| Centre No. | | | | | | Paper Reference | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 6 6 7 / 0 1 R | Signature | |

Paper Reference(s)

**6667/01R**

# Edexcel GCE

## Further Pure Mathematics FP1

## Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| Total | |

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Pink) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 3 1 5 2 A 0 1 2 8

***Turn over***

PEARSON

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**1.** The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are $z_1$, $z_2$ and $z_3$

Given that $z_1 = 1 + 2\text{i}$, find $z_2$ and $z_3$

**(5)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.** | $\text{f}(z) = 2z^3 - 3z^2 + 8z + 5$ | | |
| | $1 - 2\text{i}$ (is also a root) | seen | B1 |
| | $(z-(1+2\text{i}))(z-(1-2\text{i})) = z^2 - 2z + 5$ | Attempt to expand $(z-(1+2\text{i}))(z-(1-2\text{i}))$ or any valid method to establish the quadratic factor e.g. $z = 1 \pm 2\text{i} \Rightarrow z - 1 = \pm 2\text{i} \Rightarrow z^2 - 2z + 1 = -4$<br>$z = 1 \pm \sqrt{-4} = \frac{2 \pm \sqrt{-16}}{2} \Rightarrow b = -2, c = 5$<br>Sum of roots 2, product of roots 5<br>$\therefore z^2 - 2z + 5$ | M1A1 |
| | $\text{f}(z) = (z^2 - 2z + 5)(2z + 1)$ | Attempt at **linear** factor with their *cd* in<br>$(z^2 + az + c)(2z + d) = \pm 5$<br>Or $(z^2 - 2z + 5)(2z + a) \Rightarrow 5a = 5$ | M1 |
| | $(z_3) = -\frac{1}{2}$ | | A1 |
| | | | (5) |
| | | | **Total 5** |

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**2.**

$$f(x) = 3\cos 2x + x - 2, \qquad -\pi \leqslant x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root $\alpha$ in the interval $[2, 3]$.

**(2)**

(b) Use linear interpolation once on the interval $[2, 3]$ to find an approximation to $\alpha$.

Give your answer to 3 decimal places.

**(3)**

(c) The equation $f(x) = 0$ has another root $\beta$ in the interval $[-1, 0]$. Starting with this interval, use interval bisection to find an interval of width 0.25 which contains $\beta$.

**(4)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** | $f(x) = 3\cos 2x + x - 2$ | | |
| **(a)** | f(2) = -1.9609......<br>f(3) = 3.8805...... | Attempts to evaluate both $f(2)$ and $f(3)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf. | M1 |
| | **Sign change** (and $f(x)$ is continuous) therefore a **root** $\alpha$ is between $x = 2$ and $x = 3$ | Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.96.. < 0 < 3.88..) and conclusion. | A1 |
| | | | (2) |
| **(b)** | $\dfrac{\alpha - 2}{"1.9609..."} = \dfrac{3 - \alpha}{"3.8805..."}$ | Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(3). Can be implied by working below. | M1 |
| | **If any "negative lengths" are used, score M0** | | |
| | $(3.88... + 1.96....)\alpha = 3\times 1.96 + 2\times 3.88$ | | |
| | $\alpha_2 = \dfrac{3\times 1.96.. + 2\times 3.88..}{1.96... + 3.88...}$ | Follow through their values if seen explicitly. | A1ft |
| | $\alpha_2 = 2.336$ | cao | A1 |
| | | | (3) |
| | | | |
| **(c)** | f(0) = +(1) **or** f(-1) = -(4.248) | Award for correct sign, can be in a table. | B1 |
| | f(-0.5) (= -0.879.....) | Attempt f(-0.5) | M1 |
| | f(-0.25) (= 0.382....) | Attempt **f(-0.25)** | M1 |
| | $\therefore\ -0.5 < \beta < -0.25$ | oe with no numerical errors seen | A1 |
| | | | (4) |
| | | | |
| | | | **Total 9** |

Leave blank

**3.** (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

**(2)**

The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

(b) Write down the matrix **B**.

**(1)**

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \quad \text{where } k \text{ is a positive constant.}$$

Triangle $T$ has an area of 16 square units.

Triangle $T$ is transformed onto the triangle $T'$ by the transformation represented by the matrix **M**.

Given that the area of the triangle $T'$ is 224 square units, find the value of $k$.

**(3)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.(i)(a)** | Rotation of 45 degrees anticlockwise, about the origin | B1: Rotation about (0, 0)<br>B1: 45 degrees (anticlockwise)<br>-45 or clockwise award B0 | B1B1 |
| | | | (2) |
| **(b)** | $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ | Correct matrix | B1 |
| | | | (1) |
| **(ii)** | $\frac{224}{16}(=14)$ | Correct area scale factor.<br>Allow $\pm 14$ | B1 |
| | $\det \mathbf{M} = 3\times 3 - k\times -2 = 14$ | Attempt determinant and set equal to their area scale factor | M1 |
| | | Accept $\det \mathbf{M} = 3\times 3 \pm 2k$ only | |
| | $k = 2.5$ | oe | A1 |
| | | | (3) |
| | | | **Total 6** |

Leave blank

**4.** The complex number $z$ is given by

$$z = \frac{p + 2\text{i}}{3 + p\text{i}}$$

where $p$ is an integer.

(a) Express $z$ in the form $a + b\text{i}$ where $a$ and $b$ are real. Give your answer in its simplest form in terms of $p$.

**(4)**

(b) Given that $\arg(z) = \theta$, where $\tan\theta = 1$ find the possible values of $p$.

**(5)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.(a)** | $z=\frac{p+2\text{i}}{3+p\text{i}}.\frac{3-p\text{i}}{3-p\text{i}}$ | Multiplying top and bottom by Conjugate | M1 |
| | $=\frac{3p-p^2\text{i}+6\text{i}+2p}{9+p^2}$ | At least 3 correct terms in the numerator, evidence that $\text{i}^2=-1$and denominator real. | M1 |
| | $=\frac{5p}{p^2+9},\quad +\frac{6-p^2}{p^2+9}\text{i}$ | Real + imaginary with i factored out.<br>Accept single denominator with numerator in correct form.<br>Accept 'a= ' and 'b='. | A1, A1 |
| | | | (4) |
| **(b)** | $\arg(z)=\arctan\left(\frac{\frac{6-p^2}{p^2+9}}{\frac{5p}{p^2+9}}\right)$ | Correct method for the argument. Can be implied by correct equation for *p* | M1 |
| | $\frac{6-p^2}{5p}=1$ | Their $\arg(z)$ in terms of $p=1$ | M1 |
| | $p^2+5p-6=0$ | Correct 3TQ | A1 |
| | $(p+6)(p-1)=0\Rightarrow x=$ | M1:Attempt to solve their quadratic in *p* | M1 |
| | $p=1, p=-6$ | A1:**both** | A1 |
| | | | (5) |
| | | | **Total 9** |
| **(a)**<br>**Way 2** | $a+b\text{i}=\frac{p+2\text{i}}{3+p\text{i}}$ | Equate to $a+b\text{i}$ then rearrange and equate real and imaginary parts. | M1 |
| | $3a-pb=p,\ ap+3b=2$ | Two equations for *a* and *b* in terms of *p* and attempt to solve for *a* and *b* in terms of *p* | dM1 |
| | $=\frac{5p}{p^2+9},\quad +\frac{6-p^2}{p^2+9}\text{i}$ | Real + imaginary with i factored out.<br>Accept single denominator with numerator in correct form.<br>Accept 'a= ' and 'b='. | A1, A1 |
| | | | (5) |

Leave blank

**5.** (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4}n(n+1)(n+3)(n-2)$$

**(5)**

(b) Calculate the value of $\sum_{r=10}^{50} r(r^2 - 3)$

**(3)**



P 4 3 1 5 2 A 0 1 2 2 8

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.(a)** | $r\left(r^2-3\right)=r^3-3r$ | $r^3-3r$ | B1 |
| | $\sum_{r=1}^{n} r\left(r^2-3\right)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$ | | |
| | $=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$ | M1: An attempt to use at least one of the standard formulae correctly.<br>A1: Correct expression | M1A1 |
| | $=\frac{1}{4}n(n+1)\left(n(n+1)-6\right)$ | Attempt factor of $\frac{1}{4}n(n+1)$ before given answer | M1 |
| | $=\frac{1}{4}n(n+1)\left(n^2+n-6\right)$ | | |
| | $=\frac{1}{4}n(n+1)(n+3)(n-2)$ | cso | A1 |
| | | | |
| | | | (5) |
| | | | |
| **(b)** | $\sum_{r=10}^{50} r\left(r^2-3\right)=\text{f}(50)-\text{f}(9\,or\,10)$ | Require some use of the result in part (a) for method. | M1 |
| | $=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$ | Correct expression | A1 |
| | = 1621800 - 1890 | | |
| | = 1619910 | cao | A1 |
| | | | (3) |
| | | | **Total 8** |

Leave blank

**6.**
$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$,

(a) calculate the matrix $\mathbf{M}$,

**(6)**

(b) find the matrix $\mathbf{C}$ such that $\mathbf{MC} = \mathbf{A}$.

**(4)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.(a)** | $\mathbf{A}+\mathbf{B}=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}$ | M1:Correct attempt at matrix addition with 3 elements correct<br>A1: Correct matrix | M1A1 |
| | $\mathbf{2A}-\mathbf{B}=\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$ | M1: Correct attempt to double **A** and subtract **B** 3 elements correct<br>A1: Correct matrix | M1A1 |
| | $(\mathbf{A}+\mathbf{B})(\mathbf{2A}-\mathbf{B})=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$ | | |
| | $\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$ | M1: Correct method to multiply<br>A1: cao | M1A1 |
| | | | (6) |
| **(a) Way 2** | $(\mathbf{A}+\mathbf{B})(\mathbf{2A}-\mathbf{B})=2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2$ | M1: Expands brackets with at least 3 correct terms<br>A1: Correct expansion | M1A1 |
| | $\mathbf{A}^2=\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix},\mathbf{BA}=\begin{pmatrix}-3 & -1\\ -1 & 0\end{pmatrix},$<br>$\mathbf{AB}=\begin{pmatrix}-2 & 3\\ 1 & -1\end{pmatrix},\mathbf{B}^2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ | M1: Attempts $\mathbf{A}^2$, $\mathbf{B}^2$ and **AB** or **BA**<br>A1: Correct matrices | M1A1 |
| | $2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$ | M1: Substitutes into their expansion<br>A1: Correct matrix | M1A1 |
| **(b)** | $\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$ | May be implied by later work | B1 |
| | $\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$ | An attempt at their $\frac{1}{\det \mathbf{M}}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$ | M1 |
| | $\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$ | Correct order required and an attempt to multiply | dM1 |
| | $\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5 & -2\\ 13 & 7\end{pmatrix}$ | oe | A1 |
| | | | (4) |
| | | | **Total 10** |
| **(b) Way 2** | $\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$ | Correct statement | B1 |
| | $a-c=2,\ b-d=1$<br>$-7a-2c=-1,\ -7b-2d=0$ | Multiplies correctly to obtain 4 equations | M1 |
| | $a=\frac{5}{9},b=\frac{2}{9},c=-\frac{13}{9},d=-\frac{7}{9}$ | M1: Solves to obtain values for $a$, $b$, $c$ and $d$<br>A1: Correct values | M1A1 |

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**7.** The parabola $C$ has cartesian equation $y^2 = 4ax, \ a > 0$

The points $P(ap^2, 2ap)$ and $P'(ap^2, -2ap)$ lie on $C$.

(a) Show that an equation of the normal to $C$ at the point $P$ is

$$y + px = 2ap + ap^3$$

**(5)**

(b) Write down an equation of the normal to $C$ at the point $P'$.

**(1)**

The normal to $C$ at $P$ meets the normal to $C$ at $P'$ at the point $Q$.

(c) Find, in terms of $a$ and $p$, the coordinates of $Q$.

**(2)**

Given that $S$ is the focus of the parabola,

(d) find the area of the quadrilateral $SPQP'$.

**(3)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.(a)** | $y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = 2a.\frac{1}{2ap}$ | $\frac{dy}{dx} = kx^{-\frac{1}{2}}$ or $ky\frac{dy}{dx} = c$ their $\frac{dy}{dp} \times \left(\frac{1}{\text{their } \frac{dx}{dp}}\right)$ | M1 |
| | $\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ *or* $2y\frac{dy}{dx} = 4a$ *or* $\frac{dy}{dx} = 2a.\frac{1}{2ap}$ | Correct differentiation | A1 |
| | At $P$, gradient of normal = $-p$ | Correct normal gradient with no errors seen. | A1 |
| | $y - 2ap = -p(x - ap^2)$ | Applies $y - 2ap = \text{their } m_N (x - ap^2)$ or $y = (\text{their } m_N)x + c$ using $x = ap^2$ *and* $y = 2ap$ in an attempt to find c. **Their $m_N$ must be different from their $m_T$ and must be a function of $p$.** | M1 |
| | $y + px = 2ap + ap^3$ * | cso **given answer** | A1* |
| | | | (5) |
| **(b)** | $y - px = -2ap - ap^3$ | oe | B1 |
| | | | (1) |
| **(c)** | $y = 0 \Rightarrow x = 2a + ap^2$ | M1: $y$ = 0 in either normal or solves simultaneously to find $x$<br>A1: $y$ = 0 and correct $x$ coordinate. | M1A1 |
| | | | (2) |
| **(d)** | $S$ is (a, 0) | Can be implied below | B1 |
| | Area $SPQP'$ = $\frac{1}{2} \times ("2a + ap^2" - a) \times 2ap \times 2$ | Correct method for the area of the quadrilateral. | M1 |
| | $= 2a^2 p(1 + p^2)$ | Any equivalent form | A1 |
| | | | (3) |
| | | | **Total 11** |

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**8.** The rectangular hyperbola $H$ has equation $xy = c^2$, where $c$ is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on $H$.

An equation for the tangent to $H$ at $P$ is given by

$$y = -\frac{1}{t^2}x + \frac{2c}{t}$$

The points $A$ and $B$ lie on $H$.

The tangent to $H$ at $A$ and the tangent to $H$ at $B$ meet at the point $\left(-\frac{6}{7}c, \frac{12}{7}c\right)$.

Find, in terms of $c$, the coordinates of $A$ and the coordinates of $B$.

**(5)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** | | | |
| | $\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t}$ | Substitutes $\left(-\frac{6}{7}c,\ \frac{12}{7}c\right)$ into the equation of the tangent | M1 |
| | $\frac{12}{7}c = -\frac{1}{t^2} \times -\frac{6}{7}c + \frac{2c}{t} \Rightarrow$ <br> $6t^2 - 7t - 3 = 0$ | Correct 3TQ in terms of $t$ | A1 |
| | $6t^2 - 7t - 3 = 0 \Rightarrow (3t+1)(2t-3) = 0 \Rightarrow t =$ | Attempt to solve their 3TQ for $t$ | M1 |
| | $t = -\frac{1}{3},\ t = \frac{3}{2} \Rightarrow \left(-\frac{1}{3}c,\ -3c\right),\ \left(\frac{3}{2}c,\ \frac{2}{3}c\right)$ | M1: Uses at least one of their values of $t$ to find $A$ or $B$. <br> A1: Correct coordinates. | M1A1 |
| | | | (5) |
| | | | **Total 5** |

Leave blank

**9.** (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r + 1)2^{r-1} = n2^n$$

**(5)**

(b) A sequence of numbers is defined by

$$u_1 = 0, \qquad u_2 = 32,$$

$$u_{n+2} = 6u_{n+1} - 8u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^{n+1} - 2^{n+3}$$

**(7)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9.(a)** | When n = 1, rhs = lhs = 2 | | B1 |
| | Assume true for $n = k$ so $\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k$ | | |
| | $\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+1+1)2^{k+1-1}$ | M1: Attempt to add (k + 1)th term<br>A1: Correct expression | M1A1 |
| | $= k2^k + (k+2)2^k$ | | |
| | $= 2 \times k2^k + 2 \times 2^k$ | | |
| | $= (k+1)2^{k+1}$ | At least one correct intermediate step required. | A1 |
| | If the result is **true for *n* = *k*** then it has been shown **true for *n* = *k* + 1.** As it is **true for *n* = 1** then it is **true for all *n*** (positive integers.) | cso, statements can be seen anywhere in the solution. | A1 |
| | | Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0 | |
| | | | (5) |
| | | | |
| **(b)** | When $n = 1$ $u_1 = 4^2 - 2^4 = 0$ | $4^2 - 2^4 = 0$ seen | B1 |
| | When $n = 2$ $u_2 = 4^3 - 2^5 = 32$ | $4^3 - 2^5 = 32$ seen | B1 |
| | True for *n* = 1 and *n* = 2 | | |
| | Assume $u_k = 4^{k+1} - 2^{k+3}$ and $u_{k+1} = 4^{k+2} - 2^{k+4}$ | | |
| | $u_{k+2} = 6u_{k+1} - 8u_k$<br>$= 6(4^{k+2} - 2^{k+4}) - 8(4^{k+1} - 2^{k+3})$ | M1: Attempts $u_{k+2}$ in terms of $u_{k+1}$ and $u_k$<br>A1: Correct expression | M1A1 |
| | $= 6.4^{k+2} - 6.2^{k+4} - 8.4^{k+1} + 8.2^{k+3}$ | | |
| | $= 6.4^{k+2} - 3.2^{k+5} - 2.4^{k+2} + 2.2^{k+5}$ | Attempt $u_{k+2}$ in terms of $4^{k+2}$ and $2^{k+5}$ | M1 |
| | $= 4.4^{k+2} - 2^{k+5} = 4^{k+3} - 2^{k+5}$ | | |
| | So $u_{k+2} = 4^{(k+2)+1} - 2^{(k+2)+3}$ | Correct expression | A1 |
| | If the result **is true for *n* = *k*** and ***n* = *k* + 1** then it has been shown **true for *n* = *k* + 2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** (positive integers.) | cso, statements can be seen anywhere in the solution. | A1 |
| | | Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0 | |
| | | | (7) |
| | | | **Total 12** |