Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Core Mathematics C4

Advanced

Friday 22 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
6666/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. (a) Find the binomial series expansion of

$$\sqrt{4-9x}, \quad |x| < \frac{4}{9}$$

in ascending powers of x, up to and including the term in x^2
Give each coefficient in its simplest form.

(5)

(b) Use the expansion from part (a), with a suitable value of x, to find an approximate value for $\sqrt{310}$
Show all your working and give your answer to 3 decimal places.

(3)

Question Number	Scheme	Notes	Marks
1. (a)	$\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$		
	$=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$	see notes	
	$=2-\frac{9}{4}x;-\frac{81}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\sqrt{310}=10\sqrt{3.1}=10\sqrt{(4-9(0.1))}$, so $x=0.1$	E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x=0.1$ (or uses $x=0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0	B1
	When $x=0.1$ $\sqrt{(4-9x)}\approx 2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2+\ldots$	Substitutes their *x*, where $\lvert x\rvert<\frac{4}{9}$ into all three terms of their binomial expansion	M1
	$=2-0.225-0.01265625=1.76234375$		
	So, $\sqrt{310}\approx 17.6234375=\underline{17.623}$ (3 dp)	17.623 **cao**	A1 **cao**
	Note: the calculator value of $\sqrt{310}$ is $17.60681686...$ which is 17.607 to 3 decimal places		**[3]**
			8 marks
	Question 1 Notes		

1. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion
	M1	Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where *k* is a numerical value and **where** $k\neq 1$
	A1ft	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx)
	Note	(kx), $k\neq 1$must be consistent (on the RHS, not necessarily on the LHS) in their expansion
	Note	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(-9x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$ because (kx) is not consistent
	Note	<u>**Incorrect bracketing:**</u> $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x^2}{4}\right)+\ldots\right]$ is B1M1A0 unless recovered
	A1	$2-\frac{9}{4}x$ **(simplified fractions) or** allow $2-2.25x$ or $2-2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$

		Question 1 Notes Continued
1. (a) ctd.	**SC**	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 (i.e. scores A0A0 in the final two marks to (a)) then **allow Special Case 2nd A1 for either** **SC:** $2\left[1-\frac{9}{8}x;...\right]$ or **SC:** $2\left[1+...-\frac{81}{128}x^2+...\right]$ or **SC:** $\lambda\left[1-\frac{9}{8}x-\frac{81}{128}x^2+...\right]$ or **SC:** $\left[\lambda-\frac{9\lambda}{8}x-\frac{81\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $\left[.....\right]$ is a simplified fraction or a decimal, **OR SC:** for $2-\frac{18}{8}x-\frac{162}{128}x^2+...$ (i.e. for not simplifying their correct coefficients)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$, where $k=\frac{9}{4}$ and not $-\frac{9}{4}$ and achieve $2+\frac{9}{4}x;-\frac{81}{64}x^2+...$ will get B1M1A1A0A1
	Note	**Ignore** extra terms beyond the term in x^2
	Note	You can ignore subsequent working following a correct answer
	Note	Allow B1M1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$
	Note	Allow B1M1A1A1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]=2-\frac{9}{4}x-\frac{81}{64}x^2+...$
(b)	**Note**	Give B1 M1 for $\sqrt{310}\approx10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2\right)$
	Note	**Other alternative suitable values for *x* for** $\sqrt{310}\approx\beta\sqrt{4-9(\text{their }x)}$

b	x	Estimate	b	x	Estimate
7	$-\frac{38}{147}$	17.479	14	$\frac{79}{294}$	18.256
8	$-\frac{3}{32}$	17.599	15	$\frac{118}{405}$	18.555
9	$\frac{14}{729}$	17.607	16	$\frac{119}{384}$	18.899
10	$\frac{1}{10}$	17.623	17	$\frac{94}{289}$	19.283
11	$\frac{58}{363}$	17.690	18	$\frac{493}{1458}$	19.701
12	$\frac{133}{648}$	17.819	19	$\frac{126}{361}$	20.150
13	$\frac{122}{507}$	18.009	20	$\frac{43}{120}$	20.625

	Note	Apply the scheme in the same way for their β and their x E.g. Give B1 M1 A1 for $\sqrt{310}\approx12\left(2-\frac{9}{4}\left(\frac{133}{648}\right)-\frac{81}{64}\left(\frac{133}{648}\right)^2\right)=17.819$ (3 dp)
	Note	Allow B1 M1 A1 for $\sqrt{310}\approx100\left(2-\frac{9}{4}(0.441)-\frac{81}{64}(0.441)^2\right)=76.161$ (3 dp)
	Note	Give B1 M1 A0 for $\sqrt{310}\approx10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2-\frac{729}{512}(0.1)^3\right)=17.609$ (3 dp)

	Question 1 Notes Continued	
1. (b)	**Note**	***Send to review*** using $\beta = \sqrt{155}$ and $x = \frac{2}{9}$ (which gives 17.897 (3 dp))
	Note	***Send to review*** using $\beta = \sqrt{1000}$ and $x = 0.41$ (which gives 27.346 (3 dp))

1. (a) **Alt 1**	**Alternative method 1:** Candidates can apply an alternative form of the binomial expansion $\left\{(4-9x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(-9x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(-9x)^2$	
	B1	$(4)^{\frac{1}{2}}$ or 2
	M1	Any two of three (un-simplified) terms correct
	A1	All three (un-simplified) terms correct
	A1	$2 - \frac{9}{4}x$ **(simplified fractions)** or allow $2 - 2.25x$ or $2 - 2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$
	Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(-9x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(-9x)^2$ without further working is B0M0A0

1. (a)	**Alternative Method 2: Maclaurin Expansion** $f(x) = (4-9x)^{\frac{1}{2}}$		
	$f''(x) = -\frac{81}{4}(4-9x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1
	$f'(x) = \frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	$\pm a(4-9x)^{-\frac{1}{2}}$; $a \neq \pm 1$	M1
		$\frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	A1 oe
	$\left\{\therefore f(0) = 2,\ f'(0) = -\frac{9}{4} \text{ and } f''(0) = -\frac{81}{32}\right\}$		
	So, $f(x) = 2 - \frac{9}{4}x; - \frac{81}{64}x^2 + \ldots$		A1; A1

2. The curve C has equation

$$x^2 + xy + y^2 - 4x - 5y + 1 = 0$$

(a) Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

(b) Find the x coordinates of the two points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Give exact answers in their simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Notes	Marks
2.	$x^2 + xy + y^2 - 4x - 5y + 1 = 0$		
(a)	$\left\{\frac{\xcancel{\mathrm{d}y}}{\xcancel{\mathrm{d}x}} \xcancel{=}\right\}\quad \underline{2x} + \underline{\underline{\left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right)}}\ \underline{+ 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 4 - 5\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$		M1$\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$2x + y - 4 + (x + 2y - 5)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$		dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]
(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ 2x + y - 4 = 0$		M1
	$\{y = 4 - 2x \Rightarrow\}\ x^2 + x(4 - 2x) + (4 - 2x)^2 - 4x - 5(4 - 2x) + 1 = 0$		dM1
	$x^2 + 4x - 2x^2 + 16 - 16x + 4x^2 - 4x - 20 + 10x + 1 = 0$		
	gives $3x^2 - 6x - 3 = 0$ **or** $3x^2 - 6x = 3$ **or** $x^2 - 2x - 1 = 0$	Correct 3TQ in terms of x	A1
	$(x - 1)^2 - 1 - 1 = 0$ and $x = ...$	Method mark for solving a 3TQ in x	ddM1
	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only	A1
			[5]
(b) **Alt 1**	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ 2x + y - 4 = 0$		M1
	$\left\{x = \frac{4 - y}{2} \Rightarrow\right\}\quad \left(\frac{4 - y}{2}\right)^2 + \left(\frac{4 - y}{2}\right)y + y^2 - 4\left(\frac{4 - y}{2}\right) - 5y + 1 = 0$		dM1
	$\left(\frac{16 - 8y + y^2}{2}\right) + \left(\frac{4y - y^2}{2}\right) + y^2 - 2(4 - y) - 5y + 1 = 0$		
	gives $3y^2 - 12y - 12 = 0$ **or** $3y^2 - 12y = 12$ **or** $y^2 - 4y - 4 = 0$	Correct 3TQ in terms of y	A1
	$(y - 2)^2 - 4 - 4 = 0$ and $y = ...$ $x = \frac{4 - (2 + 2\sqrt{2})}{2}$, $x = \frac{4 - (2 - 2\sqrt{2})}{2}$	Solves a 3TQ in y **and finds at least one value for** x	ddM1
	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only	A1
			[5]
			10
(a) **Alt 1**	$\left\{\frac{\xcancel{\mathrm{d}x}}{\xcancel{\mathrm{d}y}} \xcancel{=}\right\}\quad \underline{2x\frac{\mathrm{d}x}{\mathrm{d}y}} + \underline{\underline{\left(y\frac{\mathrm{d}x}{\mathrm{d}y} + x\right)}}\ \underline{+ 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5} = \underline{0}$		M1$\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$x + 2y - 5 + (2x + y - 4)\frac{\mathrm{d}x}{\mathrm{d}y} = 0$		dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]

		Question 2 Notes
2. (a)	**M1**	Differentiates implicitly to include either $x\frac{dy}{dx}$ or $y^2 \to 2y\frac{dy}{dx}$ or $-5y \to -5\frac{dy}{dx}$. $\left(\text{Ignore } \frac{dy}{dx} = ...\right)$
	A1	$x^2 \to 2x$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx} = 0$
	B1	$xy \to y + x\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0
	Note	$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx} \;\to\; 2x + y - 4 = -x\frac{dy}{dx} - 2y\frac{dy}{dx} + 5\frac{dy}{dx}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are **at least two terms** in $\frac{dy}{dx}$.
	A1 **cso**	$\frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(b)	**M1** **Note** **Note**	Sets the numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e. This mark can also be gained by setting $\frac{dy}{dx}$ equal to zero in their differentiated equation from (a) **If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	dM1	**dependent on the previous M mark** Substitutes their *x* or their *y* (from their numerator = 0) into the printed equation to give an equation in one variable only
	A1 **Note**	For obtaining the correct 3TQ. E.g.: either $3x^2 - 6x - 3\,\{=0\}$ or $-3x^2 + 6x + 3\,\{=0\}$ This mark can also be awarded for a correct 3 term equation. E.g. either $3x^2 - 6x = 3$ $x^2 - 2x - 1 = 0$ or $x^2 = 2x + 1$ are all fine for A1
	ddM1	**dependent on the previous 2 M marks** See page 6: Method mark for solving THEIR 3-term quadratic in one variable **<u>Quadratic Equation to solve:</u>** $3x^2 - 6x - 3 = 0$ **<u>Way 1:</u>** $x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-3)}}{2(3)}$ **<u>Way 2:</u>** $x^2 - 2x - 1 = 0 \Rightarrow (x-1)^2 - 1 - 1 = 0 \Rightarrow x = ...$ **<u>Way 3:</u>** Or writes down at least one ***exact*** correct *x*-root **(*or one correct x-root to 2 dp*)** from ***their*** quadratic equation. This is usually found on their calculator. **<u>Way 4:</u> (Only allowed if their 3TQ can be factorised)** • $(x^2 + bx + c) = (x + p)(x + q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to $x = ...$ • $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = a$, leading to $x = ...$
	Note	If a candidate applies ***the alternative method*** then they also need to use their $x = \frac{4 - y}{2}$ to find **at least one value** for *x* in order to gain the final M mark.
	A1	Exact values of $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ (or $1 \pm \sqrt{2}$) , **cao** Apply isw if *y*-values are also found.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 5 marks in part (b)

		Question 2 Notes
2. (a) **Alt 1**	**M1**	Differentiates implicitly to include either $y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-4x \to -4\frac{\mathrm{d}x}{\mathrm{d}y}$. $\left(\text{Ignore } \frac{\mathrm{d}x}{\mathrm{d}y} = ...\right)$
	A1	$x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 = 0$
	B1	$xy \to y\frac{\mathrm{d}x}{\mathrm{d}y} + x$
	Note	If an extra term appears then award 1st A0
	Note	$2x\frac{\mathrm{d}x}{\mathrm{d}y} + y\frac{\mathrm{d}x}{\mathrm{d}y} + x + 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 \;\to\; x + 2y - 5 = -2x\frac{\mathrm{d}x}{\mathrm{d}y} - y\frac{\mathrm{d}x}{\mathrm{d}y} + 4\frac{\mathrm{d}x}{\mathrm{d}y}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are **at least two terms** in $\frac{\mathrm{d}x}{\mathrm{d}y}$
	A1 **cso**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(a)	**Note**	Writing down ***from no working*** • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ scores M1 A1 B1 M1 A1 • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{x + 2y - 5}$ scores M1 A0 B1 M1 A0
	Note	Writing $2x\,\mathrm{d}x + y\,\mathrm{d}x + x\,\mathrm{d}y + 2y\,\mathrm{d}y - 4\mathrm{d}x - 5\mathrm{d}y = 0$ scores M1 A1 B1

3. (i) Given that

$$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x, \quad x > -\frac{1}{2}$$

(3)

(ii) Find

$$\int (\mathrm{e}^x + 1)^3\,\mathrm{d}x$$

(3)

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x, \quad x > 0$$

(4)

Question Number	Scheme	Notes	Marks
3. (i)	$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$		
(a)	$B=6,\ C=1$	At least one of $B=6$ or $C=1$	B1
		Both $B=6$ and $C=1$	B1
	$13-4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ $x=-3 \Rightarrow 25 = 25C \Rightarrow C=1$ $x=-\frac{1}{2} \Rightarrow 13--2 = \frac{5}{2}B \Rightarrow 15=2.5B \Rightarrow B=6$	Writes down a correct identity and attempts to find the value of either one of A or B or C	M1
	Either $x^2: 0=2A+4C$, constant: $13=3A+3B+C$, $x: -4=7A+B+4C$ or $x=0 \Rightarrow 13=3A+3B+C$ leading to $A=-2$	Using a correct identity to find $A=-2$	A1
			[4]
(b)	$\int \frac{13-4x}{(2x+1)^2(x+3)}\,dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,dx$		
	$= \frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ o.e. $\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\}$	See notes	M1
		At least two terms correctly integrated	A1ft
		Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line Ignore the absence of '$+c$'	A1
			[3]
(ii)	$\{(e^x+1)^3 =\}\ e^{3x}+3e^{2x}+3e^x+1$	$e^{3x}+3e^{2x}+3e^x+1$, simplified or un-simplified	B1
	$\left\{\int (e^x+1)^3\,dx\right\} = \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	At least 3 examples (see notes) of correct ft integration	M1
		$\frac{1}{3}e^{3x}+\frac{3}{2}e^{2x}+3e^x+x$, simplified or un-simplified with or without $+c$	A1
			[3]
(iii)	$\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx,\ x>0;\ u^3=x$		
	$3u^2\frac{du}{dx}=1$	$3u^2\frac{du}{dx}=1$ or $\frac{dx}{du}=3u^2$ or $\frac{du}{dx}=\frac{1}{3}x^{-\frac{2}{3}}$ or $3u^2du=dx$ o.e.	B1
	$=\int \frac{1}{4u^3+5u}.3u^2\,du\ \left\{=\int\frac{3u}{4u^2+5}\,du\right\}$	Expression of the form $\int \frac{\pm ku^2}{4u^3\pm5u}\{du\}$, $k\neq0$ Does not have to include integral sign or du Can be implied by later working	M1
	$=\frac{3}{8}\ln(4u^2+5)\ \{+c\}$	**dependent on the previous M mark** $\pm\lambda\ln(4u^2+5)$; λ is a constant; $\lambda\neq0$	dM1
	$=\frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	Correct answer in x with or without $+c$	A1
			[4]
			14

	Question 3 Notes		
3. (iii) **Alt 1**	**Alternative method 1 for part (iii)**		
	$\left\{\int \frac{1}{4x+5x^{\frac{1}{3}}}\,\mathrm{d}x\right\} = \int \frac{x^{-\frac{1}{3}}}{4x^{\frac{2}{3}}+5}\,\mathrm{d}x$	Attempts to multiply numerator and denominator by $x^{-\frac{1}{3}}$	M1
		Expression of the form $\int \frac{\pm kx^{-\frac{1}{3}}}{4x^{\frac{2}{3}}\pm 5}\mathrm{d}x,\ k \neq 0$ Does not have to include integral sign or $\mathrm{d}u$ Can be implied by later working	M1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\{+c\}$	$\pm\lambda\ln(4x^{\frac{2}{3}}+5)$; λ is a constant; $\lambda \neq 0$	dM1
		Correct answer in x with or without $+c$	A1
			[4]

3. (i) (a)	**M1**	Writes down ***a correct identity*** (although this can be implied) and attempts ***to find the value of at least one*** of either A or B or C. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients.
	Note	The correct partial fraction from no working scores B1B1M1A1
(i) (b)	**M1**	At least 2 of either $\pm\frac{P}{(2x+1)} \to \pm D\ln(2x+1)$ or $\pm D\ln(x+\frac{1}{2})$ or $\pm\frac{Q}{(2x+1)^2} \to \pm E(2x+1)^{-1}$ or $\pm\frac{R}{(x+3)} \to \pm F\ln(x+3)$ for their constants P, Q, R.
	A1ft **Note**	At least two terms from any of $\pm\frac{P}{(2x+1)}$ or $\pm\frac{Q}{(2x+1)^2}$ or $\pm\frac{R}{(x+3)}$ correctly integrated. Can be un-simplified for the A1ft mark.
	A1	Correct answer of $\frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)$ $\{+c\}$ simplified or un-simplified. with or without '+ c'.
	Note	Allow final A1 for equivalent answers, e.g. $\ln\left(\frac{x+3}{2x+1}\right) - \frac{3}{2x+1}\{+c\}$ or $\ln\left(\frac{2x+6}{2x+1}\right) - \frac{3}{2x+1}\{+c\}$
	Note	**Beware that** $\int \frac{-2}{(2x+1)}\,\mathrm{d}x = \int \frac{-1}{(x+\frac{1}{2})}\,\mathrm{d}x = -\ln(x+\frac{1}{2})\{+c\}$ is correct integration
	Note	E.g. Allow M1 A1ft A1 for a correct un-simplified $\ln(x+3) - \ln(x+\frac{1}{2}) - \frac{3}{2}(x+\frac{1}{2})^{-1}\{+c\}$
	Note	Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $-\ln 2x+1 - 3(2x+1)^{-1} + \ln x+3\{+c\}$ unless recovered
(ii)	**Note**	Give B1 for an un-simplified $e^{3x} + 2e^{2x} + e^{2x} + 2e^{x} + e^{x} + 1$
	M1	At least 3 of either $ae^{3x} \to \frac{a}{3}e^{3x}$ **or** $be^{2x} \to \frac{b}{2}e^{2x}$ **or** $de^{x} \to de^{x}$ **or** $\mu \to \mu x$; $\alpha, \beta, \delta, \mu \neq 0$
	Note	Give A1 for an un-simplified $\frac{1}{3}e^{3x} + e^{2x} + \frac{1}{2}e^{2x} + 2e^{x} + e^{x} + x$, with or without $+c$
(iii)	**Note**	1st M1 can be implied by $\int \frac{\pm ku}{4u^2 \pm 5}\{\mathrm{d}u\},\ k \neq 0$. Does not have to include integral sign or $\mathrm{d}u$
	Note	Condone 1st M1 for expressions of the form $\int\left(\frac{\pm 1}{4u^3 \pm 5u}\cdot\frac{\pm k}{u^{-2}}\right)\{\mathrm{d}u\},\ k \neq 0$
	Note	Give 2nd M0 for $\frac{3u}{8u}\ln(4u^2+5)\{+c\}$ (u's not cancelled) unless recovered in later working
	Note	E.g. Give 2nd M0 for integration leading to $\frac{3}{4}u\ln(4u^2+5)$ as this is not in the form $\pm\lambda\ln(4u^2+5)$

	Note	Condone 2nd M1 for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $\frac{3}{8}\ln 4x^{\frac{2}{3}}+5\ \{+c\}$ unless recovered

Question Number	Scheme	Notes	Marks
3. (ii) **Alt 1**	$\int(\mathrm{e}^x+1)^3\,\mathrm{d}x;\quad u=\mathrm{e}^x+1\ \Rightarrow\ \frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$		
	$\left\{=\int\frac{u^3}{(u-1)}\mathrm{d}u\ =\right\}\int\left(u^2+u+1+\frac{1}{u-1}\right)\mathrm{d}u$	$\int\left(u^2+u+1+\frac{1}{u-1}\right)\{\mathrm{d}u\}$ where $u=\mathrm{e}^x+1$	B1
	$=\frac{1}{3}u^3+\frac{1}{2}u^2+u+\ln(u-1)\ \{+c\}$	At least 3 of either $\alpha u^2\to\frac{\alpha}{3}u^3$ **or** $\beta u\to\frac{\beta}{2}u^2$ **or** $\delta\to\delta u$ **or** $\frac{\lambda}{u-1}\to\lambda\ln(u-1)$; $\alpha,\beta,\delta,\lambda\neq 0$	M1
	$=\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+\ln(\mathrm{e}^x+1-1)\ \{+c\}$		
	$=\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+x\ \{+c\}$	$\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+x$ or $\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+\mathrm{e}^x+x$ simplified or un-simplified with or without $+c$ **Note:** $\ln(\mathrm{e}^x+1-1)$ needs to be simplified to x for this mark	A1
			[3]
3. (ii) **Alt 2**	$\int(\mathrm{e}^x+1)^3\,\mathrm{d}x;\quad u=\mathrm{e}^x\ \Rightarrow\ \frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$		
	$\left\{=\int\frac{(u+1)^3}{u}\mathrm{d}u\ =\right\}\int\left(u^2+3u+3+\frac{1}{u}\right)\mathrm{d}u$	$\int\left(u^2+3u+3+\frac{1}{u}\right)\{\mathrm{d}u\}$ where $u=\mathrm{e}^x$	B1
	$=\frac{1}{3}u^3+\frac{3}{2}u^2+3u+\ln u\ \{+c\}$	At least 3 of either $\alpha u^2\to\frac{\alpha}{3}u^3$ **or** $\beta u\to\frac{\beta}{2}u^2$ **or** $\delta\to\delta u$ **or** $\frac{\lambda}{u}\to\lambda\ln u$; $\alpha,\beta,\delta,\lambda\neq 0$	M1
	$=\frac{1}{3}\mathrm{e}^{3x}+\frac{3}{2}\mathrm{e}^{2x}+3\mathrm{e}^x+x\ \{+c\}$	$\frac{1}{3}\mathrm{e}^{3x}+\frac{3}{2}\mathrm{e}^{2x}+3\mathrm{e}^x+x$, simplified or un-simplified with or without $+c$ **Note:** $\ln(\mathrm{e}^x)$ needs to be simplified to x for this mark	A1
			[3]

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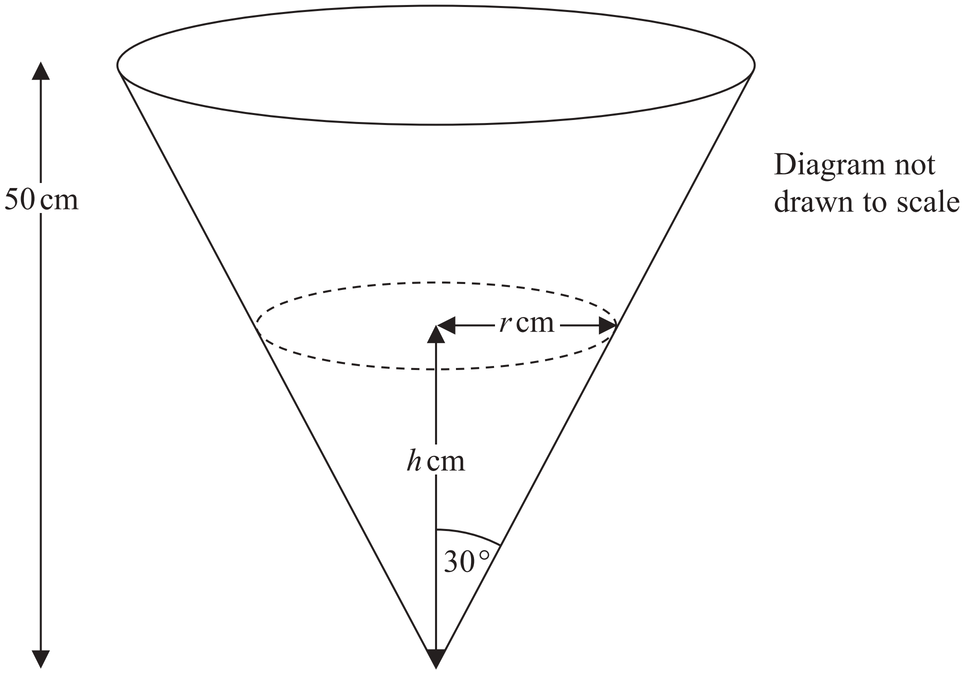

Figure 1

A water container is made in the shape of a hollow inverted right circular cone with semi-vertical angle of 30°, as shown in Figure 1. The height of the container is 50 cm.

When the depth of the water in the container is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(2)

Given that the volume of water in the container increases at a constant rate of 200 cm^3 s^{-1},

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 15$
Give your answer in its simplest form in terms of π.

(4)

Question Number	Scheme	Notes	Marks
4. (a)	$\frac{r}{h}=\tan 30 \Rightarrow r=h\tan 30 \left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{h}{r}=\tan 60 \Rightarrow r=\frac{h}{\tan 60} \left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{r}{\sin 30}=\frac{h}{\sin 60} \Rightarrow r=\frac{h\sin 30}{\sin 60} \left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $h^2+r^2=(2r)^2 \Rightarrow r^2=\frac{1}{3}h^2$	Correct use of trigonometry to find r in terms of h **or** correct use of Pythagoras to find r^2 in terms of h^2	M1
	$\left\{V=\frac{1}{3}\pi r^2 h \Rightarrow\right\} V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V=\frac{1}{9}\pi h^3$ *	Correct proof of $V=\frac{1}{9}\pi h^3$ or $V=\frac{1}{9}h^3\pi$ Or shows $\frac{1}{9}\pi h^3$ or $\frac{1}{9}h^3\pi$ with some reference to $V=$ in their solution	A1 *
			[2]
(b) **Way 1**	$\frac{\mathrm{d}V}{\mathrm{d}t}=200$		
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi h^2$	$\frac{1}{3}\pi h^2$ o.e.	B1
	Either • $\left\{\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\} \left(\frac{1}{3}\pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}=200$ • $\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}\Rightarrow\right\} \frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi h^2}$	**either** $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=200$ **or** $200\div\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$	M1
	When $h=15,\ \frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi(15)^2} \left\{=\frac{200}{75\pi}=\frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{8}{3p}$ (cm s^{-1})	$\frac{8}{3p}$	A1 **cao**
			[4]
			6
(b) **Way 2**	$\frac{\mathrm{d}V}{\mathrm{d}t}=200 \Rightarrow V=200t+c \Rightarrow \frac{1}{9}\pi h^3=200t+c$		
	$\left(\frac{1}{3}\pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}=200$	$\frac{1}{3}\pi h^2$ o.e. as in Way 1	B1 M1
	When $h=15,\ \frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi(15)^2} \left\{=\frac{200}{75\pi}=\frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{8}{3p}$ (cm s^{-1})	$\frac{8}{3p}$	A1 **cao**
			[4]

	Question 4 Notes	
4. (a)	**Note**	Allow M1 for writing down $r = h\tan 30$
	Note	Give M0 A0 for writing down $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ with no evidence of using trigonometry on *r* and *h* or Pythagoras on *r* and *h*
	Note	Give M0 (unless recovered) for evidence of $\frac{1}{3}\pi r^2 h = \frac{1}{9}\pi h^3$ leading to either $r^2 = \frac{1}{3}h^2$ or $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$
(b)	**B1** **Note**	Correct simplified or un-simplified differentiation of *V*. E.g. $\frac{1}{3}\pi h^2$ or $\frac{3}{9}\pi h^2$ $\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their *V*
	M1	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 200$ or $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$
	dM1	**dependent on the previous M mark** Substitutes $h = 15$ into an expression ***which is a result*** of either $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ or $200 \times \frac{1}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$
	A1	$\frac{8}{3p}$ (units are not required)
	Note	Give final A0 for using $\frac{\mathrm{d}V}{\mathrm{d}t} = -200$ to give $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{8}{3\pi}$, unless recovered to $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8}{3\pi}$

5.

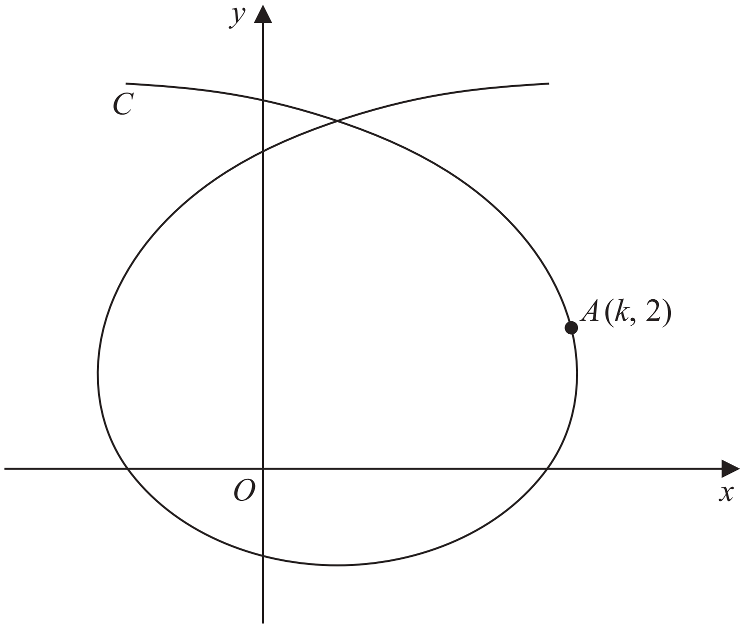

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 1 + t - 5\sin t, \quad y = 2 - 4\cos t, \quad -\pi \leqslant t \leqslant \pi$$

The point A lies on the curve C.

Given that the coordinates of A are $(k, 2)$, where $k > 0$

(a) find the exact value of k, giving your answer in a fully simplified form.

(2)

(b) Find the equation of the tangent to C at the point A.
Give your answer in the form $y = px + q$, where p and q are exact real values.

(5)

Question Number	Scheme	Notes	Marks
5.	$x=1+t-5\sin t,\ y=2-4\cos t,\ -\pi \leqslant t \leqslant \pi;\ A(k,\ 2),\ k>0$, lies on C		
(a)	{When $y=2$,} $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2},\ \frac{\pi}{2}$ $k(\text{or } x)=1+\frac{\pi}{2}-5\sin\left(\frac{\pi}{2}\right)$ **or** $k(\text{or } x)=1-\frac{\pi}{2}-5\sin\left(-\frac{\pi}{2}\right)$	Sets $y=2$ to find t and some evidence of using their t to find $x=...$	M1
	$\left\{\text{When } t=-\frac{\pi}{2},\ k>0,\right\}$ so $k=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	k (or x) $=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt}=1\text{-}\ 5\cos t,\quad \frac{dy}{dt}=4\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied)	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied)	B1
	$\frac{dy}{dx}=\frac{4\sin t}{1-5\cos t}$ at $t=-\frac{\pi}{2},\ \frac{dy}{dx}=\frac{4\sin\left(-\frac{\pi}{2}\right)}{1-5\cos\left(-\frac{\pi}{2}\right)}\quad \{=-4\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$ **Note:** their t can lie outside $-\pi \leqslant t \leqslant \pi$ for this mark	M1
	• $y-2=-4\left(x-\left(6-\frac{\pi}{2}\right)\right)$ • $2=(-4)\left(6-\frac{\pi}{2}\right)+c \Rightarrow\ y=-4x+2+4\left(6-\frac{\pi}{2}\right)$	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found using calculus **Note:** their k (or x) must be in terms of π and correct bracketing must be used or implied	M1
	$\{y-2=-4x+24-2\pi \Rightarrow\}\quad y=-4x+26-2\pi$	**dependent on all previous marks in part (b)** $y=-4x+26-2\pi$	A1 **cso**
		$(p=-4,\ q=26-2\pi)$	**[5]**
			7

	Question 5 Notes	
5. (a)	**Note**	M1 can be implied by either x or $k=6-\frac{\pi}{2}$ or awrt 4.43 or x or $k=\frac{\pi}{2}-4$ or awrt -2.43
	Note	An answer of 4.429… without reference to a correct **exact** answer is A0
	Note	M1 can be earned in part (a) by working in degrees
	Note	Give M0 for not substituting their t back into x. E.g. $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2} \Rightarrow k=-\frac{\pi}{2}$
	Note	If two values for k are found, they must identify the correct answer for A1
	Note	Condone M1 for $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2},\frac{\pi}{2} \Rightarrow x=1-\frac{\pi}{2}-5\sin\left(\frac{\pi}{2}\right)$
(b)	**Note**	The 1st M mark may be implied by their value for $\frac{dy}{dx}$ e.g. $\frac{dy}{dx}=\frac{4\sin t}{1-5\cos t}$, followed by an answer of -4 (from $t=-\frac{\pi}{2}$) or 4 (from $t=\frac{\pi}{2}$)
	Note	Give 1st M0 for **applying** their $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ even if they state $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$
	2nd M1	• **applies** $y-2=(\text{their } m_T)(x-(\text{their } k))$, • **applies** $2=(\text{their } m_T)(\text{their } k)+c$ **leading to** $y=(\text{their } m_T)x+(\text{their } c)$ where k must be in terms of π and $m_T\ (\neq m_N)$ is a numerical value found using calculus
	Note	Correct bracketing must be used for 2nd M1, but this mark can be implied by later working

	Question 5 Notes Continued	
5. (b)	**Note**	The final A mark is dependent on all previous marks in part (b) being scored. This is because the correct answer can follow from an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	The first 3 marks can be gained by using degrees in part (b)
	Note	Condone mixing a correct t with an incorrect x or an incorrect t with a correct x for the M marks
	Note	Allow final A1 for any answer in the form $y = px + q$ E.g. Allow final A1 for $y = -4x + 26 - 2\pi$, $y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ or $y = -4x + \left(\frac{52 - 4\pi}{2}\right)$
	Note	Do not apply isw in part (b). So, an incorrect answer following from a correct answer is A0
	Note	Do not allow $y = 2(-2x + 13 - \pi)$ for A1
	Note	$y = -4x + 26 - 2\pi$ followed by $y = 2(-2x + 13 - \pi)$ is condoned for final A1

6. Given that $y = 2$ when $x = -\frac{\pi}{8}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \qquad -\frac{1}{2} < x < \frac{1}{2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

Question Number	Scheme	Notes	Marks
6.	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x};\ -\frac{1}{2} < x < \frac{1}{2};\ y = 2$ at $x = -\frac{\pi}{8}$		
	$\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3\cos^2 2x}\,\mathrm{d}x$	Separates variables as shown Can be implied by a correct attempt at integration Ignore the integral signs	B1
	$\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$		
	$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)\{+c\}$	$\pm\frac{A}{y^2} \to \pm\frac{B}{y};\ A, B \neq 0$	M1
		$\pm\lambda\tan 2x$	M1
		$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$	A1
	$-\frac{1}{2} = \frac{1}{6}\tan\left(2\left(-\frac{\pi}{8}\right)\right) + c$	Use of $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated equation ***containing a constant of integration***, e.g. c	M1
	$-\frac{1}{2} = -\frac{1}{6} + c \Rightarrow c = -\frac{1}{3}$		
	$-\frac{1}{y} = \frac{1}{6}\tan 2x - \frac{1}{3} = \frac{\tan(2x) - 2}{6}$		
	$y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ or $y = \frac{6\cot 2x}{-1 + 2\cot 2x}$ $\left\{-\frac{1}{2} < x < \frac{1}{2}\right\}$		A1 o.e.
			[6]
			6

		Question 6 Notes
6.	**B1**	Separates variables as shown. $\mathrm{d}y$ and $\mathrm{d}x$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number "3" may appear on either side. E.g. $\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ or $\int \frac{3}{y^2}\,\mathrm{d}y = \int \frac{1}{\cos^2 2x}\,\mathrm{d}x$ are fine for B1
	Note	Allow e.g. $\int \frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ for B1 or condone $\int \frac{1}{y^2} = \int \frac{1}{3}\sec^2 2x$ for B1
	Note	B1 can be implied by correct integration of both sides
	M1	$\pm\frac{A}{y^2} \to \pm\frac{B}{y};\ A, B \neq 0$
	M1	$\frac{1}{\cos^2 2x}$ or $\sec^2 2x \to \pm\lambda\tan 2x;\ \lambda \neq 0$
	A1	$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$ with or without $'+c'$. E.g. $-\frac{6}{y} = \tan 2x$
	M1	Evidence of using both $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing c
	Note	This mark can be implied by the correct value of c
	Note	You may need to use your calculator to check that they have satisfied the final M mark
	Note	Condone using $x = \frac{\pi}{8}$ instead of $x = -\frac{\pi}{8}$
	A1	$y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ **or any equivalent correct answer in the form** $y = \mathrm{f}(x)$
	Note	You can ignore subsequent working, which follows from a correct answer

	Question 6 Notes Continued	
6.	**Note**	Writing $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}y^2 \sec^2 2x$ leading to e.g. • $y = \frac{1}{9}y^3\left(\frac{1}{2}\tan 2x\right)$ gets 2nd M0 for $\pm\lambda\tan 2x$ • $u = \frac{1}{3}y^2,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{3}y,\ v = \frac{1}{2}\tan 2x$ gets 2nd M0 for $\pm\lambda\tan 2x$ because the variables have not been separated

7. The point A with coordinates $(-3, 7, 2)$ lies on a line l_1
The point B also lies on the line l_1

Given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$,

(a) find the coordinates of point B.
(2)

The point P has coordinates $(9, 1, 8)$

(b) Find the cosine of the angle PAB, giving your answer as a simplified surd.
(3)

(c) Find the exact area of triangle PAB, giving your answer in its simplest form.
(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2
(2)

The point Q lies on the line l_2

Given that the line segment AP is perpendicular to the line segment BQ,

(e) find the coordinates of the point Q.
(5)

Question Number	Scheme	Notes	Marks
7.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta=$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(a)	$\left\{\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}\Rightarrow\right\}$	Attempts to add $\overrightarrow{OA}$ to $\overrightarrow{AB}$	M1
	$\overrightarrow{OB}=\begin{pmatrix}-3\\7\\2\end{pmatrix}+\begin{pmatrix}4\\-6\\2\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix}\Rightarrow B(1,1,4)$	$(1,1,4)$ or $\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\mathbf{i}+\mathbf{j}+4\mathbf{k}$	A1
	Note: M1 can be implied by at least 2 correct components for B		**[2]**
(b)	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\left\{\cos\theta=\dfrac{\overrightarrow{AP}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert\lvert\overrightarrow{AB}\rvert}\right\}=\dfrac{\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors	dM1
	$\left\{\cos\theta=\dfrac{96}{\sqrt{216}.\sqrt{56}}\Rightarrow\cos\theta\right\}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(c)	$\left\{\cos\theta=\dfrac{4}{\sqrt{21}}\right\}\Rightarrow\sin\theta=\dfrac{\sqrt{21-16}}{\sqrt{21}}=\dfrac{\sqrt{5}}{\sqrt{21}}=\dfrac{\sqrt{105}}{21}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB=\dfrac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)$ $\left\{=12\sqrt{21}\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\right\}=12\sqrt{5}$	see notes	M1
		$12\sqrt{5}$	A1 **cao**
			[3]
(d)	$\{l_2:\}\ \mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$	$\mathbf{p}+\lambda\mathbf{d}$ or $\mathbf{p}+\mu\mathbf{d}$, $\mathbf{p}\neq0$, $\mathbf{d}\neq0$ with either $\mathbf{p}=9\mathbf{i}+\mathbf{j}+8\mathbf{k}$ or $\mathbf{d}=4\mathbf{i}-6\mathbf{j}+2\mathbf{k}$ or $\mathbf{d}=$ multiple of $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$	M1
		Correct vector equation	A1
			[2]
(e)	$\overrightarrow{BQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-4\mu\\6\mu\\-4-2\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow96+48\mu+36\mu+24+12\mu=0\Rightarrow96\mu+120=0\Rightarrow\mu=-\dfrac{5}{4}$	$\mu=-\dfrac{120}{96}$ or $\mu=-\dfrac{5}{4}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+4(-1.25)\\1-6(-1.25)\\8+2(-1.25)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4,8.5,5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4,8.5,5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
			15

Question Number	Scheme	Notes	Marks
7.	$\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}$, $\overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}$, $\overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}$; $\overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$	Let $\theta=$ size of angle PAB. A, B lie on l_1 and P lies on l_2	
(e) **Alt 1**	$\overrightarrow{BQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-2\mu\\3\mu\\-4-\mu\end{pmatrix}\right\}$	Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$	M1
	$\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$	Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for μ	dM1
	$\Rightarrow 96+24\mu+18\mu+24+6\mu=0\Rightarrow 48\mu+120=0\Rightarrow\mu=-\frac{5}{2}$	$\mu=-\frac{5}{2}$	A1 o.e.
	$\overrightarrow{OQ}=\begin{pmatrix}9+2(-2.5)\\1-3(-2.5)\\8+1(-2.5)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4,8.5,5.5)$	Substitutes their value of μ into $\overrightarrow{OQ}$	ddM1
		$(4, 8.5, 5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$	A1 o.e.
			[5]
(b) **Alt 1**	**<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	$\mathbf{d}_1\times\mathbf{d}_2=\underline{\begin{pmatrix}12\\-6\\6\end{pmatrix}\times\begin{pmatrix}4\\-6\\2\end{pmatrix}}=\left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\12&-6&6\\4&-6&2\end{vmatrix}=24\mathbf{i}+0\mathbf{j}-48\mathbf{k}\right\}$		
	$\sin\theta=\dfrac{\sqrt{(24)^2+(0)^2+(-48)^2}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$	Applies vector cross product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors	dM1
	$\left\{\sin\theta=\dfrac{\sqrt{2880}}{\sqrt{216}.\sqrt{56}}=\sqrt{\dfrac{5}{21}}\right\}\left\{\Rightarrow\cos\theta\right\}=\sqrt{\dfrac{16}{21}}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]
(b) **Alt 2**	**<u>Cosine Rule</u>**		
	$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$	An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$	M1
	Note: $\left\lvert\overrightarrow{PA}\right\rvert=\sqrt{216}$, $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{56}$ and $\left\lvert\overrightarrow{PB}\right\rvert=\sqrt{80}$		
	$\left(\sqrt{80}\right)^2=\left(\sqrt{216}\right)^2+\left(\sqrt{56}\right)^2-2\left(\sqrt{216}\right)\left(\sqrt{56}\right)\cos\theta$	Applies the cosine rule the correct way round	dM1
	$\cos\theta=\dfrac{216+56-80}{2\sqrt{216}\sqrt{56}}=\dfrac{192}{2\sqrt{216}\sqrt{56}}$		
	$\left\{\Rightarrow\cos\theta\right\}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$	$\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$	A1
			[3]

	Question 7 Notes	
7. (b)	**Note**	If no "subtraction" seen, you can award 1st M1 for 2 out of 3 correct components of the difference
	Note	For dM1 the dot product formula can be applied as $\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}\cos\theta=\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}$
	Note	***Evaluation*** of the dot product for $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is not required for the dM1 mark
	A1	For either $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$
	Note	Using $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{24+18+6}{\sqrt{216}.\sqrt{14}}=\frac{48}{12\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Using $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\frac{4+3+1}{\sqrt{6}.\sqrt{14}}=\frac{8}{2\sqrt{21}}=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Give M1M1A0 for finding $\theta=\text{awrt }29.2$ without reference to $\cos\theta=\underline{\underline{\frac{4}{\sqrt{21}}}}$ or $\underline{\underline{\frac{4}{21}\sqrt{21}}}$
	Note	Condone taking the dot product between vectors the wrong way round for the M1 dM1 marks
	Note	**Vectors the wrong way round**
		• E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ ***with no other working*** is final A0 • E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\frac{4}{\sqrt{21}}$ or $-\frac{4}{21}\sqrt{21}$ **followed by** $\cos\theta=\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ or just simply writing $\frac{4}{\sqrt{21}}$ or $\frac{4}{21}\sqrt{21}$ is final A1
	Note	In part (b), give M0dM0 for finding and using $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$
(c)	**Note**	Give 1st M0 for $\sin\theta=\sin\left(\cos^{-1}\left(\frac{4\sqrt{21}}{21}\right)\right)$ or $\sin\theta=1-\left(\frac{4}{21}\sqrt{21}\right)^2$ ***unless recovered***
	M1	Give 2nd M1 for either • $\frac{1}{2}(\text{their length }AP)(\text{their length }AB)(\text{their attempt at }\sin\theta)$ • $\frac{1}{2}(\text{their length }AP)(\text{their length }AB)\sin(\text{their }29.2^\circ\text{ from part (b)})$ • $\frac{1}{2}(\text{their length }AP)(\text{their length }AB)\sin\theta;$ where $\cos\theta=...$ in part (b)
	Note	$\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(\text{awrt }29.2^\circ\text{ or awrt }150.8^\circ)$ {= awrt 26.8} without reference to finding $\sin\theta$ as an exact value if M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to finding $\sin\theta$ as an exact value is M0 M1 A0
	Note	Anything that rounds to 26.8 without reference to $12\sqrt{5}$ is A0
	Note	If they use $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ in part (b), then this can be followed through in part (c) for the 2nd M mark as e.g. $\frac{1}{2}\left(\sqrt{110}\right)\left(\sqrt{56}\right)\sin\theta$
	Note	Finding $12\sqrt{5}$ in part (c) is M1 dM1 A1, even if there is little or no evidence of finding an exact value for $\sin\theta$. So $\frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(29.2^\circ)=12\sqrt{5}$ is M1 dM1 A1

	Question 7 Notes Continued	
7. (d)	**Note**	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is not required for the M mark
	A1	Writing $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ a multiple of $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$
	Note	Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is required for the A mark
	Note	Other valid $\mathbf{p} = \begin{pmatrix}9\\1\\8\end{pmatrix}$ are e.g. $\mathbf{p} = \begin{pmatrix}13\\-5\\10\end{pmatrix}$ or $\mathbf{p} = \begin{pmatrix}5\\7\\6\end{pmatrix}$. So $\mathbf{r} = \begin{pmatrix}13\\-5\\10\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ is M1 A1
	Note	Give A0 for writing $l_2 : \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or ans $= \begin{pmatrix}9\\1\\8\end{pmatrix} + \mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ unless recovered
	Note	Using scalar parameter λ or other scalar parameters (e.g. μ or *s* or *t*) is fine for M1 and/or A1
(e)	**ddM1**	Substitutes their value of μ into $\overrightarrow{OQ}$, where $\overrightarrow{OQ}$ = their equation for l_2
	Note	If they use $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ in part (b), then this can be followed through in part (e) for the 2nd M mark and the 3rd M mark
	Note	You imply the final M mark in part (e) for at least 2 correctly followed through components for *Q* from their μ

Question Number	Scheme	Notes	Marks
7. (c) **Alt 1**	**Vector Cross Product:** Use this scheme if a vector cross product method is being applied		
	$\overrightarrow{AP}\times\overrightarrow{AB} = \underline{\begin{pmatrix}12\\-6\\6\end{pmatrix}\times\begin{pmatrix}4\\-6\\2\end{pmatrix}} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\12&-6&6\\4&-6&2\end{vmatrix} = 24\mathbf{i}+0\mathbf{j}-48\mathbf{k}\right\}$		
	Area $PAB = \frac{1}{2}\sqrt{(24)^2+(-48)^2}$	Uses a vector product and $\sqrt{(\text{"}24\text{"})^2+(\text{"}0\text{"})^2+(\text{"}-48\text{"})^2}$	M1
		Uses a vector product and $\frac{1}{2}\sqrt{(\text{"}24\text{"})^2+(\text{"}0\text{"})^2+(\text{"}-48\text{"})^2}$	M1
	$=12\sqrt{5}$	$12\sqrt{5}$	A1 **cao**
			[3]
7. (c) **Alt 2**	**Note:** $\cos APB = \frac{5}{\sqrt{30}}$ or $\frac{1}{6}\sqrt{30}$ **Note:** $\lvert\overrightarrow{PA}\rvert = \sqrt{216}$ and $\lvert\overrightarrow{PB}\rvert = \sqrt{80}$		
	$\sin\theta = \frac{\sqrt{30-25}}{\sqrt{30}} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{6}}{6}$	A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$	M1
	Area $PAB = \frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{80}\right)\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\left\{=12\sqrt{30}\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\right\}=12\sqrt{5}$	$\frac{1}{2}$(their *PA*)(their *PB*)$\sin\theta$	M1
		$12\sqrt{5}$	A1 **cao**
			[3]

8.

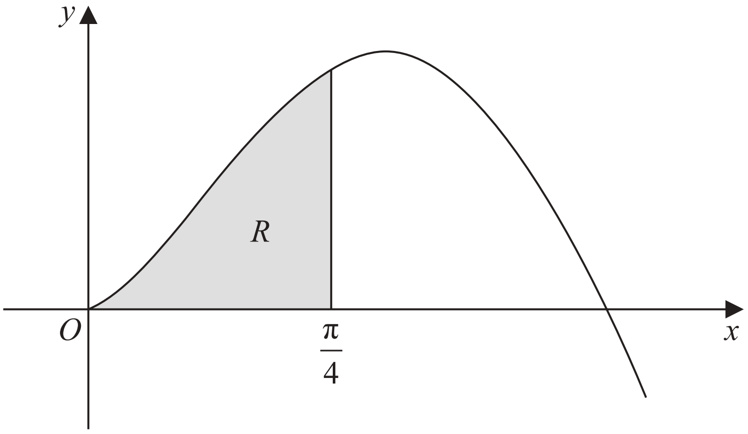

Diagram not drawn to scale

Figure 3

(a) Find $\int x \cos 4x \, \mathrm{d}x$

(3)

Figure 3 shows part of the curve with equation $y = \sqrt{x} \sin 2x, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of this solid of revolution, giving your answer in its simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

Question Number	Scheme	Notes	Marks
8. (a)	$\left\{\int x\cos 4x\,\mathrm{d}x\right\}$ $=\frac{1}{4}x\sin 4x-\int\frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$	$\pm\alpha x\sin 4x\pm\beta\int\sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$; $\alpha,\ \beta\neq 0$	M1
		$\frac{1}{4}x\sin 4x-\int\frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$ **Can be simplified or un-simplified**	A1
	$=\frac{1}{4}x\sin 4x+\frac{1}{16}\cos 4x\,\{+c\}$	$\frac{1}{4}x\sin 4x+\frac{1}{16}\cos 4x$ o.e. with or without $+c$ **Can be simplified or un-simplified**	A1
	Note: You can ignore subsequent working following on from a correct solution		**[3]**
(b) **Way 1**	$\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$	$\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x=\right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x=1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral Can be implied.	M1
		Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	A1
	$\left\{\int\left(\frac{1}{2}x-\frac{1}{2}x\cos 4x\right)\mathrm{d}x\right\}$ $=\frac{1}{4}x^2-\frac{1}{2}\left(\frac{1}{4}x\sin 4x+\frac{1}{16}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A, B,\ C\neq 0$ which can be simplified or un-simplified. **Note:** Allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)	M1
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{8}x\sin 4x-\frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2-\frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right)-\frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right)-\left(0-0-\frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$=\left(\frac{\pi^2}{64}+\frac{1}{32}\right)-\left(-\frac{1}{32}\right)=\frac{\pi^2}{64}+\frac{1}{16}$		
	So, $V=\pi\left(\frac{\pi^2}{64}+\frac{1}{16}\right)$ or $\frac{1}{64}\pi^3+\frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32}+\frac{1}{8}\right)$ o.e.	two term exact answer	A1 o.e.
			[6]
			9
	Question 8 Notes		
	SC	<u>**Special Case for the 2nd M and 3rd M mark for those who use their answer from part (a)**</u> You can apply the 2nd M and 3rd M marks for integration of the form $\pm Ax^2\pm$ (their answer to part (a)) where their answer to part (a) is in the form • $\pm Bx\sin kx\pm C\cos px$ to give $\pm Ax^2\pm Bx\sin kx\pm C\cos px$ • $\pm Bx\sin kx\pm C\sin px$ to give $\pm Ax^2\pm Bx\sin kx\pm C\sin px$ • $\pm Bx\cos kx\pm C\sin px$ to give $\pm Ax^2\pm Bx\cos kx\pm C\sin px$ • $\pm Bx\cos kx\pm C\cos px$ to give $\pm Ax^2\pm Bx\cos kx\pm C\cos px$ $k, p\neq 0,\ \ k, p$ can be 1	

Question Number	Scheme	Notes	Marks
8. (b) **Way 2**	$\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$	$\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied	B1
	$\left\{\int x\sin^2 2x\,\mathrm{d}x=\right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$	For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x=1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral. Can be implied	M1
		Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ **Note:** This mark can be implied for stating $u=x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1-\cos 4x}{2}$ **or** $u=\frac{1}{2}x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=1-\cos 4x$	A1
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\int\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)\mathrm{d}x$		
	$=x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\left(\frac{1}{4}x^2+\frac{1}{32}\cos 4x\right)\{+c\}$	Integrates to give $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A,B,\ C\neq 0$ or an expression that can be simplified to this form	M1 **(B1 on ePEN)**
	$\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{8}x\sin 4x-\frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$		
	$=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2-\frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right)-\frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right)-\left(0-0-\frac{1}{32}\cos 0\right)$	**dependent on the previous M mark** see notes	dM1
	$=\left(\frac{\pi^2}{64}+\frac{1}{32}\right)-\left(-\frac{1}{32}\right)=\frac{\pi^2}{64}+\frac{1}{16}$		
	So, $V=\pi\left(\frac{\pi^2}{64}+\frac{1}{16}\right)$ or $\frac{1}{64}\pi^3+\frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32}+\frac{1}{8}\right)$ o.e.		A1 o.e.
			[6]

	Question 8 Notes Continued	
8. (a)	**SC**	Give ***Special Case*** M1A0A0 for writing down the correct "by parts" formula and using $u=x,\ \frac{\mathrm{d}v}{\mathrm{d}x}=\cos 4x,$ but making only one error in the application of the correct formula
(b)	**Note**	You can imply B1 for seeing $\pi\int y^2\{\mathrm{d}x\}$, followed by $y^2=\left(\sqrt{x}\sin 2x\right)^2$ or $y^2=x\sin^2 2x$
	Note	If the form $\cos 4x=\cos^2 2x-\sin^2 2x$ or $\cos 4x=2\cos^2 2x-1$ is used, the 1st M cannot be gained until $\cos^2 2x$ has been replaced by $\cos^2 2x=1-\sin^2 2x$ and the result is applied to their integral
	Note	Mixing x's and e.g. θ's : Condone $\cos 4\theta=1-2\sin^2 2\theta,\ \sin^2 2\theta=\frac{1-\cos 4\theta}{2}$ or $\lambda\sin^2 2\theta=\lambda\left(\frac{1-\cos 4\theta}{2}\right)$ if recovered in their integration
	Final M1	**Complete** method of applying limits of $\frac{\pi}{4}$ and 0 to all terms of an expression of the form $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A,B,\ C\neq 0$ and subtracting the correct way round.
	Note	For the final M1 mark in Way 1, allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)

	Question 8 Notes Continued	
8. (b)	**Note**	Evidence of a proper consideration of the limit of 0 on $\cos 4x$ **where applicable** is needed for the final M mark E.g. $\left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}} =$ • $= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) + \frac{1}{32}$ is final M1 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - 0$ is final M0 • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \frac{1}{32}$ is final M0 (adding) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(\frac{1}{32}\right)$ is final M1 (condone) • $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - (0+0+0)$ is final M0
8. (b)	**Note**	<u>**Alternative Method**</u>: $\begin{Bmatrix} u = \sin^2 2x & \frac{\mathrm{d}v}{\mathrm{d}x} = x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2\sin 4x & v = \frac{1}{2}x^2 \end{Bmatrix}, \begin{Bmatrix} u = x^2 & \frac{\mathrm{d}v}{\mathrm{d}x} = \sin 4x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x & v = -\frac{1}{4}\cos 4x \end{Bmatrix}$ $\int x\sin^2 2x\,\mathrm{d}x$ $= \frac{1}{2}x^2\sin^2 2x - \int \frac{1}{2}x^2(2\sin 4x)\mathrm{d}x$ $= \frac{1}{2}x^2\sin^2 2x - \int x^2\sin 4x\,\mathrm{d}x$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x - \int 2x.\left(-\frac{1}{4}\cos 4x\right)\mathrm{d}x\right)$ $= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x + \frac{1}{2}\int x\cos 4x\,\mathrm{d}x\right)$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\int x\cos 4x\,\mathrm{d}x$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right) \{+c\}$ $= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x \{+c\}$ $V = \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e.