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Centre Number

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Candidate Number

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# Core Mathematics C4

## Advanced

Friday 24 June 2016 – Morning

**Time: 1 hour 30 minutes**

Paper Reference

**6666/01****You must have:**

Mathematical Formulae and Statistical Tables (Pink)

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. Use the binomial series to find the expansion of

$$\frac{1}{(2 + 5x)^3}, \quad |x| < \frac{2}{5}$$

in ascending powers of  $x$ , up to and including the term in  $x^3$ .

Give each coefficient as a fraction in its simplest form.

(6)

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Question Number	Scheme	Notes	Marks
<b>1.</b> <b>Way 1</b>	$\left\{ \frac{1}{(2+5x)^3} \right\} (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of $-3$	M1
	$= \frac{(2)^{-3}}{8} \left( 1 + \frac{5x}{2} \right)^{-3} = \frac{1}{8} \left( 1 + \frac{5x}{2} \right)^{-3}$	$\frac{2^{-3}}{8}$ or $\frac{1}{8}$	B1
	$= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3)(kx) + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 + \dots \right]$	see notes	M1 A1
	$= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3) \left( \frac{5x}{2} \right) + \frac{(-3)(-4)}{2!} \left( \frac{5x}{2} \right)^2 + \frac{(-3)(-4)(-5)}{3!} \left( \frac{5x}{2} \right)^3 + \dots \right]$		
	$= \frac{1}{8} \left[ 1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \dots \right]$		
	$= \frac{1}{8} \left[ 1 - 7.5x + 37.5x^2 - 156.25x^3 + \dots \right]$		
	$= \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$ or $\frac{1}{8} - \frac{15}{16}x; + 4\frac{11}{16}x^2 - 19\frac{17}{32}x^3 + \dots$		A1; A1
			[6]
			6
<b>Way 2</b>	$f(x) = (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of $-3$	M1
	$f''(x) = 300(2+5x)^{-5}, f'''(x) = -7500(2+5x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
	$f'(x) = -15(2+5x)^{-4}$	$\pm a(2+5x)^{-4}, a \neq \pm 1$	M1
		$-15(2+5x)^{-4}$	A1 oe
	$\left\{ \therefore f(0) = \frac{1}{8}, f'(0) = -\frac{15}{16}, f''(0) = \frac{75}{8} \text{ and } f'''(0) = -\frac{1875}{16} \right\}$		
	So, $f(x) = \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$	Same as in Way 1	A1; A1
			[6]
<b>Way 3</b>	$(2+5x)^{-3}$	Same as in Way 1	M1
	$= \frac{(2)^{-3}}{8} + (-3)(2)^{-4}(5x) + \frac{(-3)(-4)}{2!} (2)^{-5}(5x)^2 + \frac{(-3)(-4)(-5)}{3!} (2)^{-6}(5x)^3$	Same as in Way 1	B1
		Any two terms correct	M1
		All four terms correct	A1
	$= \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$	Same as in Way 1	A1; A1
	<b>Note:</b> Terms can be simplified or un-simplified for B1 2 <sup>nd</sup> M1 1 <sup>st</sup> A1		[6]
	<b>Note:</b> The terms in C need to be evaluated So $^{-3}C_0(2)^{-3} + ^{-3}C_1(2)^{-4}(5x) + ^{-3}C_2(2)^{-5}(5x)^2 + ^{-3}C_3(2)^{-6}(5x)^3$ without further working is B0 1 <sup>st</sup> M0 1 <sup>st</sup> A0		

Question 1 Notes		
1.	1 <sup>st</sup> M1	mark can be implied by a constant term of $(2)^{-3}$ or $\frac{1}{8}$ .
	B1	$2^{-3}$ or $\frac{1}{8}$ outside brackets or $\frac{1}{8}$ as candidate's constant term in their binomial expansion.
	2 <sup>nd</sup> M1	Expands $(\dots + kx)^{-3}$ , $k = \text{a value} \neq 1$ , to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1 + (-3)(kx)$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1 + \dots + \frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.
	1 <sup>st</sup> A1	A correct simplified or un-simplified $1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent $(kx)$ . Note that $(kx)$ must be consistent and $k = \text{a value} \neq 1$ . (on the RHS, not necessarily the LHS) in a candidate's expansion.
	Note	You would award B1M1A0 for $\frac{1}{8} \left[ 1 + (-3) \left( \frac{5x}{2} \right) + \frac{(-3)(-4)}{2!} (5x)^2 + \frac{(-3)(-4)(-5)}{3!} \left( \frac{5x}{2} \right)^3 + \dots \right]$ because $(kx)$ is not consistent.
	Note	<b>Incorrect bracketing:</b> $= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3) \left( \frac{5x}{2} \right) + \frac{(-3)(-4)}{2!} \left( \frac{5x^2}{2} \right) + \frac{(-3)(-4)(-5)}{3!} \left( \frac{5x^3}{2} \right) + \dots \right]$ is M1A0 unless recovered.
	2 <sup>nd</sup> A1	For $\frac{1}{8} - \frac{15}{16}x$ ( <b>simplified</b> ) or also allow $0.125 - 0.9375x$ .
	3 <sup>rd</sup> A1	Accept only $\frac{75}{16}x^2 - \frac{625}{32}x^3$ or $4\frac{11}{16}x^2 - 19\frac{17}{32}x^3$ or $4.6875x^2 - 19.53125x^3$
	SC	If a candidate <b>would otherwise score</b> 2 <sup>nd</sup> A0, 3 <sup>rd</sup> A0 then <b>allow Special Case 2<sup>nd</sup> A1 for either</b>
		<b>SC:</b> $\frac{1}{8} \left[ 1 - \frac{15}{2}x; \dots \right]$ or <b>SC:</b> $\frac{1}{8} \left[ 1 + \dots + \frac{75}{2}x^2 + \dots \right]$ or <b>SC:</b> $\frac{1}{8} \left[ 1 + \dots - \frac{625}{4}x^3 + \dots \right]$
		<b>SC:</b> $\lambda \left[ 1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \dots \right]$ or <b>SC:</b> $\left[ \lambda - \frac{15\lambda}{2}x + \frac{75\lambda}{2}x^2 - \frac{625\lambda}{4}x^3 + \dots \right]$
		(where $\lambda$ can be 1 or omitted), where each term in the $[\dots]$ is a simplified fraction or a decimal
	SC	<b>Special case for the 2<sup>nd</sup> M1 mark</b> Award Special Case 2 <sup>nd</sup> M1 for a correct simplified or un-simplified $1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2 + \frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n \neq -3$ , $n \neq \text{positive integer}$ and a consistent $(kx)$ . Note that $(kx)$ must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. <b>Note</b> that $k \neq 1$ .
	Note	Ignore extra terms beyond the term in $x^3$
	Note	You can ignore subsequent working following a correct answer.

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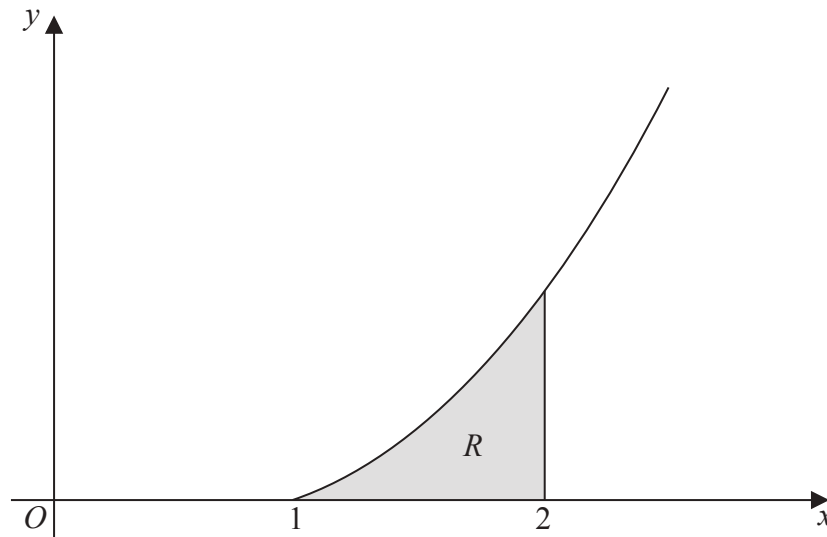


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = x^2 \ln x$ ,  $x \geq 1$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the line  $x = 2$

The table below shows corresponding values of  $x$  and  $y$  for  $y = x^2 \ln x$

$x$	1	1.2	1.4	1.6	1.8	2
$y$	0	0.2625		1.2032	1.9044	2.7726

- (a) Complete the table above, giving the missing value of  $y$  to 4 decimal places. (1)
- (b) Use the trapezium rule with all the values of  $y$  in the completed table to obtain an estimate for the area of  $R$ , giving your answer to 3 decimal places. (3)
- (c) Use integration to find the exact value for the area of  $R$ . (5)

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Question Number	Scheme										Marks
2.	$\frac{x}{y}$	1	1.2	1.4	1.6	1.8	2		$y = x^2 \ln x$		
(a)	0	0.2625	0.659485...	1.2032	1.9044	2.7726				0.6595	B1 <b>cao</b>
	{At $x = 1.4$ ,} $y = 0.6595$ (4 dp)										[1]
(b)	$\frac{1}{2} \times (0.2) \times [0 + 2.7726 + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)]$  { <b>Note:</b> The “0” does not have to be included in [.....]}								Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$	B1 o.e.	
									For structure of [.....]	M1	
	$\left\{ = \frac{1}{10}(10.8318) \right\} = 1.08318 = 1.083$ (3 dp)					anything that rounds to 1.083				A1	
											[3]
(c) <b>Way 1</b>	$\left\{ I = \int x^2 \ln x \, dx \right\}, \left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{array} \right\}$										
	$= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left( \frac{1}{x} \right) \{dx\}$				Either $x^2 \ln x \rightarrow \pm \lambda x^3 \ln x - \int \mu x^3 \left( \frac{1}{x} \right) \{dx\}$ or $\pm \lambda x^3 \ln x - \int \mu x^2 \{dx\}$ , where $\lambda, \mu > 0$					M1	
					$x^2 \ln x \rightarrow \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left( \frac{1}{x} \right) \{dx\}$ , simplified or un-simplified					A1	
	$= \frac{x^3}{3} \ln x - \frac{x^3}{9}$				$\frac{x^3}{3} \ln x - \frac{x^3}{9}$ , simplified or un-simplified					A1	
	$\text{Area}(R) = \left\{ \left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \right\} = \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left( 0 - \frac{1}{9} \right)$					<b>dependent on the previous M mark.</b> Applies limits of 2 and 1 and subtracts the correct way round					dM1
	$= \frac{8}{3} \ln 2 - \frac{7}{9}$					$\frac{8}{3} \ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24 \ln 2 - 7)$					A1 oe <b>cso</b>
											[5]
(c) <b>Way 2</b>	$I = x^2(x \ln x - x) - \int 2x(x \ln x - x) \, dx$				$\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = \ln x \Rightarrow v = x \ln x - x \end{array} \right\}$						
	$\text{So, } 3I = x^2(x \ln x - x) + \int 2x^2 \{dx\}$										
	$\text{and } I = \frac{1}{3}x^2(x \ln x - x) + \frac{1}{3} \int 2x^2 \{dx\}$				A full method of applying $u = x^2, v' = \ln x$ to give $\pm \lambda x^2(x \ln x - x) \pm \mu \int x^2 \{dx\}$					M1	
					$\frac{1}{3}x^2(x \ln x - x) + \frac{1}{3} \int 2x^2 \{dx\}$ simplified or un-simplified					A1	
	$= \frac{1}{3}x^2(x \ln x - x) + \frac{2}{9}x^3$				$\frac{x^3}{3} \ln x - \frac{x^3}{9}$ , simplified or un-simplified					A1	
					<b>Then award dM1A1 in the same way as above</b>					M1 A1	
											[5]
											9

Question 2 Notes		
2. (a)	<b>B1</b>	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	<b>B1</b>	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{2} \times \frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	<b>M1</b>	For structure of trapezium rule [ ..... ]
	<b>Note</b>	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	<b>A1</b>	anything that rounds to 1.083
	<b>Note</b>	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704...)
	<b>Note</b>	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	<b>Note</b>	Award B1M1A1 for $\frac{1}{10}(2.7726) + \frac{1}{5}(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) = \text{awrt } 1.083$
	<b>Bracketing mistake: Unless the final answer implies that the calculation has been done correctly</b>	
	Award B1M0A0 for $\frac{1}{2}(0.2) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) + 2.7726$ (answer of 10.9318)	
	Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)$ (answer of 8.33646)	
	<b>Alternative method: Adding individual trapezia</b>	
	Area $\approx 0.2 \times \left[ \frac{0 + 0.2625}{2} + \frac{0.2625 + "0.6595"}{2} + \frac{"0.6595" + 1.2032}{2} + \frac{1.2032 + 1.9044}{2} + \frac{1.9044 + 2.7726}{2} \right] = 1.08318...$	
	<b>B1</b>	0.2 and a divisor of 2 on all terms inside brackets
	<b>M1</b>	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2
	<b>A1</b>	anything that rounds to 1.083
(c)	<b>A1</b>	Exact answer needs to be a two term expression in the form $a \ln b + c$
	<b>Note</b>	Give A1 e.g. $\frac{8}{3} \ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24 \ln 2 - 7)$ or $\frac{4}{3} \ln 4 - \frac{7}{9}$ or $\frac{1}{3} \ln 256 - \frac{7}{9}$ or $-\frac{7}{9} + \frac{8}{3} \ln 2$ or $\ln 2^{\frac{8}{3}} - \frac{7}{9}$ or equivalent.
	<b>Note</b>	Give final A0 for a final answer of $\frac{8 \ln 2 - \ln 1}{3} - \frac{7}{9}$ or $\frac{8 \ln 2}{3} - \frac{1}{3} \ln 1 - \frac{7}{9}$ or $\frac{8 \ln 2}{3} - \frac{8}{9} + \frac{1}{9}$ or $\frac{8}{3} \ln 2 - \frac{7}{9} + c$
	<b>Note</b>	$\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	<b>Note</b>	Give dM0A0 for $\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \rightarrow \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \frac{1}{9}$ (adding rather than subtracting)
	<b>Note</b>	Allow dM1A0 for $\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \rightarrow \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left( 0 + \frac{1}{9} \right)$
	<b>SC</b>	A candidate who uses $u = \ln x$ and $\frac{dv}{dx} = x^2$ , $\frac{du}{dx} = \frac{\alpha}{x}$ , $v = \beta x^3$ , writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1 <sup>st</sup> M1.

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3. The curve  $C$  has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(5)

The point  $P$  with coordinates  $\left(3, \frac{1}{2}\right)$  lies on  $C$ .

The normal to  $C$  at  $P$  meets the  $x$ -axis at the point  $A$ .

(b) Find the  $x$  coordinate of  $A$ , giving your answer in the form  $\frac{a\pi + b}{c\pi + d}$ ,  
where  $a, b, c$  and  $d$  are integers to be determined.

(4)

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Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) Way 1	$\left\{ \frac{dy}{dx} \right\} \times \left( 4xy + 2x^2 \frac{dy}{dx} \right) + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dy}{dx} (2x^2 + 4 + \pi \sin(\pi y)) + 4xy + 2 = 0$		dM1
	$\left\{ \frac{dy}{dx} \right\} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
(b)	At $\left( 3, \frac{1}{2} \right)$ , $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi \sin(\frac{1}{2}\pi)} \left\{ = \frac{-8}{22 + \pi} \right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical $m_N$ <b>Can be implied by later working</b>	M1
	<ul style="list-style-type: none"> <li><math>y - \frac{1}{2} = \left( \frac{22 + \pi}{8} \right)(x - 3)</math></li> <li><math>\frac{1}{2} = \left( \frac{22 + \pi}{8} \right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}</math></li> <li><math>\Rightarrow y = \left( \frac{22 + \pi}{8} \right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}</math></li> </ul>	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical $m_N (\neq m_T)$ where $m_N$ is in terms of $\pi$ and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{ x = \frac{-4}{22 + \pi} + 3 \Rightarrow \right\} x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) Way 2	$\left\{ \frac{dx}{dy} \right\} \times \left( 4xy \frac{dx}{dy} + 2x^2 \right) + 2 \frac{dx}{dy} + 4 + \pi \sin(\pi y) = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dx}{dy} (4xy + 2) + 2x^2 + 4 + \pi \sin(\pi y) = 0$		dM1
	$\frac{dx}{dy} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
Question 3 Notes			
3. (a)	<b>Note</b> Writing down <i>from no working</i> <ul style="list-style-type: none"> <li><math>\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}</math> scores M1A1B1M1A1</li> <li><math>\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi \sin(\pi y)}</math> scores M1A0B1M1A0</li> </ul>		
	<b>Note</b> Few candidates will write $4xydx + 2x^2dy + 2dx + 4dy + \pi \sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$ or equivalent. This should get full marks.		

Question 3 Notes Continued		
3. (a) Way 1	<b>M1</b>	Differentiates implicitly to include either $2x^2 \frac{dy}{dx}$ or $4y \rightarrow 4 \frac{dy}{dx}$ or $-\cos(\pi y) \rightarrow \pm \lambda \sin(\pi y) \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$ ). $\lambda$ is a constant which can be 1.
	<b>1<sup>st</sup> A1</b>	$2x + 4y - \cos(\pi y) = 17 \rightarrow 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$
	<b>Note</b>	$4xy + 2x^2 \frac{dy}{dx} + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} \rightarrow 2x^2 \frac{dy}{dx} + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = -4xy - 2$ will get 1 <sup>st</sup> A1 (implied) as the "= 0" can be implied by the rearrangement of their equation.
	<b>B1</b>	$2x^2 y \rightarrow 4xy + 2x^2 \frac{dy}{dx}$
	<b>Note</b>	If an extra term appears then award 1 <sup>st</sup> A0.
	<b>dM1</b>	<b>Dependent on the first method mark being awarded.</b> An attempt to factorise out <b>all the terms in</b> $\frac{dy}{dx}$ as long as there are <b>at least two terms</b> in $\frac{dy}{dx}$ . ie. $\frac{dy}{dx}(2x^2 + 4 + \pi \sin(\pi y)) + \dots = \dots$
	<b>Note</b>	Writing down an extra $\frac{dy}{dx} = \dots$ and then including it in their factorisation is fine for dM1.
	<b>Note</b>	<b>Final A1 cso:</b> If the candidate's solution is not completely correct, then do not give this mark.
	<b>Note</b>	<b>Final A1 isw:</b> You can, however, ignore subsequent working following on from correct solution.
(a)	<b>Way 2</b>	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	<b>1<sup>st</sup> M1</b>	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$ . E.g. " $-4xy$ " $\rightarrow$ " $-6$ " in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}$ , $y = 3$ .
	<b>3<sup>rd</sup> M1</b>	<b>is dependent on the first M1.</b>
	<b>Note</b>	The 2 <sup>nd</sup> M1 mark can be implied by later working. <b>Eg. Award 2<sup>nd</sup> M1 3<sup>rd</sup> M1</b> for $\frac{\frac{1}{2}}{3-x} = \frac{-1}{\text{their } m_r}$
	<b>Note</b>	We can accept $\sin \pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2 <sup>nd</sup> M1 mark.  But, $\sin \pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of $\pi$ for the 3 <sup>rd</sup> M1 mark.  The 3 <sup>rd</sup> M1 can be accessed for terms containing $\pi \sin\left(\frac{\pi}{2}\right)$ .

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- $$\frac{dx}{dt} = -\frac{5}{2}x, \quad t \geq 0$$

Given that  $x = 60$  when  $t = 0$ ,

- (4)

- (3)

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geq 0$		
(a) Way 1	$\int \frac{1}{x} dx = \int -\frac{5}{2} dt$	Separates variables as shown. $dx$ and $dt$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give <b>either</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$ , including "+c"	A1
	$\{t=0, x=60 \Rightarrow\} \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}} \text{ or } \underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 cso
			[4]
(a) Way 2	$\frac{dt}{dx} = -\frac{2}{5x} \text{ or } t = \int -\frac{2}{5x} dx$	<b>Either</b> $\frac{dt}{dx} = -\frac{2}{5x}$ <b>or</b> $t = \int -\frac{2}{5x} dx$	B1
	$t = -\frac{2}{5} \ln x + c$	Integrates both sides to give <b>either</b> $t = \dots$ <b>or</b> $\pm \alpha \ln px; \alpha \neq 0, p > 0$	M1
		$t = -\frac{2}{5} \ln x + c$ , including "+c"	A1
	$\{t=0, x=60 \Rightarrow\} c = \frac{2}{5} \ln 60 \Rightarrow t = -\frac{2}{5} \ln x + \frac{2}{5} \ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}} \text{ or } \underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 cso
			[4]
(a) Way 3	$\int_{60}^x \frac{1}{x} dx = \int_0^t -\frac{5}{2} dt$	Ignore limits	B1
	$[\ln x]_{60}^x = \left[-\frac{5}{2}t\right]_0^t$	Integrates both sides to give <b>either</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^x = \left[-\frac{5}{2}t\right]_0^t$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}} \text{ or } \underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]
(b)	$20 = 60e^{-\frac{5}{2}t} \text{ or } \ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of <b>either</b> $x = \pm \lambda e^{\pm \mu t} \pm \beta$ <b>or</b> $x = \pm \lambda e^{\pm \mu t \pm \alpha \ln \delta x}$ <b>or</b> $\pm \alpha \ln \delta x = \pm \mu t \pm \beta$ <b>or</b> $t = \pm \lambda \ln \delta x \pm \beta$ ; $\alpha, \lambda, \mu, \delta \neq 0$ and $\beta$ can be 0	M1
	$t = -\frac{2}{5} \ln \left(\frac{20}{60}\right)$ $\{= 0.4394449... \text{ (days)}\}$ <b>Note: <math>t</math> must be greater than 0</b>	<b>dependent on the previous M mark</b> Uses correct algebra to achieve an equation of the form of either $t = A \ln \left(\frac{60}{20}\right)$ or $A \ln \left(\frac{20}{60}\right)$ or $A \ln 3$ or $A \ln \left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ( $A \in \mathbb{R}, t > 0$ )	dM1
	$\Rightarrow t = 632.8006... = 633 \text{ (to the nearest minute)}$	awrt 633 <b>or</b> 10 hours and awrt 33 minutes	A1 cso
	<b>Note:</b> dM1 can be implied by $t = \text{awrt } 0.44$ from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geq 0$		
(a) Way 4	$\int \frac{2}{5x} dx = -\int dt$	Separates variables as shown. $dx$ and $dt$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5} \ln(5x) = -t + c$	Integrates both sides to give <b>either</b> $\pm \alpha \ln(px)$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ); $k, \alpha \neq 0$ ; $p > 0$	M1
		$\frac{2}{5} \ln(5x) = -t + c$ , including "+c"	A1
	$\left\{ t = 0, x = 60 \Rightarrow \right\} \quad \frac{2}{5} \ln 300 = c$ $\frac{2}{5} \ln(5x) = -t + \frac{2}{5} \ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 cso
			[4]
(a) Way 5	$\left\{ \frac{dt}{dx} = -\frac{2}{5x} \Rightarrow \right\} \quad t = \int_{60}^x -\frac{2}{5x} dx$	Ignore limits	B1
	$t = \left[ -\frac{2}{5} \ln x \right]_{60}^x$	Integrates both sides to give <b>either</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ) <b>or</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ ; $k, \alpha \neq 0$	M1
		$t = \left[ -\frac{2}{5} \ln x \right]_{60}^x$ including the correct limits	A1
	$t = -\frac{2}{5} \ln x + \frac{2}{5} \ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]
Question 4 Notes			
4. (a)	<b>B1</b>	For the correct separation of variables. E.g. $\int \frac{1}{5x} dx = \int -\frac{1}{2} dt$	
	<b>Note</b>	B1 can be implied by seeing <b>either</b> $\ln x = -\frac{5}{2}t + c$ <b>or</b> $t = -\frac{2}{5} \ln x + c$ with or without $+c$	
	<b>Note</b>	B1 can also be implied by seeing $[\ln x]_{60}^x = \left[ -\frac{5}{2}t \right]_0^t$	
	<b>Note</b>	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen	
	<b>Note</b>	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \rightarrow x = 60e^{-\frac{5}{2}t}$	
	<b>Note</b>	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$ )	
	<b>Note</b>	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.	
	<b>Note</b>	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.	
(b)	<b>A1</b>	You can apply <b>cso</b> for the work only seen in part (b).	
	<b>Note</b>	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t = \text{awrt } 633$ from no incorrect working.	
	<b>Note</b>	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0	

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### Figure 2

Figure 2 shows a sketch of the curve  $C$  with parametric equations

$$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$$

The point  $P$  lies on  $C$  and has coordinates  $\left(4\sqrt{3}, \frac{15}{2}\right)$ .

- (a) Find the exact value of  $\frac{dy}{dx}$  at the point  $P$ .

Give your answer as a simplified surd.

(4)

The point  $Q$  lies on the curve  $C$ , where  $\frac{dy}{dx} = 0$

- (b) Find the exact coordinates of the point  $Q$ .

(2)



Question Number	Scheme		Notes	Marks
5.	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$			
(a) Way 1	$\frac{dx}{dt} = 4 \sec^2 t, \quad \frac{dy}{dt} = 10\sqrt{3} \cos 2t$  $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} \quad \left\{ = \frac{5}{2} \sqrt{3} \cos 2t \cos^2 t \right\}$		<b>Either both</b> $x$ and $y$ are differentiated correctly with respect to $t$ <b>or</b> their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ <b>or</b> applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
			Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{ \text{At } P\left(4\sqrt{3}, \frac{15}{2}\right), t = \frac{\pi}{3} \right\}$			
	$\frac{dy}{dx} = \frac{10\sqrt{3} \cos\left(\frac{2\pi}{3}\right)}{4 \sec^2\left(\frac{\pi}{3}\right)}$		<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$		$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
				[4]
(b)	$\left\{ 10\sqrt{3} \cos 2t = 0 \Rightarrow t = \frac{\pi}{4} \right\}$			
	So $x = 4 \tan\left(\frac{\pi}{4}\right), y = 5\sqrt{3} \sin\left(2\left(\frac{\pi}{4}\right)\right)$		At least one of either $x = 4 \tan\left(\frac{\pi}{4}\right)$ <b>or</b> $y = 5\sqrt{3} \sin\left(2\left(\frac{\pi}{4}\right)\right)$ <b>or</b> $x = 4$ <b>or</b> $y = 5\sqrt{3}$ <b>or</b> $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$		$(4, 5\sqrt{3})$ <b>or</b> $x = 4, y = 5\sqrt{3}$	A1
				[2]
6				
Question 5 Notes				
5. (a)	1 <sup>st</sup> A1	Correct $\frac{dy}{dx}$ . E.g. $\frac{10\sqrt{3} \cos 2t}{4 \sec^2 t}$ or $\frac{5}{2}\sqrt{3} \cos 2t \cos^2 t$ or $\frac{5}{2}\sqrt{3} \cos^2 t (\cos^2 t - \sin^2 t)$ or any equivalent form.		
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ <b>without reference to</b> $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$		
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$		
(b)	Note	Also allow M1 for either $x = 4 \tan(45)$ or $y = 5\sqrt{3} \sin(2(45))$		
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)		
	Note	Give A0 for stating more than one set of coordinates for $Q$ .		
	Note	Writing $x = 4, y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.		

Question Number	Scheme	Notes	Marks
5.	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 2	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{ \begin{array}{l} u = 40\sqrt{3}x \quad v = x^2 + 16 \\ \frac{du}{dx} = 40\sqrt{3} \quad \frac{dv}{dx} = 2x \end{array} \right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{ = \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2} \right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$ ; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
			[4]
(a) Way 3	$y = 5\sqrt{3} \sin\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right) \left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$ ; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}(\sqrt{3})\right) \left(\frac{2}{1+3}\right) \left(\frac{1}{4}\right) \left\{ = 5\sqrt{3} \left(-\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{4}\right) \right\}$	<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
			[4]



DO NOT WRITE IN THIS AREA

- giving your answer in the form  $a\pi + b$ , where  $a$  and  $b$  are exact constants.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)} dy, y > 0$ , (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx, x = 4\sin^2 \theta$		
(i) Way 1	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A = -2$ or their $B = 9$	A1
	$y = -\frac{2}{3} \Rightarrow -6 = -\frac{2}{3}B \Rightarrow B = 9$	Both their $A = -2$ and their $B = 9$	A1
	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{-2}{y} + \frac{9}{(3y+2)} dy$	Integrates to give at least one of either $\frac{A}{y} \rightarrow \pm \lambda \ln y$ or $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $A \neq 0, B \neq 0$	M1
	$= -2 \ln y + 3 \ln(3y+2) \{+c\}$	At least one term correctly followed through from their A or from their B	A1 ft
		$-2 \ln y + 3 \ln(3y+2)$ or $-2 \ln y + 3 \ln(y + \frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 cao
			[6]
(ii) (a) Way 1	$\{x = 4\sin^2 \theta \Rightarrow \frac{dx}{d\theta} = 8\sin \theta \cos \theta \text{ or } \frac{dx}{d\theta} = 4\sin 2\theta \text{ or } dx = 8\sin \theta \cos \theta d\theta\}$		B1
	$\int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 8\sin \theta \cos \theta \{d\theta\} \text{ or } \int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 4\sin 2\theta \{d\theta\}$		M1
	$= \int \underline{\tan \theta} \cdot 8\sin \theta \cos \theta \{d\theta\} \text{ or } \int \underline{\tan \theta} \cdot 4\sin 2\theta \{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \rightarrow \pm K \tan \theta \text{ or } \pm K \left(\frac{\sin \theta}{\cos \theta}\right)$	<u>M1</u>
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1
	$3 = 4\sin^2 \theta \text{ or } \frac{3}{4} = \sin^2 \theta \text{ or } \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x = 0 \rightarrow \theta = 0\}$	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$= \{8\} \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta \quad \left\{ = \int (4-4\cos 2\theta) d\theta \right\}$	Applies $\cos 2\theta = 1 - 2\sin^2 \theta$ to their integral. (See notes)	M1
	$= \{8\} \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right) \quad \{ = 4\theta - 2\sin 2\theta \}$	For $\pm \alpha \theta \pm \beta \sin 2\theta, \alpha, \beta \neq 0$	M1
		$\sin^2 \theta \rightarrow \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{ \int_0^{\frac{\pi}{3}} 8\sin^2 \theta d\theta = 8 \left[ \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right]_0^{\frac{\pi}{3}} \right\} = 8 \left( \left( \frac{\pi}{6} - \frac{1}{4} \left( \frac{\sqrt{3}}{2} \right) \right) - (0+0) \right)$		
	$= \frac{4}{3}\pi - \sqrt{3}$	“two term” exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$	A1 o.e.
			[4]
			15

Question 6 Notes		
6. (i)	1 <sup>st</sup> M1	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied <i>for writing down</i> either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	<b>Give</b> 2 <sup>nd</sup> M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm \alpha \ln(3y^2+2y)$
	Note	<b>...but allow</b> 2 <sup>nd</sup> M1 for either $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$
6. (ii)(a)	1 <sup>st</sup> M1	Substitutes $x = 4\sin^2 \theta$ and their $dx$ (from their correctly rearranged $\frac{dx}{d\theta}$ ) into $\sqrt{\left(\frac{x}{4-x}\right)} dx$
	Note	$dx \neq \lambda d\theta$ . For example $dx \neq d\theta$
	Note	Allow substituting $dx = 4\sin 2\theta$ for the 1 <sup>st</sup> M1 after a correct $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 4\sin 2\theta d\theta$
	2 <sup>nd</sup> M1	Applying $x = 4\sin^2 \theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K \tan \theta$ or $\pm K \left(\frac{\sin \theta}{\cos \theta}\right)$
	Note	Integral sign is not needed for this mark.
	1 <sup>st</sup> A1	Simplifies to give $\int 8\sin^2 \theta d\theta$ including $d\theta$
	2 <sup>nd</sup> B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
(ii)(b)	Note	Allow 2 <sup>nd</sup> B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ <b>and</b> $x = 4\sin^2 0 = 0$
	Note	Allow 2 <sup>nd</sup> B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3, \theta = \frac{\pi}{3}; x = 0, \theta = 0$
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2 \theta$ <b>E.g.:</b> $\cos 2\theta = 1 - 2\sin^2 \theta$ or $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ or $K \sin^2 \theta = K \left(\frac{1 - \cos 2\theta}{2}\right)$ and <i>applies</i> it to their integral. <b>Note:</b> Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm \alpha \theta \pm \beta \sin 2\theta$ or $k(\pm \alpha \theta \pm \beta \sin 2\theta)$ , $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	1 <sup>st</sup> A1	Integrating $\sin^2 \theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$ , un-simplified or simplified. Correct solution only. Can be implied by $k \sin^2 \theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2 <sup>nd</sup> A1	A <b>correct solution in part (ii)</b> leading to a “two term” exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397... (without a correct <b>exact</b> answer) is A0.
	Note	Candidates can work in terms of $\lambda$ (note that $\lambda$ is not given in (ii)) and gain the 1 <sup>st</sup> three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2 \theta d\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$ ) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
6. (i) Way 2	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{6y+2}{3y^2+2y} dy - \int \frac{3y+6}{y(3y+2)} dy$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 6=2A \Rightarrow A=3$	At least one of their $A=3$ <b>or</b> their $B=-6$	A1
	$y=-\frac{2}{3} \Rightarrow 4=-\frac{2}{3}B \Rightarrow B=-6$	Both their $A=3$ <b>and</b> their $B=-6$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$  $= \int \frac{6y+2}{3y^2+2y} dy - \int \frac{3}{y} dy + \int \frac{6}{(3y+2)} dy$  $= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2) \{+c\}$	Integrates to give at least one of <b>either</b> $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
		$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>
			[6]
6. (i) Way 3	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{3y+1}{3y^2+2y} dy - \int \frac{5}{y(3y+2)} dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 5=2A \Rightarrow A=\frac{5}{2}$	At least one of their $A=\frac{5}{2}$ <b>or</b> their $B=-\frac{15}{2}$	A1
	$y=-\frac{2}{3} \Rightarrow 5=-\frac{2}{3}B \Rightarrow B=-\frac{15}{2}$	Both their $A=\frac{5}{2}$ <b>and</b> their $B=-\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$  $= \int \frac{3y+1}{3y^2+2y} dy - \int \frac{\frac{5}{2}}{y} dy + \int \frac{\frac{15}{2}}{(3y+2)} dy$  $= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2) \{+c\}$	Integrates to give at least one of <b>either</b> $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
		$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>
			[6]

	Scheme	Notes	
<b>6. (i) Way 4</b>	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{3y}{y(3y+2)} dy - \int \frac{4}{y(3y+2)} dy$		
	$= \int \frac{3}{(3y+2)} dy - \int \frac{4}{y(3y+2)} dy$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 4=2A \Rightarrow A=2$	At least one of their $A=2$ <b>or</b> their $B=-6$	A1
	$y=-\frac{2}{3} \Rightarrow 4=-\frac{2}{3}B \Rightarrow B=-6$	Both their $A=2$ <b>and</b> their $B=-6$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$	Integrates to give at least one of <b>either</b> $\frac{C}{(3y+2)} \rightarrow \pm \alpha \ln(3y+2)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ , $A \neq 0, B \neq 0, C \neq 0$	M1
	$= \int \frac{3}{3y+2} dy - \int \frac{2}{y} dy + \int \frac{6}{(3y+2)} dy$	At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2) \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>
			<b>[6]</b>
	Alternative methods for B1M1M1A1 in (ii)(a)		
<b>(ii)(a) Way 2</b>	$\{x = 4\sin^2 \theta \Rightarrow \frac{dx}{d\theta} = 8\sin \theta \cos \theta\}$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 8\sin \theta \cos \theta \{d\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2 \theta}{(1-\sin^2 \theta)}} \cdot 8\cos \theta \sin \theta \{d\theta\}$		
	$= \int \frac{\sin \theta}{\sqrt{(1-\sin^2 \theta)}} \cdot 8\sqrt{(1-\sin^2 \theta)} \sin \theta \{d\theta\}$		
	$= \int \sin \theta \cdot 8\sin \theta \{d\theta\}$	Correct method leading to $\sqrt{(1-\sin^2 \theta)}$ being cancelled out	M1
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1 <b>cso</b>
<b>(ii)(a) Way 3</b>	$\{x = 4\sin^2 \theta \Rightarrow \frac{dx}{d\theta} = 4\sin 2\theta\}$	As in Way 1	B1
	$x = 4\sin^2 \theta = 2 - 2\cos 2\theta, \quad 4-x = 2+2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}} \cdot 4\sin 2\theta \{d\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}} \cdot \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}} 4\sin 2\theta \{d\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}} \cdot 4\sin 2\theta \{d\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta} \cdot 4\sin 2\theta \{d\theta\} = \int 2(2-2\cos 2\theta) \cdot \{d\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1 <b>cso</b>

7. (a) Find

$$\int (2x - 1)^{\frac{3}{2}} dx$$

giving your answer in its simplest form.

(2)

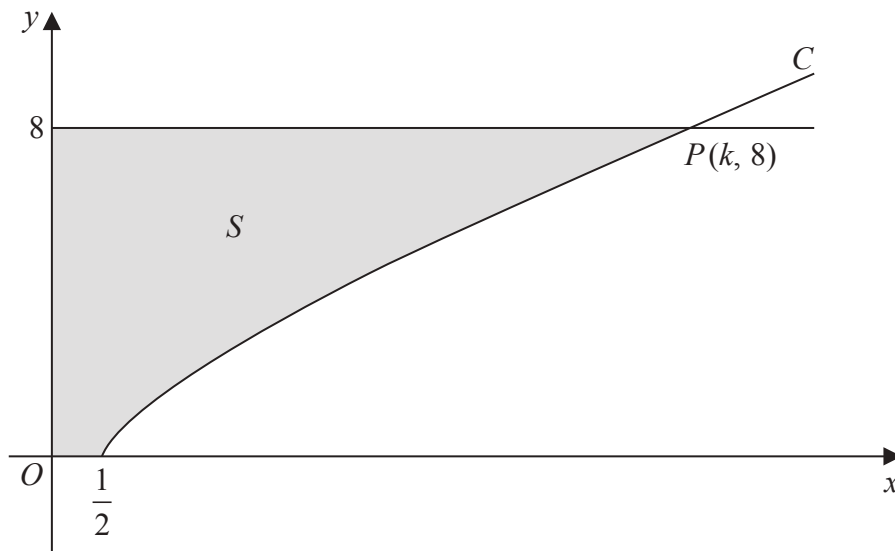


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x - 1)^{\frac{3}{4}}, \quad x \geq \frac{1}{2}$$

The curve C cuts the line  $y = 8$  at the point P with coordinates  $(k, 8)$ , where  $k$  is a constant.

(b) Find the value of  $k$ .

(2)

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the  $x$ -axis, the  $y$ -axis and the line  $y = 8$ . This region is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

(c) Find the exact value of the volume of the solid generated.

(4)

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Question Number	Scheme	Notes	Marks
7.	$y = (2x - 1)^{\frac{3}{4}}, \quad x \geq \frac{1}{2}$ passes through $P(k, 8)$		
(a)	$\left\{ \int (2x - 1)^{\frac{3}{2}} dx \right\} = \frac{1}{5}(2x - 1)^{\frac{5}{2}} \{ + c \}$	$(2x \pm 1)^{\frac{3}{2}} \rightarrow \pm \lambda(2x \pm 1)^{\frac{5}{2}} \text{ or } \pm \lambda u^{\frac{5}{2}}$ where $u = 2x \pm 1; \lambda \neq 0$	M1
		$\frac{1}{5}(2x - 1)^{\frac{5}{2}}$ with or without $+ c$ . Must be simplified.	A1
			[2]
(b)	$\{P(k, 8) \Rightarrow\} 8 = (2k - 1)^{\frac{3}{4}} \Rightarrow k = \frac{8^{\frac{4}{3}} + 1}{2}$	Sets $8 = (2k - 1)^{\frac{3}{4}}$ or $8 = (2x - 1)^{\frac{3}{4}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	M1
	So, $k = \frac{17}{2}$	$k$ (or $x$ ) = $\frac{17}{2}$ or 8.5	A1
			[2]
(c)	$\pi \int \left( (2x - 1)^{\frac{3}{4}} \right)^2 dx$	For $\pi \int \left( (2x - 1)^{\frac{3}{4}} \right)^2$ or $\pi \int (2x - 1)^{\frac{3}{2}}$ Ignore limits and $dx$ . Can be implied.	B1
	$\left\{ \int_{\frac{1}{2}}^{\frac{17}{2}} y^2 dx \right\} = \left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_{\frac{1}{2}}^{\frac{17}{2}} = \left( \left( \frac{16^{\frac{5}{2}}}{5} \right) - (0) \right) \left\{ = \frac{1024}{5} \right\}$ <b>Note:</b> It is not necessary to write the "-0"	Applies $x$ -limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm \beta(2x - 1)^{\frac{5}{2}}; \beta \neq 0$ and subtracts the correct way round.	M1
	$\left\{ V_{\text{cylinder}} \right\} = \pi(8)^2 \left( \frac{17}{2} \right) \left\{ = 544\pi \right\}$	$\pi(8)^2$ (their answer to part (b)) $V_{\text{cylinder}} = 544\pi$ implies this mark	B1 ft
	$\left\{ \text{Vol}(S) = 544\pi - \frac{1024\pi}{5} \right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$	An exact correct answer in the form $k\pi$ E.g. $\frac{1696}{5}\pi, \frac{3392}{10}\pi$ or $339.2\pi$	A1
			[4]
Alt. (c)	$\text{Vol}(S) = \pi(8)^2 \left( \frac{1}{2} \right) + \pi \int_{0.5}^{8.5} \left( 8^2 - \underline{\underline{(2x - 1)^{\frac{3}{2}}}} \right) dx$	For $\pi \int \dots \underline{\underline{(2x - 1)^{\frac{3}{2}}}}$ Ignore limits and $dx$ .	B1
	$= \pi(8)^2 \left( \frac{1}{2} \right) + \pi \left[ 64x - \frac{1}{5}(2x - 1)^{\frac{5}{2}} \right]_{0.5}^{8.5}$		
	$= \pi(8)^2 \left( \frac{1}{2} \right) + \pi \left( \left( \underline{\underline{64("8.5")}} - \frac{1}{5}(2(8.5) - 1)^{\frac{5}{2}} \right) - \left( \underline{\underline{64(0.5)}} - \frac{1}{5}(2(0.5) - 1)^{\frac{5}{2}} \right) \right)$	as above	M1
			<u>B1</u>
	$\left\{ = 32\pi + \pi \left( \left( 544 - \frac{1024}{5} \right) - (32 - 0) \right) \right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$		A1
			[4]
			8

Question 7 Notes			
7. (b)	SC	Allow Special Case SC M1 for a candidate who sets $8 = (2k - 1)^{\frac{3}{2}}$ or $8 = (2x - 1)^{\frac{3}{2}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	
7. (c)	M1	Can also be given for applying $u$ -limits of “16” ( $2(\text{"part (b)"} - 1)$ ) and 0 to an expression of the form $\pm \beta u^{\frac{5}{2}}$ ; $\beta \neq 0$ and subtracts the correct way round.	
	Note	You can give M1 for $\left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_{-\frac{1}{2}}^{\frac{17}{2}} = \frac{1024}{5}$	
	Note	Give M0 for $\left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_0^{\frac{17}{2}} = \left( \left( \frac{16^{\frac{5}{2}}}{5} \right) - (0) \right)$	
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height $k$ .	
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi \int_0^{8.5} 8^2 \, dx = \pi [64x]_0^{8.5}$ is <b>not sufficient</b> for B1 but $\pi(64(8.5) - 0)$ is <b>sufficient</b> for B1.	
7.	MISREADING IN BOTH PARTS (B) AND (C)		
	Apply the misread rule (MR) for candidates who apply $y = (2x - 1)^{\frac{3}{2}}$ to <b>both</b> parts (b) and (c)		
(b)	$\{P(k, 8) \Rightarrow\} 8 = (2k - 1)^{\frac{3}{2}} \Rightarrow k = \frac{8^{\frac{2}{3}} + 1}{2}$	Sets $8 = (2k - 1)^{\frac{3}{2}}$ or $8 = (2x - 1)^{\frac{3}{2}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	M1
	So, $k = \frac{5}{2}$	$k$ (or $x$ ) = $\frac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi \int \left( (2x - 1)^{\frac{3}{2}} \right)^2 \, dx$	For $\pi \int \left( (2x - 1)^{\frac{3}{2}} \right)^2$ or $\pi \int (2x - 1)^3$ Ignore limits and $dx$ . Can be implied.	B1
	$\left\{ \int_{\frac{1}{2}}^{\frac{17}{2}} y^2 \, dx \right\} = \left[ \frac{(2x - 1)^4}{8} \right]_{\frac{1}{2}}^{\frac{17}{2}} = \left( \left( \frac{4^4}{8} \right) - (0) \right) \{ = 32 \}$	Applies $x$ -limits of “2.5” (their answer to part (b)) and 0.5 to an expression of the form $\pm \beta (2x - 1)^4$ ; $\beta \neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}} = \pi(8)^2 \left( \frac{5}{2} \right) \{ = 160\pi \}$	$\pi(8)^2$ (their answer to part (b)) Sight of $160\pi$ implies this mark	B1 ft
	$\{ \text{Vol}(S) = 160\pi - 32\pi \} \Rightarrow \text{Vol}(S) = 128\pi$	An exact correct answer in the form $k\pi$ E.g. $128\pi$	A1
			[4]
	Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0	
	Note	If a candidate uses $y = (2x - 1)^{\frac{3}{4}}$ in part (b) and then uses $y = (2x - 1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).	



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8. With respect to a fixed origin  $O$ , the line  $l_1$  is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where  $\mu$  is a scalar parameter.

The point  $A$  lies on  $l_1$  where  $\mu = 1$

- (a) Find the coordinates of  $A$ .

(1)

The point  $P$  has position vector  $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ .

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$

- (b) Write down a vector equation for the line  $l_2$

(2)

- (c) Find the exact value of the distance  $AP$ .

Give your answer in the form  $k\sqrt{2}$ , where  $k$  is a constant to be determined.

(2)

The acute angle between  $AP$  and  $l_2$  is  $\theta$ .

- (d) Find the value of  $\cos \theta$

(3)

A point  $E$  lies on the line  $l_2$   
Given that  $AP = PE$ ,

- (e) find the area of triangle  $APE$ ,

(2)

- (f) find the coordinates of the two possible positions of  $E$ .

(5)

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Question Number	Scheme	Notes	Marks
8.	$l_1: \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ . $\overrightarrow{OA}$ occurs when $\mu = 1$ . $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2: \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda \mathbf{d}$ or $\mathbf{a} + \mu \mathbf{d}$ , $\mathbf{a} + t \mathbf{d}$ , $\mathbf{a} \neq 0$ , $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ , or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ or $l =$ or $l_2 =$	A1
	$\mathbf{d}_2$ is the direction vector of $l_2$	Do not allow $l_2: \text{or } l_2 \rightarrow \text{or } l_1 =$ for the A1 mark.	[2]
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding $AP$	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos \theta = \frac{\overrightarrow{AP} \cdot \mathbf{d}_2}{ \overrightarrow{AP}   \mathbf{d}_2 } = \frac{\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}}{\sqrt{(-2)^2 + (0)^2 + (2)^2} \cdot \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	dependent on the previous M mark. Applies dot product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos \theta\} = \frac{\pm(10+0+6)}{\sqrt{8} \cdot \sqrt{50}} = \frac{4}{\sqrt{5}}$	$\{\cos \theta\} = \frac{4}{5}$ or 0.8 or $\frac{8}{10}$ or $\frac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE\} = \frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin \theta$	$\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin \theta$ or $\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	$2.4$ or $\frac{12}{5}$ or $\frac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE = \text{their } 2\sqrt{2}$ from part (c)		
	$\{PE^2 =\} (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \frac{4}{25} \Rightarrow\} \lambda = \pm \frac{2}{5}$	Either $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$	A1
	$l_2: \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \frac{2}{5} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	dependent on the previous M mark Substitutes at least one of their values of $\lambda$ into $l_2$ .	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix} 3 \\ \frac{17}{5} \\ \frac{4}{5} \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}$ , $\{\overrightarrow{OE}\} = \begin{pmatrix} -1 \\ \frac{33}{5} \\ \frac{16}{5} \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15

Question 8 Notes			
8. (a)	B1	Allow $A(3, 5, 0)$ or $3\mathbf{i} + 5\mathbf{j}$ or $3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$ or $\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$ or benefit of the doubt	3 5 0
(b)	A1	Correct vector equation using $\mathbf{r} =$ or $l =$ or $l_2 =$ or Line 2 = i.e. Writing $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \mathbf{d}$ , where $\mathbf{d}$ is a multiple of $\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ .	
	Note	Allow the use of parameters $\mu$ or $t$ instead of $\lambda$ .	
(c)	M1	Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find $AP$	
	Note	Allow M1A1 for $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ .	
(d)	Note	For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a).	
	Note	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1	
	Note	Evaluating the dot product (i.e. $(-2)(-5) + (0)(4) + (2)(3)$ ) is not required for M1 and dM1 marks.	
	Note	In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$	
	Note	$\cos\theta = \frac{-10+0-6}{\sqrt{8}\cdot\sqrt{50}} = -\frac{4}{5}$ followed by $\cos\theta = \frac{4}{5}$ is fine for A1 cso	
	Note	Give M1dM1A1 for $\{\cos\theta\} = \frac{\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ 8 \\ 6 \end{pmatrix}}{\sqrt{8}\cdot 10\sqrt{2}} = \frac{20+12}{40} = \frac{4}{5}$	
	Note	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869\dots^\circ$	
<b>Alternative Method: Vector Cross Product</b>			
<b>Only apply this scheme if it is clear that a candidate is applying a vector cross product method.</b>			
		$\overrightarrow{AP} \times \mathbf{d}_2 = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 2 \\ -5 & 4 & 3 \end{vmatrix} = -8\mathbf{i} - 4\mathbf{j} - 8\mathbf{k} \right\}$	Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ M1
	$\sin\theta = \frac{\sqrt{(-8)^2 + (-4)^2 + (-8)^2}}{\sqrt{(-2)^2 + (0)^2 + (2)^2} \cdot \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\sin\theta = \frac{12}{\sqrt{8}\cdot\sqrt{50}} = \frac{3}{5} \Rightarrow \cos\theta = \frac{4}{5}$	$\cos\theta = \frac{4}{5}$ or $0.8$ or $\frac{8}{10}$ or $\frac{16}{20}$	A1
(e)	Note	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2 \sin(36.869\dots^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2 \sin(180^\circ - 36.869\dots^\circ)$ ; = awrt 2.40	
	Note	Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin\theta = \frac{3}{5}$ from their $\cos\theta = \frac{4}{5}$	

Question 8 Notes Continued		
8. (f)	<b>Note</b>	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working
	<b>SC</b>	Allow special case 1 <sup>st</sup> M1 for $\lambda = 2.5$ from comparing lengths or from no working
	<b>Note</b>	Give 1 <sup>st</sup> M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$
	<b>Note</b>	Give 1 <sup>st</sup> M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent
	<b>Note</b>	Give 1 <sup>st</sup> M1 for $\lambda = \frac{\text{their } AP = "2\sqrt{2}"}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1 <sup>st</sup> A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$
	<b>Note</b>	So $\left\{ \hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \text{"vector"} = \frac{2\sqrt{2}}{5\sqrt{2}} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \right\}$ is M1A1
	<b>Note</b>	The 2 <sup>nd</sup> dM1 in part (f) can be implied for at least 2 (out of 6) correct x, y, z ordinates from their values of $\lambda$ .
	<b>Note</b>	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.
	<b>CAREFUL</b>	<div>Putting <math>l_2</math> equal to A gives</div> <div> <math display="block">\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = \frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}</math> </div> <div>Give M0 dM0 for finding and using <math>\lambda = \frac{2}{5}</math> from this incorrect method.</div>
	<b>CAREFUL</b>	<div>Putting <math>\lambda \mathbf{d}_2 = \overline{AP}</math> gives</div> <div> <math display="block">\lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = -\frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}</math> </div> <div>Give M0 dM0 for finding and using <math>\lambda = -\frac{2}{5}</math> from this incorrect method.</div>
	<b>General</b>	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1
	<b>General</b>	You can follow through their $\mathbf{a}_2$ in part (b) for (d) M1dM1, (f) M1dM1.