Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1 R	Signature	

Paper Reference(s)

6666/01R

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
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7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

(4)

Question Number	Scheme		Marks
1.	$\frac{5x+3}{(2x+1)(x+1)^2} \equiv \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2, \quad C = 2$	Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.	B1 **cso**
	$5x + 3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: \; 0 = A + 2B, \quad$ constant$: \; 3 = A + B + C$ $x: \; 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 **cso**
			[4] **4**
	So, $\frac{5x+3}{(2x+1)(x+1)^2} \equiv \frac{2}{(2x+1)} - \frac{1}{(x+1)} + \frac{2}{(x+1)^2}$		

Notes for Question 1

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or***
comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.

Note: If a candidate does not give partial fraction decomposition then:

- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

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2. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

Question Number	Scheme		Marks
2.	$3^{x-1} + xy - y^2 + 5 = 0$		
		$3^{x-1} \to 3^{x-1}\ln 3$	B1 oe
	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ (ignore) $3^{x-1}\ln 3 + \left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$xy \to +y + x\frac{dy}{dx}$	B1
		$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	A1
	$\{(1, 3) \Rightarrow\}\ 3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression.	dM1*
	$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5\frac{dy}{dx}$		
	$\frac{dy}{dx} = \frac{3 + \ln 3}{5}$		dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 2

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \to 3^{x-1}\ln 3$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) \to \frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1} = e^{(x-1)\ln 3} \to \ln 3\, e^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) = \frac{1}{3}e^{x\ln 3} \to \frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

B1: $xy \to +y + x\frac{dy}{dx}$

1st A1: $\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$= 0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3 + y + x\frac{dy}{dx} - 2y\frac{dy}{dx}$ leading to $3^{x-1}\ln 3 + y = 2y\frac{dy}{dx} - x\frac{dy}{dx}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{dy}{dx} = \frac{y + 3^{x-1}\ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$

2nd A1: **cso.** Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$, $\left(= \frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda = 5 \text{ and } \mu = 3\right)$

Note: $3 = \ln e^3$ needs to be seen in their proof.

Notes for Question 2 Continued

2.	**Alternative Method:** <u>Multiplying both sides by 3</u>	
	$3^{x-1} + xy - y^2 + 5 = 0$	
	$3^x + 3xy - 3y^2 + 15 = 0$	
	$3^x \rightarrow 3^x \ln 3$	B1
***Aliter* Way 2**	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ $3^x \ln 3 + \left(3y + 3x\frac{dy}{dx}\right) - 6y\frac{dy}{dx} = 0$ (ignore) — Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
	$3xy \rightarrow +3y + 3x\frac{dy}{dx}$	B1
	$... + 3y + 3x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$	A1
	$\{(1, 3) \Rightarrow\}$ $3^1 \ln 3 + 3(3) + (3)(1)\frac{dy}{dx} - 6(3)\frac{dy}{dx} = 0$ — Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression.	dM1*
	$3\ln 3 + 9 + 3\frac{dy}{dx} - 18\frac{dy}{dx} = 0 \Rightarrow 9 + 3\ln 3 = 15\frac{dy}{dx}$	
	$\frac{dy}{dx} = \frac{9 + 3\ln 3}{15}$ $\left\{= \frac{3 + \ln 3}{5}\right\}$	dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right)$	
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$ — **Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
		[7] **7**
	NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3. **NOTE:** For reference, $\frac{dy}{dx} = \frac{3y + 3^x \ln 3}{6y - 3x}$ **NOTE:** If the candidate applies this method then $3xy \rightarrow +3y + 3x\frac{dy}{dx}$ must be seen for the 2nd B1 mark.	

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3. Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x + 1)}}\,\mathrm{d}x$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

Question Number	Scheme		Marks
3.	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,dx\,,\;\; u = 2+\sqrt{(2x+1)}$		
	$\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = u-2$	Either $\frac{du}{dx} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = \pm\lambda(u-2)$	M1
		Either $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,dx\right\} = \int \frac{1}{u}(u-2)\,du$	Correct substitution (Ignore integral sign and du).	A1
	$= \int \left(1-\frac{2}{u}\right)du$	An attempt to divide each term by u.	dM1
		$\pm Au \pm B\ln u$	ddM1
	$= u - 2\ln u$	$u - 2\ln u$	A1 ft
	$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 **cao cso**
			[8] **8**

Notes for Question 3

M1: Also allow $du = \pm\lambda\frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

Note: The expressions must contain du and dx. They can be simplified or un-simplified.

A1: Also allow $du = \frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

Note: The expressions must contain du and dx. They can be simplified or un-simplified.

A1: $\int \frac{1}{u}(u-2)\,du$**.** (Ignore integral sign and du).

dM1: An attempt to divide each term by u.
Note that this mark is dependent on the previous M1 mark being awarded.
Note that this mark can be implied by later working.

ddM1: $\pm Au \pm B\ln u\,,\; A \neq 0,\, B \neq 0$
Note that this mark is dependent on the two previous M1 marks being awarded.

A1ft: $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0,\, B \neq 0$

M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.

A1: **cso** and **cao**. $2+2\ln\left(\frac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B, \text{ so } A=2,\, B=\frac{3}{5}\right)$

Note: $2-2\ln\left(\frac{3}{5}\right)$ is A0.

Notes for Question 3 Continued	
3. ctd	**Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = u - 2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1. but **Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

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4. (a) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(3)

Question Number	Scheme		Marks
4. (a)	$\left\{\sqrt[3]{(8-9x)}\right\} = (8-9x)^{\frac{1}{3}}$	Power of $\frac{1}{3}$	M1
	$= \underline{(8)^{\frac{1}{3}}}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$	$\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$		
	$= 2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$	See notes below!	
	$= 2-\frac{3}{4}x;-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$		A1; A1
			[6]
(b)	$\left\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\right\}$ so $x=0.1$	Writes down or uses $x=0.1$	B1
	When $x=0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+\ldots$ $= 2-0.075-0.0028125-0.00017578125$ $= 1.922011719$		M1
	So, $\sqrt[3]{7100} = 19.220117919\ldots = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**
			[3] **9**

Notes for Question 4

(a)

M1: Writes or uses $\frac{1}{3}$. This mark can be implied by a constant term of $8^{\frac{1}{3}}$ or 2.

<u>**B1**</u>: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.

M1: Expands $(\ldots+kx)^{\frac{1}{3}}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+\left(\frac{1}{3}\right)(kx)$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ or $1+\ \ldots\ldots +\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2$

or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ **where** $k \neq 1$ are fine for M1.

A1: A correct simplified or un-simplified $1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$

expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$.

You would award B1M1A0 for $2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$

because (kx) is not consistent.

Notes for Question 4 Continued	
4. (a) ctd	***"Incorrect bracketing"*** $=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x^2}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x^3}{8}\right)+\ldots}\right]$ is M1A0 unless recovered. **A1:** For $2-\frac{3}{4}x$ **(simplified please)** or also allow $2-0.75x$. **Allow Special Case A1A0 for either SC:** $=2\left[1-\frac{3}{8}x;\ \ldots\right]$ **or SC:** $K\left[1-\frac{3}{8}x-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$ (where K can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal. **A1:** Accept only $-\frac{9}{32}x^2-\frac{45}{256}x^3$ or $-0.28125x^2-0.17578125x^3$ Candidates who write $=2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{9x}{8}\right)^3+\ldots}\right]$ where $k=\frac{9}{8}$ and not $-\frac{9}{8}$ and achieve $2+\frac{3}{4}x;-\frac{9}{32}x^2+\frac{45}{256}x^3+\ldots$ will get B1M1A1A0A0. <u>**Note for final two marks:**</u> $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2+\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1. $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2-\frac{3}{4}-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1 <u>***Alternative method:***</u> Candidates can apply an alternative form of the binomial expansion. $\left\{\sqrt[3]{(8-9x)}\right\}=(8-9x)^{\frac{1}{3}}=(8)^{\frac{1}{3}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-9x)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-9x)^3$ **B1:** $(8)^{\frac{1}{3}}$ or 2 **M1:** Any two of four (un-simplified or simplified) terms correct. **A1:** All four (un-simplified or simplified) terms correct. **A1:** $2-\frac{3}{4}x$ **A1:** $-\frac{9}{32}x^2-\frac{45}{256}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{\frac{1}{3}}C_0(8)^{\frac{1}{3}}+{}^{\frac{1}{3}}C_1(8)^{-\frac{2}{3}}(-9x)+{}^{\frac{1}{3}}C_2(8)^{-\frac{5}{3}}(-9x)^2+{}^{\frac{1}{3}}C_3(8)^{-\frac{8}{3}}(-9x)^3$ without further working is B0M0A0.
(b)	**B1:** Writes down or uses $x=0.1$ **M1:** Substitutes their x, where $\lvert x\rvert<\frac{8}{9}$ into at least two terms of their binomial expansion. **A1:** 19.2201 **cao** **Be Careful!** The binomial answer is 19.22011719 and the calculated $\sqrt[3]{7100}$ is 19.21997343... which is 19.2200 to 4 decimal places.

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5.

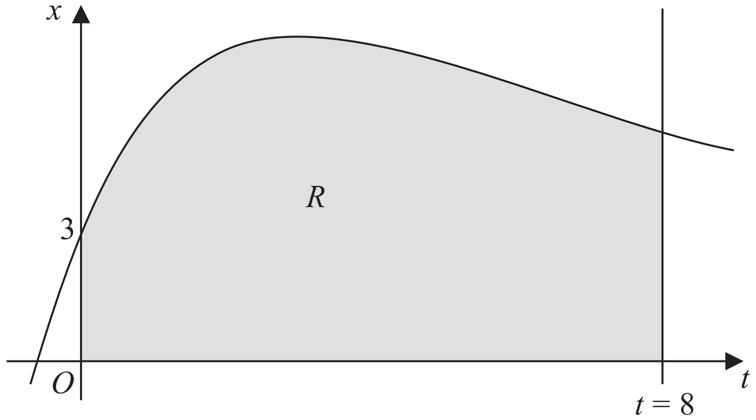


Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

(b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places.

(3)

(c) Use calculus to find the exact value for the area of R.

(6)

(d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

(1)

Question Number	Scheme		Marks
5. (a)	$6.248046798... = 6.248$ (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2}\times 2\,;\times\underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int\left(4t\,\mathrm{e}^{-\frac{1}{3}t}+3\right)\mathrm{d}t\right\} = -12t\,\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$	$\pm At\,\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0$, $B\neq 0$	M1
		See notes.	A1
	$+\,3t$	$3\to 3t$	B1
	$= -12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}\,\{+3t\}$	$-12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$	A1
	$\left[-12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}+3t\right]_0^8 =$		
	$=\left(-12(8)\mathrm{e}^{-\frac{1}{3}(8)} - 36\mathrm{e}^{-\frac{1}{3}(8)}+3(8)\right)-\left(-12(0)\mathrm{e}^{-\frac{1}{3}(0)} - 36\mathrm{e}^{-\frac{1}{3}(0)}+3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t}\pm\mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t}\pm\mu\mathrm{e}^{-\frac{1}{3}t}+Bt$ and subtracts the correct way round.	dM1
	$=\left(-96\mathrm{e}^{-\frac{8}{3}} - 36\mathrm{e}^{-\frac{8}{3}}+24\right)-\left(0-36+0\right)$		
	$= 60 - 132\mathrm{e}^{-\frac{8}{3}}$	$60 - 132\mathrm{e}^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $=\left\lvert 60-132\mathrm{e}^{-\frac{8}{3}} - 49.37\right\rvert = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1
			[1]
			11

Notes for Question 5

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times 2$ or 1
	M1: <u>For structure of trapezium rule</u>$\left[\ \underline{............}\ \right]$. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37)
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 50.828…
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $1+3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223$ (nb: answer of 50.369).

Notes for Question 5 Continued	
5. (b) ctd	*Alternative method for part (b): Adding individual trapezia*
	Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]=49.369$
	B1: 2 and a divisor of 2 on all terms inside brackets.
	M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1: anything that rounds to 49.37
(c)	**M1:** For $4t\mathrm{e}^{-\frac{1}{3}t} \to \pm At\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0,\ B\neq 0$
	A1: For $t\mathrm{e}^{-\frac{1}{3}t} \to \left(-3t\mathrm{e}^{-\frac{1}{3}t} - \int -3\mathrm{e}^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark).
	or $4t\mathrm{e}^{-\frac{1}{3}t} \to 4\left(-3t\mathrm{e}^{-\frac{1}{3}t} - \int -3\mathrm{e}^{-\frac{1}{3}t}\right)$ or $-12t\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}$ or $12\left(-t\mathrm{e}^{-\frac{1}{3}t} - \int -\mathrm{e}^{-\frac{1}{3}t}\right)$
	These results can be implied. They can be simplified or un-simplified.
	B1: $3 \to 3t$ or $3 \to 3x$ (bod) .
	Note: Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4.
	Be careful some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \to 4\left(....+\frac{3}{4}t\right)$
	which would then be fine for B1.
	Note: Allow B1 for $\int_0^8 3\,\mathrm{d}t = 24$
	A1: For correct integration of $4t\mathrm{e}^{-\frac{1}{3}t}$ to give $-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$ or $4\left(-3t\mathrm{e}^{-\frac{1}{3}t} - 9\mathrm{e}^{-\frac{1}{3}t}\right)$ or equivalent.
	This can be simplified or un-simplified.
	dM1: Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or
	$\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.
	Note: Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0.
	A1: An exact answer of $60 - 132\mathrm{e}^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0.
	Note: A decimal answer of 50.82818444... without a correct exact answer is A0.
	Note: If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0.
	IMPORTANT: that is fine for candidates to work in terms of x rather than t in part (c).
	Note: The "$3t$" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46.
	Candidates may give correct decimal answers of 1.458184439... or 1.459184439...
	Note: You can award this mark whether or not the candidate has answered part (c) correctly.

Leave blank

6. Relative to a fixed origin O, the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a, b and c are constants and λ is a parameter.

Given that the point A lies on the line l,

(a) find the value of a.

(3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the values of b and c,

(5)

(c) find the distance AB.

(2)

The image of the point B after reflection in the line l is the point B'.

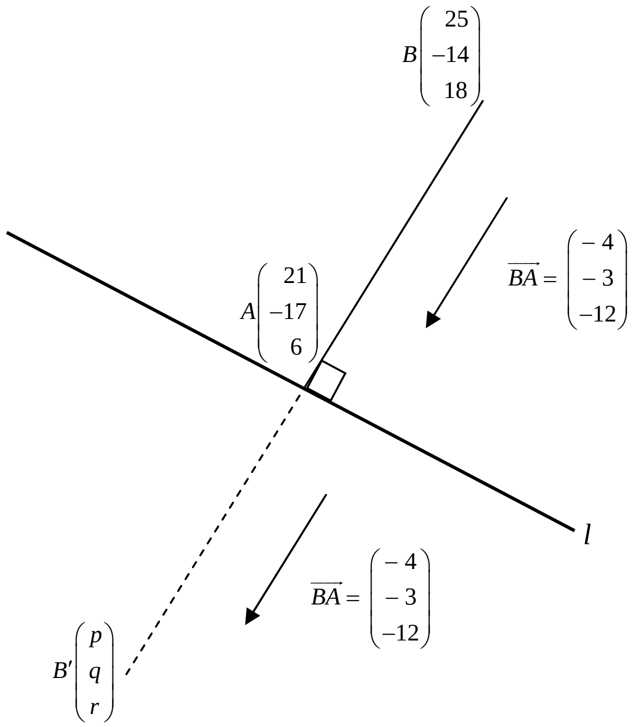
(d) Find the position vector of the point B'.

(2)

Question Number	Scheme		Marks
6.	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overrightarrow{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$		
(a)	A is on l, so $\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$		
	$\{\mathbf{k}: \ 10 - \lambda = 6 \Rightarrow\} \ \lambda = 4$	$\lambda = 4$	B1
	$\{\mathbf{i}: \ a + 6\lambda = 21 \Rightarrow\} \ a + 6(4) = 21$	Substitutes their value of λ into $a + 6\lambda = 21$	M1
	$a = -3$	$a = -3$	A1 **cao**
			[3]
(b)	$\{\overrightarrow{AB}\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.	M1
	$\{\overrightarrow{AB}\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$		
	$\left\{\overrightarrow{AB} \perp l \Rightarrow \overrightarrow{AB} \bullet \mathbf{d} = 0\right\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$	See notes.	M1; A1 ft
	$\{\mathbf{j}: \ b + c\lambda = -17 \Rightarrow\} \ b + (-4)(4) = -17\ ; \Rightarrow \ b = -1$	See notes.	ddM1; A1 **cso cao**
			[5]
(c)	$\lvert AB \rvert = \sqrt{4^2 + 3^2 + 12^2}$ or $\lvert AB \rvert = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$	See notes.	M1
	So, $\lvert AB \rvert = 13$		A1 **cao**
			[2]
(d)	$\overrightarrow{OB'} \left\{= \overrightarrow{OA} + \overrightarrow{BA}\right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	See notes for alternative methods.	M1;A1 **cao**
			[2]
			12

Notes for Question 6

(a)	**B1:** $\lambda = 4$ seen or implied. **M1:** Substitutes their value of λ into $a + 6\lambda = 21$ **A1:** $a = -3$. Note: Award B1M1A1 if the candidate states $a = -3$ from no working. ***<u>Alternative Method Using Simultaneous equations for part (a).</u>*** **B1:** For $60 - 6\lambda = 36$ **M1:** $60 - 6\lambda = 36$ and $a + 6\lambda = 21$ solved simultaneously to give $a = ...$ **A1:** $a = -3$, **cao**.

Notes for Question 6 Continued	
6. (b) ctd	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **M1:** ***Applies*** the formula $\overrightarrow{AB} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ or $\overrightarrow{BA} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ correctly to give a linear equation in c which is set equal to zero. **Note:** The dot product can also be with $\pm k \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$. **A1ft:** $c = -4$ **or** for finding a correct follow through c. **ddM1:** Substitutes their value of λ and their value of c into $b + c\lambda = -17$ **Note** that this mark is dependent on the two previous method marks being awarded. **A1:** $b = -1$
(c)	**M1:** An attempt to apply a three term Pythagoras in order to find $\lvert AB \rvert$, so taking the square root is required here. **A1:** 13 **cao** **Note:** Don't recover work for part (b) in part (c).
(d)	**M1:** For a full ***applied*** method of finding the coordinates of B'. **Note:** You can give M1 for 2 out of 3 correct components of B'. **A1:** For either $\begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$ or $17\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}$ or $(17, -20, -6)$ **cao**. ***Helpful diagram!*** 

Notes for Question 6 Continued	
	Acceptable Methods for the Method mark in part (d)
Way 1	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}+\overrightarrow{BA}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}+\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 2	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}-\overrightarrow{AB}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 3	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}+2\overrightarrow{BA}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}+2\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 4	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}-2\overrightarrow{AB}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}-2\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 5	$\begin{pmatrix}25\\-14\\18\end{pmatrix}\rightarrow\begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix}\rightarrow\begin{pmatrix}21\\-17\\6\end{pmatrix}\rightarrow\begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix}\left\{\rightarrow\begin{pmatrix}17\\-20\\-6\end{pmatrix}\right\}$, so $\overrightarrow{OA}$ + their $\overrightarrow{BA}$
Way 6	$\overrightarrow{OB'}\left\{=2\overrightarrow{OA}-\overrightarrow{OB}\right\}=2\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}25\\-14\\18\end{pmatrix}$
Way 7	$\overrightarrow{OB}=25\mathbf{i}-14\mathbf{j}+18\mathbf{k}$, $\overrightarrow{OA}=21\mathbf{i}-17\mathbf{j}+6\mathbf{k}$ and $\overrightarrow{OB'}=p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$, $(21,-17,6)=\left(\frac{25+p}{2},\frac{-14+q}{2},\frac{18+r}{2}\right)$ $p=21(2)-25=17$ $q=-17(2)+14=-20$ $r=6(2)-18=-6$ **M1:** Writing down any two equations correctly and an attempt to find at least two of p, q or r.

Leave blank

7.

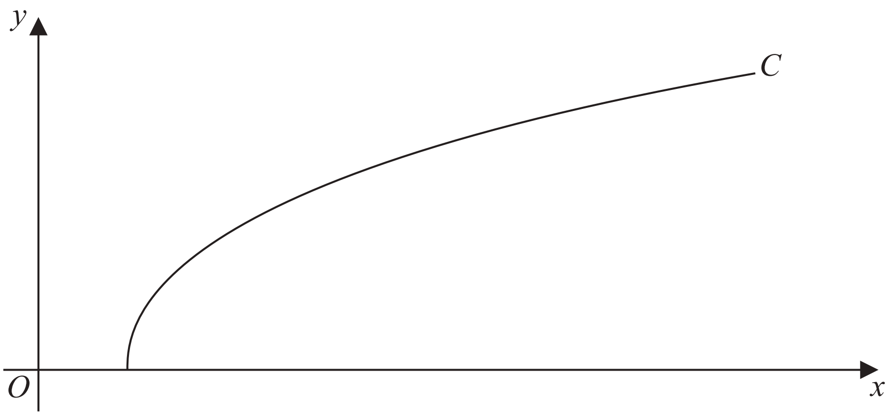


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$

(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}, \qquad a \leqslant x \leqslant b$$

stating the values of a and b.

(3)

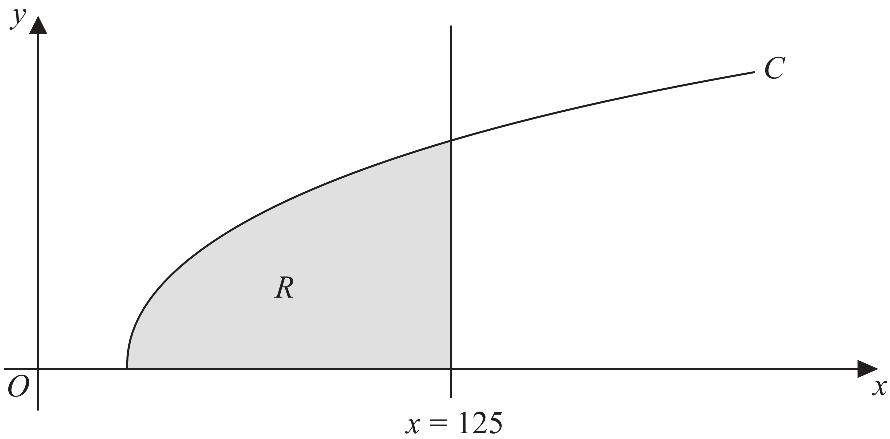


Figure 3

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.

(5)

P 4 2 9 5 4 A 0 2 2 2 8

Question Number	Scheme		Marks
7.	$x = 27\sec^3 t,\quad y = 3\tan t,\quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 81\sec^2 t\sec t\tan t\,,\quad \frac{\mathrm{d}y}{\mathrm{d}t} = 3\sec^2 t$	At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct.	B1
		Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2 t}{81\sec^3 t\tan t}\left\{= \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 **cao cso**
			[4]
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 \mathrm{d}x$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)\mathrm{d}x$	For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$	Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5] **12**

Notes for Question 7

(a)	**B1:** At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working.
	B1: Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working.
	M1: Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$, where both $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ are trigonometric functions of t.
	A1: $\frac{4}{72}$ or any equivalent correct rational answer not involving surds.
	Allow $0.0\dot{5}$ with the recurring symbol.

	Notes for Question 7 Continued
	Note: Please check that their $\frac{dx}{dt}$ is differentiated correctly. Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$ is correct.
(b)	**M1:** Either: • Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only. • Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity. • Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity. **A1*:** For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$. **Note** this result is printed on the Question Paper, so no incorrect working is allowed. **B1:** Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.
(c)	**B1:** For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and dx. Can be implied. **M1:** Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe **A1:** $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe. **dM1:** Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round. **Note:** that this mark is dependent upon the previous method mark being awarded. **A1:** A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** A decimal answer of 2661.557... without a correct exact answer is A0. **Note:** If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.
(a) **Way 2**	***Alternative response using the Cartesian equation in part (a)*** $\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ — $\frac{dy}{dx} = \pm K x^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$ **M1**; $\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe **A1** At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$ — Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{dy}{dx}$. **dM1** So, $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$ — $\frac{1}{18}$ **A1 cao cso** **Note:** Way 2 is marked as M1 A1 dM1 A1 **Note:** For way 2 the second M1 mark is dependent on the first M1 being gained.

Notes for Question 7 Continued			
7. (b) **Way 2**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS**		
	$\{\text{RHS} =\}\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}}-9} = \sqrt{9\sec^2 t-9} = \sqrt{9\tan^2 t}$	For applying $1+\tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$=3\tan t = y\ \{= \text{LHS}\}$ **cso**	Correct proof from $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
7. (b) **Way 3**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**		
	$\{\text{LHS} =\}\ y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t-9}$	For applying $1+\tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\sec^2 t-9}$	M1
	$= \sqrt{9\left(\frac{x}{27}\right)^{\frac{2}{3}}-9} = \sqrt{9\left(\frac{x^{\frac{2}{3}}}{9}\right)-9} = \left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t-9}$ by using a correct trigonometric identity.		
7. (c) **Way 2**	***Alternative response for part (c) using parametric integration***		
	$V = \pi\int 9\tan^2 t\left(81\sec^2 t\sec t\tan t\right)\,\mathrm{d}t$	$\pi\int 3\tan t\left(81\sec^2 t\sec t\tan t\right)\,\mathrm{d}t$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\int 729\sec^2 t\tan^2 t\sec t\tan t\ \mathrm{d}t$		
	$= \{\pi\}\int 729\sec^2 t\left(\sec^2 t-1\right)\sec t\tan t\ \mathrm{d}t$		
	$= \{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ \mathrm{d}t$		
	$= \{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ \mathrm{d}t$		
	$= \{\pi\}\left[729\left(\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t\right)\right]$	$\pm A\sec^5 t \pm B\sec^3 t$	M1
		$729\left(\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t\right)$	A1
	$V = \{\pi\}\left[729\left(\frac{1}{5}\left(\frac{5}{3}\right)^5 - \frac{1}{3}\left(\frac{5}{3}\right)^3\right) - 729\left(\frac{1}{5}1^5 - \frac{1}{3}1^3\right)\right]$	Substitutes $\sec t = \frac{5}{3}$ and $\sec t = 1$ into an integrated function and subtracts the correct way round.	dM1
	$= 729\pi\left[\left(\frac{250}{243}\right) - \left(-\frac{2}{15}\right)\right]$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5]

Leave blank

8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent.
(2)

Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t.
(6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.
(4)

Question Number	Scheme		Marks
8.	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x)$, where M is a constant		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}$ is the <u>rate of increase</u> of the <u>mass of waste</u> products.	Any one correct explanation.	B1
	M is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u> (or the <u>initial mass</u> of <u>unburned fuel</u>)	Both explanations are correct.	B1
			[2]
(b)	$\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$ or $\int \frac{1}{k(M-x)}\,\mathrm{d}x = \int \mathrm{d}t$		B1
	$-\ln(M-x) = kt \ \{+c\}$ or $-\frac{1}{k}\ln(M-x) = t \ \{+c\}$	See notes	M1 A1
	$\{t=0,\ x=0 \Rightarrow\}\ -\ln(M-0) = k(0) + c$	See notes	M1
	$c = -\ln M \Rightarrow -\ln(M-x) = kt - \ln M$		
	then either...	***or...***	
	$-kt = \ln(M-x) - \ln M$	$kt = \ln M - \ln(M-x)$	
	$-kt = \ln\left(\frac{M-x}{M}\right)$	$kt = \ln\left(\frac{M}{M-x}\right)$	
	$\mathrm{e}^{-kt} = \frac{M-x}{M}$	$\mathrm{e}^{kt} = \frac{M}{M-x}$	ddM1
	$M\mathrm{e}^{-kt} = M - x$	$(M-x)\mathrm{e}^{kt} = M$ $M - x = M\mathrm{e}^{-kt}$	
	leading to $x = M - M\mathrm{e}^{-kt}$ or $x = M(1-\mathrm{e}^{-kt})$ oe		A1 * **cso**
			[6]
(c)	$\left\{x = \frac{1}{2}M,\ t = \ln 4 \Rightarrow\right\}\ \frac{1}{2}M = M(1-\mathrm{e}^{-k\ln 4})$		M1
	$\Rightarrow \frac{1}{2} = 1 - \mathrm{e}^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$		
	So $k = \frac{1}{2}$		A1
	$x = M\left(1 - \mathrm{e}^{-\frac{1}{2}\ln 9}\right)$		dM1
	$x = \frac{2}{3}M$	$x = \frac{2}{3}M$	A1 **cso**
			[4] **12**

Notes for Question 8 Continued	
8. (a)	**B1:** At least one explanation correct. **B1:** Both explanations are correct. $\frac{\mathrm{d}x}{\mathrm{d}t}$ is the rate of increase of the mass of waste products. or the rate of change of the mass of waste products. M is the total mass of unburned fuel and waste fuel or the initial mass of unburned fuel or the total mass of rocket fuel and waste fuel or the initial mass of rocket fuel or the initial mass of fuel or the total mass of waste and unburned products.
(b)	**B1:** Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1: Both** $\pm\lambda\ln(M-x)$ or $\pm\lambda\ln(x-M)$ **and** $\pm\mu t$ where λ and μ are any constants. **A1:** For $-\ln(M-x)=kt$ or $-\ln(x-M)=kt$ or $-\frac{1}{k}\ln(M-x)=t$ or $-\frac{1}{k}\ln(x-M)=t$ or $-\frac{1}{k}\ln(kM-kx)=t$ or $-\frac{1}{k}\ln(kx-kM)=t$ **Note:** $+c$ is not needed for this mark. **IMPORTANT:** $+c$ can be on either side of their equation for the 1st A1 mark. **M1:** Substitutes $t=0$ AND $x=0$ in an integrated or changed equation containing c (or A or $\ln A$, etc.) **Note** that this mark can be implied by the correct value of c. **ddM1:** Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on both previous method marks being awarded. **A1:** $x=M-Me^{-kt}$ or $x=M(1-e^{-kt})$ or $x=\frac{M(e^{kt}-1)}{e^{kt}}$ or equivalent where x is the subject. **Note:** Please check their working as incorrect working can lead to a correct answer. **Note:** $\left\{\frac{\mathrm{d}x}{\mathrm{d}t}=k(M-x)\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{kM-kx}\Rightarrow\right\}$ $x=-\frac{1}{k}\ln(kM-kx)\ \{+c\}$ is B1(Implied) M1A1.
(c)	**M1:** Substitutes $x=\frac{1}{2}M$ and $t=\ln 4$ into one of their earlier equations connecting x and t. **A1:** $k=\frac{1}{2}$, which can be an un-simplified equivalent numerical value. i.e. $k=\frac{\ln 2}{\ln 4}$ is fine for A1. **dM1:** Substitutes $t=\ln 4$ and their evaluated k (which must be a numerical value) into one of their earlier equations connecting x and t. **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (c). **A1:** $x=\frac{2}{3}M$ **cso**. **Note:** Please check their working as incorrect working can lead to a correct answer.

Notes for Question 8 Continued		
***Aliter* 8. (b) Way 2**	$\int \frac{1}{M-x}\,dx = \int k\,dt$	B1
	$-\ln(M-x) = kt\ \{+c\}$ See notes	M1 A1
	$\ln(M-x) = -kt + c$	
	$M - x = Ae^{-kt}$	
	$\{t = 0,\ x = 0 \Rightarrow\}\ M - 0 = Ae^{-k(0)}$	M1
	$\Rightarrow M = A$	
	$M - x = Me^{-kt}$	ddM1
	So, $x = M - Me^{-kt}$	A1
		[6]
(b)	**B1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $x = 0$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **ddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on both previous method marks being awarded. **Note:** $\ln(M-x) = -kt + c$ leading to $\ln(M-x) = e^{-kt} + e^{c}$ or $\ln(M-x) = e^{-kt} + A$ would be dddM0. **A1:** Same as the original scheme.	
***Aliter* 8. (b) Way 3**	$\int_0^x \frac{1}{M-x}\,dx = \int_0^t k\,dt$	B1
	$\left[-\ln(M-x)\right]_0^x = \left[kt\right]_0^t$	M1 A1
	$-\ln(M-x) - (-\ln M) = kt$ Applies limits of	M1
	$-\ln(M-x) + \ln M = kt$	
	and then follows the original scheme.	
(a)	**B1M1A1:** Mark as in the original scheme (ignoring the limits). **ddM1:** Applies limits 0 and x on their integrated LHS and limits of 0 and t. **M1A1:** Same as the original scheme.	

Notes for Question 8 Continued

Aliter 8. (b) Way 4	$\int \frac{1}{M-x}\,dx = \int k\,dt \quad \left\{\Rightarrow \int \frac{-1}{x-M}\,dx = \int k\,dt\right\}$		B1
	$-\ln\lvert x-M\rvert = kt + c$	*Modulus not required for 1st A1.*	M1 A1
	$\{t=0,\ x=0 \Rightarrow\}\ -\ln\lvert 0-M\rvert = k(0)+c$	*Modulus not required here!*	M1
	$\Rightarrow c = -\ln M \Rightarrow -\ln\lvert x-M\rvert = kt - \ln M$		
	then either... $-kt = \ln\lvert x-M\rvert - \ln M$; $-kt = \ln\left\lvert\frac{x-M}{M}\right\rvert$ ***or...*** $kt = \ln M - \ln\lvert x-M\rvert$; $kt = \ln\left\lvert\frac{M}{x-M}\right\rvert$		
	As $x < M$		
	$-kt = \ln\left(\frac{M-x}{M}\right)$; $e^{-kt} = \frac{M-x}{M}$ **or** $kt = \ln\left(\frac{M}{M-x}\right)$; $e^{kt} = \frac{M}{M-x}$	*Understanding of modulus is required here!*	**ddM1**
	$Me^{-kt} = M-x$ **or** $(M-x)e^{kt} = M$; $M-x = Me^{-kt}$		
	leading to $x = M - Me^{-kt}$ or $x = M(1-e^{-kt})$ oe		**A1 * cso**
			[6]
	B1: Mark as in the original scheme. **M1A1M1:** Mark as in the original scheme ignoring the modulus. **ddM1:** Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert x-M\rvert$ to $\ln(M-x)$ in their working. **Note:** This mark is dependent on both the previous method marks being awarded. **A1:** Mark as in the original scheme.		

Aliter 8. (b) Way 5	***Use of an integrating factor*** **(I.F.)**	
	$\frac{dx}{dt} = k(M-x) \Rightarrow \frac{dx}{dt} + kx = kM$	
	$\text{I.F.} = e^{kt}$	B1
	$\frac{d}{dt}\left(e^{kt}x\right) = kMe^{kt}$,	
	$e^{kt}x = Me^{kt} + c$	M1A1
	$x = M + ce^{-kt}$	
	$\{t=0,\ x=0 \Rightarrow\}\ 0 = M + ce^{-k(0)}$	M1
	$\Rightarrow c = -M$	
	$x = M - Me^{-kt}$	ddM1A1