





Question Number	Scheme	Marks
1.	$\frac{5x + 3}{(2x + 1)(x + 1)^2} \equiv \frac{A}{(2x + 1)} + \frac{B}{(x + 1)} + \frac{C}{(x + 1)^2}$ $A = 2, C = 2$ $5x + 3 \equiv A(x + 1)^2 + B(2x + 1)(x + 1) + C(2x + 1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$ <p>Either <math>x^2: 0 = A + 2B</math>, constant: <math>3 = A + B + C</math>  <math>x: 5 = 2A + 3B + 2C</math></p> <p>leading to <math>B = -1</math></p> <p>So, <math display="block">\frac{5x + 3}{(2x + 1)(x + 1)^2} \equiv \frac{2}{(2x + 1)} - \frac{1}{(x + 1)} + \frac{2}{(x + 1)^2}</math></p>	<p>At least one of "A" or "C" are correct. B1</p> <p>Breaks up their partial fraction correctly into three terms <b>and</b> both "A" = 2 and "C" = 2. B1 cso</p> <p>Writes down <b>a correct identity</b> and attempts to find the value of either one "A" or "B" or "C". M1</p> <p>Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition. A1 cso</p> <p style="text-align: right;"><b>[4]</b> <b>4</b></p>

**Notes for Question 1**

**BE CAREFUL!** Candidates will assign *their own* "A, B and C" for this question.

**B1:** At least one of "A" or "C" are correct.

**B1:** Breaks up their partial fraction correctly into three terms **and** both "A" = 2 and "C" = 2.

**M1:** Writes down **a correct identity** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".  
 This can be achieved by **either** substituting values into their identity **or** comparing coefficients and solving the resulting equations simultaneously.

**A1:** Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.

**Note:** If a candidate does not give partial fraction decomposition then:

- the 2<sup>nd</sup> B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

**Note:** The correct partial fraction from no working scores B1B1M1A1.

**Note:** A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.



Question Number	Scheme	Marks
2.	$3^{x-1} + xy - y^2 + 5 = 0$ $\left\{ \begin{array}{l} \cancel{\frac{dy}{dx}} \\ \cancel{\frac{dy}{dx}} \end{array} \right\} \times \left\{ \begin{array}{l} \times \\ \times \end{array} \right\} 3^{x-1} \ln 3 + \left( y + x \frac{dy}{dx} \right) - 2y \frac{dy}{dx} = 0$ <p>(ignore)</p> $\{(1, 3) \Rightarrow\} 3^{(1-1)} \ln 3 + 3 + (1) \frac{dy}{dx} - 2(3) \frac{dy}{dx} = 0$ $\ln 3 + 3 + \frac{dy}{dx} - 6 \frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{3 + \ln 3}{5}$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3) = \frac{1}{5} \ln(3e^3)$	$3^{x-1} \rightarrow 3^{x-1} \ln 3$ B1 oe Differentiates implicitly to include either $\pm \lambda x \frac{dy}{dx}$ or $\pm ky \frac{dy}{dx}$ . M1* $xy \rightarrow + y + x \frac{dy}{dx}$ B1 $\dots + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$ A1 Substitutes $x = 1, y = 3$ into their differentiated equation or expression. dM1* dM1* Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$ A1 cso [7] 7

Notes for Question 2

**B1:** Correct differentiation of  $3^{x-1}$ . I.e.  $3^{x-1} \rightarrow 3^{x-1} \ln 3$  or  $3^{x-1} = \frac{1}{3}(3^x) \rightarrow \frac{1}{3}(3^x) \ln 3$

or  $3^{x-1} = e^{(x-1)\ln 3} \rightarrow \ln 3 e^{(x-1)\ln 3}$  or  $3^{x-1} = \frac{1}{3}(3^x) = \frac{1}{3}e^{x \ln 3} \rightarrow \frac{1}{3}(\ln 3)e^{x \ln 3}$

**M1:** Differentiates implicitly to include either  $\pm \lambda x \frac{dy}{dx}$  or  $\pm ky \frac{dy}{dx}$ . (Ignore  $\left(\frac{dy}{dx} = \right)$ ).

**B1:**  $xy \rightarrow + y + x \frac{dy}{dx}$

**1<sup>st</sup> A1:**  $\dots + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$  **Note:** The 1<sup>st</sup> A0 follows from an award of the 2<sup>nd</sup> B0.

**Note:** The "= 0" can be implied by rearrangement of their equation.

ie:  $3^{x-1} \ln 3 + y + x \frac{dy}{dx} - 2y \frac{dy}{dx}$  leading to  $3^{x-1} \ln 3 + y = 2y \frac{dy}{dx} - x \frac{dy}{dx}$  will get A1 (implied).

**2<sup>nd</sup> M1:** **Note:** This method mark is dependent upon the 1<sup>st</sup> M1\* mark being awarded.

Substitutes  $x = 1, y = 3$  into their differentiated equation or expression. Allow one slip.

**3<sup>rd</sup> M1:** **Note:** This method mark is dependent upon the 1<sup>st</sup> M1\* mark being awarded.

Candidate has two differentiated terms in  $\frac{dy}{dx}$  and rearranges to make  $\frac{dy}{dx}$  the subject.

**Note:** It is possible to gain the 3<sup>rd</sup> M1 mark before the 2<sup>nd</sup> M1 mark.

**Eg:** Candidate may write  $\frac{dy}{dx} = \frac{y + 3^{x-1} \ln 3}{2y - x}$  before substituting in  $x = 1$  and  $y = 3$

**2<sup>nd</sup> A1:** cso. Uses  $3 = \ln e^3$  to achieve  $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$ ,  $\left( = \frac{1}{\lambda} \ln(\mu e^3) \right)$ ,  $\lambda = 5$  and  $\mu = 3$

**Note:**  $3 = \ln e^3$  needs to be seen in their proof.

Notes for Question 2 Continued

<p>2.</p> <p><i>Aliter</i> <b>Way 2</b></p>	<p><b>Alternative Method: Multiplying both sides by 3</b></p> $3^{x-1} + xy - y^2 + 5 = 0$ $3^x + 3xy - 3y^2 + 15 = 0$ $\left\{ \begin{array}{l} \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right\} \times 3^x \ln 3 + \left( 3y + 3x \frac{dy}{dx} \right) - 6y \frac{dy}{dx} = 0$ <p>(ignore)</p> $\{(1, 3) \Rightarrow\} 3^1 \ln 3 + 3(3) + (3)(1) \frac{dy}{dx} - 6(3) \frac{dy}{dx} = 0$ $3 \ln 3 + 9 + 3 \frac{dy}{dx} - 18 \frac{dy}{dx} = 0 \Rightarrow 9 + 3 \ln 3 = 15 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{9 + 3 \ln 3}{15} \left\{ = \frac{3 + \ln 3}{5} \right\}$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3)$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3) = \frac{1}{5} \ln(3e^3)$	<p><math>3^x \rightarrow 3^x \ln 3</math> B1</p> <p>Differentiates implicitly to include either <math>\pm \lambda x \frac{dy}{dx}</math> or <math>\pm ky \frac{dy}{dx}</math>. M1*</p> <p><math>3xy \rightarrow + 3y + 3x \frac{dy}{dx}</math> B1</p> <p><math>\dots + 3y + 3x \frac{dy}{dx} - 6y \frac{dy}{dx} = 0</math> A1</p> <p>Substitutes <math>x = 1, y = 3</math> into their differentiated equation or expression. dM1*</p> <p>Uses <math>3 = \ln e^3</math> to achieve <math>\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)</math> A1 cso</p> <p>[7] 7</p>
<p><b>NOTE:</b> Only apply this scheme if the candidate has multiplied both sides of their equation by 3.</p> <p><b>NOTE:</b> For reference, <math>\frac{dy}{dx} = \frac{3y + 3^x \ln 3}{6y - 3x}</math></p> <p><b>NOTE:</b> If the candidate applies this method then <math>3xy \rightarrow + 3y + 3x \frac{dy}{dx}</math> must be seen for the 2<sup>nd</sup> B1 mark.</p>		



Question Number	Scheme	Marks
3.	$\int_0^4 \frac{1}{2 + \sqrt{2x+1}} dx, \quad u = 2 + \sqrt{2x+1}$ $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}} \quad \text{or} \quad \frac{dx}{du} = u-2$ $\left\{ \int \frac{1}{2 + \sqrt{2x+1}} dx \right\} = \int \frac{1}{u} (u-2) du$ $= \int \left( 1 - \frac{2}{u} \right) du$ $= u - 2 \ln u$ $\left\{ \text{So } [u - 2 \ln u]_3^5 \right\} = (5 - 2 \ln 5) - (3 - 2 \ln 3)$ $= 2 + 2 \ln \left( \frac{3}{5} \right)$	<p>M1</p> <p>A1</p> <p>A1 Correct substitution (Ignore integral sign and <math>du</math>).</p> <p>dM1 An attempt to divide each term by <math>u</math>.</p> <p>ddM1 <math>\pm Au \pm B \ln u</math></p> <p>A1 ft <math>u - 2 \ln u</math></p> <p>M1 Applies limits of 5 and 3 in <math>u</math> or 4 and 0 in <math>x</math> in their integrated function and subtracts the correct way round.</p> <p>A1 cao cso <math>2 + 2 \ln \left( \frac{3}{5} \right)</math></p> <p>[8] 8</p>
<b>Notes for Question 3</b>		
<p><b>M1:</b> Also allow <math>du = \pm \lambda \frac{1}{(u-2)} dx</math> or <math>(u-2)du = \pm \lambda dx</math> <b>Note:</b> The expressions must contain <math>du</math> and <math>dx</math>. They can be simplified or un-simplified.</p> <p><b>A1:</b> Also allow <math>du = \frac{1}{(u-2)} dx</math> or <math>(u-2)du = \pm \lambda dx</math> <b>Note:</b> The expressions must contain <math>du</math> and <math>dx</math>. They can be simplified or un-simplified.</p> <p><b>A1:</b> <math>\int \frac{1}{u} (u-2) du</math>. (Ignore integral sign and <math>du</math>).</p> <p><b>dM1:</b> An attempt to divide each term by <math>u</math>. <b>Note</b> that this mark is dependent on the previous M1 mark being awarded. <b>Note</b> that this mark can be implied by later working.</p> <p><b>ddM1:</b> <math>\pm Au \pm B \ln u</math>, <math>A \neq 0</math>, <math>B \neq 0</math> <b>Note</b> that this mark is dependent on the two previous M1 marks being awarded.</p> <p><b>A1ft:</b> <math>u - 2 \ln u</math> or <math>\pm Au \pm B \ln u</math> being correctly followed through, <math>A \neq 0</math>, <math>B \neq 0</math></p> <p><b>M1:</b> Applies limits of 5 and 3 in <math>u</math> or 4 and 0 in <math>x</math> in their integrated function and subtracts the correct way round.</p> <p><b>A1:</b> cso and cao. <math>2 + 2 \ln \left( \frac{3}{5} \right)</math> or <math>2 + 2 \ln(0.6)</math>, <math>\left( = A + 2 \ln B, \text{ so } A = 2, B = \frac{3}{5} \right)</math></p> <p><b>Note:</b> <math>2 - 2 \ln \left( \frac{3}{5} \right)</math> is A0.</p>		



Notes for Question 3 Continued

3. ctd

Note:  $\int \frac{1}{u} (u - 2) du = u - 2 \ln u$  with no working is 2<sup>nd</sup> M1, 3<sup>rd</sup> M1, 3<sup>rd</sup> A1.

but Note:  $\int \frac{1}{u} (u - 2) du = (u - 2) \ln u$  with no working is 2<sup>nd</sup> M0, 3<sup>rd</sup> M0, 3<sup>rd</sup> A0.



Question Number	Scheme	Marks
<p>4. (a)</p>	$\left\{ \sqrt[3]{(8-9x)} \right\} = (8-9x)^{\frac{1}{3}}$ $= \underline{(8)^{\frac{1}{3}}} \left( 1 - \frac{9x}{8} \right)^{\frac{1}{3}} = \underline{2} \left( 1 - \frac{9x}{8} \right)^{\frac{1}{3}}$ $= \{2\} \left[ 1 + \left( \frac{1}{3} \right) (kx) + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right)}{2!} (kx)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{3!} (kx)^3 + \dots \right]$ $= \{2\} \left[ 1 + \left( \frac{1}{3} \right) \left( \frac{-9x}{8} \right) + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{2!} \left( \frac{-9x}{8} \right)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right) \left( -\frac{5}{3} \right)}{3!} \left( \frac{-9x}{8} \right)^3 + \dots \right]$ $= 2 \left[ 1 - \frac{3}{8}x; -\frac{9}{64}x^2 - \frac{45}{512}x^3 + \dots \right]$ $= 2 - \frac{3}{4}x; -\frac{9}{32}x^2 - \frac{45}{256}x^3 + \dots$	<p>Power of <math>\frac{1}{3}</math> M1</p> <p><math>(8)^{\frac{1}{3}}</math> or <math>\underline{2}</math> B1</p> <p>see notes M1 A1</p> <p>See notes below!</p> <p>A1; A1 [6]</p>
<p>(b)</p>	$\left\{ \sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)}, \right\} \text{ so } x = 0.1$ <p>When <math>x = 0.1</math>, <math>\sqrt[3]{(8-9x)} \approx 2 - \frac{3}{4}(0.1) - \frac{9}{32}(0.1)^2 - \frac{45}{256}(0.1)^3 + \dots</math></p> $= 2 - 0.075 - 0.0028125 - 0.00017578125$ $= 1.922011719$ <p>So, <math>\sqrt[3]{7100} = 19.220117919\dots = \underline{19.2201}</math> (4 dp)</p>	<p>Writes down or uses <math>x = 0.1</math> B1</p> <p>M1</p> <p>19.2201 cso A1 cao [3]</p> <p>9</p>

Notes for Question 4

<p>(a)</p>	<p><b>M1:</b> Writes or uses <math>\frac{1}{3}</math>. This mark can be implied by a constant term of <math>8^{\frac{1}{3}}</math> or 2.</p> <p><b>B1:</b> <math>(8)^{\frac{1}{3}}</math> or <math>\underline{2}</math> outside brackets or <math>\underline{2}</math> as candidate's constant term in their binomial expansion.</p> <p><b>M1:</b> Expands <math>(\dots + kx)^{\frac{1}{3}}</math> to give any 2 terms out of 4 terms simplified or un-simplified,</p> <p>Eg: <math>1 + \left( \frac{1}{3} \right) (kx)</math> or <math>\frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right)}{2!} (kx)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{3!} (kx)^3</math> or <math>1 + \dots + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right)}{2!} (kx)^2</math></p> <p>or <math>\frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right)}{2!} (kx)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{3!} (kx)^3</math> where <math>k \neq 1</math> are fine for M1.</p> <p><b>A1:</b> A correct simplified or un-simplified <math>1 + \left( \frac{1}{3} \right) (kx) + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right)}{2!} (kx)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{3!} (kx)^3</math> expansion with consistent <math>(kx)</math>. Note that <math>(kx)</math> must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. <b>Note</b> that <math>k \neq 1</math>.</p> <p>You would award B1M1A0 for <math>2 \left[ 1 + \left( \frac{1}{3} \right) \left( \frac{-9x}{8} \right) + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right)}{2!} (-9x)^2 + \frac{\left( \frac{1}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{5}{3} \right) \left( -\frac{5}{3} \right)}{3!} \left( \frac{-9x}{8} \right)^3 + \dots \right]</math> because <math>(kx)</math> is not consistent.</p>
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Notes for Question 4 Continued

4. (a) ctd

“Incorrect bracketing” =  $\{2\} \left[ 1 + \left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{-9x^2}{8}\right)}{2!} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{-9x^3}{8}\right)}{3!} + \dots \right]$

is M1A0 unless recovered.

A1: For  $2 - \frac{3}{4}x$  (simplified please) or also allow  $2 - 0.75x$ .

Allow Special Case A1A0 for either SC:  $= 2 \left[ 1 - \frac{3}{8}x; \dots \right]$  or SC:  $K \left[ 1 - \frac{3}{8}x - \frac{9}{64}x^2 - \frac{45}{512}x^3 + \dots \right]$

(where K can be 1 or omitted), with each term in the [.....] either a simplified fraction or a decimal.

A1: Accept only  $-\frac{9}{32}x^2 - \frac{45}{256}x^3$  or  $-0.28125x^2 - 0.17578125x^3$

Candidates who write  $= 2 \left[ 1 + \left(\frac{1}{3}\right)\left(\frac{9x}{8}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{9x^2}{8}\right)}{2!} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{9x^3}{8}\right)}{3!} + \dots \right]$  where  $k = \frac{9}{8}$

and not  $-\frac{9}{8}$  and achieve  $2 + \frac{3}{4}x; -\frac{9}{32}x^2 + \frac{45}{256}x^3 + \dots$  will get B1M1A1A0A0.

**Note for final two marks:**

$2 \left[ 1 - \frac{3}{8}x; -\frac{9}{64}x^2 - \frac{45}{512}x^3 + \dots \right] = 2 + \frac{3}{4}x - \frac{9}{32}x^2 - \frac{45}{256}x^3 + \dots$  scores final A0A1.

$2 \left[ 1 - \frac{3}{8}x; -\frac{9}{64}x^2 - \frac{45}{512}x^3 + \dots \right] = 2 - \frac{3}{4} - \frac{9}{32}x^2 - \frac{45}{256}x^3 + \dots$  scores final A0A1

**Alternative method:** Candidates can apply an alternative form of the binomial expansion.

$\left\{ \sqrt[3]{(8-9x)} \right\} = (8-9x)^{\frac{1}{3}} = (8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-9x) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(-9x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(-9x)^3$

B1:  $(8)^{\frac{1}{3}}$  or 2

M1: Any two of four (un-simplified or simplified) terms correct.

A1: All four (un-simplified or simplified) terms correct.

A1:  $2 - \frac{3}{4}x$

A1:  $-\frac{9}{32}x^2 - \frac{45}{256}x^3$

Note: The terms in C need to be evaluated,

so  ${}^{\frac{1}{3}}C_0(8)^{\frac{1}{3}} + {}^{\frac{1}{3}}C_1(8)^{-\frac{2}{3}}(-9x) + {}^{\frac{1}{3}}C_2(8)^{-\frac{5}{3}}(-9x)^2 + {}^{\frac{1}{3}}C_3(8)^{-\frac{8}{3}}(-9x)^3$  without further working is B0M0A0.

(b) B1: Writes down or uses  $x = 0.1$

M1: Substitutes their  $x$ , where  $|x| < \frac{8}{9}$  into at least two terms of their binomial expansion.

A1: 19.2201 cao

**Be Careful!** The binomial answer is 19.22011719

and the calculated  $\sqrt[3]{7100}$  is 19.21997343... which is 19.2200 to 4 decimal places.



Question Number	Scheme	Marks
5. (a)	6.248046798... = 6.248 (3dp) <span style="float: right;">6.248 or awrt 6.248</span>	B1 [1]
(b)	$\text{Area} \approx \frac{1}{2} \times 2 \times [3 + 2(7.107 + 7.218 + \text{their } 6.248) + 5.223]$ $= 49.369 = 49.37 \text{ (2 dp)}$ <span style="float: right;">49.37 or awrt 49.37</span>	B1; M1 A1 [3]
(c)	$\left\{ \int (4te^{-\frac{1}{3}t} + 3) dt \right\} = -12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t} \{dt\} + 3t$ $= -12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} \{+ 3t\}$ $\left[ -12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + 3t \right]_0^8 =$ $= \left( -12(8)e^{-\frac{1}{3}(8)} - 36e^{-\frac{1}{3}(8)} + 3(8) \right) - \left( -12(0)e^{-\frac{1}{3}(0)} - 36e^{-\frac{1}{3}(0)} + 3(0) \right)$ $= \left( -96e^{-\frac{8}{3}} - 36e^{-\frac{8}{3}} + 24 \right) - (0 - 36 + 0)$ $= 60 - 132e^{-\frac{8}{3}}$ <span style="float: right;">60 - 132e<sup>-8/3</sup></span>	$\pm Ate^{-\frac{1}{3}t} \pm B \int e^{-\frac{1}{3}t} \{dt\}, A \neq 0, B \neq 0$ See notes. 3 → 3t -12te <sup>-1/3t</sup> - 36e <sup>-1/3t</sup> M1 A1 B1 A1 Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm \lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm \lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round. dM1 A1 [6]
(d)	Difference = $\left  60 - 132e^{-\frac{8}{3}} - 49.37 \right  = 1.458184439... = 1.46 \text{ (2 dp)}$ <span style="float: right;">1.46 or awrt 1.46</span>	B1 [1]

**Notes for Question 5**

(a)	<b>B1:</b> 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	<b>B1:</b> Outside brackets $\frac{1}{2} \times 2$ or 1 <b>M1:</b> For structure of trapezium rule [ ..... ]. Allow one miscopy of their values. <b>A1:</b> 49.37 or anything that rounds to 49.37 <b>Note:</b> It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37) <b>Note:</b> Working must be seen to demonstrate the use of the trapezium rule. <b>Note:</b> actual area is 50.828... <b>Bracketing mistake:</b> <b>Unless the final answer implies that the calculation has been done correctly,</b> Award B1M0A0 for $1 + 3 + 2(7.107 + 7.218 + \text{their } 6.248) + 5.223$ (nb: answer of 50.369).

[1]  
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## Notes for Question 5 Continued

5. (b) ctd

*Alternative method for part (b): Adding individual trapezia*

$$\text{Area} \approx 2 \times \left[ \frac{3+7.107}{2} + \frac{7.107+7.218}{2} + \frac{7.218+6.248}{2} + \frac{6.248+5.223}{2} \right] = 49.369$$

**B1:** 2 and a divisor of 2 on all terms inside brackets.**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.**A1:** anything that rounds to 49.37(c) **M1:** For  $4te^{-\frac{1}{3}t} \rightarrow \pm Ate^{-\frac{1}{3}t} \pm B \int e^{-\frac{1}{3}t} \{dt\}$ ,  $A \neq 0$ ,  $B \neq 0$ **A1:** For  $te^{-\frac{1}{3}t} \rightarrow \left( -3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t} \right)$  (some candidates lose the 4 and this is fine for the first A1 mark).

$$\text{or } 4te^{-\frac{1}{3}t} \rightarrow 4 \left( -3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t} \right) \text{ or } -12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t} \text{ or } 12 \left( -te^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t} \right)$$

These results can be implied. They can be simplified or un-simplified.

**B1:**  $3 \rightarrow 3t$  or  $3 \rightarrow 3x$  (bod) .**Note:** Award B0 for 3 integrating to  $12t$  (implied), which is a common error when taking out a factor of 4.**Be careful** some candidates will factorise out 4 and have  $4 \left( \dots + \frac{3}{4} \right) \rightarrow 4 \left( \dots + \frac{3}{4}t \right)$ 

which would then be fine for B1.

**Note:** Allow B1 for  $\int_0^8 3dt = 24$ **A1:** For correct integration of  $4te^{-\frac{1}{3}t}$  to give  $-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$  or  $4 \left( -3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t} \right)$  or equivalent.

This can be simplified or un-simplified.

**dm1:** Substitutes limits of 8 and 0 into an integrated function of the form of either  $\pm \lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$  or  $\pm \lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$  and subtracts the correct way round.**Note:** Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dm1. So, just subtracting zero is M0.**A1:** An exact answer of  $60 - 132e^{-\frac{8}{3}}$ . A decimal answer of 50.82818444... without a correct answer is A0.**Note:** A decimal answer of 50.82818444... without a correct exact answer is A0.**Note:** If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0.**IMPORTANT:** that is fine for candidates to work in terms of  $x$  rather than  $t$  in part (c).**Note:** The "3t" is needed for B1 and the final A1 mark.(d) **B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46.

Candidates may give correct decimal answers of 1.458184439... or 1.459184439...

**Note:** You can award this mark whether or not the candidate has answered part (c) correctly.





Question Number	Scheme	Marks
<p><b>6.</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> <p><b>(d)</b></p>	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overline{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overline{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$ <p>A is on <math>l</math>, so <math>\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}</math></p> <p>{<b>k</b>: <math>10 - \lambda = 6 \Rightarrow \lambda = 4</math></p> <p>{<b>i</b>: <math>a + 6\lambda = 21 \Rightarrow a + 6(4) = 21</math> <math>a = -3</math></p> $\left\{ \overline{AB} \right\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} \quad \left\{ \overline{BA} \right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$ $\left\{ \overline{AB} \right\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \quad \left\{ \overline{BA} \right\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$ <p>{<math>\overline{AB} \perp l \Rightarrow \overline{AB} \cdot \mathbf{d} = 0</math>} <math>\Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4</math></p> <p>{<b>j</b>: <math>b + c\lambda = -17 \Rightarrow b + (-4)(4) = -17; \Rightarrow b = -1</math></p> <p><math> AB  = \sqrt{4^2 + 3^2 + 12^2}</math> or <math> AB  = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}</math> So, <math> AB  = 13</math></p> $\overline{OB'} \left\{ = \overline{OA} + \overline{BA} \right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	<p><math>\lambda = 4</math> B1</p> <p>Substitutes their value of <math>\lambda</math> into <math>a + 6\lambda = 21</math> M1</p> <p><math>a = -3</math> A1 <b>cao</b></p> <p>Finds the difference between <math>\overline{OA}</math> and <math>\overline{OB}</math>. Ignore labelling. M1</p> <p>See notes. M1; A1 ft</p> <p>See notes. ddM1; A1 <b>cso cao</b></p> <p>See notes. M1</p> <p>See notes for alternative methods. M1; A1 <b>cao</b></p> <p>[3]</p> <p>[5]</p> <p>[2]</p> <p>[2]</p> <p><b>12</b></p>
<b>Notes for Question 6</b>		
<b>(a)</b>	<p><b>B1:</b> <math>\lambda = 4</math> seen or implied.</p> <p><b>M1:</b> Substitutes their value of <math>\lambda</math> into <math>a + 6\lambda = 21</math></p> <p><b>A1:</b> <math>a = -3</math>.</p> <p>Note: Award B1M1A1 if the candidate states <math>a = -3</math> from no working.</p> <p><u>Alternative Method Using Simultaneous equations for part (a).</u></p> <p><b>B1:</b> For <math>60 - 6\lambda = 36</math></p> <p><b>M1:</b> <math>60 - 6\lambda = 36</math> and <math>a + 6\lambda = 21</math> solved simultaneously to give <math>a = \dots</math></p> <p><b>A1:</b> <math>a = -3</math>, <b>cao</b>.</p>	

Notes for Question 6 Continued

6. (b)  
ctd

**M1:** Finds the difference between  $\overline{OA}$  and  $\overline{OB}$ . Ignore labelling.

If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.

**M1:** *Applies* the formula  $\overline{AB} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$  or  $\overline{BA} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$  correctly to give a linear equation in  $c$  which is set equal

to zero. **Note:** The dot product can also be with  $\pm k \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ .

**A1ft:**  $c = -4$  or for finding a correct follow through  $c$ .

**ddM1:** Substitutes their value of  $\lambda$  and their value of  $c$  into  $b + c\lambda = -17$

**Note** that this mark is dependent on the two previous method marks being awarded.

**A1:**  $b = -1$

(c) **M1:** An attempt to apply a three term Pythagoras in order to find  $|AB|$ ,  
so taking the square root is required here.

**A1:** 13 cao

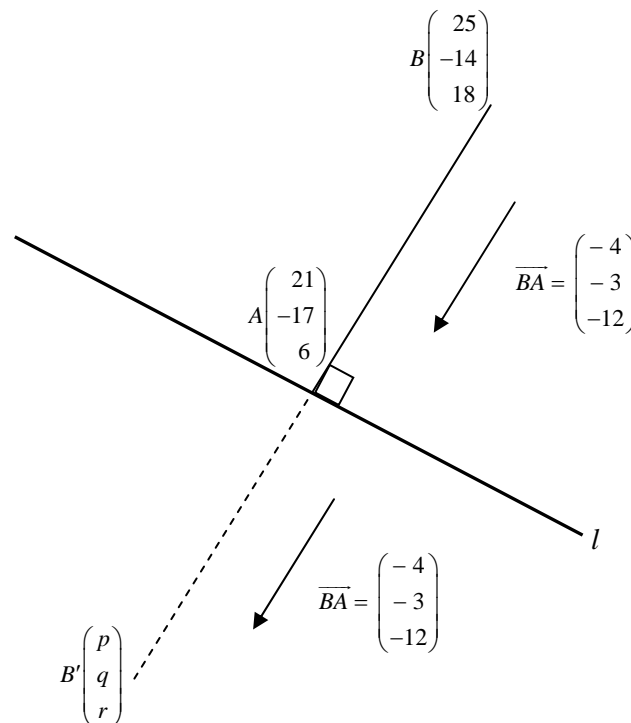
**Note:** Don't recover work for part (b) in part (c).

(d) **M1:** For a full *applied* method of finding the coordinates of  $B'$ .

**Note:** You can give M1 for 2 out of 3 correct components of  $B'$ .

**A1:** For either  $\begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$  or  $17\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}$  or  $(17, -20, -6)$  cao.

*Helpful diagram!*



Notes for Question 6 Continued

Acceptable Methods for the Method mark in part (d)

<b>Way 1</b>	$\overline{OB'} \{ = \overline{OA} + \overline{BA} \} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$ (using their $\overline{BA}$ )
<b>Way 2</b>	$\overline{OB'} \{ = \overline{OA} - \overline{AB} \} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ (using their $\overline{AB}$ )
<b>Way 3</b>	$\overline{OB'} \{ = \overline{OB} + 2\overline{BA} \} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} + 2 \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$ (using their $\overline{BA}$ )
<b>Way 4</b>	$\overline{OB'} \{ = \overline{OB} - 2\overline{AB} \} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ (using their $\overline{AB}$ )
<b>Way 5</b>	$\begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} \rightarrow \begin{pmatrix} \text{Minus 4} \\ \text{Minus 3} \\ \text{Minus 12} \end{pmatrix} \rightarrow \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} \text{Minus 4} \\ \text{Minus 3} \\ \text{Minus 12} \end{pmatrix} \left\{ \rightarrow \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix} \right\}$ , so $\overline{OA}$ + their $\overline{BA}$
<b>Way 6</b>	$\overline{OB'} \{ = 2\overline{OA} - \overline{OB} \} = 2 \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$
<b>Way 7</b>	$\overline{OB} = 25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$ , $\overline{OA} = 21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and $\overline{OB'} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$ , $(21, -17, 6) = \left( \frac{25 + p}{2}, \frac{-14 + q}{2}, \frac{18 + r}{2} \right)$ $p = 21(2) - 25 = 17$ $q = -17(2) + 14 = -20$ $r = 6(2) - 18 = -6$
	<b>M1:</b> Writing down any two equations correctly and an attempt to find at least two of $p, q$ or $r$ .

7.

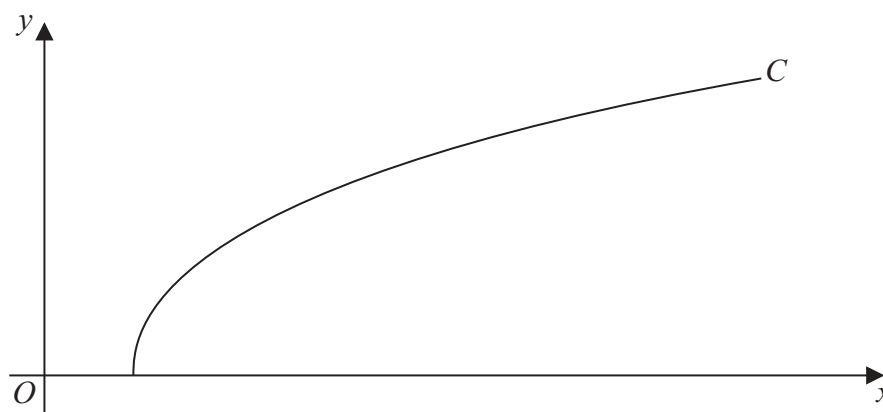


Figure 2

Figure 2 shows a sketch of the curve  $C$  with parametric equations

$$x = 27 \sec^3 t, \quad y = 3 \tan t, \quad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve  $C$  at the point where  $t = \frac{\pi}{6}$  (4)

(b) Show that the cartesian equation of  $C$  may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \quad a \leq x \leq b$$

stating the values of  $a$  and  $b$ . (3)

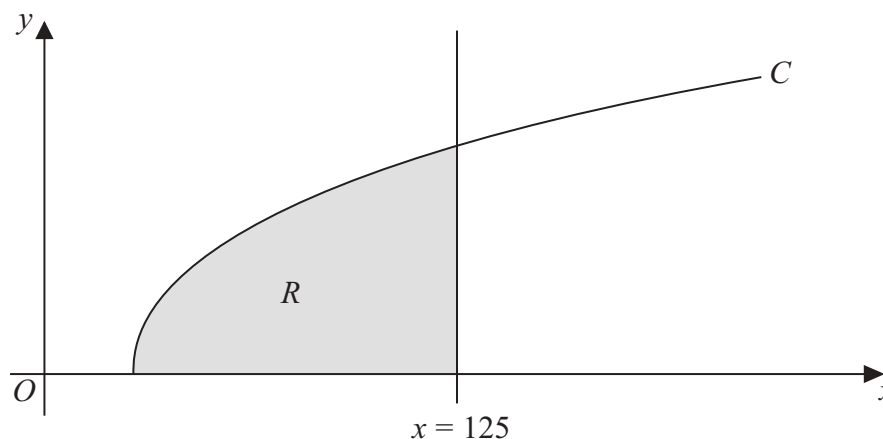


Figure 3

The finite region  $R$  which is bounded by the curve  $C$ , the  $x$ -axis and the line  $x = 125$  is shown shaded in Figure 3. This region is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution. (5)



Question Number	Scheme	Marks
7.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leq t \leq \frac{\pi}{3}$	
(a)	$\frac{dx}{dt} = 81\sec^2 t \sec t \tan t, \quad \frac{dy}{dt} = 3\sec^2 t$ $\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t \tan t} \left\{ = \frac{1}{27\sec t \tan t} = \frac{\cos t}{27 \tan t} = \frac{\cos^2 t}{27 \sin t} \right\}$ At $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{3\sec^2(\frac{\pi}{6})}{81\sec^3(\frac{\pi}{6})\tan(\frac{\pi}{6})} = \frac{4}{72} \left\{ = \frac{3}{54} = \frac{1}{18} \right\}$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. B1 Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. B1 Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ M1; $\frac{4}{72}$ A1 cao cso [4]
(b)	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\frac{x}{27}}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$ $\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} *$ $a = 27$ and $b = 216$ or $27 \leq x \leq 216$	$a = 27$ and $b = 216$ B1 M1 A1 * cso [3]
(c)	$V = \pi \int_{27}^{125} \left( \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \right)^2 dx$ or $\pi \int_{27}^{125} \left(x^{\frac{2}{3}} - 9\right) dx$ $= \{\pi\} \left[ \frac{3}{5}x^{\frac{5}{3}} - 9x \right]_{27}^{125}$ $= \{\pi\} \left( \left( \frac{3}{5}(125)^{\frac{5}{3}} - 9(125) \right) - \left( \frac{3}{5}(27)^{\frac{5}{3}} - 9(27) \right) \right)$ $= \{\pi\} ((1875 - 1125) - (145.8 - 243))$ $= \frac{4236\pi}{5}$ or $847.2\pi$	For $\pi \int \left( \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \right)^2$ or $\pi \int \left(x^{\frac{2}{3}} - 9\right)$ B1 Ignore limits and dx. Can be implied. Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe M1 $\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe A1 Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round. dM1 $\frac{4236\pi}{5}$ or $847.2\pi$ A1 [5]

Notes for Question 7

(a)	<p><b>B1:</b> At least one of <math>\frac{dx}{dt}</math> or <math>\frac{dy}{dt}</math> correct. <b>Note:</b> that this mark can be implied from their working.</p> <p><b>B1:</b> Both <math>\frac{dx}{dt}</math> and <math>\frac{dy}{dt}</math> are correct. <b>Note:</b> that this mark can be implied from their working.</p> <p><b>M1:</b> Applies their <math>\frac{dy}{dt}</math> divided by their <math>\frac{dx}{dt}</math>, where both <math>\frac{dy}{dt}</math> and <math>\frac{dx}{dt}</math> are trigonometric functions of <math>t</math>.</p> <p><b>A1:</b> <math>\frac{4}{72}</math> or any equivalent correct rational answer not involving surds.</p> <p>Allow 0.05 with the recurring symbol.</p>
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Notes for Question 7 Continued

**Note:** Please check that their  $\frac{dx}{dt}$  is differentiated correctly.

Eg. Note that  $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$  is correct.

- (b) **M1:** Either:
- Applying a correct trigonometric identity (usually  $1 + \tan^2 t = \sec^2 t$ ) to give a Cartesian equation in  $x$  and  $y$  only.
  - Starting from the RHS and goes on to achieve  $\sqrt{9\tan^2 t}$  by using a correct trigonometric identity.
  - Starts from the LHS and goes on to achieve  $\sqrt{9\sec^2 t - 9}$  by using a correct trigonometric identity.

**A1\*:** For a correct proof of  $y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}$ .

**Note** this result is printed on the Question Paper, so no incorrect working is allowed.

**B1:** Both  $a = 27$  and  $b = 216$ . **Note** that  $27 \leq x \leq 216$  is also fine for B1.

- (c) **B1:** For a correct statement of  $\pi \int \left( (x^{\frac{2}{3}} - 9)^{\frac{1}{2}} \right)^2$  or  $\pi \int (x^{\frac{2}{3}} - 9)$ . Ignore limits and  $dx$ . Can be implied.

**M1:** Either integrates to give  $\pm Ax^{\frac{5}{3}} \pm Bx$ ,  $A \neq 0$ ,  $B \neq 0$  or integrates  $x^{\frac{2}{3}}$  correctly to give  $\frac{3}{5}x^{\frac{5}{3}}$  oe

**A1:**  $\frac{3}{5}x^{\frac{5}{3}} - 9x$  or  $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$  oe.

**dM1:** Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.  
**Note:** that this mark is dependent upon the previous method mark being awarded.

**A1:** A correct exact answer of  $\frac{4236\pi}{5}$  or  $847.2\pi$ .

**Note:** The  $\pi$  in the volume formula is only required for the B1 mark and the final A1 mark.

**Note:** A decimal answer of 2661.557... without a correct exact answer is A0.

**Note:** If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.

(a) Alternative response using the Cartesian equation in part (a)

Way 2

$$\left\{ y = \left( x^{\frac{2}{3}} - 9 \right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2} \left( x^{\frac{2}{3}} - 9 \right)^{-\frac{1}{2}} \left( \frac{2}{3} x^{-\frac{1}{3}} \right) \right.$$

$$\text{At } t = \frac{\pi}{6}, x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \left( (24\sqrt{3})^{\frac{2}{3}} - 9 \right)^{-\frac{1}{2}} \left( \frac{2}{3} (24\sqrt{3})^{-\frac{1}{3}} \right)$$

$$\text{So, } \Rightarrow \frac{dy}{dx} = \frac{1}{2} \left( \frac{1}{\sqrt{3}} \right) \left( \frac{1}{3\sqrt{3}} \right) = \frac{1}{18}$$

**Note:** Way 2 is marked as M1 A1 dM1 A1

**Note:** For way 2 the second M1 mark is dependent on the first M1 being gained.

$$\frac{dy}{dx} = \pm K x^{-\frac{1}{3}} \left( x^{\frac{2}{3}} - 9 \right)^{-\frac{1}{2}} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{1}{2} \left( x^{\frac{2}{3}} - 9 \right)^{-\frac{1}{2}} \left( \frac{2}{3} x^{-\frac{1}{3}} \right) \quad \text{oe} \quad \text{A1}$$

Uses  $t = \frac{\pi}{6}$  to find  $x$  and substitutes their  $x$  into an expression for  $\frac{dy}{dx}$ . dM1

$$\frac{1}{18} \quad \text{A1 cao cso}$$

Notes for Question 7 Continued

<p>7. (b) Way 2</p>	<p><b>Alternative responses for M1A1 in part (b): STARTING FROM THE RHS</b></p> $\{\text{RHS}=\} \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} = \sqrt{\left(27 \sec^3 t\right)^{\frac{2}{3}} - 9} = \sqrt{9 \sec^2 t - 9} = \sqrt{9 \tan^2 t}$ $= 3 \tan t = y \{\text{LHS}\} \quad \text{cso}$ <p>For applying <math>1 + \tan^2 t = \sec^2 t</math> oe to achieve <math>\sqrt{9 \tan^2 t}</math> M1</p> <p>Correct proof from <math>\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}</math> to <math>y</math>. A1*</p> <p><b>M1:</b> Starts from the RHS and goes on to achieve <math>\sqrt{9 \tan^2 t}</math> by using a correct trigonometric identity.</p>
<p>7. (b) Way 3</p>	<p><b>Alternative responses for M1A1 in part (b): STARTING FROM THE LHS</b></p> $\{\text{LHS}=\} y = 3 \tan t = \sqrt{\left(9 \tan^2 t\right)} = \sqrt{9 \sec^2 t - 9}$ $= \sqrt{9 \left(\frac{x}{27}\right)^{\frac{2}{3}} - 9} = \sqrt{9 \left(\frac{x^{\frac{2}{3}}}{9}\right) - 9} = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \quad \text{cso}$ <p>For applying <math>1 + \tan^2 t = \sec^2 t</math> oe to achieve <math>\sqrt{9 \sec^2 t - 9}</math> M1</p> <p>Correct proof from <math>y</math> to <math>\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}</math>. A1*</p> <p><b>M1:</b> Starts from the LHS and goes on to achieve <math>\sqrt{9 \sec^2 t - 9}</math> by using a correct trigonometric identity.</p>
<p>7. (c) Way 2</p>	<p><b>Alternative response for part (c) using parametric integration</b></p> $V = \pi \int 9 \tan^2 t (81 \sec^2 t \sec t \tan t) dt$ $= \{\pi\} \int 729 \sec^2 t \tan^2 t \sec t \tan t dt$ $= \{\pi\} \int 729 \sec^2 t (\sec^2 t - 1) \sec t \tan t dt$ $= \{\pi\} \int 729 (\sec^4 t - \sec^2 t) \sec t \tan t dt$ $= \{\pi\} \int 729 (\sec^4 t - \sec^2 t) \sec t \tan t dt$ $= \{\pi\} \left[ 729 \left( \frac{1}{5} \sec^5 t - \frac{1}{3} \sec^3 t \right) \right]$ $V = \{\pi\} \left[ 729 \left( \frac{1}{5} \left(\frac{5}{3}\right)^5 - \frac{1}{3} \left(\frac{5}{3}\right)^3 \right) - 729 \left( \frac{1}{5} 1^5 - \frac{1}{3} 1^3 \right) \right]$ $= 729 \pi \left[ \left( \frac{250}{243} \right) - \left( -\frac{2}{15} \right) \right]$ $= \frac{4236 \pi}{5} \quad \text{or} \quad 847.2 \pi$ <p><math>\pi \int 3 \tan t (81 \sec^2 t \sec t \tan t) dt</math> B1</p> <p>Ignore limits and <math>dx</math>. Can be implied.</p> <p><math>\pm A \sec^5 t \pm B \sec^3 t</math> M1</p> <p><math>729 \left( \frac{1}{5} \sec^5 t - \frac{1}{3} \sec^3 t \right)</math> A1</p> <p>Substitutes <math>\sec t = \frac{5}{3}</math> and <math>\sec t = 1</math> into an integrated function and subtracts the correct way round. dM1</p> <p><math>\frac{4236 \pi}{5} \quad \text{or} \quad 847.2 \pi</math> A1</p>





Question Number	Scheme	Marks										
<p>8.</p> <p>(a)</p>	<p><math>\frac{dx}{dt} = k(M - x)</math>, where <math>M</math> is a constant</p> <p><math>\frac{dx}{dt}</math> is the <u>rate of increase</u> of the <u>mass of waste</u> products.</p> <p><math>M</math> is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u> (or the <u>initial mass</u> of <u>unburned fuel</u>)</p>	<p>Any one correct explanation. B1</p> <p>Both explanations are correct. B1</p>										
<p>(b)</p>	<p><math>\int \frac{1}{M-x} dx = \int k dt</math> or <math>\int \frac{1}{k(M-x)} dx = \int dt</math></p> <p><math>-\ln(M-x) = kt + c</math> or <math>-\frac{1}{k} \ln(M-x) = t + c</math></p> <p><math>\{t=0, x=0 \Rightarrow\} -\ln(M-0) = k(0) + c</math></p> <p><math>c = -\ln M \Rightarrow -\ln(M-x) = kt - \ln M</math></p> <table border="1" style="width:100%; border-collapse: collapse;"> <tr> <td style="width:50%; padding: 5px;"><i>then either...</i></td> <td style="width:50%; padding: 5px;"><i>or...</i></td> </tr> <tr> <td style="padding: 5px;"><math>-kt = \ln(M-x) - \ln M</math></td> <td style="padding: 5px;"><math>kt = \ln M - \ln(M-x)</math></td> </tr> <tr> <td style="padding: 5px;"><math>-kt = \ln\left(\frac{M-x}{M}\right)</math></td> <td style="padding: 5px;"><math>kt = \ln\left(\frac{M}{M-x}\right)</math></td> </tr> <tr> <td style="padding: 5px;"><math>e^{-kt} = \frac{M-x}{M}</math></td> <td style="padding: 5px;"><math>e^{kt} = \frac{M}{M-x}</math></td> </tr> <tr> <td style="padding: 5px;"><math>Me^{-kt} = M-x</math></td> <td style="padding: 5px;"><math>(M-x)e^{kt} = M</math> <math>M-x = Me^{-kt}</math></td> </tr> </table> <p>leading to <math>x = M - Me^{-kt}</math> or <math>x = M(1 - e^{-kt})</math> oe</p>	<i>then either...</i>	<i>or...</i>	$-kt = \ln(M-x) - \ln M$	$kt = \ln M - \ln(M-x)$	$-kt = \ln\left(\frac{M-x}{M}\right)$	$kt = \ln\left(\frac{M}{M-x}\right)$	$e^{-kt} = \frac{M-x}{M}$	$e^{kt} = \frac{M}{M-x}$	$Me^{-kt} = M-x$	$(M-x)e^{kt} = M$ $M-x = Me^{-kt}$	<p>[2]</p> <p>B1</p> <p>See notes M1 A1</p> <p>See notes M1</p> <p>ddM1</p> <p>A1 * cso</p>
<i>then either...</i>	<i>or...</i>											
$-kt = \ln(M-x) - \ln M$	$kt = \ln M - \ln(M-x)$											
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<p>(c)</p>	<p><math>\left\{x = \frac{1}{2}M, t = \ln 4 \Rightarrow\right\} \frac{1}{2}M = M(1 - e^{-k \ln 4})</math></p> <p><math>\Rightarrow \frac{1}{2} = 1 - e^{-k \ln 4} \Rightarrow e^{-k \ln 4} = \frac{1}{2} \Rightarrow -k \ln 4 = -\ln 2</math></p> <p>So <math>k = \frac{1}{2}</math></p> <p><math>x = M\left(1 - e^{-\frac{1}{2} \ln 9}\right)</math></p> <p><math>x = \frac{2}{3}M</math></p>	<p>[6]</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p><math>x = \frac{2}{3}M</math> A1 cso</p> <p>[4]</p> <p>12</p>										

## Notes for Question 8 Continued

8. (a)

**B1:** At least one explanation correct.**B1:** Both explanations are correct.

$\frac{dx}{dt}$  is the rate of increase of the mass of waste products.  
or the rate of change of the mass of waste products.

$M$  is the total mass of unburned fuel and waste fuel  
or the initial mass of unburned fuel  
or the total mass of rocket fuel and waste fuel  
or the initial mass of rocket fuel  
or the initial mass of fuel  
or the total mass of waste and unburned products.

(b)

**B1:** Separates variables as shown.  $dx$  and  $dt$  should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.**M1: Both**  $\pm \lambda \ln(M - x)$  or  $\pm \lambda \ln(x - M)$  **and**  $\pm \mu t$  where  $\lambda$  and  $\mu$  are any constants.

**A1:** For  $-\ln(M - x) = kt$  or  $-\ln(x - M) = kt$  or  $-\frac{1}{k} \ln(M - x) = t$  or  $-\frac{1}{k} \ln(x - M) = t$   
or  $-\frac{1}{k} \ln(kM - kx) = t$  or  $-\frac{1}{k} \ln(kx - kM) = t$

**Note:**  $+c$  is not needed for this mark.**IMPORTANT:**  $+c$  can be on either side of their equation for the 1<sup>st</sup> A1 mark.**M1:** Substitutes  $t = 0$  AND  $x = 0$  in an integrated or changed equation containing  $c$  (or  $A$  or  $\ln A$ , etc.)**Note** that this mark can be implied by the correct value of  $c$ .**ddM1:** Uses their value of  $c$  which must be a  $\ln$  term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on both previous method marks being awarded.**A1:**  $x = M - Me^{-kt}$  or  $x = M(1 - e^{-kt})$  or  $x = \frac{M(e^{kt} - 1)}{e^{kt}}$  or equivalent where  $x$  is the subject.**Note:** Please check their working as incorrect working can lead to a correct answer.**Note:**  $\left\{ \frac{dx}{dt} = k(M - x) \Rightarrow \frac{dx}{dt} = \frac{1}{kM - kx} \Rightarrow \right\} x = -\frac{1}{k} \ln(kM - kx) \{+c\}$  is B1(Implied) M1A1.

(c)

**M1:** Substitutes  $x = \frac{1}{2}M$  and  $t = \ln 4$  into one of their earlier equations connecting  $x$  and  $t$ .**A1:**  $k = \frac{1}{2}$ , which can be an un-simplified equivalent numerical value. i.e.  $k = \frac{\ln 2}{\ln 4}$  is fine for A1.**dM1:** Substitutes  $t = \ln 4$  and their evaluated  $k$  (which must be a numerical value) into one of their earlier equations connecting  $x$  and  $t$ .**Note:** that the 2<sup>nd</sup> Method mark is dependent on the 1<sup>st</sup> Method mark being awarded in part (c).**A1:**  $x = \frac{2}{3}M$  cso.**Note:** Please check their working as incorrect working can lead to a correct answer.

Notes for Question 8 Continued

<p><i>Aliter</i> 8. (b) Way 2</p>	$\int \frac{1}{M-x} dx = \int k dt$ $-\ln(M-x) = kt \{+ c\}$ $\ln(M-x) = -kt + c$ $M-x = Ae^{-kt}$ $\{t=0, x=0 \Rightarrow\} M-0 = Ae^{-k(0)}$ $\Rightarrow M = A$ $M-x = Me^{-kt}$ <p>So, <math>x = M - Me^{-kt}</math></p>	<p>B1</p> <p>See notes M1 A1</p> <p>M1</p> <p>ddM1</p> <p>A1</p> <p style="text-align: right;"><b>[6]</b></p>
<p>(b)</p>	<p><b>B1M1A1:</b> Mark as in the original scheme.  <b>M1:</b> Substitutes <math>t = 0</math> AND <math>x = 0</math> in an integrated equation containing their constant of integration which could be <math>c</math> or <math>A</math>. <b>Note</b> that this mark can be implied by the correct value of <math>c</math> or <math>A</math>.  <b>ddM1:</b> Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.  <b>Note:</b> This mark is dependent on both previous method marks being awarded.  <b>Note:</b> <math>\ln(M-x) = -kt + c</math> leading to <math>\ln(M-x) = e^{-kt} + e^c</math> or <math>\ln(M-x) = e^{-kt} + A</math> would be dddM0.  <b>A1:</b> Same as the original scheme.</p>	
<p><i>Aliter</i> 8. (b) Way 3</p>	$\int_0^x \frac{1}{M-x} dx = \int_0^t k dt$ $[-\ln(M-x)]_0^x = [kt]_0^t$ $-\ln(M-x) - (-\ln M) = kt$ $-\ln(M-x) + \ln M = kt$ <p>and then follows the original scheme.</p>	<p>B1</p> <p>M1 A1</p> <p>Applies limits of M1</p>
<p>(a)</p>	<p><b>B1M1A1:</b> Mark as in the original scheme (ignoring the limits).  <b>ddM1:</b> Applies limits 0 and <math>x</math> on their integrated LHS and limits of 0 and <math>t</math>.  <b>M1A1:</b> Same as the original scheme.</p>	

Notes for Question 8 Continued

<p><b>Aliter</b> <b>8. (b)</b> <b>Way 4</b></p>	$\int \frac{1}{M-x} dx = \int k dt \quad \left\{ \Rightarrow \int \frac{-1}{x-M} dx = \int k dt \right\}$ $-\ln x-M  = kt + c$ $\{t=0, x=0 \Rightarrow\} -\ln 0-M  = k(0) + c$ $\Rightarrow c = -\ln M \Rightarrow -\ln x-M  = kt - \ln M$	<p><i>Modulus not required for 1<sup>st</sup> A1.</i> <i>Modulus not required here!</i></p>	<p>B1 M1 A1 M1</p>										
<table border="1" style="width:100%; border-collapse: collapse;"> <tr> <td style="width:50%; padding: 5px;"> <p><i>then either...</i></p> <math display="block">-kt = \ln x-M  - \ln M</math> <math display="block">-kt = \ln \left  \frac{x-M}{M} \right </math> </td> <td style="width:50%; padding: 5px;"> <p><i>or...</i></p> <math display="block">kt = \ln M - \ln x-M </math> <math display="block">kt = \ln \left  \frac{M}{x-M} \right </math> </td> </tr> <tr> <td colspan="2" style="text-align: center; padding: 5px;"> <p>As <math>x &lt; M</math></p> </td> </tr> <tr> <td style="padding: 5px;"> <math display="block">-kt = \ln \left( \frac{M-x}{M} \right)</math> <math display="block">e^{-kt} = \frac{M-x}{M}</math> </td> <td style="padding: 5px;"> <math display="block">kt = \ln \left( \frac{M}{M-x} \right)</math> <math display="block">e^{kt} = \frac{M}{M-x}</math> </td> </tr> <tr> <td style="padding: 5px;"> <math display="block">Me^{-kt} = M-x</math> </td> <td style="padding: 5px;"> <math display="block">(M-x)e^{kt} = M</math> <math display="block">M-x = Me^{-kt}</math> </td> </tr> <tr> <td colspan="2" style="padding: 5px;"> <p>leading to <math>x = M - Me^{-kt}</math> or <math>x = M(1 - e^{-kt})</math> oe</p> </td> </tr> </table>		<p><i>then either...</i></p> $-kt = \ln x-M  - \ln M$ $-kt = \ln \left  \frac{x-M}{M} \right $	<p><i>or...</i></p> $kt = \ln M - \ln x-M $ $kt = \ln \left  \frac{M}{x-M} \right $	<p>As <math>x &lt; M</math></p>		$-kt = \ln \left( \frac{M-x}{M} \right)$ $e^{-kt} = \frac{M-x}{M}$	$kt = \ln \left( \frac{M}{M-x} \right)$ $e^{kt} = \frac{M}{M-x}$	$Me^{-kt} = M-x$	$(M-x)e^{kt} = M$ $M-x = Me^{-kt}$	<p>leading to <math>x = M - Me^{-kt}</math> or <math>x = M(1 - e^{-kt})</math> oe</p>		<p><i>Understanding of modulus is required here!</i></p>	<p><b>ddM1</b>  <b>A1 * cso</b></p>
<p><i>then either...</i></p> $-kt = \ln x-M  - \ln M$ $-kt = \ln \left  \frac{x-M}{M} \right $	<p><i>or...</i></p> $kt = \ln M - \ln x-M $ $kt = \ln \left  \frac{M}{x-M} \right $												
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<p><b>B1:</b> Mark as in the original scheme.  <b>M1A1M1:</b> Mark as in the original scheme ignoring the modulus.  <b>ddM1:</b> Mark as in the original scheme <b>AND</b> the candidate must demonstrate that they have converted <math>\ln x-M </math> to <math>\ln(M-x)</math> in their working.  <b>Note:</b> This mark is dependent on both the previous method marks being awarded.  <b>A1:</b> Mark as in the original scheme.</p>													
<p><b>Aliter</b> <b>8. (b)</b> <b>Way 5</b></p>	<p><i>Use of an integrating factor (I.F.)</i></p> $\frac{dx}{dt} = k(M-x) \Rightarrow \frac{dx}{dt} + kx = kM$ <p>I.F. = <math>e^{kt}</math></p> $\frac{d}{dt}(e^{kt}x) = kMe^{kt},$ $e^{kt}x = Me^{kt} + c$ $x = M + ce^{-kt}$ $\{t=0, x=0 \Rightarrow\} 0 = M + ce^{-k(0)}$ $\Rightarrow c = -M$ $x = M - Me^{-kt}$	<p>B1  M1A1  M1  ddM1A1</p>											

[6]