Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1 R	Signature	

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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9	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are z_1, z_2 and z_3

Given that $z_1 = 1 + 2\text{i}$, find z_2 and z_3

(5)

Question Number	Scheme		Marks
1.	$f(z) = 2z^3 - 3z^2 + 8z + 5$		
	$1 - 2i$ (is also a root)	seen	B1
	$(z-(1+2i))(z-(1-2i)) = z^2 - 2z + 5$	Attempt to expand $(z-(1+2i))(z-(1-2i))$ or any valid method to establish the quadratic factor e.g. $z = 1 \pm 2i \Rightarrow z - 1 = \pm 2i \Rightarrow z^2 - 2z + 1 = -4$ $z = 1 \pm \sqrt{-4} = \frac{2 \pm \sqrt{-16}}{2} \Rightarrow b = -2, c = 5$ Sum of roots 2, product of roots 5 $\therefore z^2 - 2z + 5$	M1A1
	$f(z) = (z^2 - 2z + 5)(2z + 1)$	Attempt at **linear** factor with their *cd* in $(z^2 + az + c)(2z + d) = \pm 5$ Or $(z^2 - 2z + 5)(2z + a) \Rightarrow 5a = 5$	M1
	$(z_3) = -\frac{1}{2}$		A1
			(5)
			Total 5

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2.

$$f(x) = 3\cos 2x + x - 2, \qquad -\pi \leqslant x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2, 3].

(2)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α.

Give your answer to 3 decimal places.

(3)

(c) The equation $f(x) = 0$ has another root β in the interval [–1, 0]. Starting with this interval, use interval bisection to find an interval of width 0.25 which contains β.

(4)

Question Number	Scheme		Marks
2.	$f(x) = 3\cos 2x + x - 2$		
(a)	f(2) = -1.9609...... f(3) = 3.8805......	Attempts to evaluate both $f(2)$ and $f(3)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $f(x)$ is continuous) therefore a **root** α is between $x = 2$ and $x = 3$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.96.. < 0 < 3.88..) and conclusion.	A1
			(2)
(b)	$\dfrac{\alpha - 2}{\text{"1.9609..."}} = \dfrac{3 - \alpha}{\text{"3.8805..."}}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(3). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$(3.88... + 1.96....)\alpha = 3 \times 1.96 + 2 \times 3.88$		
	$\alpha_2 = \dfrac{3 \times 1.96.. + 2 \times 3.88..}{1.96... + 3.88...}$	Follow through their values if seen explicitly.	A1ft
	$\alpha_2 = 2.336$	cao	A1
			(3)
(c)	f(0) = +(1) **or** f(-1) = -(4.248)	Award for correct sign, can be in a table.	B1
	f(-0.5) (= -0.879.....)	Attempt f(-0.5)	M1
	f(-0.25) (= 0.382....)	Attempt **f(-0.25)**	M1
	$\therefore\ -0.5 < \beta < -0.25$	oe with no numerical errors seen	A1
			(4)
			Total 9

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3. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

(b) Write down the matrix **B**.

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix},$$ where k is a positive constant.

Triangle T has an area of 16 square units.

Triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of the triangle T' is 224 square units, find the value of k.

(3)

Question Number	Scheme		Marks
3.(i)(a)	Rotation of 45 degrees anticlockwise, about the origin	B1: Rotation about (0, 0) B1: 45 degrees (anticlockwise) -45 or clockwise award B0	B1B1
			(2)
(b)	$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Correct matrix	B1
			(1)
(ii)	$\frac{224}{16}(=14)$	Correct area scale factor. Allow ± 14	B1
	$\det \mathbf{M} = 3\times 3 - k\times -2 = 14$	Attempt determinant and set equal to their area scale factor	M1
		Accept $\det \mathbf{M} = 3\times 3 \pm 2k$ only	
	$k = 2.5$	oe	A1
			(3)
			Total 6

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4. The complex number z is given by

$$z = \frac{p + 2\text{i}}{3 + p\text{i}}$$

where p is an integer.

(a) Express z in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.
(4)

(b) Given that $\arg(z) = \theta$, where $\tan \theta = 1$ find the possible values of p.
(5)

Question Number	Scheme		Marks
4.(a)	$z=\frac{p+2\text{i}}{3+p\text{i}}.\frac{3-p\text{i}}{3-p\text{i}}$	Multiplying top and bottom by Conjugate	M1
	$=\frac{3p-p^2\text{i}+6\text{i}+2p}{9+p^2}$	At least 3 correct terms in the numerator, evidence that $\text{i}^2=-1$and denominator real.	M1
	$=\frac{5p}{p^2+9}, \quad +\frac{6-p^2}{p^2+9}\text{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\arg(z)=\arctan\left(\frac{\frac{6-p^2}{p^2+9}}{\frac{5p}{p^2+9}}\right)$	Correct method for the argument. Can be implied by correct equation for *p*	M1
	$\frac{6-p^2}{5p}=1$	Their $\arg(z)$ in terms of $p=1$	M1
	$p^2+5p-6=0$	Correct 3TQ	A1
	$(p+6)(p-1)=0 \Rightarrow x=$	M1:Attempt to solve their quadratic in *p*	M1
	$p=1, p=-6$	A1:**both**	A1
			(5)
			Total 9
(a) Way 2	$a+b\text{i}=\frac{p+2\text{i}}{3+p\text{i}}$	Equate to $a+b\text{i}$ then rearrange and equate real and imaginary parts.	M1
	$3a-pb=p,\ ap+3b=2$	Two equations for *a* and *b* in terms of *p* and attempt to solve for *a* and *b* in terms of *p*	dM1
	$=\frac{5p}{p^2+9}, \quad +\frac{6-p^2}{p^2+9}\text{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(5)

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5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4}n(n+1)(n+3)(n-2)$$

(5)

(b) Calculate the value of $\sum_{r=10}^{50} r(r^2 - 3)$

(3)

P 4 3 1 5 2 A 0 1 2 2 8

Question Number	Scheme		Marks
5.(a)	$r\left(r^2-3\right)=r^3-3r$	r^3-3r	B1
	$\sum_{r=1}^{n} r\left(r^2-3\right)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$		
	$=\frac{1}{4}n^2\left(n+1\right)^2-\frac{3}{2}n\left(n+1\right)$	M1: An attempt to use at least one of the standard formulae correctly. A1: Correct expression	M1A1
	$=\frac{1}{4}n\left(n+1\right)\left(n\left(n+1\right)-6\right)$	Attempt factor of $\frac{1}{4}n\left(n+1\right)$ before given answer	M1
	$=\frac{1}{4}n\left(n+1\right)\left(n^2+n-6\right)$		
	$=\frac{1}{4}n\left(n+1\right)\left(n+3\right)\left(n-2\right)$	cso	A1
			(5)
(b)	$\sum_{r=10}^{50} r\left(r^2-3\right)=\text{f}(50)-\text{f}(9\,or\,10)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$	Correct expression	A1
	= 1621800 - 1890		
	= 1619910	cao	A1
			(3)
			Total 8

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6. $$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$,

(a) calculate the matrix $\mathbf{M}$,

(6)

(b) find the matrix $\mathbf{C}$ such that $\mathbf{MC} = \mathbf{A}$.

(4)

Question Number	Scheme		Marks
6.(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}$	M1:Correct attempt at matrix addition with 3 elements correct A1: Correct matrix	M1A1
	$2\mathbf{A}-\mathbf{B}=\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$	M1: Correct attempt to double **A** and subtract **B** 3 elements correct A1: Correct matrix	M1A1
	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$		
	$\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$	M1: Correct method to multiply A1: cao	M1A1
			(6)
(a) Way 2	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2$	M1: Expands brackets with at least 3 correct terms A1: Correct expansion	M1A1
	$\mathbf{A}^2=\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix},\mathbf{BA}=\begin{pmatrix}-3 & -1\\ -1 & 0\end{pmatrix},$ $\mathbf{AB}=\begin{pmatrix}-2 & 3\\ 1 & -1\end{pmatrix},\mathbf{B}^2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	M1: Attempts $\mathbf{A}^2$, $\mathbf{B}^2$ and **AB** or **BA** A1: Correct matrices	M1A1
	$2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$	M1: Substitutes into their expansion A1: Correct matrix	M1A1
(b)	$\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$	May be implied by later work	B1
	$\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$	An attempt at their $\frac{1}{\det\mathbf{M}}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$	M1
	$\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$	Correct order required and an attempt to multiply	dM1
	$\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5 & -2\\ 13 & 7\end{pmatrix}$	oe	A1
			(4)
			Total 10
(b) Way 2	$\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$	Correct statement	B1
	$a-c=2,\ b-d=1$ $-7a-2c=-1,\ -7b-2d=0$	Multiplies correctly to obtain 4 equations	M1
	$a=\frac{5}{9},b=\frac{2}{9},c=-\frac{13}{9},d=-\frac{7}{9}$	M1: Solves to obtain values for a, b, c and d A1: Correct values	M1A1

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7. The parabola C has cartesian equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $P'(ap^2, -2ap)$ lie on C.

(a) Show that an equation of the normal to C at the point P is

$$y + px = 2ap + ap^3$$

(5)

(b) Write down an equation of the normal to C at the point P'.

(1)

The normal to C at P meets the normal to C at P' at the point Q.

(c) Find, in terms of a and p, the coordinates of Q.

(2)

Given that S is the focus of the parabola,

(d) find the area of the quadrilateral $SPQP'$.

(3)

Question Number	Scheme		Marks
7.(a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = 2a.\frac{1}{2ap}$	$\frac{dy}{dx} = kx^{-\frac{1}{2}}$ or $ky\frac{dy}{dx} = c$ their $\frac{dy}{dp} \times \left(\frac{1}{\text{their}\frac{dx}{dp}}\right)$	M1
	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ *or* $2y\frac{dy}{dx} = 4a$ *or* $\frac{dy}{dx} = 2a.\frac{1}{2ap}$	Correct differentiation	A1
	At *P*, gradient of normal = -*p*	Correct normal gradient with no errors seen.	A1
	$y - 2ap = -p(x - ap^2)$	Applies $y - 2ap = \text{their } m_N(x - ap^2)$ or $y = (\text{their } m_N)x + c$ using $x = ap^2$ *and* $y = 2ap$ in an attempt to find c. **Their m_N must be different from their m_T and must be a function of *p*.**	M1
	$y + px = 2ap + ap^3$ *	cso **given answer**	A1*
			(5)
(b)	$y - px = -2ap - ap^3$	oe	B1
			(1)
(c)	$y = 0 \Rightarrow x = 2a + ap^2$	M1: *y* = 0 in either normal or solves simultaneously to find *x* A1: *y* = 0 and correct *x* coordinate.	M1A1
			(2)
(d)	*S* is (a, 0)	Can be implied below	B1
	Area *SPQP'* = $\frac{1}{2} \times ("2a + ap^2" - a) \times 2ap \times 2$	Correct method for the area of the quadrilateral.	M1
	$= 2a^2p(1 + p^2)$	Any equivalent form	A1
			(3)
			Total 11

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8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

An equation for the tangent to H at P is given by

$$y = -\frac{1}{t^2}x + \frac{2c}{t}$$

The points A and B lie on H.

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{6}{7}c, \frac{12}{7}c\right)$.

Find, in terms of c, the coordinates of A and the coordinates of B.

(5)

Question Number	Scheme		Marks
8.			
	$\frac{12}{7}c = -\frac{1}{t^2}\times -\frac{6}{7}c + \frac{2c}{t}$	Substitutes $\left(-\frac{6}{7}c,\ \frac{12}{7}c\right)$ into the equation of the tangent	M1
	$\frac{12}{7}c = -\frac{1}{t^2}\times -\frac{6}{7}c + \frac{2c}{t} \Rightarrow$ $6t^2 - 7t - 3 = 0$	Correct 3TQ in terms of t	A1
	$6t^2 - 7t - 3 = 0 \Rightarrow (3t+1)(2t-3) = 0 \Rightarrow t =$	Attempt to solve their 3TQ for t	M1
	$t = -\frac{1}{3},\ t = \frac{3}{2} \Rightarrow \left(-\frac{1}{3}c,\ -3c\right),\ \left(\frac{3}{2}c,\ \frac{2}{3}c\right)$	M1: Uses at least one of their values of t to find A or B. A1: Correct coordinates.	M1A1
			(5)
			Total 5

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9. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n}(r+1)2^{r-1} = n2^n$$

(5)

(b) A sequence of numbers is defined by

$$u_1 = 0, \qquad u_2 = 32,$$

$$u_{n+2} = 6u_{n+1} - 8u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^{n+1} - 2^{n+3}$$

(7)

Question Number	Scheme		Marks
9.(a)	When n = 1, rhs = lhs = 2		B1
	Assume true for $n = k$ so $\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k$		
	$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+1+1)2^{k+1-1}$	M1: Attempt to add (k + 1)th term A1: Correct expression	M1A1
	$= k2^k + (k+2)2^k$		
	$= 2\times k2^k + 2\times 2^k$		
	$=(k+1)2^{k+1}$	At least one correct intermediate step required.	A1
	If the result is **true for *n* = *k*** then it has been shown **true for *n* = *k* + 1.** As it is **true for *n* = 1** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(5)
(b)	When $n = 1$ $u_1 = 4^2 - 2^4 = 0$	$4^2 - 2^4 = 0$ seen	B1
	When $n = 2$ $u_2 = 4^3 - 2^5 = 32$	$4^3 - 2^5 = 32$ seen	B1
	True for $n = 1$ and $n = 2$		
	Assume $u_k = 4^{k+1} - 2^{k+3}$ and $u_{k+1} = 4^{k+2} - 2^{k+4}$		
	$u_{k+2} = 6u_{k+1} - 8u_k$ $= 6(4^{k+2} - 2^{k+4}) - 8(4^{k+1} - 2^{k+3})$	M1: Attempts u_{k+2} in terms of u_{k+1} and u_k A1: Correct expression	M1A1
	$= 6.4^{k+2} - 6.2^{k+4} - 8.4^{k+1} + 8.2^{k+3}$		
	$= 6.4^{k+2} - 3.2^{k+5} - 2.4^{k+2} + 2.2^{k+5}$	Attempt u_{k+2} in terms of 4^{k+2} and 2^{k+5}	M1
	$= 4.4^{k+2} - 2^{k+5} = 4^{k+3} - 2^{k+5}$		
	So $u_{k+2} = 4^{(k+2)+1} - 2^{(k+2)+3}$	Correct expression	A1
	If the result **is true for *n* = *k*** and ***n* = *k* + 1** then it has been shown **true for *n* = *k* + 2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** (positive integers.)	cso, statements can be seen anywhere in the solution.	A1
		Do not award final A if *n* defined incorrectly e.g. '*n* is an integer' award A0	
			(7)
			Total 12