrac{9}{4},\ y = \dfrac{3}{\left(-\frac{3}{4}\right)} = -4 \ \Rightarrow \left(-\dfrac{9}{4}, -4\right)$	Both sets of values correct.	A1
			[5] **8**
(b)	**<u>Alt Method:</u>** An attempt to eliminate either x or y from $xy = 9$ and $6y = 4x - 15$ 1st M1: A full method to obtain a quadratic equation in either x or y. 1st A1: For either $4x^2 - 15x - 54 = 0$ or $6y^2 + 15y - 36 = 0$ or equivalent. 2nd M1: A valid attempt at solving the quadratic. 2nd A1: For either $x = 6, -\dfrac{9}{4}$ or $y = \dfrac{3}{2}, -4$ 3rd A1: Both $\left(6, \dfrac{3}{2}\right)$ and $\left(-\dfrac{9}{4}, -4\right)$.		

Leave blank

6. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(a) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(b) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(c) Find the matrix **Q**.

(2)

Question Number	Scheme		Marks
6.	$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$		
(a)	$\mathbf{P} = \mathbf{AB}\ \left\{= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\right\}$	$\mathbf{P} = \mathbf{AB}$, seen or implied.	M1
	$\mathbf{P} = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$	Correct answer.	A1
			[2]
(b)	$\det \mathbf{P} = 1(-3) - (4)(-2)\ \{= -3 + 8 = 5\}$	Applies "$ad - bc$".	M1
	$\text{Area}(T) = \underline{\frac{24}{5}}\ (\text{units})^2$	$\frac{24}{\text{their } \det \mathbf{P}}$, dependent on previous M	dM1
		$\underline{\frac{24}{5}}$ or $\underline{4.8}$	A1ft
			[3]
(c)	$\mathbf{QP} = \mathbf{I} \Rightarrow \mathbf{QPP}^{-1} = \mathbf{IP}^{-1} \Rightarrow \mathbf{Q} = \mathbf{P}^{-1}$		
	$\mathbf{Q} = \mathbf{P}^{-1} = \frac{1}{5}\begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix}$	$\mathbf{Q} = \mathbf{P}^{-1}$ stated or an attempt to find $\mathbf{P}^{-1}$.	M1
		Correct ft inverse matrix.	A1ft
			[2] **7**
	Using **BA,** area is the same in (b) and inverse is $\frac{1}{5}\begin{pmatrix} 1 & -2 \\ 4 & -3 \end{pmatrix}$ in (c) and could gain ft marks.		

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(b) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(c) show that PQ is perpendicular to SQ.

(3)

Question Number	Scheme		Marks
7.	$y^2 = 4ax$, at $P(at^2, 2at)$.		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$ or (implicitly) $2y\frac{dy}{dx} = 4a$ or (chain rule) $\frac{dy}{dx} = 2a \times \frac{1}{2at}$	$\frac{dy}{dx} = \pm k\,x^{-\frac{1}{2}}$ or $ky\frac{dy}{dx} = c$ or $\dfrac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	M1
	When $x = at^2$, $\frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{at^2}} = \frac{\sqrt{a}}{\sqrt{a}\,t} = \frac{1}{t}$ or $\frac{dy}{dx} = \frac{4a}{2(2at)} = \frac{1}{t}$	$\frac{dy}{dx} = \frac{1}{t}$	A1
	T: $y - 2at = \frac{1}{t}(x - at^2)$	Applies $y - 2at =$ their $m_T(x - at^2)$ **Their m_T must be a function of t from calculus.**	M1
	T: $ty - 2at^2 = x - at^2$		
	T: $ty = x + at^2$	Correct solution.	A1 **cso** * **[4]**
(b)	At Q, $x = 0 \Rightarrow y = \frac{at^2}{t} = at \Rightarrow Q(0, at)$	$y = at$ or $Q(0, at)$	B1 **[1]**
(c)	$S(a, 0)$		
	$\text{m}(PQ) = \frac{at - 2at}{0 - at^2} = \frac{-at}{-at^2} = \frac{1}{t}$	A correct method for finding either $\text{m}(PQ)$ or $\text{m}(SQ)$ for their Q or S.	M1
	$\text{m}(SQ) = \frac{at - 0}{0 - a} = \frac{at}{-a} = -t$	$\text{m}(PQ) = \frac{1}{t}$ and $\text{m}(SQ) = -t$	A1
	$\text{m}(PQ) \times \text{m}(SQ) = \frac{1}{t} \times -t = -1 \Rightarrow PQ \perp SQ$	Shows $\text{m}(PQ) \times \text{m}(SQ) = -1$ and conclusion.	A1 **cso** **[3]** **8**

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8. (a) Prove by induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

(b) Hence, show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \frac{1}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Question Number	Scheme		Marks
8. (a)	$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$		
	$n=1$; LHS $= \sum_{r=1}^{1} r(2r-1) = 1$ RHS $= \frac{1}{6}(1)(2)(3) = 1$ As LHS = RHS, the summation formula is true for $n = 1$.	$\frac{1}{6}(1)(2)(3) = 1$ seen	B1
	Assume that the summation formula is true for $n = k$.		
	ie. $\sum_{r=1}^{k} r(2r-1) = \frac{1}{6}k(k+1)(4k-1)$.		
	With $n = k+1$ terms the summation formula becomes:		
	$\sum_{r=1}^{k+1} r(2r-1) = \underline{\frac{1}{6}k(k+1)(4k-1) + (k+1)(2(k+1)-1)}$	$S_{k+1} = S_k + u_{k+1}$ with $S_k = \frac{1}{6}k(k+1)(4k-1)$.	M1
	$= \frac{1}{6}k(k+1)(4k-1) + (k+1)(2k+1)$		
	$= \frac{1}{6}(k+1)\left(k(4k-1) + 6(2k+1)\right)$	Factorise by $\frac{1}{6}(k+1)$	dM1
	$= \frac{1}{6}(k+1)\left(4k^2 + 11k + 6\right)$	$\left(4k^2 + 11k + 6\right)$ or equivalent quadratic seen	A1
	$= \frac{1}{6}(k+1)(k+2)(4k+3)$		
	$= \frac{1}{6}(k+1)(k+1+1)(4(k+1)-1)$	Correct completion to S_{k+1} in terms of $k+1$ dependent on both Ms.	dM1
	If the summation formula is <u>true for</u> $\underline{n = k}$, then it is shown to be <u>true for</u> $\underline{n = k+1}$. As the result is <u>true for</u> $\underline{n = 1}$, it is now also <u>true for all</u> $\underline{n}$ and $n \in \mathbb{Z}^+$ by mathematical induction.	Conclusion with all 4 underlined elements that can be seen anywhere in the solution	A1 cso
			[6]

Question Number	Scheme	Marks
8. (b)	$\sum_{r=n+1}^{3n} r(2r-1) = S_{3n} - S_n$	
	$= \frac{1}{6}.3n(3n+1)(12n-1) - \frac{1}{6}n(n+1)(4n-1)$ — Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once. Correct un-simplified expression.	M1 A1
	$= \frac{1}{6}n\left\{3(3n+1)(12n-1) - (n+1)(4n-1)\right\}$	
	$= \frac{1}{6}n\left\{3(36n^2 + 9n - 1) - (4n^2 + 3n - 1)\right\}$ — Factorises out $\frac{1}{6}n$ or $\frac{1}{3}n$ and an attempt to open up the brackets.	dM1
	$= \frac{1}{6}n\left\{108n^2 + 27n - 3 - 4n^2 - 3n + 1\right\}$	
	$= \frac{1}{6}n\left\{104n^2 + 24n - 2\right\}$	
	$= \frac{1}{3}n\left(52n^2 + 12n - 1\right)$ — $= \frac{1}{3}n\left(52n^2 + 12n - 1\right)$	A1
	$\{a = 52, b = 12, c = -1\}$	**[4]** **10**

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9. The complex number w is given by

$$w = 10 - 5\text{i}$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places.

(2)

The complex numbers z and w satisfy the equation

$$(2 + \text{i})(z + 3\text{i}) = w$$

(c) Use algebra to find z, giving your answer in the form $a + b\text{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\text{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

(d) find the value of λ.

(2)

Question Number	Scheme		Marks
9.	$w = 10 - 5\mathrm{i}$		
(a)	$\lvert w\rvert = \left\{\sqrt{10^2 + (-5)^2}\right\} = \sqrt{125}$ or $5\sqrt{5}$ or $11.1803...$	$\underline{\sqrt{125}}$ or $\underline{5\sqrt{5}}$ or <u>awrt 11.2</u>	B1
			[1]
(b)	$\arg w = -\tan^{-1}\left(\frac{5}{10}\right)$	Use of $\tan^{-1}$ or $\tan$	M1
	$= -0.463647609... = -0.46$ (2 dp)	awrt -0.46 or awrt 5.82	A1 oe
			[2]
	$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$		
(c)	$z + 3\mathrm{i} = \dfrac{10 - 5\mathrm{i}}{(2 + \mathrm{i})}$	Simplifies to give $* = \dfrac{\text{complex no.}}{(2+\mathrm{i})}$	B1
	$z + 3\mathrm{i} = \dfrac{(10 - 5\mathrm{i})}{(2 + \mathrm{i})} \times \dfrac{(2 - \mathrm{i})}{(2 - \mathrm{i})}$	Multiplies by $\dfrac{\text{their } (2-\mathrm{i})}{\text{their } (2-\mathrm{i})}$	M1
	$z + 3\mathrm{i} = \dfrac{20 - 10\mathrm{i} - 10\mathrm{i} - 5}{1 + 4}$	Simplifies realising that a real number is needed on the denominator and applies $\mathrm{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$z + 3\mathrm{i} = \dfrac{15 - 20\mathrm{i}}{5}$		
	$z + 3\mathrm{i} = 3 - 4\mathrm{i}$ $z = 3 - 7\mathrm{i}$ (Note: $a = 3, b = -7$.)	$z = 3 - 7\mathrm{i}$	A1
			[4]
	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4}$		
(d)	$\lambda + 9\mathrm{i} + w = \lambda + 9\mathrm{i} + 10 - 5\mathrm{i} = (\lambda + 10) + 4\mathrm{i}$		
	$\arg(\lambda + 9\mathrm{i} + w) = \dfrac{\pi}{4} \Rightarrow \lambda + 10 = 4$	Combines real and imaginary parts and puts "Real part = Imaginary part" i.e. $\dfrac{\lambda+10}{4} = 1$ or $\dfrac{4}{\lambda+10} = 1$ o.e.	M1
	So, $\lambda = -6$	-6	A1
			[2] **9**
(c) (c)	<u>**Alt 1: Scheme as above:**</u> $(2 + \mathrm{i})z + 6\mathrm{i} + 3\mathrm{i}^2 = 10 - 5\mathrm{i} \Rightarrow (2 + \mathrm{i})z = 13 - 11\mathrm{i}$ B1 for $z = \dfrac{13 - 11\mathrm{i}}{2 + \mathrm{i}}$; M1 for $z = \dfrac{(13 - 11\mathrm{i})}{(2 + \mathrm{i})} \times \dfrac{(2 - \mathrm{i})}{(2 - \mathrm{i})}$; M1 for $z = \dfrac{26 - 13\mathrm{i} - 22\mathrm{i} - 11}{4 + 1}$; A1 for $z = 3 - 7\mathrm{i}$ <u>**Alt 2:**</u> **Let** $z = a + \mathrm{i}b$ gives $(2+i)(a + \mathrm{i}b + 3\mathrm{i}) = 10 - 5\mathrm{i}$ for B1 Equating real and imaginary parts to form two equations both involving a and b for M1 Solves simultaneous equations as far as a= or b= for M1 a=3, b=-7 or $z = 3 - 7\mathrm{i}$ for A1		

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10. (i) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to evaluate

$$\sum_{r=1}^{24}(r^3 - 4r)$$

(2)

(ii) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=0}^{n}(r^2 - 2r + 2n + 1) = \frac{1}{6}(n+1)(n+a)(bn+c)$$

for all integers $n \geqslant 0$, where a, b and c are constant integers to be found.

(6)

Question Number	Scheme		Marks
10.	$\sum_{r=1}^{24}\left(r^3 - 4r\right)$		
(i)	$= \frac{1}{4}24^2(24+1)^2 - 4.\frac{1}{2}24(24+1)$	An attempt to use at least one of the standard formulae correctly and substitute 24.	M1
	$\{= 90000 - 1200\}$		
	$= 88800$	88800	A1 **cao**
			[2]
(ii)	$\sum_{r=0}^{n}\left(r^2 - 2r + 2n + 1\right)$		
	$= \underline{\frac{1}{6}n(n+1)(2n+1) - 2.\frac{1}{2}n(n+1)} + 2n(n+1) + (n+1)$	An attempt to use at least one of the standard formulae correctly.	M1
		Correct underlined expression.	A1
		$2n \rightarrow 2n(n+1)$	B1
		$1 \rightarrow (n+1)$	B1
	$= \frac{1}{6}(n+1)\left\{2n^2 + n - 6n + 12n + 6\right\}$	An attempt to factorise out $\frac{1}{6}(n+1)$ or $\frac{1}{6}n$.	M1
	$= \frac{1}{6}(n+1)\left\{2n^2 + 7n + 6\right\}$		
	$= \frac{1}{6}(n+1)(n+2)(2n+3)$	Correct answer. (Note: $a = 2$, $b = 2$, $c = 3$.)	A1
			[6]
			8