Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1 R	Signature	

Paper Reference(s)

6666/01R

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Find the binomial expansion of

$$\frac{1}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(b) Hence, or otherwise, find the expansion of

$$\frac{3+x}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(3)

Question Number	Scheme		Marks
1. (a)	$\left\{\frac{1}{\sqrt{(9-10x)}} =\right\} (9-10x)^{-\frac{1}{2}}$	$(9-10x)^{-\frac{1}{2}}$ or uses power of $-\frac{1}{2}$	B1
	$= \underline{(9)^{-\frac{1}{2}}}\left(1-\frac{10x}{9}\right)^{-\frac{1}{2}} = \underline{\frac{1}{3}}\left(1-\frac{10x}{9}\right)^{-\frac{1}{2}}$	$\underline{(9)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{3}}$	<u>B1</u>
	$=\left\{\frac{1}{3}\right\}\left[1+\left(-\frac{1}{2}\right)(kx)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2+\dots\right]$	At least two correct terms. **See notes**	M1
	$=\left\{\frac{1}{3}\right\}\left[1+\left(-\frac{1}{2}\right)\left(\frac{-10x}{9}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{-10x}{9}\right)^2+\dots\right]$		
	$=\frac{1}{3}\left[1+\frac{5}{9}x+\frac{25}{54}x^2+\dots\right]$		
	$=\frac{1}{3}+\frac{5}{27}x;+\frac{25}{162}x^2+\dots$		A1; A1
			[5]
(b)	$\frac{3+x}{\sqrt{(9-10x)}} = (3+x)(9-10x)^{-\frac{1}{2}}$		
	$=(3+x)\left(\frac{1}{3}+\frac{5}{27}x+\left\{\frac{25}{162}x^2+\right\}\dots\right)$	*Can be implied by later work* **See notes**	M1
	$= 1+\frac{5}{9}x+\frac{25}{54}x^2+\frac{1}{3}x+\frac{5}{27}x^2+\dots$	Multiplies out to give exactly one constant term, exactly 2 terms in x and exactly 2 terms in x^2. Ignore terms in x^3. Can be implied.	M1
	$= 1+\frac{8}{9}x+\frac{35}{54}x^2+\dots$		A1
			[3] **8**

Question 1 Notes

(a)	**B1**	Writes down $(9-10x)^{-\frac{1}{2}}$ or uses power of $-\frac{1}{2}$.
		This mark can be implied by a constant term of $(9)^{-\frac{1}{2}}$ or $\frac{1}{3}$.
	<u>**B1**</u>	$\underline{(9)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{3}}$ outside brackets or $\frac{1}{3}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\dots+kx)^{-\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or an un-simplified,
		$1+(-\frac{1}{2})(kx)$ or $(-\frac{1}{2})(kx)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2$ or $1+\dots\dots+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(kx)^2$, **where** $k \neq 1$**.**
	A1	$\frac{1}{3}+\frac{5}{27}x$ **(simplified fractions)**
	A1	Accept only $\frac{25}{162}x^2$

1. (a) ctd	**Note**	**You cannot recover correct work for part (a) in part (b).** i.e. if the correct answer to (a) appears as part of their solution in part (b), it cannot be credited in part (a).
	SC	If a candidate ***would otherwise score*** A0A0 then **allow Special Case 1st A1 for either**
		SC: $\frac{1}{3}\left[1+\frac{5}{9}x; ...\right]$ or **SC**: $\lambda\left[1+\frac{5}{9}x+\frac{25}{54}x^2+...\right]$ or **SC**: $\left[\lambda+\frac{5\lambda}{9}x+\frac{25\lambda}{54}x^2+...\right]$
		(where λ can be 1 or omitted), with each term in the $[........]$ is a simplified fraction
	SC	**<u>Special case for the M1 mark</u>**
		Award Special Case M1 for a correct simplified or un-simplified $1+n(kx)+\frac{n(n-1)}{2!}(kx)^2$ expansion with a value of $n\neq-\frac{1}{2}$, $n\neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion.
		Note that $k\neq1$**.**
	Note	Candidates who write $\left\{\frac{1}{3}\right\}\left[1+\left(-\frac{1}{2}\right)\left(\frac{10x}{9}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{10x}{9}\right)^2+...\right]$
		where $k=\frac{10}{9}$ and not $-\frac{10}{9}$ and achieve $\frac{1}{3}-\frac{5}{27}x;+\frac{25}{162}x^2+...$ will get B1B1M1A0A1.
(b)		
	M1	Writes down $(3+x)($their part (a) answer, at least 2 of the 3 terms.$)$
	Note	$(3+x)\left(\frac{1}{4}+\frac{5}{4}x+...\right)$ or $(3+x)\left(\frac{1}{3}+\frac{5}{27}x+\frac{25}{162}x^2+...\right)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x.
	M1	Multiplies out to give exactly one constant term, exactly 2 terms in x and exactly 2 terms in x^2.
	Note	This M1 mark can be implied. You can also ignore x^3 terms.
	A1	$1+\frac{8}{9}x+\frac{35}{54}x^2+...$

<u>Alternative Methods for part (a)</u>
<u>Alternative method 1:</u> Candidates can apply an alternative form of the binomial expansion.

$$\left\{\frac{1}{\sqrt{(9-10x)}}=\right\}(9-10x)^{-\frac{1}{2}}=(9)^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)(9)^{-\frac{3}{2}}(-10x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(9)^{-\frac{5}{2}}(-10x)^2$$

B1	Writes down $(9-10x)^{-\frac{1}{2}}$ or uses power of $-\frac{1}{2}$.
B1	$9^{-\frac{1}{2}}$ or $\frac{1}{3}$
M1	Any two of three (un-simplified or simplified) terms correct.
A1	$\frac{1}{3}+\frac{5}{27}x$
A1	$\frac{25}{162}x^2$
Note	The terms in C need to be evaluated, so ${}^{-\frac{1}{2}}C_0(9)^{-\frac{1}{2}}+{}^{-\frac{1}{2}}C_1(9)^{-\frac{3}{2}}(-10x)+{}^{-\frac{1}{2}}C_2(9)^{-\frac{5}{2}}(-10x)^2$ without further working is B1B0M0A0A0.

1. (a)	**Alternative Method 2: *Maclaurin Expansion*** Let $\mathrm{f}(x) = \dfrac{1}{\sqrt{(9-10x)}}$		
	$\{\mathrm{f}(x) =\}\ (9-10x)^{-\frac{1}{2}}$	$(9-10x)^{-\frac{1}{2}}$	B1
	$\mathrm{f}''(x) = 75(9-10x)^{-\frac{5}{2}}$	Correct $\mathrm{f}''(x)$	B1 oe
	$\mathrm{f}'(x) = \left(-\tfrac{1}{2}\right)(9-10x)^{-\frac{3}{2}}(-10)$	$\pm a(9-10x)^{-\frac{3}{2}};\ a \neq \pm 1$	M1
	$\left\{\therefore \mathrm{f}(0) = \dfrac{1}{3},\ \mathrm{f}'(0) = \dfrac{5}{27} \text{ and } \mathrm{f}''(0) = \dfrac{75}{243} = \dfrac{25}{81}\right\}$		
	$\mathrm{f}(x) = \dfrac{1}{3} + \dfrac{5}{27}x; + \dfrac{25}{162}x^2 + \ldots$		A1; A1

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2.

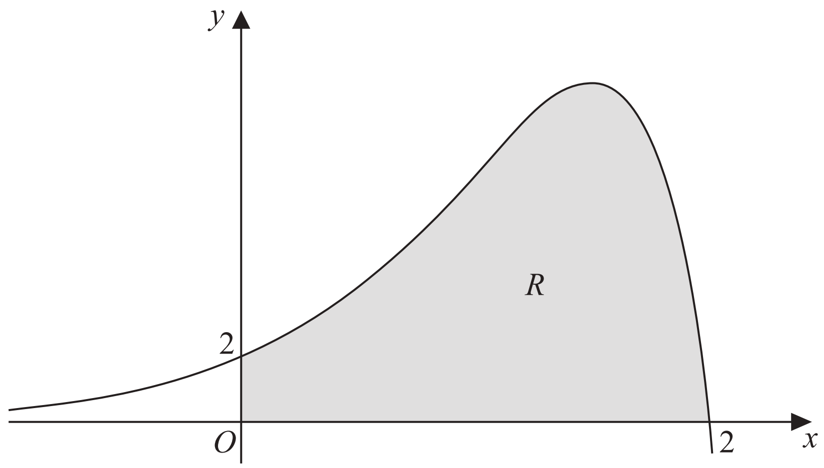

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(a) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.
(3)

(b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.
(1)

(c) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.
(5)

Question Number	Scheme		Marks
2. (a)	Area $\approx \frac{1}{2} \times 0.5\,; \times \left[\underline{2 + 2\left(4.077 + 7.389 + 10.043\right) + 0}\right]$		B1; <u>M1</u>
	$= \frac{1}{4} \times 45.018 \;=\; 11.2545 \;= 11.25\,(2\text{ dp})$	11.25	A1 **cao**
			[3]
(b)	Any one of • Increase the number of strips • Use more trapezia • Make h smaller • Increase the **number of** x and/or y values used • Shorter /smaller intervals for x • More values of y. • More intervals of x • Increase n		B1
			[1]
(c)	$\left\{\int (2-x)\mathrm{e}^{2x}\,\mathrm{d}x\right\}$, $\left\{\begin{matrix} u = 2 - x & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = -1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{2x} & \Rightarrow & v = \frac{1}{2}\mathrm{e}^{2x} \end{matrix}\right\}$		
	$= \frac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\frac{1}{2}\mathrm{e}^{2x}\,\{\mathrm{d}x\}$	Either $(2-x)\mathrm{e}^{2x} \to \pm\lambda(2-x)\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\,\{\mathrm{d}x\}$ or $\pm x\mathrm{e}^{2x} \to \pm\lambda x\mathrm{e}^{2x} \pm \int \mu\mathrm{e}^{2x}\,\{\mathrm{d}x\}$	M1
		$(2-x)\mathrm{e}^{2x} \to \frac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\frac{1}{2}\mathrm{e}^{2x}\,\{\mathrm{d}x\}$	A1
	$= \frac{1}{2}(2-x)\mathrm{e}^{2x} + \frac{1}{4}\mathrm{e}^{2x}$	$\frac{1}{2}(2-x)\mathrm{e}^{2x} + \frac{1}{4}\mathrm{e}^{2x}$	A1 oe
	Area $= \left\{\left[\frac{1}{2}(2-x)\mathrm{e}^{2x} + \frac{1}{4}\mathrm{e}^{2x}\right]_0^2\right\}$		
	$= \left(0 + \frac{1}{4}\mathrm{e}^4\right) - \left(\frac{1}{2}(2)\mathrm{e}^0 + \frac{1}{4}\mathrm{e}^0\right)$	Applies limits of 2 and 0 ***to all terms*** and subtracts the correct way round.	dM1
	$= \frac{1}{4}\mathrm{e}^4 - \frac{5}{4}$	$\frac{1}{4}\mathrm{e}^4 - \frac{5}{4}$ or $\frac{\mathrm{e}^4 - 5}{4}$ **cao**	A1 oe
			[5] **9**

Question 2 Notes

(a)	**B1**	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$.
	M1	<u>For structure of trapezium rule</u>$[\;\underline{\ldots\ldots\ldots\ldots}\;]$. Condone missing 0.
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	11.25 **cao**
	Note	Working must be seen to demonstrate the use of the trapezium rule. The actual area is 12.39953751…
	Note	Award B1M1A1 for $\frac{0.5}{2}(2+0) + \frac{1}{2}\left(4.077 + 7.389 + 10.043\right) = 11.25$

2. (a) contd		**<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly.**
		Award B1M0A0 for $\frac{1}{2} \times 0.5 + 2 + 2\left(4.077 + 7.389 + 10.043\right) + 0$ (nb: answer of 45.268).
		<u>Alternative method for part (a): Adding individual trapezia</u>
		Area $\approx 0.5 \times \left[\frac{2+4.077}{2} + \frac{4.077+7.389}{2} + \frac{7.389+10.043}{2} + \frac{10.043+0}{2}\right] = 11.2545 = 11.25$ (2 dp) cao
	B1	0.5 and a divisor of 2 on all terms inside brackets.
	M1	First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.
	A1	11.25 **cao**
(b)	**B0**	Give B0 for • smaller values of x and/or y. • use more decimal places
(c)	**M1**	Either $(2-x)\mathrm{e}^{2x} \to \pm\lambda(2-x)\mathrm{e}^{2x} \pm \int \mu \mathrm{e}^{2x}\{\mathrm{d}x\}$ or $\pm x\mathrm{e}^{2x} \to \pm\lambda x\mathrm{e}^{2x} \pm \int \mu \mathrm{e}^{2x}\{\mathrm{d}x\}$
	A1	$(2-x)\mathrm{e}^{2x} \to \frac{1}{2}(2-x)\mathrm{e}^{2x} - \int -\frac{1}{2}\mathrm{e}^{2x}\{\mathrm{d}x\}$ either un-simplified or simplified.
	A1	Correct expression, i.e. $\frac{1}{2}(2-x)\mathrm{e}^{2x} + \frac{1}{4}\mathrm{e}^{2x}$ or $\frac{5}{4}\mathrm{e}^{2x} - x\mathrm{e}^{2x}$ (or equivalent)
	dM1	**which is dependent on the 1st M1 mark being awarded.** **Complete method** of applying limits of 2 and 0 **to all terms** and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.
	A1	$\frac{1}{4}\mathrm{e}^4 - \frac{5}{4}$ or $\frac{\mathrm{e}^4 - 5}{4}$. Do not allow $\frac{1}{4}\mathrm{e}^4 - \frac{5}{4}\mathrm{e}^0$ unless simplified to give $\frac{1}{4}\mathrm{e}^4 - \frac{5}{4}$
	Note	12.39953751... **without seeing** $\frac{1}{4}\mathrm{e}^4 - \frac{5}{4}$ is A0.
	Note	12.39953751... from **NO** working is M0A0A0M0A0.

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3. $$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(a) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer.
(5)

(b) Find the values of y for which $\frac{dy}{dx} = 0$
(5)

Question Number	Scheme		Marks
3.	$x^2 + y^2 + 10x + 2y - 4xy = 10$		
(a)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \cancel{\times} \right\}$ $\underline{2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx}} - \underline{\underline{\left(4y + 4x\frac{dy}{dx}\right)}} = \underline{0}$	**See notes**	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	**Dependent on the first M1 mark.**	dM1
	$\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$		
	Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{ = \frac{-x - 5 + 2y}{-2x + y + 1} \right\}$		A1 **cso oe**
			[5]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\}$ $x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35 \{= 0\}$ **see notes**	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = ...$	Method mark for solving a quadratic equation.	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\} \frac{7}{3}, 5$	A1 **cao**
			[5]
	<u>Alternative method for part (b)</u>		
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\}$ $x + 5 - 2y = 0$		M1
	So $y = \frac{x + 5}{2}$,		
	$x^2 + \left(\frac{x + 5}{2}\right)^2 + 10x + 2\left(\frac{x + 5}{2}\right) - 4x\left(\frac{x + 5}{2}\right) = 10$		M1
	$x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$		
	$4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$		
	gives $-3x^2 + 14x + 5 = 0$ or $3x^2 - 14x - 5 = 0$	$3x^2 - 14x - 5 \{= 0\}$ **see notes**	A1 oe
	$(3x + 1)(x - 5) = 0$, $x = ...$ $y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$	Solves a quadratic **and finds at least one value for *y*.**	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\} \frac{7}{3}, 5$	A1 **cao**
			[5]
			10

		Question 3 Notes
3. (a)	**M1**	Differentiates implicitly to include either $\pm 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2y \to 2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1	$x^2 + y^2 + 10x + 2y \to 2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 10 + 2\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $10 \to 0$
	M1	$-4xy \to \pm 4y \pm 4x\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	If an extra term appears then award 1st A0.
	Note	$2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 10 + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 4y - 4x\frac{\mathrm{d}y}{\mathrm{d}x} \to 2x + 10 - 4y = -2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the $"=0"$ can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	A1	$\frac{x + 5 - 2y}{2x - y - 1}$ or $\frac{-x - 5 + 2y}{-2x + y + 1}$ (must be simplified).
	cso:	If the candidate's solution is not completely correct, then do not give this mark.
(b)	**M1**	Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) oe.
	NOTE	**If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	M1	Substitutes their x or their y into the printed equation to give an equation in one variable only.
	A1	For obtaining either $-3y^2 + 22y - 35\ \{= 0\}$ or $3y^2 - 22y + 35\ \{= 0\}$
	Note	This mark can also awarded for a correct three term equation, eg. either $-3y^2 + 22y = 35$ $3y^2 - 22y = -35$ or $3y^2 + 35 = 22y$ are all fine for A1.
	ddM1	**Dependent on the previous 2 M marks.** **See notes at the beginning of the mark scheme: Method mark for solving a 3 term quadratic** • $(3y - 7)(y - 5) = 0 \Rightarrow y = \dots$ • $y = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}$ • $y^2 - \frac{22}{3}y - \frac{35}{3} = 0 \Rightarrow \left(y - \frac{11}{3}\right)^2 - \frac{121}{9} + \frac{35}{3} = 0 \Rightarrow y = \dots$ • Or writes down at least one correct y- root from their quadratic equation. This is usually found from their calculator.
	Note	If a candidate applies ***the alternative method*** then they also need to use their $y = \frac{x + 5}{2}$ in order to find **at least one value** for y in order to gain the final M1.
	A1	$y = \frac{7}{3}, 5$. **cao.** (2.33 or 2.3 without reference to $\frac{7}{3}$ or $2\frac{1}{3}$ is not allowed for this mark.)
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{\mathrm{d}y}{\mathrm{d}x}$) to gain all 5 marks in part (b).

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4. (a) Express $\dfrac{25}{x^2(2x+1)}$ in partial fractions.

(4)

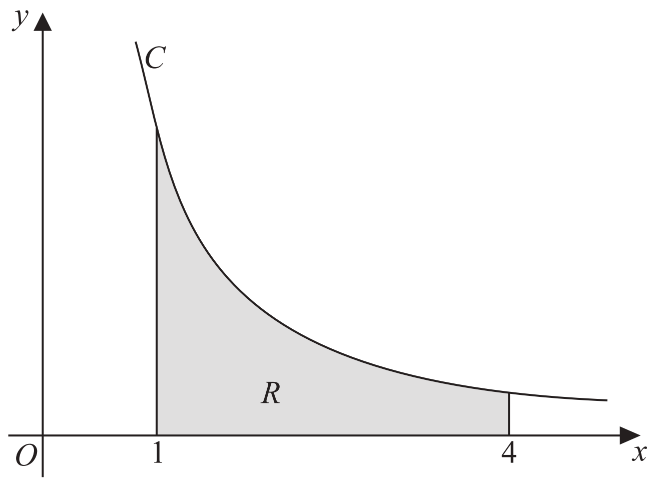

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = \dfrac{5}{x\sqrt{(2x+1)}}$, $x > 0$

The finite region R is bounded by the curve C, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$

This region is shown shaded in Figure 2

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form $a + b\ln c$, where a, b and c are constants.

(6)

Question Number	Scheme		Marks
4. (a)	$\dfrac{25}{x^2(2x+1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(2x+1)}$		
	$B = 25,\ C = 100$	At least one of "B" or "C" correct.	B1
		Breaks up their partial fraction correctly into ***three terms*** **and** both "B" $= 25$ and "C" $= 100$. **See notes.**	B1 **cso**
	$25 \equiv Ax(2x+1) + B(2x+1) + Cx^2$ $x = 0,\quad 25 = B$ $x = -\dfrac{1}{2},\quad 25 = \dfrac{1}{4}C \Rightarrow C = 100$ x^2 terms: $0 = 2A + C$ $0 = 2A + 100 \Rightarrow A = -50$ $x^2: 0 = 2A + C,\ \ x: 0 = A + 2B,$ constant: $25 = B$	Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	leading to $A = -50$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1
	$\left\{\dfrac{25}{x^2(2x+1)} \equiv -\dfrac{50}{x} + \dfrac{25}{x^2} + \dfrac{100}{(2x+1)}\right\}$		**[4]**
(b)	$V = \pi\displaystyle\int_1^4 \left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$	For $\pi\displaystyle\int\left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	For their partial fraction		
	$\left\{\displaystyle\int \dfrac{25}{x^2(2x+1)}\mathrm{d}x = \int -\dfrac{50}{x} + \dfrac{25}{x^2} + \dfrac{100}{(2x+1)}\mathrm{d}x\right\}$ $= -50\ln x + \dfrac{25x^{-1}}{(-1)} + \dfrac{100}{2}\ln(2x+1)\ \{+c\}$	Either $\pm\dfrac{A}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\dfrac{B}{x^2} \to \pm bx^{-1}$ or $\dfrac{C}{(2x+1)} \to \pm c\ln(2x+1)$	M1
		At least two terms correctly integrated	A1ft
		All three terms correctly integrated.	A1ft
	$\left\{\displaystyle\int_1^4 \dfrac{25}{x^2(2x+1)}\mathrm{d}x = \left[-50\ln x - \dfrac{25}{x} + 50\ln(2x+1)\right]_1^4\right\}$ $= \left(-50\ln 4 - \dfrac{25}{4} + 50\ln 9\right) - (0 - 25 + 50\ln 3)$	Applies limits of 4 and 1 and subtracts the correct way round.	dM1
	$= 50\ln 9 - 50\ln 4 - 50\ln 3 - \dfrac{25}{4} + 25$ $= 50\ln\left(\dfrac{3}{4}\right) + \dfrac{75}{4}$		
	So, $V = \dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$ or allow $\pi\left(\dfrac{75}{4} + 50\ln\left(\dfrac{3}{4}\right)\right)$		A1 oe
			[6] **10**

		Question 4 Notes
4. (a)		**BE CAREFUL!** Candidates will assign ***their own*** "*A*, *B* and *C*" for this question.
	B1	At least one of "*B*" or "*C*" are correct.
	B1	Breaks up their partial fraction correctly into three terms **and** both $"B" = 25$ and $"C" = 100$.
	Note	If a candidate does not give partial fraction decomposition then: • the 2nd B1 mark can follow from a correct identity.
	M1	Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "*A*" or "*B*" or "*C*". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
	A1	Correct value for "*A*" which is found using a correct identity and follows from their partial fraction decomposition.
	Note	If a candidate does not give partial fraction decomposition then the final A1 mark can be awarded for a correct "*A*" if a candidate writes out their partial fractions at the end.
	Note	The correct partial fraction from no working scores B1B1M1A1.
	Note	A number of candidates will start this problem by writing out the correct identity and then attempt to find "*A*" or "*B*" or "*C*". Therefore the B1 marks can be awarded from this method.
	Note	Award SC B1B0M0A0 for $\dfrac{25}{x^2(2x+1)} \equiv \dfrac{B}{x^2} + \dfrac{C}{(2x+1)}$ leading to $"B" = 25$ **or** $"C" = 100$
(b)	**B1**	For a correct statement of $\pi\int\left(\dfrac{5}{x\sqrt{(2x+1)}}\right)^2$ or $\pi\int\dfrac{25}{x^2(2x+1)}$. Ignore limits and d*x*. Can be implied.
	Note	**The π can only be recovered later from a correct expression.**
		For their partial fraction, (not $\sqrt{\text{their partial fraction}}$), where *A*, *B*, *C* are "their" part (a) constants
	M1	Either $\pm\dfrac{A}{x} \to \pm a\ln x$ or $\pm\dfrac{B}{x^2} \to \pm b\,x^{-1}$ or $\dfrac{C}{(2x+1)} \to \pm c\ln(2x+1)$.
	Note	$\sqrt{\dfrac{B}{x^2}} \to \dfrac{\sqrt{B}}{x}$ which integrates to $\sqrt{B}\ln x$ is **not** worthy of M1.
	A1ft	At least two terms from any of $\pm\dfrac{A}{x}$ or $\pm\dfrac{B}{x^2}$ or $\dfrac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	A1ft	All 3 terms from $\pm\dfrac{A}{x}$, $\pm\dfrac{B}{x^2}$ and $\dfrac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	Note	The 1st A1 and 2nd A1 marks in part (b) are both follow through accuracy marks.
	dM1	**Dependent on the previous M mark.** Applies limits of 4 and 1 and subtracts the correct way round.
	A1	Final correct **exact** answer in the form $a + b\ln c$. i.e. either $\dfrac{75}{4}\pi + 50\pi\ln\left(\dfrac{3}{4}\right)$ or $50\pi\ln\left(\dfrac{3}{4}\right) + \dfrac{75}{4}\pi$ or $50\pi\ln\left(\dfrac{9}{12}\right) + \dfrac{75}{4}\pi$ or $\dfrac{75}{4}\pi - 50\pi\ln\left(\dfrac{4}{3}\right)$ or $\dfrac{75}{4}\pi + 25\pi\ln\left(\dfrac{9}{16}\right)$ etc. Also allow $\pi\left(\dfrac{75}{4} + 50\ln\left(\dfrac{3}{4}\right)\right)$ or equivalent.
	Note	A candidate who achieves full marks in (a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of B1M1A1A0M1A0 in part (b).
	Note	The π in the volume formula is only required for the B1 mark and the final A1 mark.

4. (b)	**Alternative method of integration**		
	$V = \pi \int_1^4 \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$	B1	For $\pi \int \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.
	$\int \frac{25}{x^2(2x+1)}\,\mathrm{d}x\ ;\ u = \frac{1}{x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}$		
	$= \int \frac{-25}{\left(\frac{2}{u}+1\right)}\,\mathrm{d}u = \int \frac{-25}{\left(\frac{2+u}{u}\right)}\,\mathrm{d}u = \int \frac{-25u}{(2+u)}\,\mathrm{d}u = -25\int \frac{2+u-2}{(2+u)}\,\mathrm{d}u$		
	$= -25\int 1 - \frac{2}{(2+u)}\,\mathrm{d}u = -25\,(u - 2\ln(2+u))$	M1	Achieves $\pm\alpha \pm \frac{\beta}{(k+u)}$ **and** integrates to give either $\pm\alpha u$ or $\pm\beta\ln(k+u)$
		A1	**Dependent on the M mark.** Either $-25u$ or $50\ln(2+u)$
		A1	$-25\,(u - 2\ln(2+u))$
	$\left\{\int_1^4 \frac{25}{x^2(2x+1)}\,\mathrm{d}x = \left[-25u + 50\ln(2+u)\right]_1^{\frac{1}{4}}\right\}$		
	$= \left(-\frac{25}{4} + 50\ln\left(\frac{9}{4}\right)\right) - \left(-25 + 50\ln 3\right)$	dM1	Applies limits of $\frac{1}{4}$ and 1 in u or 4 and 1 in x in their integrated function and subtracts the correct way round.
	$= 50\ln\left(\frac{9}{4}\right) - 50\ln 3 - \frac{25}{4} + 25$		
	$= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$		
	So, $V = \frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$	A1	$\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$

Leave blank

5. At time t seconds the radius of a sphere is r cm, its volume is V cm^3 and its surface area is S cm^2.

$\left[\textit{You are given that } V = \frac{4}{3}\pi r^3 \textit{ and that } S = 4\pi r^2\right]$

The volume of the sphere is increasing uniformly at a constant rate of 3 cm^3 s^{-1}.

(a) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

(2)

Question Number	Scheme		Marks
5. (a)	From question, $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$, $\frac{dV}{dt} = 3$		
	$\left\{V = \frac{4}{3}\pi r^3 \Rightarrow\right\} \frac{dV}{dr} = 4\pi r^2$	$\frac{dV}{dr} = 4\pi r^2$ (Can be implied)	B1 oe
	$\left\{\frac{dV}{dr} \times \frac{dr}{dt} = \frac{dV}{dt} \Rightarrow\right\}$ $(4\pi r^2)\frac{dr}{dt} = 3$ $\left\{\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} \Rightarrow\right\}$ $\frac{dr}{dt} = (3)\frac{1}{4\pi r^2}$; $\left\{= \frac{3}{4\pi r^2}\right\}$	$\left(\text{Candidate's } \frac{dV}{dr}\right) \times \frac{dr}{dt} = 3$ or $3 \div \text{Candidate's } \frac{dV}{dr}$;	M1 oe
	When $r = 4\text{cm}$, $\frac{dr}{dt} = \frac{3}{4\pi(4)^2}$ $\left\{= \frac{3}{64\pi}\right\}$	**dependent on previous M1. see notes**	dM1
	Hence, $\frac{dr}{dt} = 0.01492077591...(\text{cm}^2\ \text{s}^{-1})$	anything that rounds to 0.0149	A1
			[4]
(b)	$\left\{\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = \right\} \Rightarrow \frac{dS}{dt} = 8\pi r \times \frac{3}{4\pi r^2}$ $\left\{\text{or } \frac{6}{r} \text{ or } 8\pi r \times 0.0149...\right\}$	$8\pi r \times \text{Candidate's } \frac{dr}{dt}$	M1; oe
	When $r = 4\text{cm}$, $\frac{dr}{dt} = 8\pi(4) \times \frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4) \times 0.0149...$		
	Hence, $\frac{dS}{dt} = 1.5\ (\text{cm}^2\ \text{s}^{-1})$	anything that rounds to 1.5	A1 **cso**
			[2] **6**

Question 5 Notes

(a)	**B1**	$\frac{dV}{dr} = 4\pi r^2$ Can be implied by later working.
	M1	$\left(\text{Candidate's } \frac{dV}{dr}\right) \times \frac{dr}{dt} = 3$ or $3 \div \text{Candidate's } \frac{dV}{dr}$
	dM1	**(dependent on the previous method mark)** Substitutes $r = 4$ into an expression which is a result of a quotient of "3" and their $\frac{dV}{dr}$.
	A1	anything that rounds to 0.0149 (units are not required)
(b)	**M1**	$8\pi r \times \text{Candidate's } \frac{dr}{dt}$
	A1	anything that rounds to 1.5 (units are not required). **Correct solution only.**
	Note	Using $\frac{dr}{dt} = 0.0149$ gives $\frac{dS}{dt} = 1.4979...$ which is fine for A1.

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6. With respect to a fixed origin, the point A with position vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ lies on the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \text{where } \lambda \text{ is a scalar parameter,}$$

and the point B with position vector $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p is a constant, lies on the line l_2 with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}, \quad \text{where } \mu \text{ is a scalar parameter.}$$

(a) Find the value of the constant p.

(1)

(b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C.

(4)

(c) Find the size of the angle ACB, giving your answer in degrees to 3 significant figures.

(3)

(d) Find the area of the triangle ABC, giving your answer to 3 significant figures.

(2)

Question Number	Scheme		Marks
6.	$l_1 : \mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix}7\\0\\7\end{pmatrix} + \mu\begin{pmatrix}3\\-5\\4\end{pmatrix}$ $\overrightarrow{OA} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}4\\p\\3\end{pmatrix}$	A lies on l_1 and B lies on l_2	
(a)	$\{B \text{ lies on } l_2 \Rightarrow \mu = -1 \Rightarrow\}$ $p = 5$	$p = 5$	B1 **[1]**
(b)	$\{l_1 = l_2 \Rightarrow\}$ $\begin{Bmatrix}\mathbf{i}: & 1 = 7 + 3\mu \\ \mathbf{j}: & 2 + 2\lambda = -5\mu \\ \mathbf{k}: & 3 - \lambda = 7 + 4\mu\end{Bmatrix}$		
	e.g. $\mathbf{i}$: $7 + 3\mu = 1$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$	$\mu = -2$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1 **[4]**
(b)	<u>**Alternative Method:**</u> Solving $\mathbf{j}$ and $\mathbf{k}$ simultaneously gives		
	$8 = 14 + 3\mu$ or $23 + 3\lambda = 35$	Writes down an equation involving only one parameter.	M1
	So, $\mu = -2$ or $\lambda = 4$	Either $\mu = -2$ or $\lambda = 4$	A1
	Point of intersection is $\overrightarrow{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}$	$\mathbf{i} + 10\mathbf{j} - \mathbf{k}$	B1
	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the $\mathbf{i}$ component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$		B1 **[4]**
(c)	$\overrightarrow{AC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\8\\-4\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}1\\10\\-1\end{pmatrix} - \begin{pmatrix}4\\5\\3\end{pmatrix} = \begin{pmatrix}-3\\5\\-4\end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\cos ACB = \dfrac{\overrightarrow{AC} \bullet \overrightarrow{BC}}{\lvert\overrightarrow{AC}\rvert . \lvert\overrightarrow{BC}\rvert} = \dfrac{\pm\left(\begin{pmatrix}0\\8\\-4\end{pmatrix} \bullet \begin{pmatrix}-3\\5\\-4\end{pmatrix}\right)}{\sqrt{(0)^2 + (8)^2 + (-4)^2} . \sqrt{(-3)^2 + (5)^2 + (-4)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\left\{\cos ACB = \dfrac{0 + 40 + 16}{\sqrt{80}.\sqrt{50}} = \dfrac{56}{\sqrt{4000}} \Rightarrow\right\}$ $ACB = 27.69446... = 27.7$ (3 sf)	Anything that rounds to 27.7	A1 **[3]**
(d)	Area $ACB = \dfrac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{50}\right)\sin 27.69446...^{\circ} = 14.696888...$	See notes Anything that rounds to 14.7	M1 A1 **[2]** **10**

	Question 6: Alternative Methods for Part (c)		
6. (c)	**<u>Alternative Method 1: Using the direction vectors of Line 1 and Line 2</u>**		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$		
	$\cos\theta = \dfrac{\mathbf{d}_1 \bullet \mathbf{d}_1}{\lvert\mathbf{d}_1\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}}{\sqrt{(0)^2+(2)^2+(-1)^2} . \sqrt{(3)^2+(-5)^2+(4)^2}}$	Applies dot product formula between their $\mathbf{d}_1$ and $\mathbf{d}_2$	M2
	$\left\{\cos\theta = \dfrac{0-10-4}{\sqrt{5}.\sqrt{50}} = \dfrac{-7\sqrt{10}}{25} \Rightarrow\right\} \theta = 152.3054385...$		
	Angle $ACB = 180 - 152.3054385... = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 2: The Cosine Rule</u>		
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	Also $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$		
	Note $\lvert\overrightarrow{AC}\rvert = \sqrt{80}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{50}$ and $\lvert\overrightarrow{AB}\rvert = \sqrt{18}$		
	$\left(\sqrt{18}\right)^2 = \left(\sqrt{80}\right)^2 + \left(\sqrt{50}\right)^2 - 2\left(\sqrt{80}\right)\left(\sqrt{50}\right)\cos\theta$	Applies the cosine rule the correct way round.	M1 oe
	$\left\{\cos\theta = \dfrac{7\sqrt{10}}{25}\right\} \Rightarrow \theta = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]
	<u>Alternative Method 3: Vector Cross Product</u> **Only apply this scheme if it is clear that a candidate is applying a vector cross product method.**		
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\overrightarrow{AC} \times \overrightarrow{BC} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -4 \\ -3 & 5 & -4 \end{vmatrix} = 24\mathbf{i} + 12\mathbf{j} + 24\mathbf{k} \right\}$ $\sin ACB = \dfrac{\sqrt{(24)^2+(12)^2+(12)^2}}{\sqrt{(0)^2+(8)^2+(-4)^2} . \sqrt{(-3)^2+(5)^2+(-4)^2}}$	Full method for applying the vector cross product formula between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.	M1
	$\left\{\sin ACB = \dfrac{\sqrt{864}}{\sqrt{80}.\sqrt{50}} = \dfrac{3\sqrt{15}}{25} \Rightarrow\right\} \theta = 27.69446145... = 27.7 \text{ (3 sf)}$	Anything that rounds to 27.7	A1
			[3]

		Question 6 Notes
6. (a)	**B1**	$p = 5$ (Ignore working.)
(b)		**Method 1**
	M1	Writes down an equation involving only one parameter. This equation will usually be $7 + 3\mu = 1$ which is found from equating the **i** components of l_1 and l_2.
	A1	Finds $\mu = -2$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\lambda = 4$ and $\mu = -2$ is true for the third component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(b)		**Alternative Method**
	M1	Writes down an equation involving only one parameter. Solving the **j** and **k** components simultaneously will usually give either $8 = 14 + 3\mu$ or $23 + 3\lambda = 35$
	A1	Finds either $\mu = -2$ or $\lambda = 4$
	B1	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$. Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$.
	B1	Finds $\lambda = 4$ and either • checks $\mu = -2$ is true for the **i** component. • substitutes $\mu = -2$ into l_1 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ **and** substitutes $\lambda = 4$ into l_2 to give $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$
(c)	**M1**	An attempt to find both the vectors $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$ by subtracting.
	M1	Applies dot product **formula** between their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$ **and** their $\left(\overrightarrow{BC} \text{ or } \overrightarrow{CB}\right)$.
	A1	anything that rounds to 27.7
	Note	An answer of 0.48336... in radians without the correct answer in degrees is A0.
	Note	Some candidates will apply the dot product formula between vectors which are the wrong way round and achieve $152.3054385...^\circ$. If they give the acute equivalent of awrt 27.7 then award A1.
(d)	**M1**	$\frac{1}{2}(\text{their length } AC)(\text{their length } BC)\sin\left(\text{their } 27.7^\circ \text{ from part (c)}\right)$
	A1	anything that rounds to 14.7. Also allow $6\sqrt{6}$.
	Note	Area $ACB = \frac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{50}\right)\sin(152.3054385...^\circ) =$ awrt 14.7 is M1A1.

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7. The rate of increase of the number, N, of fish in a lake is modelled by the differential equation

$$\frac{dN}{dt} = \frac{(kt - 1)(5000 - N)}{t} \qquad t > 0, \quad 0 < N < 5000$$

In the given equation, the time t is measured in years from the start of January 2000 and k is a positive constant.

(a) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where A is a positive constant.

(5)

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

(b) Find the exact value of the constant A and the exact value of the constant k.

(4)

(c) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish.

(1)

Question Number	Scheme		Marks
7.	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{(kt-1)(5000-N)}{t}, \quad t>0,\ 0<N<5000$		
(a)	$\displaystyle\int \frac{1}{5000-N}\,\mathrm{d}N = \int \frac{(kt-1)}{t}\,\mathrm{d}t \quad \left\{\text{or } = \int\left(k-\frac{1}{t}\right)\mathrm{d}t\right\}$	See notes	B1
	$-\ln(5000-N) = kt - \ln t;\ +c$	See notes	M1 A1; A1
	then eg either... $-kt+c = \ln(5000-N) - \ln t$; $-kt+c = \ln\left(\dfrac{5000-N}{t}\right)$; $\mathrm{e}^{-kt+c} = \dfrac{5000-N}{t}$ ***or...*** $kt+c = \ln t - \ln(5000-N)$; $kt+c = \ln\left(\dfrac{t}{5000-N}\right)$; $\mathrm{e}^{kt+c} = \dfrac{t}{5000-N}$ ***or...*** $\ln(5000-N) = -kt + \ln t + c$; $5000-N = \mathrm{e}^{-kt+\ln t+c}$; $5000-N = t\,\mathrm{e}^{-kt+c}$		
	leading to $N = 5000 - Ate^{-kt}$ with **no incorrect working/statements**. **See notes**		A1 * **cso**
			[5]
(b)	$\{t=1,\ N=1200 \Rightarrow\}\ \ 1200 = 5000 - A\mathrm{e}^{-k}$ $\{t=2,\ N=1800 \Rightarrow\}\ \ 1800 = 5000 - 2A\mathrm{e}^{-2k}$	At least one correct statement written down using the boundary conditions	B1
	So $A\mathrm{e}^{-k} = 3800$ and $2A\mathrm{e}^{-2k} = 3200$ or $A\mathrm{e}^{-2k} = 1600$		
	Eg. $\dfrac{\mathrm{e}^{-k}}{2\mathrm{e}^{-2k}} = \dfrac{3800}{3200}$ or $\dfrac{2\mathrm{e}^{-2k}}{\mathrm{e}^{-k}} = \dfrac{3200}{3800}$ So $\dfrac{1}{2}\mathrm{e}^{k} = \dfrac{3800}{3200}$ or $2\mathrm{e}^{-k} = \dfrac{3200}{3800}$	An attempt to eliminate A by producing an equation in only k.	M1
	$k = \ln\left(\dfrac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\dfrac{19}{8}\right)\right\}$	At least one of $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
	$\left\{A = 3800(\mathrm{e}^{k}) = 3800\left(\dfrac{19}{8}\right) \Rightarrow\right\}\ A = 9025$	Both $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
			[4]
	<u>Alternative Method for the M1 mark in (b)</u>		
	$\mathrm{e}^{-k} = \dfrac{3800}{A}$		
	$2A\left(\dfrac{3800}{A}\right)^2 = 3200$	An attempt to eliminate k by producing an equation in only A	M1
(c)	$\left\{t=5,\ N = 5000 - 9025(5)\mathrm{e}^{-5\ln\left(\frac{19}{8}\right)}\right\}$		
	$N = 4402.828401... = 4400$ (fish) (nearest 100)	anything that rounds to 4400	B1
			[1] **10**

		Question 7 Notes
7. (a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda\neq 0$ is a constant.
	A1	For $-\ln(5000-N)=kt-\ln t$ or $\ln(5000-N)=-kt+\ln t$ or $-\frac{1}{k}\ln(5000-N)=t-\frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^{c}$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or "$\ln e^{c}$" "$\ln c$" or and applies a fully correct method to prove the result $N=5000-Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000-N=e^{\ln t-kt+c}$ • **or** $5000-N=te^{-kt+c}$ • **or** $5000-N=te^{-kt}e^{c}$ or equivalent needs to be stated before achieving $N=5000-Ate^{-kt}$
(b)	**B1**	At least one of **either** $1200=5000-Ae^{-k}$ (or equivalent) **or** $1800=5000-2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • **or** an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k=\ln\left(\frac{19}{8}\right)$ or $k=-\ln\left(\frac{8}{19}\right)$ or $k=\ln 7600-\ln 3200$ or $k=-\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k=0.8649...$ without a correct exact equivalent is A0.
(c)	**B1**	anything that rounds to 4400

Leave blank

8.

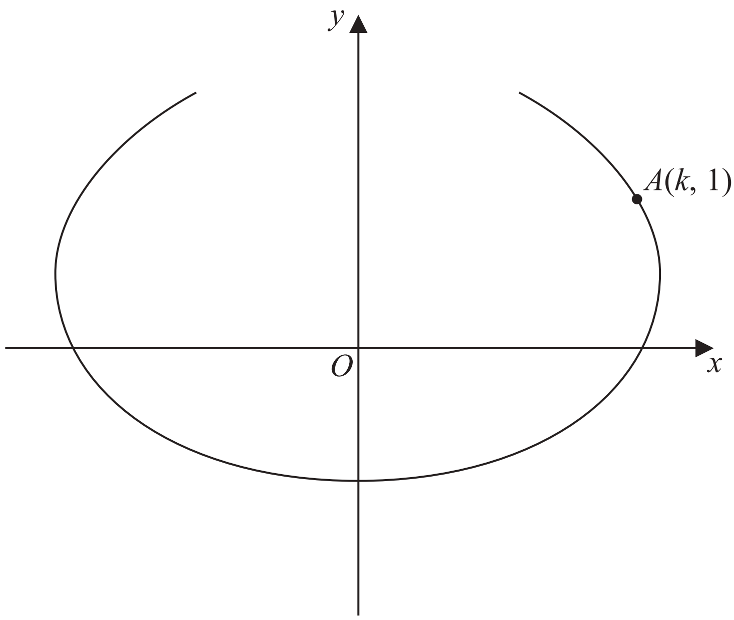

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4\sin t, \quad y = 1 - 2\cos t, \qquad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

Question Number	Scheme		Marks
8.	$x = t - 4\sin t,\quad y = 1 - 2\cos t,\quad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}\quad A(k, 1)$ lies on the curve, $k > 0$		
(a)	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$		
	k (or x) $= \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)$ **or** $x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \frac{\pi}{2}$ or $\frac{8-\pi}{2}$	x or $k = 4 - \frac{\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 4\cos t,\quad \frac{dy}{dt} = 2\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$.	M1;
	At $t = -\frac{\pi}{2}$, $\frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{2}\right)}$; $= -2$	Correct value for $\frac{dy}{dx}$ of -2	A1 **cao cso**
			[4]
(c)	$\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	gives $4\sin t - 4\cos t = -1$	**See notes**	A1
	So $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$; $= -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$; $= -1$	**See notes**	M1; A1
	$t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}$ or $t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$	**See notes**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] **12**

	Question 8 Notes	
(c)	**NOTE**	**<u>VERY IMPORTANT NOTE FOR PART (c)</u>** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

	Question 8: Alternative Methods for Part (c)		
8. (c)	**Alternative Method 1:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
		Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$ or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.	M1
	• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
	• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]
8. (c)	**Alternative Method 2:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
	So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$		
	leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
		$16 - 16\sin 2t = 1$ or equivalent.	A1
	$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\}\ t = \frac{\sin^{-1}(....)}{2}$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]

		Question 8 Notes
8. (a)	**M1**	Sets $y = 1$ to find t and uses their t to find x.
	Note	M1 can be implied by either x or $k = 4 - \frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2} - 4$ or $-2.429...$
	A1	x or $k = 4 - \frac{\pi}{2}$ or $\frac{8-\pi}{2}$
	Note	A decimal answer of 2.429... (without a correct **exact** answer) is A0.
	Note	Allow A1 for a candidate using $t = \frac{\pi}{2}$ to find $x = \frac{\pi}{2} - 4$ **and** then stating that k must be $4 - \frac{\pi}{2}$ o.e.
(b)	**B1**	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.
	B1	Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.
	M1	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute their t into their expression for $\frac{dy}{dx}$.
	Note	This mark may be implied by their final answer. i.e. $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$ followed by an answer of -2 (from $t = -\frac{\pi}{2}$) or 2 (from $t = \frac{\pi}{2}$)
	Note	**Applying** $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.
	A1	Using $t = -\frac{\pi}{2}$ $\left(\text{and not } t = \frac{3\pi}{2}\right)$ to find a correct $\frac{dy}{dx}$ of -2 ***by correct solution only.***
(c)		
	NOTE	If a candidate uses an incorrect $\frac{dy}{dx}$ expression in part (c) then the accuracy marks are not obtainable.
	1st M1	Sets their $\frac{dy}{dx} = -\frac{1}{2}$
	1st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ **on the same side.** eg. $4\sin t - 4\cos t = -1$ or $4\cos t - 4\sin t = 1$ or $\sin t - \cos t = -\frac{1}{4}$ or $\cos t - \sin t = \frac{1}{4}$ or $4\sin t - 4\cos t + 1 = 0$ or $4\cos t - 4\sin t - 1 = 0$ or $\sin t - \cos t + \frac{1}{4} = 0$ etc. are fine for A1.
	2nd M1	Rewrites $\pm\lambda\sin t \pm \mu\cos t$ in the form of either $R\cos(t \pm \alpha)$ or $R\sin(t \pm \alpha)$ where $R \neq 1$ or 0 and $\alpha \neq 0$
	2nd A1	Correct equation. Eg. $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right) = -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right) = -1$ or $\sqrt{2}\sin\left(t - \frac{\pi}{4}\right) = -\frac{1}{4}$ or $\sqrt{2}\cos\left(t + \frac{\pi}{4}\right) = \frac{1}{4}$, etc.
	Note	Unless recovered, give A0 for $4\sqrt{2}\sin\left(t - 45^\circ\right) = -1$ or $-4\sqrt{2}\cos\left(t + 45^\circ\right) = -1$, etc.
	3rd M1	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$
	4th A1	anything that rounds to 0.6077
	Note	**Do not give** the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$.
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$.