Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Core Mathematics C4

Advanced

Friday 24 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6666/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

Question Number	Scheme	Notes	Marks
1. Way 1	$\left\{\dfrac{1}{(2+5x)^3} = \right\}(2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$= \underline{(2)^{-3}}\left(1+\dfrac{5x}{2}\right)^{-3} = \underline{\dfrac{1}{8}}\left(1+\dfrac{5x}{2}\right)^{-3}$	$\underline{2^{-3}}$ or $\underline{\dfrac{1}{8}}$	<u>B1</u>
	$= \left\{\dfrac{1}{8}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$= \left\{\dfrac{1}{8}\right\}\left[1+(-3)\left(\dfrac{5x}{2}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{5x}{2}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{5x}{2}\right)^3+\ldots\right]$		
	$= \dfrac{1}{8}\left[1-\dfrac{15}{2}x+\dfrac{75}{2}x^2-\dfrac{625}{4}x^3+\ldots\right]$		
	$= \dfrac{1}{8}\left[1-7.5x+37.5x^2-156.25x^3+\ldots\right]$		
	$= \dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$ **or** $\dfrac{1}{8}-\dfrac{15}{16}x;+4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3+\ldots$		A1; A1
			[6]
			6
Way 2	$f(x)=(2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$f''(x)=300(2+5x)^{-5},\ f'''(x)=-7500(2+5x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
	$f'(x)=-15(2+5x)^{-4}$	$\pm a(2+5x)^{-4},\ a\neq\pm1$	M1
		$-15(2+5x)^{-4}$	A1 oe
	$\left\{\therefore f(0)=\dfrac{1}{8},\ f'(0)=-\dfrac{15}{16},\ f''(0)=\dfrac{75}{8}\ \text{and}\ f'''(0)=-\dfrac{1875}{16}\right\}$		
	So, $f(x)=\dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
			[6]
Way 3	$(2+5x)^{-3}$	Same as in Way 1	M1
	$= \underline{(2)^{-3}}+(-3)(2)^{-4}(5x)+\dfrac{(-3)(-4)}{2!}(2)^{-5}(5x)^2+\dfrac{(-3)(-4)(-5)}{3!}(2)^{-6}(5x)^3$	Same as in Way 1	<u>B1</u>
		Any two terms correct	M1
		All four terms correct	A1
	$= \dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
	Note: Terms can be simplified or un-simplified for B1 2nd M1 1st A1		**[6]**
	Note: The terms in C need to be evaluated So ${}^{-3}C_0(2)^{-3}+{}^{-3}C_1(2)^{-4}(5x)+{}^{-3}C_2(2)^{-5}(5x)^2+{}^{-3}C_3(2)^{-6}(5x)^3$ without further working is B0 1st M0 1st A0		

		Question 1 Notes
1.	**1st M1**	mark can be implied by a constant term of $(2)^{-3}$ or $\frac{1}{8}$.
	B1	2^{-3} or $\frac{1}{8}$ outside brackets or $\frac{1}{8}$ as candidate's constant term in their binomial expansion.
	2nd M1	Expands $(\ldots + kx)^{-3}$, k = a value $\neq 1$, to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1 + (-3)(kx)$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1 + \ldots + \frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.
	1st A1	A correct simplified or un-simplified $1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent and k = a value $\neq 1$. (on the RHS, not necessarily the LHS) in a candidate's expansion.
	Note	You would award B1M1A0 for $\frac{1}{8}\left[1 + (-3)\left(\frac{5x}{2}\right) + \frac{(-3)(-4)}{2!}(5x)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3 + \ldots\right]$ because (kx) is not consistent.
	Note	**Incorrect bracketing:** $= \left\{\frac{1}{8}\right\}\left[1 + (-3)\left(\frac{5x}{2}\right) + \frac{(-3)(-4)}{2!}\left(\frac{5x^2}{2}\right) + \frac{(-3)(-4)(-5)}{3!}\left(\frac{5x^3}{2}\right) + \ldots\right]$ is M1A0 unless recovered.
	2nd A1	For $\frac{1}{8} - \frac{15}{16}x$ **(simplified)** or also allow $0.125 - 0.9375x$.
	3rd A1	Accept only $\frac{75}{16}x^2 - \frac{625}{32}x^3$ or $4\frac{11}{16}x^2 - 19\frac{17}{32}x^3$ or $4.6875x^2 - 19.53125x^3$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either**
		SC: $\frac{1}{8}\left[1 - \frac{15}{2}x\,;\ldots\right]$ or **SC:** $\frac{1}{8}\left[1 + \ldots + \frac{75}{2}x^2 + \ldots\right]$ or **SC:** $\frac{1}{8}\left[1 + \ldots - \frac{625}{4}x^3 + \ldots\right]$
		SC: $\lambda\left[1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \ldots\right]$ or **SC:** $\left[\lambda - \frac{15\lambda}{2}x + \frac{75\lambda}{2}x^2 - \frac{625\lambda}{4}x^3 + \ldots\right]$
		(where λ can be 1 or omitted), where each term in the $[\ldots..]$ is a simplified fraction or a decimal
	SC	**Special case for the 2nd M1 mark** Award Special Case 2nd M1 for a correct simplified or un-simplified $1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2 + \frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n \neq -3$, $n \neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$ **.**
	Note	Ignore extra terms beyond the term in x^3
	Note	You can ignore subsequent working following a correct answer.

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2.

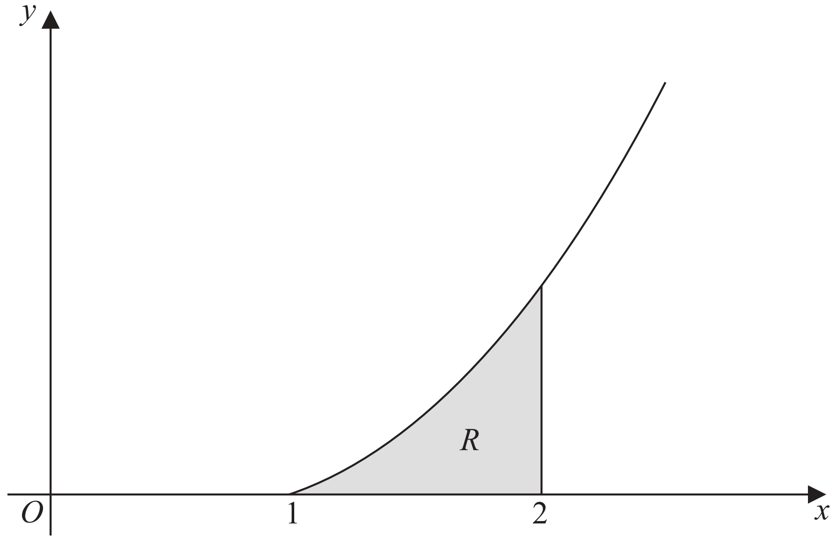

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geqslant 1$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The table below shows corresponding values of x and y for $y = x^2 \ln x$

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places.
(1)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.
(3)

(c) Use integration to find the exact value for the area of R.
(5)

Question Number	Scheme		Marks
2.	x: 1, 1.2, 1.4, 1.6, 1.8, 2; y: 0, 0.2625, ***0.659485...***, 1.2032, 1.9044, 2.7726	$y = x^2 \ln x$	
(a)	$\{\text{At } x = 1.4,\}\ y = 0.6595\ (4\text{ dp})$	0.6595	B1 **cao**
			[1]
(b)	$\frac{1}{2} \times (0.2) \times \left[0 + 2.7726 + 2\left(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044\right)\right]$	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$	B1 o.e.
	{**Note:** The "0" does not have to be included in [……]}	For structure of [...............]	M1
	$\left\{= \frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083\ (3\text{ dp})$	anything that rounds to 1.083	A1
			[3]
(c) **Way 1**	$\left\{I = \int x^2 \ln x \, dx\right\}$, $\left\{u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x};\ \frac{dv}{dx} = x^2 \Rightarrow v = \frac{1}{3}x^3\right\}$		
	$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{dx\}$	Either $x^2 \ln x \to \pm\lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{dx\}$ or $\pm\lambda x^3 \ln x - \int \mu x^2 \{dx\}$, where $\lambda, \mu > 0$	M1
		$x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{dx\}$, simplified or un-simplified	A1
	$= \frac{x^3}{3}\ln x - \frac{x^3}{9}$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
	$\text{Area}(R) = \left\{\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2\right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$	**dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round	dM1
	$= \frac{8}{3}\ln 2 - \frac{7}{9}$	$\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$	A1 oe **cso**
			[5]
(c) **Way 2**	$I = x^2(x\ln x - x) - \int 2x(x\ln x - x)dx$	$\left\{u = x^2 \Rightarrow \frac{du}{dx} = 2x;\ \frac{dv}{dx} = \ln x \Rightarrow v = x\ln x - x\right\}$	
	So, $3I = x^2(x\ln x - x) + \int 2x^2\{dx\}$		
	and $I = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$	A full method of applying $u = x^2, v' = \ln x$ to give $\pm\lambda x^2(x\ln x - x) \pm \mu\int x^2\{dx\}$	M1
		$\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$ simplified or un-simplified	A1
	$= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]
			9

		Question 2 Notes
2. (a)	**B1**	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	M1	For structure of trapezium rule $[\;\ldots\ldots\ldots\ldots\;]$
	Note	No errors are allowed [eg. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y* ordinate].
	A1	anything that rounds to 1.083
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…)
	Note	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	Note	Award B1M1A1 for $\frac{1}{10}(2.7726)+\frac{1}{5}(0.2625+\text{their } 0.6595+1.2032+1.9044)=\text{awrt } 1.083$
	Bracketing mistake: Unless the final answer implies that the calculation has been done correctly	
	Award B1M0A0 for $\frac{1}{2}(0.2)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)+2.7726$ (answer of 10.9318)	
	Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)$ (answer of 8.33646)	
	Alternative method: Adding individual trapezia	
	$\text{Area}\approx 0.2\times\left[\frac{0+0.2625}{2}+\frac{0.2625+\text{"}0.6595\text{"}}{2}+\frac{\text{"}0.6595\text{"}+1.2032}{2}+\frac{1.2032+1.9044}{2}+\frac{1.9044+2.7726}{2}\right]=1.08318...$	
	B1	0.2 and a divisor of 2 on all terms inside brackets
	M1	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2
	A1	anything that rounds to 1.083
(c)	A1	Exact answer needs to be a two term expression in the form $a\ln b+c$
	Note	Give A1 e.g. $\frac{8}{3}\ln 2-\frac{7}{9}$ or $\frac{1}{9}(24\ln 2-7)$ or $\frac{4}{3}\ln 4-\frac{7}{9}$ or $\frac{1}{3}\ln 256-\frac{7}{9}$ or $-\frac{7}{9}+\frac{8}{3}\ln 2$ or $\ln 2^{\frac{8}{3}}-\frac{7}{9}$ or equivalent.
	Note	Give final A0 for a final answer of $\frac{8\ln 2-\ln 1}{3}-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{1}{3}\ln 1-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{8}{9}+\frac{1}{9}$ or $\frac{8}{3}\ln 2-\frac{7}{9}+c$
	Note	$\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	Note	Give dM0A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\frac{1}{9}$ (adding rather than subtracting)
	Note	Allow dM1A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\left(0+\frac{1}{9}\right)$
	SC	A candidate who uses $u=\ln x$ and $\frac{dv}{dx}=x^2$, $\frac{du}{dx}=\frac{\alpha}{x}$, $v=\beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1.

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3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) **Way 1**	$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\}$ $\left(4xy + 2x^2\frac{dy}{dx}\right) + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$		M1 A1 B1
	$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{dy}{dx} =\right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)}$ $\left\{= \frac{-8}{22 + \pi}\right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\}$ $x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \times\right\}$ $\left(4xy\frac{dx}{dy} + 2x^2\right) + 2\frac{dx}{dy} + 4 + \pi\sin(\pi y) = 0$		M1 A1 B1
	$\frac{dx}{dy}(4xy + 2) + 2x^2 + 4 + \pi\sin(\pi y) = 0$		dM1
	$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]

Question 3 Notes

3. (a)	**Note**	Writing down ***from no working*** • $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi\sin(\pi y)}$ scores M1A0B1M1A0
	Note	Few candidates will write $4xy\,dx + 2x^2dy + 2dx + 4dy + \pi\sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or equivalent. This should get full marks.

		Question 3 Notes Continued
3. (a) **Way 1**	**M1**	Differentiates implicitly to include either $2x^2\frac{dy}{dx}$ or $4y \to 4\frac{dy}{dx}$ or $-\cos(\pi y) \to \pm\lambda\sin(\pi y)\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1	$2x + 4y - \cos(\pi y) = 17 \quad \to \quad 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$
	Note	$4xy + 2x^2\frac{dy}{dx} + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} \to 2x^2\frac{dy}{dx} + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = -4xy - 2$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	B1	$2x^2y \to 4xy + 2x^2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	dM1	**Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. ie. $\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + \ ... \ = \ ...$
	Note	Writing down an extra $\frac{dy}{dx} = ...$ and then including it in their factorisation is fine for dM1.
	Note	**Final A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from correct solution.
(a)	**Way 2**	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	**1st M1**	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$. E.g. $"-4xy" \to "-6"$ in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}$, $y = 3$.
	3rd M1	***is dependent on the first M1.***
	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 2nd M1 3rd M1** for $\frac{\frac{1}{2}}{3 - x} = \frac{-1}{\text{their } m_T}$
	Note	We can accept $\sin\pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2nd M1 mark. But, $\sin\pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3rd M1 mark. The 3rd M1 can be accessed for terms containing $\pi\sin\left(\frac{\pi}{2}\right)$.

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4. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

Question Number	Scheme	Notes	Marks
4.	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 2**	$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,\mathrm{d}x$	**Either** $\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ **or** $t = \int -\frac{2}{5x}\,\mathrm{d}x$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = ...$ **or** $\pm\alpha\ln px;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 3**	$\int_{60}^{x} \frac{1}{x}\,\mathrm{d}x = \int_{0}^{t} -\frac{5}{2}\,\mathrm{d}t$	Ignore limits	B1
	$\left[\ln x\right]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\left[\ln x\right]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]
(b)	$20 = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda\mathrm{e}^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda\mathrm{e}^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449...\text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. $(A \in \square, t > 0)$	dM1
	$\Rightarrow t = 632.8006... = 633$ (to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 **cso**
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
4.	$\dfrac{dx}{dt} = -\dfrac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\displaystyle\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\dfrac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\dfrac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $\dfrac{2}{5}\ln 300 = c$ $\dfrac{2}{5}\ln(5x) = -t + \dfrac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \dfrac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 5**	$\left\{\dfrac{dt}{dx} = -\dfrac{2}{5x} \Rightarrow\right\}$ $t = \displaystyle\int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\dfrac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\dfrac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\dfrac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\dfrac{2}{5}\ln x + \dfrac{2}{5}\ln 60 \Rightarrow -\dfrac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]

Question 4 Notes

4. (a)	**B1**	For the correct separation of variables. E.g. $\displaystyle\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$
	Note	B1 can be implied by seeing **either** $\ln x = -\dfrac{5}{2}t + c$ **or** $t = -\dfrac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \dfrac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \to x = 60e^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \dfrac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\dfrac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

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5.

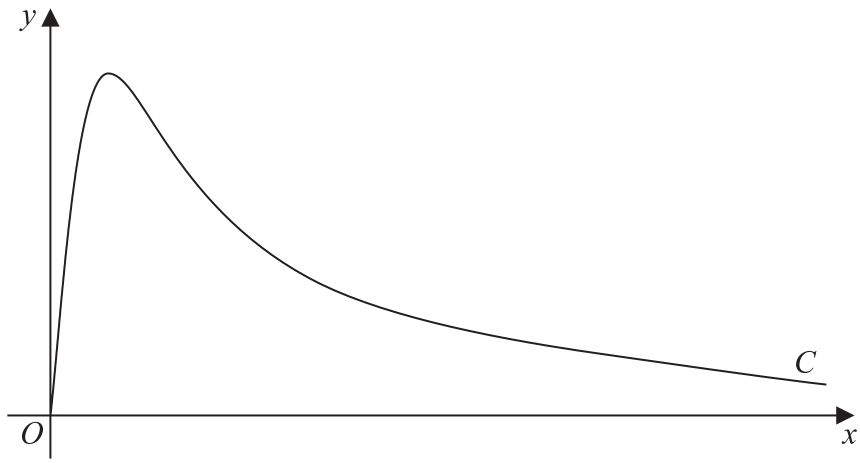

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q.

(2)

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = $ awrt 8.7	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

Question 5 Notes

5. (a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin\left(2(45)\right)$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \ \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

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6. (i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx = \lambda\int_0^{\frac{\pi}{3}} \sin^2\theta\ d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)}\,dy,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx,\ x=4\sin^2\theta$		
(i) **Way 1**	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y}+\frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{A}{y}\to\pm\lambda\ln y$ **or** $\frac{B}{(3y+2)}\to\pm\mu\ln(3y+2)$; $A\neq 0,\ B\neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y+3\ln(3y+2)\ \{+c\}$	$-2\ln y+3\ln(3y+2)$ **or** $-2\ln y+3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]
(ii) (a) **Way 1**	$\{x=4\sin^2\theta \Rightarrow\}\ \frac{dx}{d\theta}=8\sin\theta\cos\theta$ or $\frac{dx}{d\theta}=4\sin 2\theta$ or $dx=8\sin\theta\cos\theta\,d\theta$		B1
	$\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{d\theta\}$ or $\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,4\sin 2\theta\ \{d\theta\}$		M1
	$=\int \underline{\underline{\tan\theta}}\,.\,8\sin\theta\cos\theta\ \{d\theta\}$ or $\int \underline{\underline{\tan\theta}}\,.\,4\sin 2\theta\ \{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$=\int 8\sin^2\theta\,d\theta$	$\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1
	$3=4\sin^2\theta$ **or** $\frac{3}{4}=\sin^2\theta$ **or** $\sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$ $\{x=0 \to \theta=0\}$	Writes down a correct equation involving $x=3$ leading to $\theta=\frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$=\{8\}\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta \quad \left\{=\int(4-4\cos 2\theta)\,d\theta\right\}$	Applies $\cos 2\theta = 1-2\sin^2\theta$ to their integral. (See notes)	M1
	$=\{8\}\left(\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta\right) \quad \{=4\theta-2\sin 2\theta\}$	For $\pm\alpha\theta\pm\beta\sin 2\theta,\ \alpha,\beta\neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,d\theta = 8\left[\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6}-\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right)-(0+0)\right)$		
	$=\frac{4}{3}\pi-\sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi-\sqrt{3}$ or $\frac{1}{3}(4\pi-3\sqrt{3})$	A1 o.e.
			[4]
			15

		Question 6 Notes
6. (i)	**1st M1**	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied ***for writing down*** either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
6. (ii)(a)	**1st M1**	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\int\sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\,\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3, \theta = \frac{\pi}{3}; x = 0, \theta = 0$
(ii)(b)	**M1**	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\frac{1-\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A ***correct solution in part (ii)*** leading to a "two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397... (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
6. (i) **Way 2**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]
6. (i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]

	Scheme	Notes	
6. (i) **Way 4**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$= \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0,\ B \neq 0,\ C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]
	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) **Way 2**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sin\theta\,.\,8\sin\theta\ \{\mathrm{d}\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso
(ii)(a) **Way 3**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta,\ \ 4 - x = 2 + 2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}4\sin 2\theta\ \{\mathrm{d}\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int 2(2-2\cos 2\theta).\ \{\mathrm{d}\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso

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7. (a) Find

$$\int (2x-1)^{\frac{3}{2}}\,\mathrm{d}x$$

giving your answer in its simplest form.

(2)

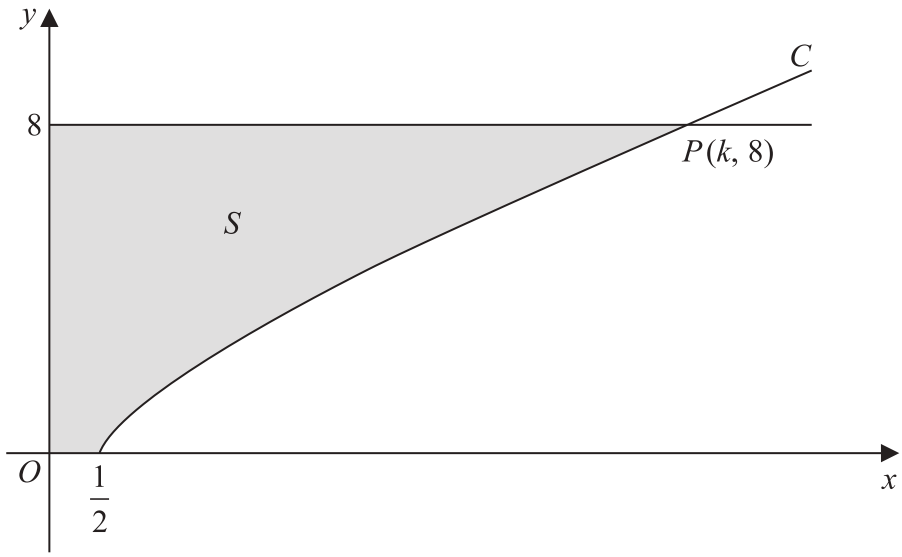

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x-1)^{\frac{3}{4}}, \qquad x \geqslant \frac{1}{2}$$

The curve C cuts the line $y = 8$ at the point P with coordinates $(k, 8)$, where k is a constant.

(b) Find the value of k.

(2)

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line $y = 8$. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the exact value of the volume of the solid generated.

(4)

Question Number	Scheme	Notes	Marks
7.	$y = (2x-1)^{\frac{3}{4}},\quad x \geqslant \frac{1}{2}$ passes though $P(k, 8)$		
(a)	$\left\{\int (2x-1)^{\frac{3}{2}}\,dx\right\} = \frac{1}{5}(2x-1)^{\frac{5}{2}}\ \{+c\}$	$(2x\pm1)^{\frac{3}{2}} \to \pm\lambda(2x\pm1)^{\frac{5}{2}}$ **or** $\pm\lambda u^{\frac{5}{2}}$ where $u = 2x\pm1$; $\lambda \neq 0$	M1
		$\frac{1}{5}(2x-1)^{\frac{5}{2}}$ with or without $+c$. Must be simplified.	A1
			[2]
(b)	$\{P(k,8) \Rightarrow\}\ 8 = (2k-1)^{\frac{3}{4}} \Rightarrow k = \dfrac{8^{\frac{4}{3}}+1}{2}$	Sets $8 = (2k-1)^{\frac{3}{4}}$ or $8 = (2x-1)^{\frac{3}{4}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k = \frac{17}{2}$	k (or x) $= \frac{17}{2}$ or 8.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2 dx$	For $\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2$ or $\pi\int(2x-1)^{\frac{3}{2}}$. Ignore limits and dx. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}} y^2\,dx\right\} = \left[\frac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}} = \left(\left(\frac{16^{\frac{5}{2}}}{5}\right) - (0)\right)\ \left\{=\frac{1024}{5}\right\}$ **Note:** It is not necessary to write the "-0"	Applies x-limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^{\frac{5}{2}}$; $\beta\neq0$ and subtracts the correct way round.	M1
	$\{V_{\text{cylinder}}\} = \pi(8)^2\left(\frac{17}{2}\right)\ \{=544\pi\}$	$\pi(8)^2$(their answer to part (b)). $V_{\text{cylinder}} = 544\pi$ implies this mark	B1 ft
	$\left\{\text{Vol}(S) = 544\pi - \frac{1024\pi}{5}\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$	An exact correct answer in the form $k\pi$ E.g. $\frac{1696}{5}\pi$, $\frac{3392}{10}\pi$ or 339.2π	A1
			[4]
Alt. (c)	$\text{Vol}(S) = \pi(8)^2\left(\frac{1}{2}\right) + \underline{\underline{\pi}}\int_{0.5}^{8.5}\left(8^2 - \underline{\underline{(2x-1)^{\frac{3}{2}}}}\right)dx$	For $\underline{\underline{\pi}}\int \ldots\ \underline{\underline{(2x-1)^{\frac{3}{2}}}}$ Ignore limits and dx.	B1
	$= \pi(8)^2\left(\frac{1}{2}\right) + \pi\left[64x - \frac{1}{5}(2x-1)^{\frac{5}{2}}\right]_{0.5}^{8.5}$		
	$= \underline{\underline{\pi(8)^2\left(\frac{1}{2}\right)}} + \underline{\underline{\pi}}\left(\left(\underline{\underline{64(\text{"}8.5\text{"})}} - \frac{1}{5}(2(8.5)-1)^{\frac{5}{2}}\right) - \left(\underline{\underline{64(0.5)}} - \frac{1}{5}(2(0.5)-1)^{\frac{5}{2}}\right)\right)$	as above	M1
			<u>B1</u>
	$\left\{= 32\pi + \pi\left(\left(544 - \frac{1024}{5}\right) - (32-0)\right)\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$		A1
			[4]
			8

		Question 7 Notes
7. (b)	**SC**	Allow Special Case SC M1 for a candidate who sets $8=(2k-1)^{\frac{3}{2}}$ or $8=(2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.
7. (c)	**M1**	Can also be given for applying u-limits of "16" $(2(\text{"part }(b)\text{"})-1)$ and 0 to an expression of the form $\pm\beta u^{\frac{5}{2}};\ \beta\neq 0$ and subtracts the correct way round.
	Note	You can give M1 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}}=\dfrac{1024}{5}$
	Note	Give M0 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{0}^{\frac{17}{2}}=\left(\left(\dfrac{16^{\frac{5}{2}}}{5}\right)-(0)\right)$
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height k.
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi\int_0^{8.5}8^2\,dx=\pi\left[64x\right]_0^{8.5}$ is **not sufficient** for B1 but $\pi(64(8.5)-0)$ **is sufficient** for B1.

7.	**MISREADING IN BOTH PARTS (B) AND (C)**		
	Apply the misread rule (MR) for candidates who apply $y=(2x-1)^{\frac{3}{2}}$ to **both** parts (b) and (c)		
(b)	$\{P(k,8)\Rightarrow\}\ 8=(2k-1)^{\frac{3}{2}}\Rightarrow k=\dfrac{8^{\frac{2}{3}}+1}{2}$	Sets $8=(2k-1)^{\frac{3}{2}}$ or $8=(2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k=\dfrac{5}{2}$	k (or x) $=\dfrac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2dx$	For $\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2$ or $\pi\int(2x-1)^3$ Ignore limits and dx. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}}y^2\,dx\right\}=\left[\dfrac{(2x-1)^4}{8}\right]_{\frac{1}{2}}^{\frac{5}{2}}=\left(\left(\dfrac{4^4}{8}\right)-(0)\right)\ \{=32\}$	Applies x-limits of "2.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^4;\ \beta\neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}}=\pi(8)^2\left(\dfrac{5}{2}\right)\ \{=160\pi\}$	$\pi(8)^2(\text{their answer to part }(b))$ Sight of 160π implies this mark	B1 ft
	$\{\text{Vol}(S)=160\pi-32\pi\}\Rightarrow\text{Vol}(S)=128\pi$	An exact correct answer in the form $k\pi$ E.g. 128π	A1
			[4]
	Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0	
	Note	If a candidate uses $y=(2x-1)^{\frac{3}{4}}$ in part (b) and then uses $y=(2x-1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).	

Leave blank

8. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A. **(1)**

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2 **(2)**

(c) Find the exact value of the distance AP.
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined. **(2)**

The acute angle between AP and l_2 is θ.

(d) Find the value of $\cos\theta$ **(3)**

A point E lies on the line l_2
Given that $AP = PE$,

(e) find the area of triangle APE, **(2)**

(f) find the coordinates of the two possible positions of E. **(5)**

Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix}8\\1\\-3\end{pmatrix} + \mu\begin{pmatrix}-5\\4\\3\end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix}-5\\4\\3\end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu = 1$. $\overrightarrow{OP} = \begin{pmatrix}1\\5\\2\end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$	A1
	$\mathbf{d}_2$ is the direction vector of l_2	Do not allow $l_2 :$ **or** $l_2 \rightarrow$ **or** $l_1 =$ for the A1 mark.	**[2]**
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix}1\\5\\2\end{pmatrix} - \begin{pmatrix}3\\5\\0\end{pmatrix} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding AP	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overrightarrow{AP} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix} \bullet \begin{pmatrix}-5\\4\\3\end{pmatrix}$	Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AP} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix}-2\\0\\2\end{pmatrix} \bullet \begin{pmatrix}-5\\4\\3\end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2} . \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	**dependent on the previous M mark.** Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos\theta\} = \dfrac{\pm(10 + 0 + 6)}{\sqrt{8}.\sqrt{50}} = \dfrac{4}{5}$	$\{\cos\theta\} = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$	$\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ or $\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c)		
	$\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \dfrac{4}{25} \Rightarrow\right\}\ \lambda = \pm\dfrac{2}{5}$	Either $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} \pm \dfrac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$	**dependent on the previous M mark** Substitutes at least one of their values of λ into l_2.	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix}3\\ \frac{17}{5}\\ \frac{4}{5}\end{pmatrix}$ or $\begin{pmatrix}3\\3.4\\0.8\end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix}-1\\ \frac{33}{5}\\ \frac{16}{5}\end{pmatrix}$ or $\begin{pmatrix}-1\\6.6\\3.2\end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15

	Question 8 Notes		
8. (a)	**B1**	Allow $A(3, 5, 0)$ or $3\mathbf{i}+5\mathbf{j}$ or $3\mathbf{i}+5\mathbf{j}+0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$	
(b)	**A1**	Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 = i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.	
	Note	Allow the use of parameters μ or t instead of λ.	
(c)	**M1**	Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find AP	
	Note	Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2+(0)^2+(2)^2} = \sqrt{8} = 2\sqrt{2}$.	
(d)	**Note**	For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a).	
	Note	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1	
	Note	***Evaluating*** the dot product (i.e. $(-2)(-5)+(0)(4)+(2)(3)$) is not required for M1 and dM1 marks.	
	Note	In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$	
	Note	$\cos\theta = \dfrac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\dfrac{4}{5}$ followed by $\cos\theta = \dfrac{4}{5}$ is fine for A1 cso	
	Note	Give M1dM1A1 for $\{\cos\theta =\} = \dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \dfrac{20+12}{40} = \dfrac{4}{5}$	
	Note	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^\circ$	
	<u>Alternative Method: Vector Cross Product</u>		
	Only apply this scheme if it is clear that a candidate is applying a vector cross product method.		
	$\overrightarrow{AP}\times\mathbf{d}_2 = \begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 0 & 2\\-5 & 4 & 3\end{vmatrix} = -8\mathbf{i}-4\mathbf{j}-8\mathbf{k}\right\}$	Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\sin\theta = \dfrac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$	Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\sin\theta = \dfrac{12}{\sqrt{8}.\sqrt{50}} = \dfrac{3}{5} \Rightarrow \cos\theta = \dfrac{4}{5}$	$\cos\theta = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$	A1
(e)	**Note**	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^\circ - 36.869...^\circ)$; = awrt 2.40	
	Note	Candidates must use their θ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$	

	Question 8 Notes Continued	
8. (f)	**Note**	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working
	SC	Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working
	Note	Give 1st M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$
	Note	Give 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent
	Note	Give 1st M1 for $\lambda = \frac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$
	Note	So $\left\{\hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ "vector" $= \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1
	Note	The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct x, y, z ordinates from their values of λ.
	Note	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.
	CAREFUL	Putting l_2 equal to A gives $\begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}3\\5\\0\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = \frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ — Give M0 dM0 for finding and using $\lambda = \frac{2}{5}$ from this incorrect method.
	CAREFUL	Putting $\lambda\mathbf{d}_2 = \overrightarrow{AP}$ gives $\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}2\\0\\-2\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = -\frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ — Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method.
	General	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1
	General	You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1.