





Question Number	Scheme		Marks
<p>1. (a)</p> <p>(b)</p>	$\left\{ \frac{1}{\sqrt{(9-10x)}} \right\} (9-10x)^{-\frac{1}{2}}$ $= (9)^{-\frac{1}{2}} \left(1 - \frac{10x}{9}\right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 - \frac{10x}{9}\right)^{-\frac{1}{2}}$ $= \left\{ \frac{1}{3} \right\} \left[ 1 + \left(-\frac{1}{2}\right)(kx) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (kx)^2 + \dots \right]$ $= \left\{ \frac{1}{3} \right\} \left[ 1 + \left(-\frac{1}{2}\right) \left(\frac{-10x}{9}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} \left(\frac{-10x}{9}\right)^2 + \dots \right]$ $= \frac{1}{3} \left[ 1 + \frac{5}{9}x + \frac{25}{54}x^2 + \dots \right]$ $= \frac{1}{3} + \frac{5}{27}x + \frac{25}{162}x^2 + \dots$ $\frac{3+x}{\sqrt{(9-10x)}} = (3+x)(9-10x)^{-\frac{1}{2}}$ $= (3+x) \left( \frac{1}{3} + \frac{5}{27}x + \left\{ \frac{25}{162}x^2 + \dots \right\} \right)$ $= 1 + \frac{5}{9}x + \frac{25}{54}x^2 + \frac{1}{3}x + \frac{5}{27}x^2 + \dots$ $= 1 + \frac{8}{9}x + \frac{35}{54}x^2 + \dots$	<p><math>(9-10x)^{-\frac{1}{2}}</math> or uses power of <math>-\frac{1}{2}</math></p> <p><math>(9)^{-\frac{1}{2}}</math> or <math>\frac{1}{3}</math></p> <p>At least two correct terms. <b>See notes</b></p> <p><i>Can be implied by later work</i> <b>See notes</b></p> <p>Multiplies out to give exactly one constant term, exactly 2 terms in <math>x</math> and exactly 2 terms in <math>x^2</math>. Ignore terms in <math>x^3</math>. Can be implied.</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1; A1</p> <p>[5]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>8</p>
<b>Question 1 Notes</b>			
(a)	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Writes down <math>(9-10x)^{-\frac{1}{2}}</math> or uses power of <math>-\frac{1}{2}</math>.</p> <p>This mark can be implied by a constant term of <math>(9)^{-\frac{1}{2}}</math> or <math>\frac{1}{3}</math>.</p> <p><math>(9)^{-\frac{1}{2}}</math> or <math>\frac{1}{3}</math> outside brackets or <math>\frac{1}{3}</math> as candidate's constant term in their binomial expansion.</p> <p>Expands <math>(\dots + kx)^{-\frac{1}{2}}</math> to give any 2 terms out of 3 terms simplified or an un-simplified,  <math>1 + (-\frac{1}{2})(kx)</math> or <math>(-\frac{1}{2})(kx) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (kx)^2</math> or <math>1 + \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (kx)^2</math>, <b>where</b> <math>k \neq 1</math>.</p> <p><math>\frac{1}{3} + \frac{5}{27}x</math> (<b>simplified fractions</b>)</p> <p>Accept only <math>\frac{25}{162}x^2</math></p>	

1. (a) ctd	<b>Note</b>	<p><b>You cannot recover correct work for part (a) in part (b).</b> i.e. if the correct answer to (a) appears as part of their solution in part (b), it cannot be credited in part (a).</p>
	<b>SC</b>	<p>If a candidate <i>would otherwise score</i> A0A0 then <b>allow Special Case 1<sup>st</sup> A1 for either</b></p>
		<p><b>SC:</b> <math>\frac{1}{3}\left[1 + \frac{5}{9}x; \dots\right]</math> or <b>SC:</b> <math>\lambda\left[1 + \frac{5}{9}x + \frac{25}{54}x^2 + \dots\right]</math> or <b>SC:</b> <math>\left[\lambda + \frac{5\lambda}{9}x + \frac{25\lambda}{54}x^2 + \dots\right]</math></p>
		<p>(where <math>\lambda</math> can be 1 or omitted), with each term in the [.....] is a simplified fraction</p>
	<b>SC</b>	<p><b>Special case for the M1 mark</b></p>
		<p>Award Special Case M1 for a correct simplified or un-simplified <math>1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2</math> expansion with a value of <math>n \neq -\frac{1}{2}</math>, <math>n \neq</math> <b>positive integer</b> and a consistent <math>(kx)</math>. Note that <math>(kx)</math> must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion.  <b>Note that <math>k \neq 1</math>.</b></p>
	<b>Note</b>	<p>Candidates who write <math>\left\{\frac{1}{3}\right\}\left[1 + \left(-\frac{1}{2}\right)\left(\frac{10x}{9}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{10x}{9}\right)^2 + \dots\right]</math> where <math>k = \frac{10}{9}</math> and not <math>-\frac{10}{9}</math> and achieve <math>\frac{1}{3} - \frac{5}{27}x; + \frac{25}{162}x^2 + \dots</math> will get B1B1M1A0A1.</p>
(b)	<b>M1</b>	<p>Writes down <math>(3 + x)</math>(their part (a) answer, at least 2 of the 3 terms.)</p>
	<b>Note</b>	<p><math>(3 + x)\left(\frac{1}{4} + \frac{5}{4}x + \dots\right)</math> or <math>(3 + x)\left(\frac{1}{3} + \frac{5}{27}x + \frac{25}{162}x^2 + \dots\right)</math> are fine for M1.</p>
	<b>Note</b>	<p>This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in <math>x</math>.</p>
	<b>M1</b>	<p>Multiplies out to give exactly one constant term, exactly 2 terms in <math>x</math> and exactly 2 terms in <math>x^2</math>.</p>
	<b>Note</b>	<p>This M1 mark can be implied. You can also ignore <math>x^3</math> terms.</p>
	<b>A1</b>	<p><math>1 + \frac{8}{9}x + \frac{35}{54}x^2 + \dots</math></p>
<p><b>Alternative Methods for part (a)</b></p>		
<p><b>Alternative method 1:</b> Candidates can apply an alternative form of the binomial expansion.</p>		
<p><math>\left\{\frac{1}{\sqrt{(9-10x)}}\right\} = (9-10x)^{-\frac{1}{2}} = (9)^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)(9)^{-\frac{3}{2}}(-10x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(9)^{-\frac{5}{2}}(-10x)^2</math></p>		
	<b>B1</b>	<p>Writes down <math>(9-10x)^{-\frac{1}{2}}</math> or uses power of <math>-\frac{1}{2}</math>.</p>
	<b>B1</b>	<p><math>9^{-\frac{1}{2}}</math> or <math>\frac{1}{3}</math></p>
	<b>M1</b>	<p>Any two of three (un-simplified or simplified) terms correct.</p>
	<b>A1</b>	<p><math>\frac{1}{3} + \frac{5}{27}x</math></p>
	<b>A1</b>	<p><math>\frac{25}{162}x^2</math></p>
	<b>Note</b>	<p>The terms in C need to be evaluated, so <math>^{-\frac{1}{2}}C_0(9)^{-\frac{1}{2}} + ^{-\frac{1}{2}}C_1(9)^{-\frac{3}{2}}(-10x) + ^{-\frac{1}{2}}C_2(9)^{-\frac{5}{2}}(-10x)^2</math> without further working is B1B0M0A0A0.</p>

<p>1. (a)</p>	<p><b><u>Alternative Method 2: Maclaurin Expansion</u></b></p> <p>Let <math>f(x) = \frac{1}{\sqrt{(9-10x)}}</math></p> <p><math>\{f(x) = (9-10x)^{-\frac{1}{2}}</math></p> <p><math>f''(x) = 75(9-10x)^{-\frac{5}{2}}</math></p> <p><math>f'(x) = (-\frac{1}{2})(9-10x)^{-\frac{3}{2}}(-10)</math></p> <p><math>\left\{ \therefore f(0) = \frac{1}{3}, f'(0) = \frac{5}{27} \text{ and } f''(0) = \frac{75}{243} = \frac{25}{81} \right\}</math></p> <p><math>f(x) = \frac{1}{3} + \frac{5}{27}x + \frac{25}{162}x^2 + \dots</math></p>	<p><math>(9-10x)^{-\frac{1}{2}}</math>   B1</p> <p>Correct <math>f''(x)</math>   B1 oe</p> <p><math>\pm a(9-10x)^{-\frac{3}{2}}; a \neq \pm 1</math>   M1</p> <p>A1; A1</p>
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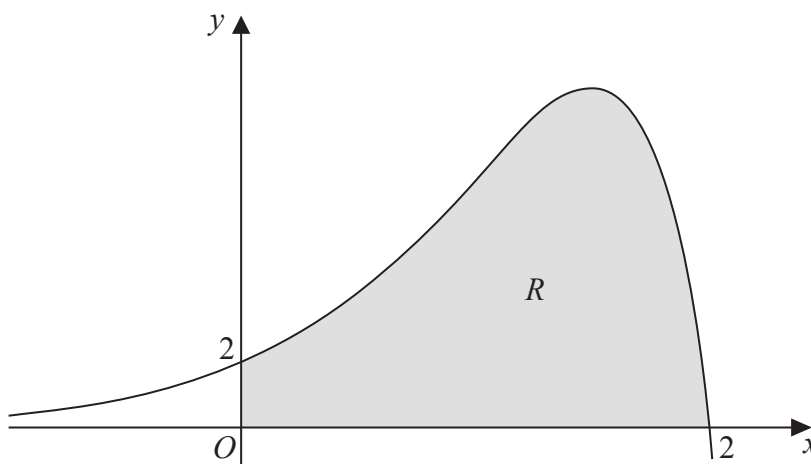


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \quad x \in \mathbb{R}$$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the  $y$ -axis.

The table below shows corresponding values of  $x$  and  $y$  for  $y = (2 - x)e^{2x}$

$x$	0	0.5	1	1.5	2
$y$	2	4.077	7.389	10.043	0

- (a) Use the trapezium rule with all the values of  $y$  in the table, to obtain an approximation for the area of  $R$ , giving your answer to 2 decimal places. (3)
- (b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of  $R$ . (1)
- (c) Use calculus, showing each step in your working, to obtain an exact value for the area of  $R$ . Give your answer in its simplest form. (5)

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Question Number	Scheme		Marks
2. (a)	$\text{Area} \approx \frac{1}{2} \times 0.5 \times [2 + 2(4.077 + 7.389 + 10.043) + 0]$ $= \frac{1}{4} \times 45.018 = 11.2545 = 11.25 \text{ (2 dp)}$		B1; M1  11.25 A1 <b>cao</b>  [3]
(b)	Any one of <ul style="list-style-type: none"> <li>• Increase the number of strips</li> <li>• Use more trapezia</li> <li>• Make <math>h</math> smaller</li> <li>• Increase the <b>number of</b> <math>x</math> and/or <math>y</math> values used</li> <li>• Shorter /smaller intervals for <math>x</math></li> <li>• More values of <math>y</math>.</li> <li>• More intervals of <math>x</math></li> <li>• Increase <math>n</math></li> </ul>		B1       [1]
(c)	$\left\{ \int (2-x)e^{2x} dx \right\}, \left\{ \begin{array}{l} u = 2-x \Rightarrow \frac{du}{dx} = -1 \\ \frac{dv}{dx} = e^{2x} \Rightarrow v = \frac{1}{2}e^{2x} \end{array} \right\}$		
$= \frac{1}{2}(2-x)e^{2x} - \int -\frac{1}{2}e^{2x} \{dx\}$			Either $(2-x)e^{2x} \rightarrow \pm \lambda(2-x)e^{2x} \pm \int \mu e^{2x} \{dx\}$ or $\pm x e^{2x} \rightarrow \pm \lambda x e^{2x} \pm \int \mu e^{2x} \{dx\}$ M1
$= \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$			$(2-x)e^{2x} \rightarrow \frac{1}{2}(2-x)e^{2x} - \int -\frac{1}{2}e^{2x} \{dx\}$ A1
$\text{Area} = \left\{ \left[ \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x} \right]_0^2 \right\}$			$\frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$ A1 oe
$= \left( 0 + \frac{1}{4}e^4 \right) - \left( \frac{1}{2}(2)e^0 + \frac{1}{4}e^0 \right)$			Applies limits of 2 and 0 <i>to all terms</i> and subtracts the correct way round. dM1
$= \frac{1}{4}e^4 - \frac{5}{4}$			$\frac{1}{4}e^4 - \frac{5}{4} \text{ or } \frac{e^4 - 5}{4} \text{ cao}$ A1 oe
<b>Question 2 Notes</b>			
(a)	<b>B1</b>  <b>M1</b> <b>Note</b> <b>A1</b> <b>Note</b> <b>Note</b>	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$ . For structure of trapezium rule [ ..... ]. Condone missing 0. No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate]. 11.25 <b>cao</b> Working must be seen to demonstrate the use of the trapezium rule. The actual area is 12.39953751... Award B1M1A1 for $\frac{0.5}{2}(2+0) + \frac{1}{2}(4.077 + 7.389 + 10.043) = 11.25$	[5] 9

2. (a) contd	<p><b>Bracketing mistake:</b> Unless the final answer implies that the calculation has been done correctly. Award B1M0A0 for <math>\frac{1}{2} \times 0.5 + 2 + 2(4.077 + 7.389 + 10.043) + 0</math> (nb: answer of 45.268).</p>
	<p><b>Alternative method for part (a): Adding individual trapezia</b> Area <math>\approx 0.5 \times \left[ \frac{2+4.077}{2} + \frac{4.077+7.389}{2} + \frac{7.389+10.043}{2} + \frac{10.043+0}{2} \right] = 11.2545 = 11.25</math> (2 dp) cao</p>
	<p><b>B1</b> 0.5 and a divisor of 2 on all terms inside brackets.</p>
	<p><b>M1</b> First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.</p>
	<p><b>A1</b> 11.25 cao</p>
(b)	<p><b>B0</b> Give B0 for</p> <ul style="list-style-type: none"> <li>• smaller values of <math>x</math> and/or <math>y</math>.</li> <li>• use more decimal places</li> </ul>
(c)	<p><b>M1</b> Either <math>(2-x)e^{2x} \rightarrow \pm \lambda(2-x)e^{2x} \pm \int \mu e^{2x} \{dx\}</math> or <math>\pm xe^{2x} \rightarrow \pm \lambda xe^{2x} \pm \int \mu e^{2x} \{dx\}</math></p> <p><b>A1</b> <math>(2-x)e^{2x} \rightarrow \frac{1}{2}(2-x)e^{2x} - \int -\frac{1}{2}e^{2x} \{dx\}</math> either un-simplified or simplified.</p> <p><b>A1</b> Correct expression, i.e. <math>\frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}</math> or <math>\frac{5}{4}e^{2x} - xe^{2x}</math> (or equivalent)</p> <p><b>dM1</b> which is dependent on the 1<sup>st</sup> M1 mark being awarded. <b>Complete method</b> of applying limits of 2 and 0 to all terms and subtracting the correct way round.</p> <p><b>Note</b> Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.</p> <p><b>A1</b> <math>\frac{1}{4}e^4 - \frac{5}{4}</math> or <math>\frac{e^4 - 5}{4}</math>. Do not allow <math>\frac{1}{4}e^4 - \frac{5}{4}e^0</math> unless simplified to give <math>\frac{1}{4}e^4 - \frac{5}{4}</math></p> <p><b>Note</b> 12.39953751... <b>without seeing</b> <math>\frac{1}{4}e^4 - \frac{5}{4}</math> is A0.</p> <p><b>Note</b> 12.39953751... from <b>NO</b> working is M0A0A0M0A0.</p>





Question Number	Scheme	Marks
<p>3.</p> <p>(a)</p> <p>(b)</p>	$x^2 + y^2 + 10x + 2y - 4xy = 10$ $\left\{ \begin{array}{l} \cancel{dx} \\ \cancel{dx} \end{array} \right\} \times \frac{2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx} - \left( 4y + 4x \frac{dy}{dx} \right)}{dx} = 0$ $2x + 10 - 4y + (2y + 2 - 4x) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$ <p>Simplifying gives <math>\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1} \left\{ = \frac{-x - 5 + 2y}{-2x + y + 1} \right\}</math></p> $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} x + 5 - 2y = 0$ <p>So <math>x = 2y - 5</math>,</p> $(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$ $4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$ <p>gives <math>-3y^2 + 22y - 35 = 0</math> or <math>3y^2 - 22y + 35 = 0</math></p> $(3y - 7)(y - 5) = 0 \text{ and } y = \dots$ $y = \frac{7}{3}, 5$	<p>See notes</p> <p>M1 A1 M1</p> <p>Dependent on the first M1 mark.</p> <p>dM1</p> <p>A1 cso oe</p> <p>[5]</p> <p>M1</p> <p>M1</p> <p>A1 oe</p> <p>see notes</p> <p>Method mark for solving a quadratic equation.</p> <p>ddM1</p> <p>A1 cao</p> <p>[5]</p>
<p>(b)</p>	<p><b>Alternative method for part (b)</b></p> $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} x + 5 - 2y = 0$ <p>So <math>y = \frac{x + 5}{2}</math>,</p> $x^2 + \left( \frac{x + 5}{2} \right)^2 + 10x + 2 \left( \frac{x + 5}{2} \right) - 4x \left( \frac{x + 5}{2} \right) = 10$ $x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$ $4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$ <p>gives <math>-3x^2 + 14x + 5 = 0</math> or <math>3x^2 - 14x - 5 = 0</math></p> $(3x + 1)(x - 5) = 0, x = \dots$ $y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$ $y = \frac{7}{3}, 5$	<p>M1</p> <p>M1</p> <p>A1 oe</p> <p>see notes</p> <p>Solves a quadratic and finds at least one value for y.</p> <p>ddM1</p> <p>A1 cao</p> <p>[5]</p>
		<b>10</b>

Question 3 Notes			
3. (a)	<b>M1</b>	Differentiates implicitly to include either $\pm 4x \frac{dy}{dx}$ or $y^2 \rightarrow 2y \frac{dy}{dx}$ or $2y \rightarrow 2 \frac{dy}{dx}$ . (Ignore $\left(\frac{dy}{dx} = \right)$ ).	
	<b>A1</b>	$x^2 + y^2 + 10x + 2y \rightarrow 2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx}$ <b>and</b> $10 \rightarrow 0$	
	<b>M1</b>	$-4xy \rightarrow \pm 4y \pm 4x \frac{dy}{dx}$	
	<b>Note</b>	If an extra term appears then award 1 <sup>st</sup> A0.	
	<b>Note</b>	$2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} \rightarrow 2x + 10 - 4y = -2y \frac{dy}{dx} - 2 \frac{dy}{dx} + 4x \frac{dy}{dx}$ will get 1 <sup>st</sup> A1 (implied) as the "= 0" can be implied by rearrangement of their equation.	
	<b>dM1</b>	<b>dependent on the first method mark being awarded.</b> An attempt to factorise out <b>all the terms in</b> $\frac{dy}{dx}$ as long as there are <b>at least two terms</b> in $\frac{dy}{dx}$ .	
	<b>A1</b>	$\frac{x + 5 - 2y}{2x - y - 1}$ or $\frac{-x - 5 + 2y}{-2x + y + 1}$ (must be simplified).	
	<b>cs0:</b>	If the candidate's solution is not completely correct, then do not give this mark.	
	(b)	<b>M1</b>	Sets the numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) oe.
		<b>NOTE</b>	<b>If the numerator involves one variable only then only the 1<sup>st</sup> M1 mark is possible in part (b).</b>
<b>M1</b>		Substitutes their $x$ or their $y$ into the printed equation to give an equation in one variable only.	
<b>A1</b>		For obtaining either $-3y^2 + 22y - 35 \{= 0\}$ or $3y^2 - 22y + 35 \{= 0\}$	
<b>Note</b>		This mark can also awarded for a correct three term equation, eg. either $-3y^2 + 22y = 35$ $3y^2 - 22y = -35$ or $3y^2 + 35 = 22y$ are all fine for A1.	
<b>ddM1</b>		<b>Dependent on the previous 2 M marks.</b> <b>See notes at the beginning of the mark scheme: Method mark for solving a 3 term quadratic</b>	
		<ul style="list-style-type: none"> <li>• <math>(3y - 7)(y - 5) = 0 \Rightarrow y = \dots</math></li> <li>• <math>y = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}</math></li> <li>• <math>y^2 - \frac{22}{3}y - \frac{35}{3} = 0 \Rightarrow \left(y - \frac{11}{3}\right)^2 - \frac{121}{9} + \frac{35}{3} = 0 \Rightarrow y = \dots</math></li> <li>• Or writes down at least one correct <math>y</math>-root from their quadratic equation. This is usually found from their calculator.</li> </ul>	
<b>Note</b>		If a candidate applies <b>the alternative method</b> then they also need to use their $y = \frac{x + 5}{2}$ in order to find <b>at least one value</b> for $y$ in order to gain the final M1.	
<b>A1</b>		$y = \frac{7}{3}, 5$ . <b>cao.</b> (2.33 or 2.3 without reference to $\frac{7}{3}$ or $2\frac{1}{3}$ is not allowed for this mark.)	
<b>Note</b>		It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$ ) to gain all 5 marks in part (b).	



Question Number	Scheme	Marks
4. (a)	$\frac{25}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$ $B = 25, C = 100$	B1 B1 cso See notes.
	$25 \equiv Ax(2x+1) + B(2x+1) + Cx^2$ $x=0, \quad 25 = B$ $x = -\frac{1}{2}, \quad 25 = \frac{1}{4}C \Rightarrow C = 100$ $x^2 \text{ terms: } 0 = 2A + C$ $0 = 2A + 100 \Rightarrow A = -50$ $x^2: 0 = 2A + C, \quad x: 0 = A + 2B,$ $\text{constant: } 25 = B$	Writes down a <b>correct identity</b> and attempts to <b>find the value</b> of either one of "A", "B" or "C". M1
	leading to $A = -50$ $\left\{ \frac{25}{x^2(2x+1)} \equiv -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)} \right\}$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition. A1 [4]
(b)	$V = \pi \int_1^4 \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2 dx$ <p><b>For their partial fraction</b></p>	For $\pi \int \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2$ Ignore limits and dx. Can be implied. B1
	$\left\{ \int \frac{25}{x^2(2x+1)} dx = \int -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)} dx \right\}$ $= -50 \ln x + \frac{25x^{-1}}{(-1)} + \frac{100}{2} \ln(2x+1) \{+ c\}$	Either $\pm \frac{A}{x} \rightarrow \pm a \ln x$ or $\pm a \ln kx$ or $\pm \frac{B}{x^2} \rightarrow \pm b x^{-1}$ or $\frac{C}{(2x+1)} \rightarrow \pm c \ln(2x+1)$ At least two terms correctly integrated All three terms correctly integrated. M1 A1ft A1ft
	$\left\{ \int_1^4 \frac{25}{x^2(2x+1)} dx = \left[ -50 \ln x - \frac{25}{x} + 50 \ln(2x+1) \right]_1^4 \right\}$ $= \left( -50 \ln 4 - \frac{25}{4} + 50 \ln 9 \right) - \left( 0 - 25 + 50 \ln 3 \right)$ $= 50 \ln 9 - 50 \ln 4 - 50 \ln 3 - \frac{25}{4} + 25$ $= 50 \ln \left( \frac{3}{4} \right) + \frac{75}{4}$ <p>So, <math>V = \frac{75}{4} \pi + 50 \pi \ln \left( \frac{3}{4} \right)</math> or allow <math>\pi \left( \frac{75}{4} + 50 \ln \left( \frac{3}{4} \right) \right)</math></p>	Applies limits of 4 and 1 and subtracts the correct way round. dM1 A1 oe [6] 10

Question 4 Notes	
4. (a)	<p><b>BE CAREFUL!</b> Candidates will assign <i>their own</i> "A, B and C" for this question.</p> <p><b>B1</b> At least one of "B" or "C" are correct.</p> <p><b>B1</b> Breaks up their partial fraction correctly into three terms <b>and</b> both "B" = 25 and "C" = 100.</p> <p><b>Note</b> If a candidate does not give partial fraction decomposition then:  <ul style="list-style-type: none"> <li>the 2<sup>nd</sup> B1 mark can follow from a correct identity.</li> </ul> </p> <p><b>M1</b> Writes down <b>a correct identity</b> (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".                      This can be achieved by <b>either</b> substituting values into their identity <b>or</b> comparing coefficients and solving the resulting equations simultaneously.</p> <p><b>A1</b> Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.</p> <p><b>Note</b> If a candidate does not give partial fraction decomposition then the final A1 mark can be awarded for a correct "A" if a candidate writes out their partial fractions at the end.</p> <p><b>Note</b> The correct partial fraction from no working scores B1B1M1A1.</p> <p><b>Note</b> A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.</p> <p><b>Note</b> Award SC B1B0M0A0 for <math>\frac{25}{x^2(2x+1)} \equiv \frac{B}{x^2} + \frac{C}{(2x+1)}</math> leading to "B" = 25 <b>or</b> "C" = 100</p>
(b)	<p><b>B1</b> For a correct statement of <math>\pi \int \left( \frac{5}{x\sqrt{2x+1}} \right)^2</math> or <math>\pi \int \frac{25}{x^2(2x+1)}</math>. Ignore limits and dx. Can be implied.</p> <p><b>Note</b> <b>The <math>\pi</math> can only be recovered later from a correct expression.</b></p>
	<p><b>For their partial fraction,</b> (not <math>\sqrt{\text{their partial fraction}}</math>), where A, B, C are "their" part (a) constants</p> <p><b>M1</b> Either <math>\pm \frac{A}{x} \rightarrow \pm a \ln x</math> or <math>\pm \frac{B}{x^2} \rightarrow \pm b x^{-1}</math> or <math>\frac{C}{(2x+1)} \rightarrow \pm c \ln(2x+1)</math>.</p> <p><b>Note</b> <math>\sqrt{\frac{B}{x^2}} \rightarrow \frac{\sqrt{B}}{x}</math> which integrates to <math>\sqrt{B} \ln x</math> is <b>not</b> worthy of M1.</p> <p><b>A1ft</b> At least two terms from any of <math>\pm \frac{A}{x}</math> or <math>\pm \frac{B}{x^2}</math> or <math>\frac{C}{(2x+1)}</math> correctly integrated. Can be un-simplified.</p> <p><b>A1ft</b> All 3 terms from <math>\pm \frac{A}{x}</math>, <math>\pm \frac{B}{x^2}</math> and <math>\frac{C}{(2x+1)}</math> correctly integrated. Can be un-simplified.</p> <p><b>Note</b> The 1<sup>st</sup> A1 and 2<sup>nd</sup> A1 marks in part (b) are both follow through accuracy marks.</p>
	<p><b>dM1</b> <b>Dependent on the previous M mark.</b>                      Applies limits of 4 and 1 and subtracts the correct way round.</p> <p><b>A1</b> Final correct <b>exact</b> answer in the form <math>a + b \ln c</math>. i.e. either <math>\frac{75}{4}\pi + 50\pi \ln\left(\frac{3}{4}\right)</math> or <math>50\pi \ln\left(\frac{3}{4}\right) + \frac{75}{4}\pi</math>                      or <math>50\pi \ln\left(\frac{9}{12}\right) + \frac{75}{4}\pi</math> or <math>\frac{75}{4}\pi - 50\pi \ln\left(\frac{4}{3}\right)</math> or <math>\frac{75}{4}\pi + 25\pi \ln\left(\frac{9}{16}\right)</math> etc.                      Also allow <math>\pi \left( \frac{75}{4} + 50 \ln\left(\frac{3}{4}\right) \right)</math> or equivalent.</p> <p><b>Note</b> A candidate who achieves full marks in (a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of B1M1A1A0M1A0 in part (b).</p> <p><b>Note</b> The <math>\pi</math> in the volume formula is only required for the B1 mark and the final A1 mark.</p>

<p>4. (b)</p>	<p><b>Alternative method of integration</b></p> $V = \pi \int_1^4 \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2 dx$ $\int \frac{25}{x^2(2x+1)} dx ; u = \frac{1}{x} \Rightarrow \frac{du}{dx} = -\frac{1}{x^2}$ $= \int \frac{-25}{\left(\frac{2}{u}+1\right)} du = \int \frac{-25}{\left(\frac{2+u}{u}\right)} du = \int \frac{-25u}{(2+u)} du = -25 \int \frac{2+u-2}{(2+u)} du$ $= -25 \int 1 - \frac{2}{(2+u)} du = -25(u - 2\ln(2+u))$ $\left\{ \int_1^4 \frac{25}{x^2(2x+1)} dx = [-25u + 50\ln(2+u)]_1^4 \right\}$ $= \left( -\frac{25}{4} + 50\ln\left(\frac{9}{4}\right) \right) - (-25 + 50\ln 3)$ $= 50\ln\left(\frac{9}{4}\right) - 50\ln 3 - \frac{25}{4} + 25$ $= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$ <p>So, <math>V = \frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)</math></p>	<table border="1"> <tr> <td data-bbox="810 235 901 369"><b>B1</b></td> <td data-bbox="901 235 1549 369">For <math>\pi \int \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2</math> Ignore limits and dx. Can be implied.</td> </tr> <tr> <td data-bbox="810 555 901 683"><b>M1</b></td> <td data-bbox="901 555 1549 683">Achieves <math>\pm \alpha \pm \frac{\beta}{(k+u)}</math> and integrates to give either <math>\pm \alpha u</math> or <math>\pm \beta \ln(k+u)</math></td> </tr> <tr> <td data-bbox="810 683 901 757"><b>A1</b></td> <td data-bbox="901 683 1549 757"><b>Dependent on the M mark.</b> Either <math>-25u</math> or <math>50\ln(2+u)</math></td> </tr> <tr> <td data-bbox="810 757 901 801"><b>A1</b></td> <td data-bbox="901 757 1549 801"><math>-25(u - 2\ln(2+u))</math></td> </tr> <tr> <td data-bbox="810 929 901 974"><b>dM1</b></td> <td data-bbox="901 929 1549 974">Applies limits of <math>\frac{1}{4}</math> and 1 in <math>u</math> or 4 and 1 in <math>x</math> in their integrated function and subtracts the correct way round.</td> </tr> <tr> <td data-bbox="810 1198 901 1243"><b>A1</b></td> <td data-bbox="901 1198 1549 1243"><math>\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)</math> or allow <math>\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)</math></td> </tr> </table>	<b>B1</b>	For $\pi \int \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2$ Ignore limits and dx. Can be implied.	<b>M1</b>	Achieves $\pm \alpha \pm \frac{\beta}{(k+u)}$ and integrates to give either $\pm \alpha u$ or $\pm \beta \ln(k+u)$	<b>A1</b>	<b>Dependent on the M mark.</b> Either $-25u$ or $50\ln(2+u)$	<b>A1</b>	$-25(u - 2\ln(2+u))$	<b>dM1</b>	Applies limits of $\frac{1}{4}$ and 1 in $u$ or 4 and 1 in $x$ in their integrated function and subtracts the correct way round.	<b>A1</b>	$\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$
<b>B1</b>	For $\pi \int \left( \frac{5}{x\sqrt{(2x+1)}} \right)^2$ Ignore limits and dx. Can be implied.													
<b>M1</b>	Achieves $\pm \alpha \pm \frac{\beta}{(k+u)}$ and integrates to give either $\pm \alpha u$ or $\pm \beta \ln(k+u)$													
<b>A1</b>	<b>Dependent on the M mark.</b> Either $-25u$ or $50\ln(2+u)$													
<b>A1</b>	$-25(u - 2\ln(2+u))$													
<b>dM1</b>	Applies limits of $\frac{1}{4}$ and 1 in $u$ or 4 and 1 in $x$ in their integrated function and subtracts the correct way round.													
<b>A1</b>	$\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$													





Question Number	Scheme	Marks
5. (a)	From question, $V = \frac{4}{3}\pi r^3$ , $S = 4\pi r^2$ , $\frac{dV}{dt} = 3$ $\left\{ V = \frac{4}{3}\pi r^3 \Rightarrow \right\} \frac{dV}{dr} = 4\pi r^2$ <span style="float: right;"><math>\frac{dV}{dr} = 4\pi r^2</math> (Can be implied)</span>	B1 oe
	$\left\{ \frac{dV}{dr} \times \frac{dr}{dt} = \frac{dV}{dt} \Rightarrow \right\} (4\pi r^2) \frac{dr}{dt} = 3$ <span style="float: right;"><math>\left( \text{Candidate's } \frac{dV}{dr} \right) \times \frac{dr}{dt} = 3</math></span> $\left\{ \frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} \Rightarrow \right\} \frac{dr}{dt} = (3) \frac{1}{4\pi r^2}; \left\{ = \frac{3}{4\pi r^2} \right\}$ <span style="float: right;">or <math>3 \div \text{Candidate's } \frac{dV}{dr}</math>;</span>	M1 oe
	When $r = 4\text{cm}$ , $\frac{dr}{dt} = \frac{3}{4\pi(4)^2} \left\{ = \frac{3}{64\pi} \right\}$ <span style="float: right;"><b>dependent on previous M1. see notes</b></span>	dM1
	Hence, $\frac{dr}{dt} = 0.01492077591\dots(\text{cm}^2 \text{ s}^{-1})$ <span style="float: right;">anything that rounds to 0.0149</span>	A1 <b>[4]</b>
(b)	$\left\{ \frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = \right\} \Rightarrow \frac{dS}{dt} = 8\pi r \times \frac{3}{4\pi r^2} \left\{ \text{or } \frac{6}{r} \text{ or } 8\pi r \times 0.0149\dots \right\}$ <span style="float: right;"><math>8\pi r \times \text{Candidate's } \frac{dr}{dt}</math></span>	M1; oe
	When $r = 4\text{cm}$ , $\frac{dr}{dt} = 8\pi(4) \times \frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4) \times 0.0149\dots$ Hence, $\frac{dS}{dt} = 1.5 (\text{cm}^2 \text{ s}^{-1})$ <span style="float: right;">anything that rounds to 1.5</span>	A1 <b>cs</b> <b>[2]</b> <b>6</b>

**Question 5 Notes**

(a)	<b>B1</b> $\frac{dV}{dr} = 4\pi r^2$ Can be implied by later working.
	<b>M1</b> $\left( \text{Candidate's } \frac{dV}{dr} \right) \times \frac{dr}{dt} = 3$ or $3 \div \text{Candidate's } \frac{dV}{dr}$
	<b>dM1</b> <b>(dependent on the previous method mark)</b> Substitutes $r = 4$ into an expression which is a result of a quotient of “3” and their $\frac{dV}{dr}$ .
	<b>A1</b> anything that rounds to 0.0149 (units are not required)
(b)	<b>M1</b> $8\pi r \times \text{Candidate's } \frac{dr}{dt}$
	<b>A1</b> anything that rounds to 1.5 (units are not required). <b>Correct solution only.</b>
	<b>Note</b> Using $\frac{dr}{dt} = 0.0149$ gives $\frac{dS}{dt} = 1.4979\dots$ which is fine for A1.



Question Number	Scheme	Marks
<p>6.</p> <p>(a)</p> <p>(b)</p>	<p><math>l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} \quad \overline{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \overline{OB} = \begin{pmatrix} 4 \\ p \\ 3 \end{pmatrix}</math></p> <p><math>\{B \text{ lies on } l_2 \Rightarrow \mu = -1 \Rightarrow\} \quad p = 5</math></p> <p><math>\{l_1 = l_2 \Rightarrow\} \begin{cases} \mathbf{i}: &amp; 1 = 7 + 3\mu \\ \mathbf{j}: &amp; 2 + 2\lambda = -5\mu \\ \mathbf{k}: &amp; 3 - \lambda = 7 + 4\mu \end{cases}</math></p> <p>e.g. <math>\mathbf{i}: 7 + 3\mu = 1</math></p> <p>So, <math>\mu = -2</math></p> <p>Point of intersection is <math>\overline{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></p> <p>Finds <math>\lambda = 4</math> and either</p> <ul style="list-style-type: none"> <li>checks <math>\lambda = 4</math> and <math>\mu = -2</math> is true for the third component.</li> <li>substitutes <math>\mu = -2</math> into <math>l_1</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></li> </ul> <p><b>and</b> substitutes <math>\lambda = 4</math> into <math>l_2</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></p>	<p><math>A</math> lies on <math>l_1</math> and <math>B</math> lies on <math>l_2</math></p> <p><math>p = 5</math></p> <p>B1</p> <p>[1]</p> <p>Writes down an equation involving only one parameter. <math>\mu = -2</math></p> <p>M1 A1 B1</p> <p>B1</p> <p>[4]</p>
<p>(b)</p>	<p><b>Alternative Method:</b> Solving <math>\mathbf{j}</math> and <math>\mathbf{k}</math> simultaneously gives</p> <p><math>8 = 14 + 3\mu</math> or <math>23 + 3\lambda = 35</math></p> <p>So, <math>\mu = -2</math> or <math>\lambda = 4</math></p> <p>Point of intersection is <math>\overline{OC} = \mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></p> <p>Finds <math>\lambda = 4</math> and either</p> <ul style="list-style-type: none"> <li>checks <math>\mu = -2</math> is true for the <math>\mathbf{i}</math> component.</li> <li>substitutes <math>\mu = -2</math> into <math>l_1</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></li> </ul> <p><b>and</b> substitutes <math>\lambda = 4</math> into <math>l_2</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></p>	<p>Writes down an equation involving only one parameter. Either <math>\mu = -2</math> or <math>\lambda = 4</math></p> <p><math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></p> <p>M1 A1 B1</p> <p>B1</p> <p>[4]</p>
<p>(c)</p> <p>(d)</p>	<p><math>\overline{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}</math> and <math>\overline{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}</math></p> <p><math>\pm \left( \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} \right)</math></p> <p><math>\cos ACB = \frac{\overline{AC} \cdot \overline{BC}}{ \overline{AC}  \cdot  \overline{BC} } = \frac{\pm \left( \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} \right)}{\sqrt{(0)^2 + (8)^2 + (-4)^2} \cdot \sqrt{(-3)^2 + (5)^2 + (-4)^2}}</math></p> <p><math>\left\{ \cos ACB = \frac{0 + 40 + 16}{\sqrt{80} \cdot \sqrt{50}} = \frac{56}{\sqrt{4000}} \Rightarrow \right\} ACB = 27.69446\dots = 27.7</math> (3 sf)</p> <p>Area <math>ACB = \frac{1}{2}(\sqrt{80})(\sqrt{50})\sin 27.69446\dots^\circ = 14.696888\dots</math></p>	<p>An attempt to find both the vectors (<math>\overline{AC}</math> or <math>\overline{CA}</math>) <b>and</b> (<math>\overline{BC}</math> or <math>\overline{CB}</math>).</p> <p>Applies dot product formula between their (<math>\overline{AC}</math> or <math>\overline{CA}</math>) <b>and</b> their (<math>\overline{BC}</math> or <math>\overline{CB}</math>).</p> <p>Anything that rounds to 27.7</p> <p>See notes</p> <p>Anything that rounds to 14.7</p> <p>M1 M1 A1 A1</p> <p>[3]</p> <p>[2]</p> <p>10</p>

**Question 6: Alternative Methods for Part (c)**

<p>6. (c)</p>	<p><b>Alternative Method 1: Using the direction vectors of Line 1 and Line 2</b></p> $\mathbf{d}_1 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$ $\cos \theta = \frac{\mathbf{d}_1 \cdot \mathbf{d}_2}{ \mathbf{d}_1   \mathbf{d}_2 } = \frac{\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}}{\sqrt{(0)^2 + (2)^2 + (-1)^2} \cdot \sqrt{(3)^2 + (-5)^2 + (4)^2}}$ $\left\{ \cos \theta = \frac{0 - 10 - 4}{\sqrt{5} \cdot \sqrt{50}} = \frac{-7\sqrt{10}}{25} \Rightarrow \right\} \theta = 152.3054385\dots$ <p>Angle <math>ACB = 180 - 152.3054385\dots = 27.69446145\dots = 27.7</math> (3 sf)</p>	<p>Applies dot product formula between their <math>\mathbf{d}_1</math> and <math>\mathbf{d}_2</math></p> <p>M2</p> <p>Anything that rounds to 27.7</p> <p>A1</p> <p>[3]</p>
	<p><b>Alternative Method 2: The Cosine Rule</b></p> $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$ <p>Also <math>\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}</math></p> <p>Note <math> \overrightarrow{AC}  = \sqrt{80}</math>, <math> \overrightarrow{BC}  = \sqrt{50}</math> and <math> \overrightarrow{AB}  = \sqrt{18}</math></p> $(\sqrt{18})^2 = (\sqrt{80})^2 + (\sqrt{50})^2 - 2(\sqrt{80})(\sqrt{50})\cos \theta$ $\left\{ \cos \theta = \frac{7\sqrt{10}}{25} \right\} \Rightarrow \theta = 27.69446145\dots = 27.7$ (3 sf)	<p>An attempt to find both the vectors (<math>\overrightarrow{AC}</math> or <math>\overrightarrow{CA}</math>) and (<math>\overrightarrow{BC}</math> or <math>\overrightarrow{CB}</math>).</p> <p>M1</p> <p>Applies the cosine rule the correct way round.</p> <p>Anything that rounds to 27.7</p> <p>M1 oe</p> <p>A1</p> <p>[3]</p>
	<p><b>Alternative Method 3: Vector Cross Product</b></p> <p><b>Only apply this scheme if it is clear that a candidate is applying a vector cross product method.</b></p> $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{BC} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix}$ $\overrightarrow{AC} \times \overrightarrow{BC} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -4 \\ -3 & 5 & -4 \end{vmatrix} = 24\mathbf{i} + 12\mathbf{j} + 24\mathbf{k} \right\}$ $\sin ACB = \frac{\sqrt{(24)^2 + (12)^2 + (12)^2}}{\sqrt{(0)^2 + (8)^2 + (-4)^2} \cdot \sqrt{(-3)^2 + (5)^2 + (-4)^2}}$ $\left\{ \sin ACB = \frac{\sqrt{864}}{\sqrt{80} \cdot \sqrt{50}} = \frac{3\sqrt{15}}{25} \Rightarrow \right\} \theta = 27.69446145\dots = 27.7$ (3 sf)	<p>An attempt to find both the vectors (<math>\overrightarrow{AC}</math> or <math>\overrightarrow{CA}</math>) and (<math>\overrightarrow{BC}</math> or <math>\overrightarrow{CB}</math>).</p> <p>M1</p> <p>Full method for applying the vector cross product formula between their (<math>\overrightarrow{AC}</math> or <math>\overrightarrow{CA}</math>) and their (<math>\overrightarrow{BC}</math> or <math>\overrightarrow{CB}</math>).</p> <p>M1</p> <p>Anything that rounds to 27.7</p> <p>A1</p> <p>[3]</p>

Question 6 Notes		
6. (a)	<b>B1</b>	$p = 5$ (Ignore working.)
(b)		<b>Method 1</b>
	<b>M1</b>	Writes down an equation involving only one parameter. This equation will usually be $7 + 3\mu = 1$ which is found from equating the <b>i</b> components of $l_1$ and $l_2$ .
	<b>A1</b>	Finds $\mu = -2$
	<b>B1</b>	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ . Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$ .
	<b>B1</b>	Finds $\lambda = 4$ and either <ul style="list-style-type: none"> <li>• checks <math>\lambda = 4</math> and <math>\mu = -2</math> is true for the third component.</li> <li>• substitutes <math>\mu = -2</math> into <math>l_1</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math> <b>and</b> substitutes <math>\lambda = 4</math> into <math>l_2</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></li> </ul>
(b)		<b>Alternative Method</b>
	<b>M1</b>	Writes down an equation involving only one parameter. Solving the <b>j</b> and <b>k</b> components simultaneously will usually give either $8 = 14 + 3\mu$ or $23 + 3\lambda = 35$
	<b>A1</b>	Finds either $\mu = -2$ or $\lambda = 4$
	<b>B1</b>	Point of intersection of $\mathbf{i} + 10\mathbf{j} - \mathbf{k}$ . Allow $(1, 10, -1)$ or $\begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$ .
	<b>B1</b>	Finds $\lambda = 4$ and either <ul style="list-style-type: none"> <li>• checks <math>\mu = -2</math> is true for the <b>i</b> component.</li> <li>• substitutes <math>\mu = -2</math> into <math>l_1</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math> <b>and</b> substitutes <math>\lambda = 4</math> into <math>l_2</math> to give <math>\mathbf{i} + 10\mathbf{j} - \mathbf{k}</math></li> </ul>
(c)	<b>M1</b>	An attempt to find both the vectors $(\overline{AC}$ or $\overline{CA})$ <b>and</b> $(\overline{BC}$ or $\overline{CB})$ by subtracting.
	<b>M1</b>	Applies dot product <b>formula</b> between their $(\overline{AC}$ or $\overline{CA})$ <b>and</b> their $(\overline{BC}$ or $\overline{CB})$ .
	<b>A1</b>	anything that rounds to 27.7
	<b>Note</b>	An answer of 0.48336... in radians without the correct answer in degrees is A0.
	<b>Note</b>	Some candidates will apply the dot product formula between vectors which are the wrong way round and achieve 152.3054385...°. If they give the acute equivalent of awrt 27.7 then award A1.
(d)	<b>M1</b>	$\frac{1}{2}(\text{their length } AC)(\text{their length } BC)\sin(\text{their } 27.7^\circ \text{ from part (c)})$
	<b>A1</b>	anything that rounds to 14.7. Also allow $6\sqrt{6}$ .
	<b>Note</b>	Area $ACB = \frac{1}{2}(\sqrt{80})(\sqrt{50})\sin(152.3054385...^\circ) = \text{awrt } 14.7$ is M1A1.



Question Number	Scheme	Marks												
7. (a)	$\frac{dN}{dt} = \frac{(kt - 1)(5000 - N)}{t}, \quad t > 0, \quad 0 < N < 5000$ $\int \frac{1}{5000 - N} dN = \int \frac{(kt - 1)}{t} dt \quad \left\{ \text{or} = \int \left( k - \frac{1}{t} \right) dt \right\}$ $-\ln(5000 - N) = kt - \ln t; + c$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%;"><i>then eg either...</i></td> <td style="width: 33%;"><i>or...</i></td> <td style="width: 33%;"><i>or...</i></td> </tr> <tr> <td><math>-kt + c = \ln(5000 - N) - \ln t</math></td> <td><math>kt + c = \ln t - \ln(5000 - N)</math></td> <td><math>\ln(5000 - N) = -kt + \ln t + c</math></td> </tr> <tr> <td><math>-kt + c = \ln\left(\frac{5000 - N}{t}\right)</math></td> <td><math>kt + c = \ln\left(\frac{t}{5000 - N}\right)</math></td> <td><math>5000 - N = e^{-kt + \ln t + c}</math></td> </tr> <tr> <td><math>e^{-kt + c} = \frac{5000 - N}{t}</math></td> <td><math>e^{kt + c} = \frac{t}{5000 - N}</math></td> <td><math>5000 - N = te^{-kt + c}</math></td> </tr> </table>	<i>then eg either...</i>	<i>or...</i>	<i>or...</i>	$-kt + c = \ln(5000 - N) - \ln t$	$kt + c = \ln t - \ln(5000 - N)$	$\ln(5000 - N) = -kt + \ln t + c$	$-kt + c = \ln\left(\frac{5000 - N}{t}\right)$	$kt + c = \ln\left(\frac{t}{5000 - N}\right)$	$5000 - N = e^{-kt + \ln t + c}$	$e^{-kt + c} = \frac{5000 - N}{t}$	$e^{kt + c} = \frac{t}{5000 - N}$	$5000 - N = te^{-kt + c}$	<p>See notes B1</p> <p>See notes M1 A1; A1</p>
<i>then eg either...</i>	<i>or...</i>	<i>or...</i>												
$-kt + c = \ln(5000 - N) - \ln t$	$kt + c = \ln t - \ln(5000 - N)$	$\ln(5000 - N) = -kt + \ln t + c$												
$-kt + c = \ln\left(\frac{5000 - N}{t}\right)$	$kt + c = \ln\left(\frac{t}{5000 - N}\right)$	$5000 - N = e^{-kt + \ln t + c}$												
$e^{-kt + c} = \frac{5000 - N}{t}$	$e^{kt + c} = \frac{t}{5000 - N}$	$5000 - N = te^{-kt + c}$												
leading to $N = 5000 - Ate^{-kt}$ with <b>no incorrect working/statements. See notes</b>		A1 * <b>cs0</b>												
(b)	$\{t = 1, N = 1200 \Rightarrow\} \quad 1200 = 5000 - Ae^{-k}$ $\{t = 2, N = 1800 \Rightarrow\} \quad 1800 = 5000 - 2Ae^{-2k}$ <p>So <math>Ae^{-k} = 3800</math></p> <p>and <math>2Ae^{-2k} = 3200</math> or <math>Ae^{-2k} = 1600</math></p> <p>Eg. <math>\frac{e^{-k}}{2e^{-2k}} = \frac{3800}{3200}</math> or <math>\frac{2e^{-2k}}{e^{-k}} = \frac{3200}{3800}</math></p> <p>So <math>\frac{1}{2}e^k = \frac{3800}{3200}</math> or <math>2e^{-k} = \frac{3200}{3800}</math></p> $k = \ln\left(\frac{7600}{3200}\right) \text{ or equivalent } \left\{ \text{eg } k = \ln\left(\frac{19}{8}\right) \right\}$ $\left\{ A = 3800(e^k) = 3800\left(\frac{19}{8}\right) \Rightarrow \right\} A = 9025$	<p>At least one correct statement written down using the boundary conditions B1</p> <p>An attempt to eliminate A by producing an equation in only k. M1</p> <p>At least one of <math>A = 9025</math> cao or <math>k = \ln\left(\frac{7600}{3200}\right)</math> or exact equivalent A1</p> <p>Both <math>A = 9025</math> cao or <math>k = \ln\left(\frac{7600}{3200}\right)</math> or exact equivalent A1</p> <p style="text-align: right;"><b>[4]</b></p>												
<b><u>Alternative Method for the M1 mark in (b)</u></b>														
$e^{-k} = \frac{3800}{A}$ $2A\left(\frac{3800}{A}\right)^2 = 3200$		<p>An attempt to eliminate k by producing an equation in only A M1</p>												
(c)	$\left\{ t = 5, N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)} \right\}$ <p><math>N = 4402.828401... = 4400</math> (fish) (nearest 100)</p>	<p>anything that rounds to 4400 B1</p> <p style="text-align: right;"><b>[1]</b></p>												
		<b>10</b>												

Question 7 Notes	
7. (a)	<p><b>B1</b> Separates variables as shown. <math>dN</math> and <math>dt</math> should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.</p> <p><b>M1</b> <b>Either</b> <math>\pm \lambda \ln(5000 - N)</math> or <math>\pm \lambda \ln(N - 5000)</math> <b>or</b> <math>kt - \ln t</math> where <math>\lambda \neq 0</math> is a constant.</p> <p><b>A1</b> For <math>-\ln(5000 - N) = kt - \ln t</math> or <math>\ln(5000 - N) = -kt + \ln t</math> or <math>-\frac{1}{k} \ln(5000 - N) = t - \frac{1}{k} \ln t</math> or</p> <p><b>A1</b> <b>which is dependent on the 1<sup>st</sup> M1 mark being awarded.</b></p> <p>For applying a constant of integration, eg. <math>+c</math> or <math>+\ln e^c</math> or <math>+\ln c</math> or <math>A</math> to their integrated equation <math>+c</math> can be on either side of their equation for the 2<sup>nd</sup> A1 mark.</p> <p><b>Note</b></p> <p><b>A1</b> Uses a constant of integration eg. "<math>c</math>" or "<math>\ln e^c</math>" "<math>\ln c</math>" or and applies a fully correct method to prove the result <math>N = 5000 - Ate^{-kt}</math> with no incorrect working seen. (Correct solution only.)</p> <p><b>NOTE IMPORTANT</b></p> <p>There needs to be an intermediate stage of justifying the <math>A</math> <b>and</b> the <math>e^{-kt}</math> in <math>Ate^{-kt}</math> by for example</p> <ul style="list-style-type: none"> <li>• <b>either</b> <math>5000 - N = e^{\ln t - kt + c}</math></li> <li>• <b>or</b> <math>5000 - N = t e^{-kt + c}</math></li> <li>• <b>or</b> <math>5000 - N = t e^{-kt} e^c</math></li> </ul> <p>or equivalent needs to be stated before achieving <math>N = 5000 - Ate^{-kt}</math></p>
(b)	<p><b>B1</b> At least one of <b>either</b> <math>1200 = 5000 - Ae^{-k}</math> (or equivalent) <b>or</b> <math>1800 = 5000 - 2Ae^{-2k}</math> (or equivalent)</p> <p><b>M1</b></p> <ul style="list-style-type: none"> <li>• <b>Either</b> an attempt to eliminate <math>A</math> by producing an equation in only <math>k</math>.</li> <li>• <b>or</b> an attempt to eliminate <math>k</math> by producing an equation in only <math>A</math></li> </ul> <p><b>A1</b> At least one of <math>A = 9025</math> or <math>k = \ln\left(\frac{7600}{3200}\right)</math> or equivalent</p> <p><b>A1</b> Both <math>A = 9025</math> or <math>k = \ln\left(\frac{7600}{3200}\right)</math> or equivalent</p> <p><b>Note</b> Alternative correct values for <math>k</math> are <math>k = \ln\left(\frac{19}{8}\right)</math> or <math>k = -\ln\left(\frac{8}{19}\right)</math> or <math>k = \ln 7600 - \ln 3200</math></p> <p>or <math>k = -\ln\left(\frac{3800}{9025}\right)</math> or equivalent.</p> <p><b>Note</b> <math>k = 0.8649...</math> without a correct exact equivalent is A0.</p>
(c)	<p><b>B1</b> anything that rounds to 4400</p>



8.

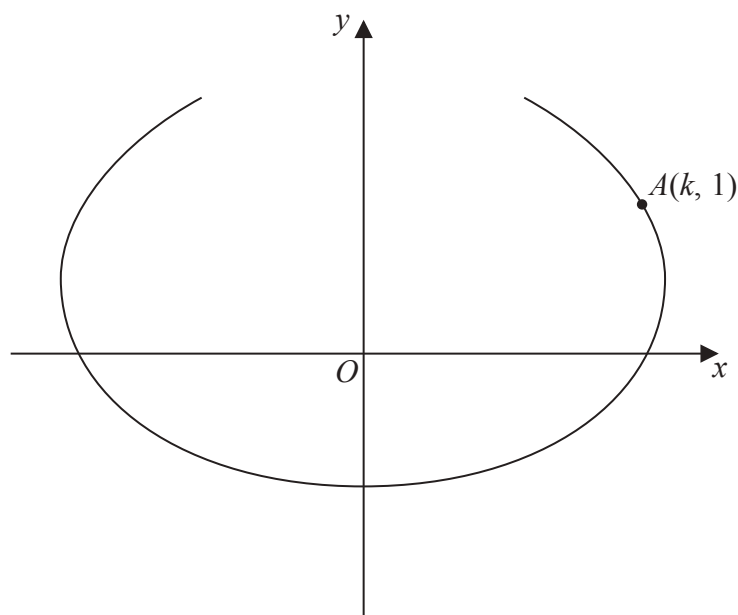


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point  $A$ , with coordinates  $(k, 1)$ , lies on the curve.

Given that  $k > 0$

(a) find the exact value of  $k$ , (2)

(b) find the gradient of the curve at the point  $A$ . (4)

There is one point on the curve where the gradient is equal to  $-\frac{1}{2}$

(c) Find the value of  $t$  at this point, showing each step in your working and giving your answer to 4 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.] (6)

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Question Number	Scheme	Marks
<p><b>8.</b></p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p><math>x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}</math> <math>A(k, 1)</math> lies on the curve, <math>k &gt; 0</math></p> <p>{When <math>y = 1</math>,} <math>1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}</math>      Sets <math>y = 1</math> to find <math>t</math> and uses their <math>t</math> to find <math>x</math>.</p> <p><math>k</math> (or <math>x</math>) <math>= \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)</math> <b>or</b> <math>x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)</math></p> <p>{When <math>t = -\frac{\pi}{2}, k &gt; 0</math>,} so <math>k = 4 - \frac{\pi}{2}</math> or <math>\frac{8 - \pi}{2}</math>      <math>x</math> or <math>k = 4 - \frac{\pi}{2}</math></p> <hr/> <p><math>\frac{dx}{dt} = 1 - 4\cos t, \quad \frac{dy}{dt} = 2\sin t</math>      At least one of <math>\frac{dx}{dt}</math> or <math>\frac{dy}{dt}</math> correct.</p> <p>Both <math>\frac{dx}{dt}</math> and <math>\frac{dy}{dt}</math> are correct.</p> <hr/> <p>So, <math>\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}</math>      Applies their <math>\frac{dy}{dt}</math> divided by their <math>\frac{dx}{dt}</math> and substitutes their <math>t</math> into their <math>\frac{dy}{dx}</math>.</p> <p>At <math>t = -\frac{\pi}{2}, \frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{2}\right)}; = -2</math>      Correct value for <math>\frac{dy}{dx}</math> of <math>-2</math></p> <hr/> <p><math>\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}</math>      Sets their <math>\frac{dy}{dx} = -\frac{1}{2}</math></p> <p>gives <math>4\sin t - 4\cos t = -1</math>      <b>See notes</b></p> <p>So <math>4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right); = -1</math> or <math>-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right); = -1</math>      <b>See notes</b></p> <p><math>t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}</math> or <math>t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}</math>      <b>See notes</b></p> <p><math>t = 0.6076875626... = 0.6077</math> (4 dp)      anything that rounds to 0.6077</p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>B1</p> <p>B1</p> <p>M1;</p> <p>A1</p> <p>cao cso</p> <p>[4]</p> <p>M1</p> <p>A1</p> <p>M1; A1</p> <p>dM1</p> <p>A1</p> <p>[6]</p> <p><b>12</b></p>
<b>Question 8 Notes</b>		
(c)	<p><b>NOTE</b></p> <p><b>NOTE</b></p>	<p><b>VERY IMPORTANT NOTE FOR PART (c)</b></p> <p>Candidates who state <math>t = 0.6077</math> with no intermediate working from <math>4\sin t - 4\cos t = -1</math> will get 2<sup>nd</sup> M0, 2<sup>nd</sup> A0, 3<sup>rd</sup> M0, 3<sup>rd</sup> A0.</p> <p>They will not express <math>4\sin t - 4\cos t</math> as either <math>4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)</math> or <math>-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)</math>.</p> <p>OR use any acceptable alternative method to achieve <math>t = 0.6077</math></p> <p>Alternative methods for part (c) are given on the next page.</p>

<b>Question 8: Alternative Methods for Part (c)</b>	
<b>8. (c)</b>	<p><b>Alternative Method 1:</b></p> $\frac{2 \sin t}{1 - 4 \cos t} = -\frac{1}{2}$ <p style="text-align: right;">Sets their <math>\frac{dy}{dx} = -\frac{1}{2}</math> M1</p> <p>eg. <math>\left(\frac{2 \sin t}{1 - 4 \cos t}\right)^2 = \frac{1}{4}</math> or <math>(4 \sin t)^2 = (4 \cos t - 1)^2</math> Squaring to give a correct equation. A1  This mark can be implied by a "squared" correct equation.</p> <p style="text-align: center;"><b>Note:</b> You can also give 1<sup>st</sup> A1 in this method for <math>4 \sin t - 4 \cos t = -1</math> as in the main scheme.</p> <p style="text-align: center;">Squares their equation, applies <math>\sin^2 t + \cos^2 t = 1</math> and achieves a <b>three term quadratic equation</b> of the form <math>\pm a \cos^2 t \pm b \cos t \pm c = 0</math> M1  or <math>\pm a \sin^2 t \pm b \sin t \pm c = 0</math> or eg. <math>\pm a \cos^2 t \pm b \cos t = \pm c</math> where <math>a \neq 0, b \neq 0</math> and <math>c \neq 0</math>.</p> <ul style="list-style-type: none"> <li>• <b>Either</b> <math>32 \cos^2 t - 8 \cos t - 15 = 0</math></li> <li>• <b>or</b> <math>32 \sin^2 t + 8 \sin t - 15 = 0</math> For a correct <b>three term quadratic equation.</b> A1</li> <li>• <b>Either</b> <math>\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1 + \sqrt{31}}{8} \Rightarrow t = \cos^{-1}(\dots)</math> <b>which is dependent on the 2<sup>nd</sup> M1 mark.</b> dM1  Uses correct algebraic processes to give <math>t = \dots</math></li> <li>• <b>or</b> <math>\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(\dots)</math> anything that rounds to 0.6077 A1  <math>t = 0.6076875626\dots = 0.6077</math> (4 dp)</li> </ul> <p style="text-align: right;"><b>[6]</b></p>
<b>8. (c)</b>	<p><b>Alternative Method 2:</b></p> $\frac{2 \sin t}{1 - 4 \cos t} = -\frac{1}{2}$ <p style="text-align: right;">Sets their <math>\frac{dy}{dx} = -\frac{1}{2}</math> M1</p> <p>eg. <math>(4 \sin t - 4 \cos t)^2 = (-1)^2</math> Squaring to give a correct equation. A1  This mark can be implied by a correct equation.</p> <p style="text-align: center;"><b>Note:</b> You can also give 1<sup>st</sup> A1 in this method for <math>4 \sin t - 4 \cos t = -1</math> as in the main scheme.</p> <p>So <math>16 \sin^2 t - 32 \sin t \cos t + 16 \cos^2 t = 1</math></p> <p style="text-align: center;">Squares their equation, applies both <math>\sin^2 t + \cos^2 t = 1</math> and <math>\sin 2t = 2 \sin t \cos t</math> and then achieves an equation of the form <math>\pm a \pm b \sin 2t = \pm c</math> M1</p> <p>leading to <math>16 - 16 \sin 2t = 1</math></p> <p style="text-align: center;"><math>16 - 16 \sin 2t = 1</math> or equivalent. A1</p> <p><math>\left\{ \sin 2t = \frac{15}{16} \Rightarrow \right\} t = \frac{\sin^{-1}(\dots)}{2}</math> <b>which is dependent on the 2<sup>nd</sup> M1 mark.</b> dM1  Uses correct algebraic processes to give <math>t = \dots</math></p> <p><math>t = 0.6076875626\dots = 0.6077</math> (4 dp) anything that rounds to 0.6077 A1</p> <p style="text-align: right;"><b>[6]</b></p>

Question 8 Notes		
8. (a)	<b>M1</b>	Sets $y = 1$ to find $t$ and uses their $t$ to find $x$ .
	<b>Note</b>	M1 can be implied by either $x$ or $k = 4 - \frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2} - 4$ or $-2.429...$
	<b>A1</b>	$x$ or $k = 4 - \frac{\pi}{2}$ or $\frac{8 - \pi}{2}$
	<b>Note</b>	A decimal answer of 2.429... (without a correct <b>exact</b> answer) is A0.
(b)	<b>Note</b>	Allow A1 for a candidate using $t = \frac{\pi}{2}$ to find $x = \frac{\pi}{2} - 4$ <b>and</b> then stating that $k$ must be $4 - \frac{\pi}{2}$ o.e.
	<b>B1</b>	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. <b>Note:</b> that this mark can be implied from their working.
	<b>B1</b>	Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. <b>Note:</b> that this mark can be implied from their working.
	<b>M1</b>	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ <b>and</b> attempts to substitute their $t$ into their expression for $\frac{dy}{dx}$ .
	<b>Note</b>	This mark may be implied by their final answer. i.e. $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$ followed by an answer of $-2$ (from $t = -\frac{\pi}{2}$ ) or $2$ (from $t = \frac{\pi}{2}$ )
	<b>Note</b>	<b>Applying</b> $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ .
(c)	<b>A1</b>	Using $t = -\frac{\pi}{2}$ (and not $t = \frac{3\pi}{2}$ ) to find a correct $\frac{dy}{dx}$ of $-2$ <b>by correct solution only</b> .
	<b>NOTE</b>	If a candidate uses an incorrect $\frac{dy}{dx}$ expression in part (c) then the accuracy marks are not obtainable.
	<b>1<sup>st</sup> M1</b>	Sets their $\frac{dy}{dx} = -\frac{1}{2}$
	<b>1<sup>st</sup> A1</b>	Rearranges to give the correct equation with $\sin t$ and $\cos t$ <b>on the same side</b> . eg. $4\sin t - 4\cos t = -1$ or $4\cos t - 4\sin t = 1$ or $\sin t - \cos t = -\frac{1}{4}$ or $\cos t - \sin t = \frac{1}{4}$ or $4\sin t - 4\cos t + 1 = 0$ or $4\cos t - 4\sin t - 1 = 0$ or $\sin t - \cos t + \frac{1}{4} = 0$ etc. are fine for A1.
	<b>2<sup>nd</sup> M1</b>	Rewrites $\pm\lambda\sin t \pm \mu\cos t$ in the form of either $R\cos(t \pm \alpha)$ or $R\sin(t \pm \alpha)$ where $R \neq 1$ or $0$ and $\alpha \neq 0$
	<b>2<sup>nd</sup> A1</b>	Correct equation. Eg. $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right) = -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right) = -1$ or $\sqrt{2}\sin\left(t - \frac{\pi}{4}\right) = -\frac{1}{4}$ or $\sqrt{2}\cos\left(t + \frac{\pi}{4}\right) = \frac{1}{4}$ , etc.
	<b>Note</b>	Unless recovered, give A0 for $4\sqrt{2}\sin(t - 45^\circ) = -1$ or $-4\sqrt{2}\cos(t + 45^\circ) = -1$ , etc.
	<b>3<sup>rd</sup> M1</b>	<b>which is dependent on the 2<sup>nd</sup> M1 mark.</b> Uses correct algebraic processes to give $t = \dots$
	<b>4<sup>th</sup> A1</b>	anything that rounds to 0.6077
	<b>Note</b>	<b>Do not give</b> the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$ .
<b>Note</b>	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$ .	