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**Pearson Edexcel GCE**

| Centre Number | Candidate Number |
|---|---|
| | |

# Core Mathematics C4

## Advanced

Friday 22 June 2018 – Morning
**Time: 1 hour 30 minutes**

Paper Reference
**6666/01**

**You must have:**
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** (a) Find the binomial series expansion of

$$\sqrt{4-9x}, \quad |x| < \frac{4}{9}$$

in ascending powers of $x$, up to and including the term in $x^2$
Give each coefficient in its simplest form.

**(5)**

(b) Use the expansion from part (a), with a suitable value of $x$, to find an approximate value for $\sqrt{310}$
Show all your working and give your answer to 3 decimal places.

**(3)**

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| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **1.** (a) | $\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ | $\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ | B1 |
| | $=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$ | see notes | M1 A1ft |
| | $=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$ | | |
| | $=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$ | see notes | |
| | $= 2-\frac{9}{4}x;\ -\frac{81}{64}x^2+\ldots$ | isw | A1; A1 |
| | | | **[5]** |
| (b) | $\sqrt{310} = 10\sqrt{3.1} = 10\sqrt{(4-9(0.1))}$, so $x = 0.1$ | E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x = 0.1$ (or uses $x = 0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0 | B1 |
| | When $x = 0.1$ $\sqrt{(4-9x)} \approx 2-\frac{9}{4}(0.1) - \frac{81}{64}(0.1)^2+\ldots$ | Substitutes their $x$, where $\lvert x\rvert < \frac{4}{9}$ into all three terms of their binomial expansion | M1 |
| | $= 2-0.225-0.01265625 = 1.76234375$ | | |
| | So, $\sqrt{310} \approx 17.6234375 = \underline{17.623}$ (3 dp) | 17.623 **cao** | A1 **cao** |
| | **Note:** the calculator value of $\sqrt{310}$ is $17.60681686\ldots$ which is $17.607$ to 3 decimal places | | **[3]** |
| | | | **8 marks** |

**Question 1 Notes**

| | | |
|---|---|---|
| **1.** (a) | **B1** | $\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion |
| | **M1** | Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where $k$ is a numerical value and **where** $k \neq 1$ |
| | **A1ft** | A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** $(kx)$ |
| | **Note** | $(kx)$, $k \neq 1$ must be consistent (on the RHS, not necessarily on the LHS) in their expansion |
| | **Note** | Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(-9x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$ because $(kx)$ is not consistent |
| | **Note** | **<u>Incorrect bracketing:</u>** $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x^2}{4}\right)+\ldots\right]$ is B1M1A0 unless recovered |
| | **A1** | $2-\frac{9}{4}x$ **(simplified fractions) or** allow $2-2.25x$ or $2-2\frac{1}{4}x$ |
| | **A1** | Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$ |

| | | **Question 1 Notes Continued** |
|---|---|---|
| **1.** (a) ctd. | **SC** | If a candidate ***would otherwise score*** 2nd A0, 3rd A0 (i.e. scores A0A0 in the final two marks to (a)) then **allow Special Case 2nd A1 for either** **SC:** $2\left[1-\frac{9}{8}x;...\right]$ or **SC:** $2\left[1+...-\frac{81}{128}x^2+...\right]$ or **SC:** $\lambda\left[1-\frac{9}{8}x-\frac{81}{128}x^2+...\right]$ or **SC:** $\left[\lambda-\frac{9\lambda}{8}x-\frac{81\lambda}{128}x^2+...\right]$ (where $\lambda$ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, **OR SC:** for $2-\frac{18}{8}x-\frac{162}{128}x^2+...$ (i.e. for not simplifying their correct coefficients) |
| | **Note** | Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$, where $k=\frac{9}{4}$ and not $-\frac{9}{4}$ and achieve $2+\frac{9}{4}x;-\frac{81}{64}x^2+...$ will get B1M1A1A0A1 |
| | **Note** | **Ignore** extra terms beyond the term in $x^2$ |
| | **Note** | You can ignore subsequent working following a correct answer |
| | **Note** | Allow B1M1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]$ |
| | **Note** | Allow B1M1A1A1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+...\right]=2-\frac{9}{4}x-\frac{81}{64}x^2+...$ |
| (b) | **Note** | Give B1 M1 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2\right)$ |
| | **Note** | **<u>Other alternative suitable values for *x* for</u>** $\underline{\sqrt{310}\approx\beta\sqrt{4-9(\text{their } x)}}$ (see table below) |
| | **Note** | Apply the scheme in the same way for their $\beta$ and their $x$ E.g. Give B1 M1 A1 for $\sqrt{310}\approx 12\left(2-\frac{9}{4}\left(\frac{133}{648}\right)-\frac{81}{64}\left(\frac{133}{648}\right)^2\right)=17.819$ (3 dp) |
| | **Note** | Allow B1 M1 A1 for $\sqrt{310}\approx 100\left(2-\frac{9}{4}(0.441)-\frac{81}{64}(0.441)^2\right)=76.161$ (3 dp) |
| | **Note** | Give B1 M1 A0 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2-\frac{729}{512}(0.1)^3\right)=17.609$ (3 dp) |

| *b* | *x* | **Estimate** | *b* | *x* | **Estimate** |
|---|---|---|---|---|---|
| 7 | $-\frac{38}{147}$ | 17.479 | 14 | $\frac{79}{294}$ | 18.256 |
| 8 | $-\frac{3}{32}$ | 17.599 | 15 | $\frac{118}{405}$ | 18.555 |
| 9 | $\frac{14}{729}$ | 17.607 | 16 | $\frac{119}{384}$ | 18.899 |
| 10 | $\frac{1}{10}$ | 17.623 | 17 | $\frac{94}{289}$ | 19.283 |
| 11 | $\frac{58}{363}$ | 17.690 | 18 | $\frac{493}{1458}$ | 19.701 |
| 12 | $\frac{133}{648}$ | 17.819 | 19 | $\frac{126}{361}$ | 20.150 |
| 13 | $\frac{122}{507}$ | 18.009 | 20 | $\frac{43}{120}$ | 20.625 |

| | | |
|---|---|---|
| | **Question 1 Notes Continued** | |
| **1.** (b) | **Note** | ***Send to review*** using $\beta = \sqrt{155}$ and $x = \dfrac{2}{9}$ (which gives 17.897 (3 dp)) |
| | **Note** | ***Send to review*** using $\beta = \sqrt{1000}$ and $x = 0.41$ (which gives 27.346 (3 dp)) |

| | | |
|---|---|---|
| **1.** (a) **Alt 1** | **<u>Alternative method 1:</u>** Candidates can apply an alternative form of the binomial expansion<br>$\left\{(4-9x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(-9x) + \dfrac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(-9x)^2$ | |
| | **B1** | $(4)^{\frac{1}{2}}$ or $2$ |
| | **M1** | Any two of three (un-simplified) terms correct |
| | **A1** | All three (un-simplified) terms correct |
| | **A1** | $2 - \dfrac{9}{4}x$ **(simplified fractions)** or allow $2 - 2.25x$ or $2 - 2\dfrac{1}{4}x$ |
| | **A1** | Accept only $-\dfrac{81}{64}x^2$ or $-1\dfrac{17}{64}x^2$ or $-1.265625x^2$ |
| | **Note** | The terms in C need to be evaluated.<br>So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(-9x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(-9x)^2$ without further working is B0M0A0 |

| | | | |
|---|---|---|---|
| **1.** (a) | **<u>Alternative Method 2: Maclaurin Expansion</u>** $\mathrm{f}(x) = (4-9x)^{\frac{1}{2}}$ | | |
| | $\mathrm{f}''(x) = -\dfrac{81}{4}(4-9x)^{-\frac{3}{2}}$ | Correct $\mathrm{f}''(x)$ | B1 |
| | $\mathrm{f}'(x) = \dfrac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$ | $\pm a(4-9x)^{-\frac{1}{2}}$; $a \neq \pm 1$ | M1 |
| | | $\dfrac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$ | A1 oe |
| | $\left\{\therefore \mathrm{f}(0) = 2,\ \mathrm{f}'(0) = -\dfrac{9}{4} \text{ and } \mathrm{f}''(0) = -\dfrac{81}{32}\right\}$ | | |
| | So, $\mathrm{f}(x) = 2 - \dfrac{9}{4}x; - \dfrac{81}{64}x^2 + \ldots$ | | A1; A1 |

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**2.** The curve $C$ has equation

$$x^2 + xy + y^2 - 4x - 5y + 1 = 0$$

(a) Use implicit differentiation to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$.

**(5)**

(b) Find the $x$ coordinates of the two points on $C$ where $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

Give exact answers in their simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2.** | $x^2 + xy + y^2 - 4x - 5y + 1 = 0$ | | |
| (a) | $\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}$ $\underline{2x} + \underline{\underline{\left(y + x\frac{dy}{dx}\right)}}$ $\underline{+ 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx}}$ $= \underline{0}$ | | M1<u>A1</u> <u><u>B1</u></u> |
| | $2x + y - 4 + (x + 2y - 5)\frac{dy}{dx} = 0$ | | dM1 |
| | $\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$ | **o.e.** | A1 **cso** |
| | | | **[5]** |
| (b) | $\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x + y - 4 = 0$ | | M1 |
| | $\{y = 4 - 2x \Rightarrow\}$ $x^2 + x(4 - 2x) + (4 - 2x)^2 - 4x - 5(4 - 2x) + 1 = 0$ | | dM1 |
| | $x^2 + 4x - 2x^2 + 16 - 16x + 4x^2 - 4x - 20 + 10x + 1 = 0$ | | |
| | gives $3x^2 - 6x - 3 = 0$ **or** $3x^2 - 6x = 3$ **or** $x^2 - 2x - 1 = 0$ | Correct 3TQ in terms of $x$ | A1 |
| | $(x - 1)^2 - 1 - 1 = 0$ and $x = ...$ | Method mark for solving a 3TQ in $x$ | ddM1 |
| | $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ | $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only | A1 |
| | | | **[5]** |
| (b) **Alt 1** | $\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x + y - 4 = 0$ | | M1 |
| | $\left\{x = \frac{4 - y}{2} \Rightarrow\right\}$ $\left(\frac{4 - y}{2}\right)^2 + \left(\frac{4 - y}{2}\right)y + y^2 - 4\left(\frac{4 - y}{2}\right) - 5y + 1 = 0$ | | dM1 |
| | $\left(\frac{16 - 8y + y^2}{2}\right) + \left(\frac{4y - y^2}{2}\right) + y^2 - 2(4 - y) - 5y + 1 = 0$ | | |
| | gives $3y^2 - 12y - 12 = 0$ **or** $3y^2 - 12y = 12$ **or** $y^2 - 4y - 4 = 0$ | Correct 3TQ in terms of $y$ | A1 |
| | $(y - 2)^2 - 4 - 4 = 0$ and $y = ...$ <br> $x = \frac{4 - (2 + 2\sqrt{2})}{2}$, $x = \frac{4 - (2 - 2\sqrt{2})}{2}$ | Solves a 3TQ in $y$ **and finds at least one value for $x$** | ddM1 |
| | $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ | $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only | A1 |
| | | | **[5]** |
| | | | **10** |
| (a) **Alt 1** | $\left\{\frac{\cancel{dx}}{\cancel{dy}} \cancel{=}\right\}$ $\underline{2x\frac{dx}{dy}} + \underline{\underline{\left(y\frac{dx}{dy} + x\right)}}$ $\underline{+ 2y - 4\frac{dx}{dy} - 5}$ $= \underline{0}$ | | M1<u>A1</u> <u><u>B1</u></u> |
| | $x + 2y - 5 + (2x + y - 4)\frac{dx}{dy} = 0$ | | dM1 |
| | $\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$ | **o.e.** | A1 **cso** |
| | | | **[5]** |

| | | **Question 2 Notes** |
|---|---|---|
| **2.** (a) | **M1** | Differentiates implicitly to include either $x\frac{dy}{dx}$ or $y^2 \to 2y\frac{dy}{dx}$ or $-5y \to -5\frac{dy}{dx}$. $\left(\text{Ignore } \frac{dy}{dx} = ...\right)$ |
| | **A1** | $x^2 \to 2x$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx} = 0$ |
| | **B1** | $xy \to y + x\frac{dy}{dx}$ |
| | **Note** | If an extra term appears then award 1st A0 |
| | **Note** | $2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx} \to 2x + y - 4 = -x\frac{dy}{dx} - 2y\frac{dy}{dx} + 5\frac{dy}{dx}$<br>will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation. |
| | **dM1** | **dependent on the previous M mark**<br>An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are **at least two terms** in $\frac{dy}{dx}$. |
| | **A1**<br><br>**cso** | $\frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$<br>If the candidate's solution is not completely correct, then do not give the final A mark |
| (b) | **M1**<br><br>**Note**<br><br>**Note** | Sets the numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.<br><br>This mark can also be gained by setting $\frac{dy}{dx}$ equal to zero in their differentiated equation from (a)<br><br>**If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).** |
| | **dM1** | **dependent on the previous M mark**<br>Substitutes their $x$ or their $y$ (from their numerator = 0) into the printed equation to give an equation in one variable only |
| | **A1**<br><br>**Note** | For obtaining the correct 3TQ. E.g.: either $3x^2 - 6x - 3 \{= 0\}$ or $-3x^2 + 6x + 3 \{= 0\}$<br>This mark can also be awarded for a correct 3 term equation. E.g. either $3x^2 - 6x = 3$<br>$x^2 - 2x - 1 = 0$ or $x^2 = 2x + 1$ are all fine for A1 |
| | **ddM1** | **dependent on the previous 2 M marks**<br>See page 6: Method mark for solving THEIR 3-term quadratic in one variable<br>**<u>Quadratic Equation to solve:</u>** $3x^2 - 6x - 3 = 0$<br>**<u>Way 1:</u>** $x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-3)}}{2(3)}$<br>**<u>Way 2:</u>** $x^2 - 2x - 1 = 0 \Rightarrow (x-1)^2 - 1 - 1 = 0 \Rightarrow x = ...$<br>**<u>Way 3:</u>** Or writes down at least one ***exact*** correct $x$-root **(*or one correct x-root to 2 dp*)** from ***their*** quadratic equation. This is usually found on their calculator.<br>**<u>Way 4:</u>** **(Only allowed if their 3TQ can be factorised)**<br>• $(x^2 + bx + c) = (x + p)(x + q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to $x = ...$<br>• $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = a$, leading to $x = ...$ |
| | **Note** | If a candidate applies ***the alternative method*** then they also need to use their $x = \frac{4 - y}{2}$ to find **at least one value** for $x$ in order to gain the final M mark. |
| | **A1** | Exact values of $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ (or $1 \pm \sqrt{2}$) , **cao** Apply isw if $y$-values are also found. |
| | **Note** | It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$ ) to gain all 5 marks in part (b) |

| | | Question 2 Notes |
|---|---|---|
| **2.** (a) **Alt 1** | **M1** | Differentiates implicitly to include either $y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-4x \to -4\frac{\mathrm{d}x}{\mathrm{d}y}$. $\left(\text{Ignore } \frac{\mathrm{d}x}{\mathrm{d}y} = ...\right)$ |
| | **A1** | $x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 = 0$ |
| | **B1** | $xy \to y\frac{\mathrm{d}x}{\mathrm{d}y} + x$ |
| | **Note** | If an extra term appears then award 1st A0 |
| | **Note** | $2x\frac{\mathrm{d}x}{\mathrm{d}y} + y\frac{\mathrm{d}x}{\mathrm{d}y} + x + 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 \;\to\; x + 2y - 5 = -2x\frac{\mathrm{d}x}{\mathrm{d}y} - y\frac{\mathrm{d}x}{\mathrm{d}y} + 4\frac{\mathrm{d}x}{\mathrm{d}y}$<br>will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation. |
| | **dM1** | **dependent on the previous M mark**<br>An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are **at least two terms** in $\frac{\mathrm{d}x}{\mathrm{d}y}$ |
| | **A1**<br><br>**cso** | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$<br>If the candidate's solution is not completely correct, then do not give the final A mark |
| (a) | **Note** | Writing down ***from no working***<br>• $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ scores M1 A1 B1 M1 A1<br>• $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{x + 2y - 5}$ scores M1 A0 B1 M1 A0 |
| | **Note** | Writing $2x\mathrm{d}x + y\mathrm{d}x + x\mathrm{d}y + 2y\mathrm{d}y - 4\mathrm{d}x - 5\mathrm{d}y = 0$ scores M1 A1 B1 |

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**3.** (i) Given that

$$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$$

(a) find the values of the constants $A$, $B$ and $C$.

**(4)**

(b) Hence find

$$\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x, \quad x > -\frac{1}{2}$$

**(3)**

(ii) Find

$$\int (\mathrm{e}^x + 1)^3\,\mathrm{d}x$$

**(3)**

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x, \quad x > 0$$

**(4)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3.** (i) | $\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$ | | |
| (a) | $B = 6,\ C = 1$ | At least one of $B = 6$ or $C = 1$ | B1 |
| | | Both $B = 6$ and $C = 1$ | B1 |
| | $13 - 4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$<br>$x = -3 \Rightarrow 25 = 25C \Rightarrow C = 1$<br>$x = -\frac{1}{2} \Rightarrow 13 - -2 = \frac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B = 6$ | Writes down a correct identity and attempts to find the value of either one of $A$ or $B$ or $C$ | M1 |
| | Either $x^2: 0 = 2A + 4C$, constant $: 13 = 3A + 3B + C$,<br>$x: -4 = 7A + B + 4C$ or $x = 0 \Rightarrow 13 = 3A + 3B + C$<br>leading to $A = -2$ | Using a correct identity to find $A = -2$ | A1 |
| | | | **[4]** |
| (b) | $\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,\mathrm{d}x$ | | |
| | $= \frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$<br>o.e.<br>$\left\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\right\}$ | See notes | M1 |
| | | ***At least two*** terms correctly integrated | A1ft |
| | | Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line<br>Ignore the absence of '$+c$' | A1 |
| | | | **[3]** |
| (ii) | $\left\{(\mathrm{e}^x+1)^3 =\right\}\ \mathrm{e}^{3x} + 3\mathrm{e}^{2x} + 3\mathrm{e}^x + 1$ | $\mathrm{e}^{3x} + 3\mathrm{e}^{2x} + 3\mathrm{e}^x + 1$, simplified or un-simplified | B1 |
| | $\left\{\int (\mathrm{e}^x+1)^3\,\mathrm{d}x\right\} = \frac{1}{3}\mathrm{e}^{3x} + \frac{3}{2}\mathrm{e}^{2x} + 3\mathrm{e}^x + x\ \{+c\}$ | At least 3 examples (see notes) of correct ft integration | M1 |
| | | $\frac{1}{3}\mathrm{e}^{3x} + \frac{3}{2}\mathrm{e}^{2x} + 3\mathrm{e}^x + x$, simplified or un-simplified with or without $+c$ | A1 |
| | | | **[3]** |
| (iii) | $\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x,\ \ x > 0;\ \ u^3 = x$ | | |
| | $3u^2\frac{\mathrm{d}u}{\mathrm{d}x} = 1$ | $3u^2\frac{\mathrm{d}u}{\mathrm{d}x} = 1$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = 3u^2$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{3}x^{-\frac{2}{3}}$<br>or $3u^2\mathrm{d}u = \mathrm{d}x$ o.e. | B1 |
| | $= \int \frac{1}{4u^3 + 5u}.3u^2\,\mathrm{d}u\ \ \left\{= \int \frac{3u}{4u^2+5}\,\mathrm{d}u\right\}$ | Expression of the form $\int \frac{\pm ku^2}{4u^3 \pm 5u}\{\mathrm{d}u\}$, $k \neq 0$<br>Does not have to include integral sign or $\mathrm{d}u$<br>Can be implied by later working | M1 |
| | $= \frac{3}{8}\ln(4u^2+5)\ \{+c\}$ | **dependent on the previous M mark**<br>$\pm\lambda\ln(4u^2+5)$; $\lambda$ is a constant; $\lambda \neq 0$ | dM1 |
| | $= \frac{3}{8}\ln\left(4x^{\frac{2}{3}} + 5\right)\ \{+c\}$ | Correct answer in $x$ with or without $+c$ | A1 |
| | | | **[4]** |
| | | | **14** |

| | | | |
|---|---|---|---|
| | **Question 3 Notes** | | |
| **3.** (iii) **Alt 1** | **Alternative method 1 for part (iii)** | | |
| | $\left\{\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx\right\} = \int \frac{x^{-\frac{1}{3}}}{4x^{\frac{2}{3}}+5}\,dx$ | Attempts to multiply numerator and denominator by $x^{-\frac{1}{3}}$ | M1 |
| | | Expression of the form $\int \frac{\pm kx^{-\frac{1}{3}}}{4x^{\frac{2}{3}}\pm 5}\,dx,\ k\neq 0$ <br> Does not have to include integral sign or $du$ <br> Can be implied by later working | M1 |
| | $= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\{+c\}$ | $\pm\lambda\ln(4x^{\frac{2}{3}}+5)$; $\lambda$ is a constant; $\lambda\neq 0$ | dM1 |
| | | Correct answer in $x$ with or without $+c$ | A1 |
| | | | **[4]** |

| | | |
|---|---|---|
| **3.** (i) (a) | **M1** | Writes down ***a correct identity*** (although this can be implied) and attempts ***to find the value of at least one*** of either $A$ or $B$ or $C$. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients. |
| | **Note** | The correct partial fraction from no working scores B1B1M1A1 |
| (i) (b) | **M1** | At least 2 of either $\pm\frac{P}{(2x+1)} \to \pm D\ln(2x+1)$ or $\pm D\ln(x+\frac{1}{2})$ or $\pm\frac{Q}{(2x+1)^2} \to \pm E(2x+1)^{-1}$ <br> or <br> $\pm\frac{R}{(x+3)} \to \pm F\ln(x+3)$ for their constants $P$, $Q$, $R$. |
| | **A1ft** | At least two terms from any of $\pm\frac{P}{(2x+1)}$ or $\pm\frac{Q}{(2x+1)^2}$ or $\pm\frac{R}{(x+3)}$ correctly integrated. |
| | **Note** | Can be un-simplified for the A1ft mark. |
| | **A1** | Correct answer of $\frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ simplified or un-simplified. <br> with or without '+ $c$'. |
| | **Note** | Allow final A1 for equivalent answers, e.g. $\ln\left(\frac{x+3}{2x+1}\right) - \frac{3}{2x+1}\ \{+c\}$ or <br> $\ln\left(\frac{2x+6}{2x+1}\right) - \frac{3}{2x+1}\ \{+c\}$ |
| | **Note** | **Beware that** $\int \frac{-2}{(2x+1)}\,dx = \int \frac{-1}{(x+\frac{1}{2})}\,dx = -\ln(x+\frac{1}{2})\ \{+c\}$ is correct integration |
| | **Note** | E.g. Allow M1 A1ft A1 for a correct un-simplified $\ln(x+3) - \ln(x+\frac{1}{2}) - \frac{3}{2}(x+\frac{1}{2})^{-1}\ \{+c\}$ |
| | **Note** | Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1 <br> E.g. Give final A0 for $-\ln 2x+1 - 3(2x+1)^{-1} + \ln x+3\ \{+c\}$ unless recovered |
| (ii) | **Note** | Give B1 for an un-simplified $e^{3x} + 2e^{2x} + e^{2x} + 2e^{x} + e^{x} + 1$ |
| | **M1** | At least 3 of either $\alpha e^{3x} \to \frac{\alpha}{3}e^{3x}$ **or** $\beta e^{2x} \to \frac{\beta}{2}e^{2x}$ **or** $\delta e^{x} \to \delta e^{x}$ **or** $\mu \to \mu x$; $\alpha, \beta, \delta, \mu \neq 0$ |
| | **Note** | Give A1 for an un-simplified $\frac{1}{3}e^{3x} + e^{2x} + \frac{1}{2}e^{2x} + 2e^{x} + e^{x} + x$, with or without $+c$ |
| (iii) | **Note** | 1st M1 can be implied by $\int \frac{\pm ku}{4u^2 \pm 5}\{du\},\ k\neq 0$. Does not have to include integral sign or $du$ |
| | **Note** | Condone 1st M1 for expressions of the form $\int\left(\frac{\pm 1}{4u^3 \pm 5u}\cdot\frac{\pm k}{u^{-2}}\right)\{du\},\ k\neq 0$ |
| | **Note** | Give 2nd M0 for $\frac{3u}{8u}\ln(4u^2+5)\ \{+c\}$ ($u$'s not cancelled) unless recovered in later working |
| | **Note** | E.g. Give 2nd M0 for integration leading to $\frac{3}{4}u\ln(4u^2+5)$ as this is not in the form <br> $\pm\lambda\ln(4u^2+5)$ |

| | | |
|---|---|---|
| | **Note** | Condone 2nd M1 for poor bracketing, but do not allow poor bracketing for the final A1<br>E.g. Give final A0 for $\frac{3}{8}\ln 4x^{\frac{2}{3}}+5\ \{+c\}$ unless recovered |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3.** (ii) **Alt 1** | $\int (\mathrm{e}^x+1)^3\,\mathrm{d}x;\quad u=\mathrm{e}^x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ | | |
| | $\left\{= \int \frac{u^3}{(u-1)}\,\mathrm{d}u\ =\right\} \int\left(u^2+u+1+\frac{1}{u-1}\right)\mathrm{d}u$ | $\int\left(u^2+u+1+\frac{1}{u-1}\right)\{\mathrm{d}u\}$ where $u=\mathrm{e}^x+1$ | B1 |
| | $=\frac{1}{3}u^3+\frac{1}{2}u^2+u+\ln(u-1)\ \{+c\}$ | At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u-1} \to \lambda\ln(u-1)$; $\alpha, \beta, \delta, \lambda \neq 0$ | M1 |
| | $=\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+\ln(\mathrm{e}^x+1-1)\ \{+c\}$ | | |
| | $=\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+x\ \{+c\}$ | $\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+(\mathrm{e}^x+1)+x$<br>or $\frac{1}{3}(\mathrm{e}^x+1)^3+\frac{1}{2}(\mathrm{e}^x+1)^2+\mathrm{e}^x+x$<br>simplified or un-simplified with or without $+\ c$<br>**Note:** $\ln(\mathrm{e}^x+1-1)$ needs to be simplified to $x$ for this mark | A1 |
| | | | **[3]** |
| **3.** (ii) **Alt 2** | $\int (\mathrm{e}^x+1)^3\,\mathrm{d}x;\quad u=\mathrm{e}^x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ | | |
| | $\left\{= \int \frac{(u+1)^3}{u}\,\mathrm{d}u\ =\right\} \int\left(u^2+3u+3+\frac{1}{u}\right)\mathrm{d}u$ | $\int\left(u^2+3u+3+\frac{1}{u}\right)\{\mathrm{d}u\}$ where $u=\mathrm{e}^x$ | B1 |
| | $=\frac{1}{3}u^3+\frac{3}{2}u^2+3u+\ln u\ \{+c\}$ | At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u} \to \lambda\ln u$; $\alpha, \beta, \delta, \lambda \neq 0$ | M1 |
| | $=\frac{1}{3}\mathrm{e}^{3x}+\frac{3}{2}\mathrm{e}^{2x}+3\mathrm{e}^x+x\ \{+c\}$ | $\frac{1}{3}\mathrm{e}^{3x}+\frac{3}{2}\mathrm{e}^{2x}+3\mathrm{e}^x+x$,<br>simplified or un-simplified with or without $+\ c$<br>**Note:** $\ln(\mathrm{e}^x)$ needs to be simplified to $x$ for this mark | A1 |
| | | | **[3]** |

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**4.**

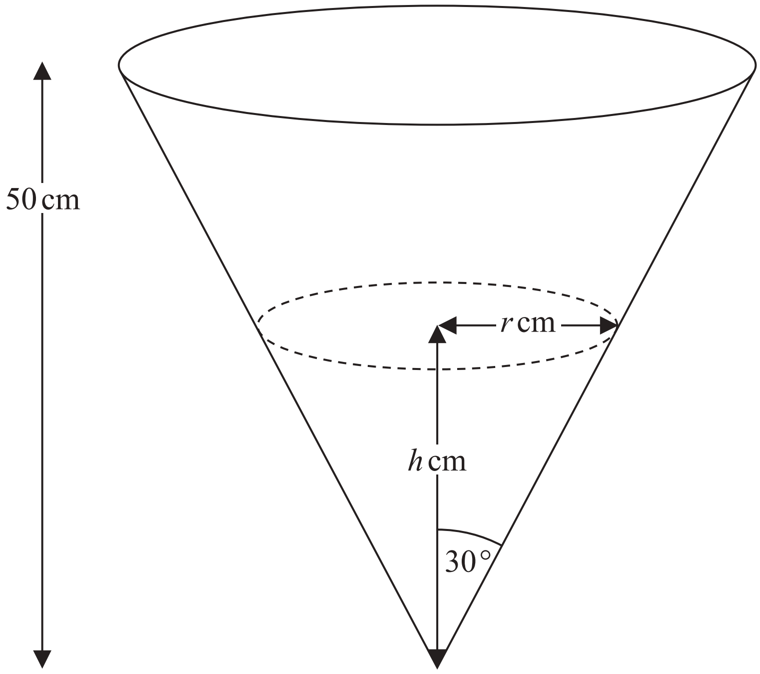


**Figure 1**

A water container is made in the shape of a hollow inverted right circular cone with semi-vertical angle of 30°, as shown in Figure 1. The height of the container is 50 cm.

When the depth of the water in the container is $h$ cm, the surface of the water has radius $r$ cm and the volume of water is $V$ cm$^3$.

(a) Show that $V = \frac{1}{9}\pi h^3$

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

**(2)**

Given that the volume of water in the container increases at a constant rate of 200 cm$^3$ s$^{-1}$,

(b) find the rate of change of the depth of the water, in cm s$^{-1}$, when $h = 15$
Give your answer in its simplest form in terms of π.

**(4)**





P 5 1 5 6 6 A 0 1 2 3 2

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **4.** (a) | $\frac{r}{h}=\tan 30 \Rightarrow r=h\tan 30 \left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ <br> **or** $\frac{h}{r}=\tan 60 \Rightarrow r=\frac{h}{\tan 60}\left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ <br> **or** $\frac{r}{\sin 30}=\frac{h}{\sin 60} \Rightarrow r=\frac{h\sin 30}{\sin 60}\left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ <br> **or** $h^2+r^2=(2r)^2 \Rightarrow r^2=\frac{1}{3}h^2$ | Correct use of trigonometry to find $r$ in terms of $h$ **or** correct use of Pythagoras to find $r^2$ in terms of $h^2$ | M1 |
| | $\left\{V=\frac{1}{3}\pi r^2 h \Rightarrow\right\} V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V=\frac{1}{9}\pi h^3$ * | Correct proof of $V=\frac{1}{9}\pi h^3$ or $V=\frac{1}{9}h^3\pi$ <br> Or shows $\frac{1}{9}\pi h^3$ or $\frac{1}{9}h^3\pi$ with some reference to $V=$ in their solution | A1 * |
| | | | **[2]** |
| (b) **Way 1** | $\frac{\mathrm{d}V}{\mathrm{d}t}=200$ | | |
| | $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi h^2$ | $\frac{1}{3}\pi h^2$ o.e. | B1 |
| | Either <br> • $\left\{\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\}\left(\frac{1}{3}\pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}=200$ <br> • $\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}\Rightarrow\right\}\frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi h^2}$ | **either** $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=200$ <br> **or** $200\div\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ | M1 |
| | When $h=15$, $\frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi(15)^2}\left\{=\frac{200}{75\pi}=\frac{600}{225\pi}\right\}$ | **dependent on the previous M mark** | dM1 |
| | $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{8}{3\pi}$ $(\mathrm{cm\,s^{-1}})$ | $\frac{8}{3\pi}$ | A1 **cao** |
| | | | **[4]** |
| | | | **6** |
| (b) **Way 2** | $\frac{\mathrm{d}V}{\mathrm{d}t}=200 \Rightarrow V=200t+c \Rightarrow \frac{1}{9}\pi h^3=200t+c$ | | |
| | $\left(\frac{1}{3}\pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}=200$ | $\frac{1}{3}\pi h^2$ o.e. | B1 |
| | | as in Way 1 | M1 |
| | When $h=15$, $\frac{\mathrm{d}h}{\mathrm{d}t}=200\times\frac{1}{\frac{1}{3}\pi(15)^2}\left\{=\frac{200}{75\pi}=\frac{600}{225\pi}\right\}$ | **dependent on the previous M mark** | dM1 |
| | $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{8}{3\pi}$ $(\mathrm{cm\,s^{-1}})$ | $\frac{8}{3\pi}$ | A1 **cao** |
| | | | **[4]** |

| | | |
|---|---|---|
| | **Question 4 Notes** | |
| **4.** (a) | **Note** | Allow M1 for writing down $r = h\tan 30$ |
| | **Note** | Give M0 A0 for writing down $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ with no evidence of using trigonometry on $r$ and $h$ or Pythagoras on $r$ and $h$ |
| | **Note** | Give M0 (unless recovered) for evidence of $\frac{1}{3}\pi r^2 h = \frac{1}{9}\pi h^3$ leading to either $r^2 = \frac{1}{3}h^2$ or $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ |
| | | |
| (b) | **B1**<br>**Note** | Correct simplified or un-simplified differentiation of $V$. E.g. $\frac{1}{3}\pi h^2$ or $\frac{3}{9}\pi h^2$<br>$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their $V$ |
| | **M1** | $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 200$ or $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ |
| | **dM1** | **dependent on the previous M mark**<br>Substitutes $h = 15$ into an expression ***which is a result***<br>of either $200 \div \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$ or $200 \times \frac{1}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$ |
| | **A1** | $\frac{8}{3p}$ (units are not required) |
| | **Note** | Give final A0 for using $\frac{\mathrm{d}V}{\mathrm{d}t} = -200$ to give $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{8}{3\pi}$, unless recovered to $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8}{3\pi}$ |

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**5.**

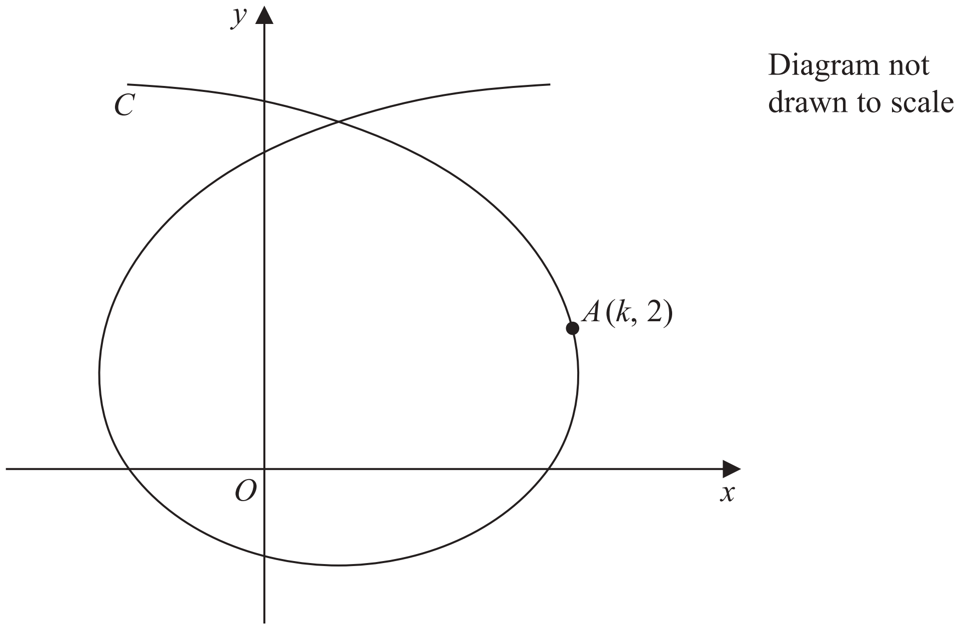


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with parametric equations

$$x = 1 + t - 5\sin t, \quad y = 2 - 4\cos t, \quad -\pi \leqslant t \leqslant \pi$$

The point $A$ lies on the curve $C$.

Given that the coordinates of $A$ are $(k, 2)$, where $k > 0$

(a) find the exact value of $k$, giving your answer in a fully simplified form.

**(2)**

(b) Find the equation of the tangent to $C$ at the point $A$.
Give your answer in the form $y = px + q$, where $p$ and $q$ are exact real values.

**(5)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5.** | $x = 1 + t - 5\sin t,\ y = 2 - 4\cos t,\ -\pi \leqslant t \leqslant \pi;\ A(k, 2),\ k > 0$, lies on $C$ | | |
| (a) | {When $y = 2$,} $2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ <br> $k\,(\text{or } x) = 1 + \frac{\pi}{2} - 5\sin\left(\frac{\pi}{2}\right)$ **or** $k\,(\text{or } x) = 1 - \frac{\pi}{2} - 5\sin\left(-\frac{\pi}{2}\right)$ | Sets $y = 2$ to find $t$ and some evidence of using their $t$ to find $x = ...$ | M1 |
| | $\left\{\text{When } t = -\frac{\pi}{2},\ k > 0,\right\}$ so $k = 6 - \frac{\pi}{2}$ or $\frac{12-\pi}{2}$ | $k$ (or $x$) $= 6 - \frac{\pi}{2}$ or $\frac{12-\pi}{2}$ | A1 |
| | | | **[2]** |
| (b) | $\frac{dx}{dt} = 1 - 5\cos t,\quad \frac{dy}{dt} = 4\sin t$ | At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied) | B1 |
| | | Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied) | B1 |
| | $\frac{dy}{dx} = \frac{4\sin t}{1 - 5\cos t}$ <br> at $t = -\frac{\pi}{2}$, $\frac{dy}{dx} = \frac{4\sin\left(-\frac{\pi}{2}\right)}{1 - 5\cos\left(-\frac{\pi}{2}\right)}$ $\{= -4\}$ | Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their $t$ into their $\frac{dy}{dx}$ <br> **Note:** their $t$ can lie outside $-\pi \leqslant t \leqslant \pi$ for this mark | M1 |
| | • $y - 2 = -4\left(x - \left(6 - \frac{\pi}{2}\right)\right)$ <br> • $2 = (-4)\left(6 - \frac{\pi}{2}\right) + c \Rightarrow y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ | Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found using calculus <br> **Note:** their $k$ (or $x$) must be in terms of $\pi$ and correct bracketing must be used or implied | M1 |
| | $\{y - 2 = -4x + 24 - 2\pi \Rightarrow\}\quad y = -4x + 26 - 2\pi$ | **dependent on all previous marks in part (b)** <br> $y = -4x + 26 - 2\pi$ | A1 **cso** |
| | | $(p = -4,\ q = 26 - 2\pi)$ | **[5]** |
| | | | **7** |

**Question 5 Notes**

| | | |
|---|---|---|
| **5.** (a) | **Note** | M1 can be implied by either $x$ or $k = 6 - \frac{\pi}{2}$ or awrt 4.43 or $x$ or $k = \frac{\pi}{2} - 4$ or awrt $-2.43$ |
| | **Note** | An answer of 4.429… without reference to a correct **exact** answer is A0 |
| | **Note** | M1 can be earned in part (a) by working in degrees |
| | **Note** | Give M0 for not substituting their $t$ back into $x$. E.g. $2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2} \Rightarrow k = -\frac{\pi}{2}$ |
| | **Note** | If two values for $k$ are found, they must identify the correct answer for A1 |
| | **Note** | Condone M1 for $2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2} \Rightarrow x = 1 - \frac{\pi}{2} - 5\sin\left(\frac{\pi}{2}\right)$ |
| (b) | **Note** | The 1st M mark may be implied by their value for $\frac{dy}{dx}$ <br> e.g. $\frac{dy}{dx} = \frac{4\sin t}{1 - 5\cos t}$, followed by an answer of $-4$ (from $t = -\frac{\pi}{2}$) or $4$ (from $t = \frac{\pi}{2}$) |
| | **Note** | Give 1st M0 for **applying** their $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ |
| | **2nd M1** | • **applies** $y - 2 = (\text{their } m_T)(x - (\text{their } k))$, <br> • **applies** $2 = (\text{their } m_T)(\text{their } k) + c$ **leading to** $y = (\text{their } m_T)x + (\text{their } c)$ <br> where $k$ must be in terms of $\pi$ and $m_T\ (\neq m_N)$ is a numerical value found using calculus |
| | **Note** | Correct bracketing must be used for 2nd M1, but this mark can be implied by later working |

| | | |
|---|---|---|
| | **Question 5 Notes Continued** | |
| **5.** (b) | **Note** | The final A mark is dependent on all previous marks in part (b) being scored.<br>This is because the correct answer can follow from an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ |
| | **Note** | The first 3 marks can be gained by using degrees in part (b) |
| | **Note** | Condone mixing a correct $t$ with an incorrect $x$ or an incorrect $t$ with a correct $x$ for the M marks |
| | **Note** | Allow final A1 for any answer in the form $y = px + q$<br>E.g. Allow final A1 for $y = -4x + 26 - 2\pi$, $y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ or<br>$y = -4x + \left(\frac{52 - 4\pi}{2}\right)$ |
| | **Note** | Do not apply isw in part (b). So, an incorrect answer following from a correct answer is A0 |
| | **Note** | Do not allow $y = 2(-2x + 13 - \pi)$ for A1 |
| | **Note** | $y = -4x + 26 - 2\pi$ followed by $y = 2(-2x + 13 - \pi)$ is condoned for final A1 |

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**6.** Given that $y = 2$ when $x = -\dfrac{\pi}{8}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \qquad -\frac{1}{2} < x < \frac{1}{2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

**(6)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| 6. | $\frac{dy}{dx} = \frac{y^2}{3\cos^2 2x}$; $-\frac{1}{2} < x < \frac{1}{2}$; $y = 2$ at $x = -\frac{\pi}{8}$ | | |
| | $\int \frac{1}{y^2}\,dy = \int \frac{1}{3\cos^2 2x}\,dx$ | Separates variables as shown<br>Can be implied by<br>a correct attempt at integration<br>Ignore the integral signs | B1 |
| | $\int \frac{1}{y^2}\,dy = \int \frac{1}{3}\sec^2 2x\,dx$ | | |
| | $-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)\{+c\}$ | $\pm\frac{A}{y^2} \to \pm\frac{B}{y}$; $A, B \neq 0$ | M1 |
| | | $\pm\lambda\tan 2x$ | M1 |
| | | $-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$ | A1 |
| | $-\frac{1}{2} = \frac{1}{6}\tan\left(2\left(-\frac{\pi}{8}\right)\right) + c$ | Use of $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated equation ***containing a constant of integration***, e.g. $c$ | M1 |
| | $-\frac{1}{2} = -\frac{1}{6} + c \Rightarrow c = -\frac{1}{3}$ | | |
| | $-\frac{1}{y} = \frac{1}{6}\tan 2x - \frac{1}{3} = \frac{\tan(2x) - 2}{6}$ | | |
| | $y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ or $y = \frac{6\cot 2x}{-1 + 2\cot 2x}$ $\left\{-\frac{1}{2} < x < \frac{1}{2}\right\}$ | | A1 o.e. |
| | | | **[6]** |
| | | | **6** |

| | | Question 6 Notes |
|---|---|---|
| 6. | **B1** | Separates variables as shown. $dy$ and $dx$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number "3" may appear on either side.<br>E.g. $\int \frac{1}{y^2}\,dy = \int \frac{1}{3}\sec^2 2x\,dx$ or $\int \frac{3}{y^2}\,dy = \int \frac{1}{\cos^2 2x}\,dx$ are fine for B1 |
| | **Note** | Allow e.g. $\int \frac{1}{y^2}\frac{dy}{dx}\,dx = \int \frac{1}{3}\sec^2 2x\,dx$ for B1 or condone $\int \frac{1}{y^2} = \int \frac{1}{3}\sec^2 2x$ for B1 |
| | **Note** | B1 can be implied by correct integration of both sides |
| | **M1** | $\pm\frac{A}{y^2} \to \pm\frac{B}{y}$; $A, B \neq 0$ |
| | **M1** | $\frac{1}{\cos^2 2x}$ or $\sec^2 2x \to \pm\lambda\tan 2x$; $\lambda \neq 0$ |
| | **A1** | $-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$ with or without '$+c$'. E.g. $-\frac{6}{y} = \tan 2x$ |
| | **M1**<br>**Note**<br>**Note**<br>**Note** | Evidence of using both $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing $c$<br>This mark can be implied by the correct value of $c$<br>You may need to use your calculator to check that they have satisfied the final M mark<br>Condone using $x = \frac{\pi}{8}$ instead of $x = -\frac{\pi}{8}$ |
| | **A1** | $y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ **or any equivalent correct answer in the form** $y = f(x)$ |
| | **Note** | You can ignore subsequent working, which follows from a correct answer |

| | Question 6 Notes Continued | |
|---|---|---|
| **6.** | **Note** | Writing $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}y^2 \sec^2 2x$ leading to e.g.<br>• $y = \frac{1}{9}y^3\left(\frac{1}{2}\tan 2x\right)$ gets 2nd M0 for $\pm\lambda \tan 2x$<br>• $u = \frac{1}{3}y^2,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{3}y,\ v = \frac{1}{2}\tan 2x$ gets 2nd M0 for $\pm\lambda \tan 2x$<br>because the variables have not been separated |

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**7.** The point $A$ with coordinates $(-3, 7, 2)$ lies on a line $l_1$
The point $B$ also lies on the line $l_1$

Given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$,

(a) find the coordinates of point $B$.
**(2)**

The point $P$ has coordinates $(9, 1, 8)$

(b) Find the cosine of the angle $PAB$, giving your answer as a simplified surd.
**(3)**

(c) Find the exact area of triangle $PAB$, giving your answer in its simplest form.
**(3)**

The line $l_2$ passes through the point $P$ and is parallel to the line $l_1$

(d) Find a vector equation for the line $l_2$
**(2)**

The point $Q$ lies on the line $l_2$

Given that the line segment $AP$ is perpendicular to the line segment $BQ$,

(e) find the coordinates of the point $Q$.
**(5)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** | $\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$ | Let $\theta=$ size of angle $PAB$. $A$, $B$ lie on $l_1$ and $P$ lies on $l_2$ | |
| (a) | $\left\{\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}\Rightarrow\right\}$ | Attempts to add $\overrightarrow{OA}$ to $\overrightarrow{AB}$ | M1 |
| | $\overrightarrow{OB}=\begin{pmatrix}-3\\7\\2\end{pmatrix}+\begin{pmatrix}4\\-6\\2\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix}\Rightarrow B(1,1,4)$ | $(1,1,4)$ or $\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\mathbf{i}+\mathbf{j}+4\mathbf{k}$ | A1 |
| | **Note:** M1 can be implied by at least 2 correct components for $B$ | | **[2]** |
| (b) | $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$ | An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$ | M1 |
| | $\left\{\cos\theta=\dfrac{\overrightarrow{AP}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert\lvert\overrightarrow{AB}\rvert}\right\}=\dfrac{\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$ | Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors | dM1 |
| | $\left\{\cos\theta=\dfrac{96}{\sqrt{216}.\sqrt{56}}\Rightarrow\cos\theta\right\}=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ | $\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ | A1 |
| | | | **[3]** |
| (c) | $\left\{\cos\theta=\dfrac{4}{\sqrt{21}}\right\}\Rightarrow\sin\theta=\dfrac{\sqrt{21-16}}{\sqrt{21}}=\dfrac{\sqrt{5}}{\sqrt{21}}=\dfrac{\sqrt{105}}{21}$ | A correct method for converting an exact value for $\cos\theta$ to an exact value for $\sin\theta$ | M1 |
| | Area $PAB=\dfrac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\left\{=12\sqrt{21}\left(\dfrac{\sqrt{5}}{\sqrt{21}}\right)\right\}=12\sqrt{5}$ | see notes | M1 |
| | | $12\sqrt{5}$ | A1 **cao** |
| | | | **[3]** |
| (d) | $\{l_2:\}\ \mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}4\\-6\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}9\\1\\8\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\1\end{pmatrix}$ | $\mathbf{p}+\lambda\mathbf{d}$ or $\mathbf{p}+\mu\mathbf{d}$, $\mathbf{p}\neq 0$, $\mathbf{d}\neq 0$ with either $\mathbf{p}=9\mathbf{i}+\mathbf{j}+8\mathbf{k}$ or $\mathbf{d}=4\mathbf{i}-6\mathbf{j}+2\mathbf{k}$ or $\mathbf{d}=$ multiple of $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ | M1 |
| | | Correct vector equation | A1 |
| | | | **[2]** |
| (e) | $\overrightarrow{BQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-4\mu\\6\mu\\-4-2\mu\end{pmatrix}\right\}$ | Applies their $\overrightarrow{OQ}-$ their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}-$ their $\overrightarrow{OQ}$ | M1 |
| | $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+4\mu\\-6\mu\\4+2\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$ | Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for $\mu$ | dM1 |
| | $\Rightarrow 96+48\mu+36\mu+24+12\mu=0\Rightarrow 96\mu+120=0\Rightarrow\mu=-\dfrac{5}{4}$ | $\mu=-\dfrac{120}{96}$ or $\mu=-\dfrac{5}{4}$ | A1 o.e. |
| | $\overrightarrow{OQ}=\begin{pmatrix}9+4(-1.25)\\1-6(-1.25)\\8+2(-1.25)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4,8.5,5.5)$ | Substitutes their value of $\mu$ into $\overrightarrow{OQ}$ | ddM1 |
| | | $(4,8.5,5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$ | A1 o.e. |
| | | | **[5]** |
| | | | **15** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** | $\overrightarrow{OA}=\begin{pmatrix}-3\\7\\2\end{pmatrix}, \overrightarrow{AB}=\begin{pmatrix}4\\-6\\2\end{pmatrix}, \overrightarrow{OP}=\begin{pmatrix}9\\1\\8\end{pmatrix}; \overrightarrow{OQ}=\begin{pmatrix}9+4\mu\\1-6\mu\\8+2\mu\end{pmatrix}$ or $\overrightarrow{OQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}$ | Let $\theta=$ size of angle $PAB$. $A$, $B$ lie on $l_1$ and $P$ lies on $l_2$ | |
| (e) **Alt 1** | $\overrightarrow{BQ}=\begin{pmatrix}9+2\mu\\1-3\mu\\8+\mu\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}\left\{=\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\right\}$ $\left\{\overrightarrow{QB}=\begin{pmatrix}-8-2\mu\\3\mu\\-4-\mu\end{pmatrix}\right\}$ | Applies their $\overrightarrow{OQ}$ − their $\overrightarrow{OB}$ **or** their $\overrightarrow{OB}$ − their $\overrightarrow{OQ}$ | M1 |
| | $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0\Rightarrow\begin{pmatrix}8+2\mu\\-3\mu\\4+\mu\end{pmatrix}\bullet\begin{pmatrix}12\\-6\\6\end{pmatrix}=0\Rightarrow\mu=...$ | Applies $\overrightarrow{BQ}\bullet\overrightarrow{AP}=0$, o.e. and ***solves*** the resulting equation to find a value for $\mu$ | dM1 |
| | $\Rightarrow 96+24\mu+18\mu+24+6\mu=0\Rightarrow 48\mu+120=0\Rightarrow\mu=-\frac{5}{2}$ | $\mu=-\frac{5}{2}$ | A1 o.e. |
| | $\overrightarrow{OQ}=\begin{pmatrix}9+2(-2.5)\\1-3(-2.5)\\8+1(-2.5)\end{pmatrix}=\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}\Rightarrow Q(4, 8.5, 5.5)$ | Substitutes their value of $\mu$ into $\overrightarrow{OQ}$ | ddM1 |
| | | $(4, 8.5, 5.5)$ or $\begin{pmatrix}4\\8.5\\5.5\end{pmatrix}$ or $4\mathbf{i}+8.5\mathbf{j}+5.5\mathbf{k}$ | A1 o.e. |
| | | | **[5]** |
| (b) **Alt 1** | **<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied | | |
| | $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$ | An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$ | M1 |
| | $\mathbf{d}_1\times\mathbf{d}_2=\underline{\begin{pmatrix}12\\-6\\6\end{pmatrix}\times\begin{pmatrix}4\\-6\\2\end{pmatrix}}=\left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\12&-6&6\\4&-6&2\end{vmatrix}=24\mathbf{i}+0\mathbf{j}-48\mathbf{k}\right\}$ | | |
| | $\sin\theta=\dfrac{\sqrt{(24)^2+(0)^2+(-48)^2}}{\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}}$ | Applies vector cross product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ or a multiple of these vectors | dM1 |
| | $\left\{\sin\theta=\dfrac{\sqrt{2880}}{\sqrt{216}.\sqrt{56}}=\sqrt{\dfrac{5}{21}}\right\}\{\Rightarrow\cos\theta\}=\sqrt{\dfrac{16}{21}}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$ | $\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ | A1 |
| | | | **[3]** |
| (b) **Alt 2** | **<u>Cosine Rule</u>** | | |
| | $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}9\\1\\8\end{pmatrix}-\begin{pmatrix}-3\\7\\2\end{pmatrix}=\begin{pmatrix}12\\-6\\6\end{pmatrix}$ or $\overrightarrow{PA}=\begin{pmatrix}-12\\6\\-6\end{pmatrix}$ | An attempt to find $\overrightarrow{AP}$ or $\overrightarrow{PA}$ | M1 |
| | Note: $\lvert\overrightarrow{PA}\rvert=\sqrt{216}$, $\lvert\overrightarrow{AB}\rvert=\sqrt{56}$ and $\lvert\overrightarrow{PB}\rvert=\sqrt{80}$ | | |
| | $\left(\sqrt{80}\right)^2=\left(\sqrt{216}\right)^2+\left(\sqrt{56}\right)^2-2\left(\sqrt{216}\right)\left(\sqrt{56}\right)\cos\theta$ | Applies the cosine rule the correct way round | dM1 |
| | $\cos\theta=\dfrac{216+56-80}{2\sqrt{216}\sqrt{56}}=\dfrac{192}{2\sqrt{216}\sqrt{56}}$ | | |
| | $\{\Rightarrow\cos\theta\}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$ | $\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ | A1 |
| | | | **[3]** |

| | | **Question 7 Notes** |
|---|---|---|
| 7. (b) | **Note** | If no "subtraction" seen, you can award 1st M1 for 2 out of 3 correct components of the difference |
| | **Note** | For dM1 the dot product formula can be applied as<br>$\sqrt{(12)^2+(-6)^2+(6)^2}.\sqrt{(4)^2+(-6)^2+(2)^2}\cos\theta=\begin{pmatrix}12\\-6\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-6\\2\end{pmatrix}$ |
| | **Note** | ***Evaluation*** of the dot product for $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is not required for the dM1 mark |
| | **A1** | For either $\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ or $\cos\theta=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ |
| | **Note** | Using $12\mathbf{i}-6\mathbf{j}+6\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\dfrac{24+18+6}{\sqrt{216}.\sqrt{14}}=\dfrac{48}{12\sqrt{21}}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$ |
| | **Note** | Using $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ & $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ gives $\cos\theta=\dfrac{4+3+1}{\sqrt{6}.\sqrt{14}}=\dfrac{8}{2\sqrt{21}}=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$ |
| | **Note** | Give M1M1A0 for finding $\theta=$ awrt 29.2 without reference to $\cos\theta=\underline{\underline{\dfrac{4}{\sqrt{21}}}}$ or $\underline{\underline{\dfrac{4}{21}\sqrt{21}}}$ |
| | **Note** | Condone taking the dot product between vectors the wrong way round for the M1 dM1 marks |
| | **Note** | **<u>Vectors the wrong way round</u>** |
| | | • E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\dfrac{4}{\sqrt{21}}$ or $-\dfrac{4}{21}\sqrt{21}$ ***with no other working*** is final A0<br>• E.g. taking the dot product between $\overrightarrow{PA}$ and $\overrightarrow{AB}$ to give $\cos\theta=-\dfrac{4}{\sqrt{21}}$ or $-\dfrac{4}{21}\sqrt{21}$ **followed by** $\cos\theta=\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ or just simply writing $\dfrac{4}{\sqrt{21}}$ or $\dfrac{4}{21}\sqrt{21}$ is final A1 |
| | **Note** | In part (b), give M0dM0 for finding and using $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ |
| (c) | **Note** | Give 1st M0 for $\sin\theta=\sin\left(\cos^{-1}\left(\dfrac{4\sqrt{21}}{21}\right)\right)$ or $\sin\theta=1-\left(\dfrac{4}{21}\sqrt{21}\right)^2$ ***unless recovered*** |
| | **M1** | Give 2nd M1 for either<br>• $\dfrac{1}{2}$(their length $AP$)(their length $AB$)(their attempt at $\sin\theta$)<br>• $\dfrac{1}{2}$(their length $AP$)(their length $AB$)sin(their 29.2° from part (b))<br>• $\dfrac{1}{2}$(their length $AP$)(their length $AB$)$\sin\theta$; where $\cos\theta=...$ in part (b) |
| | **Note** | $\dfrac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(\text{awrt } 29.2° \text{ or awrt } 150.8°)$ {= awrt 26.8} without reference to finding $\sin\theta$ as an exact value if M0 M1 A0 |
| | **Note** | Anything that rounds to 26.8 without reference to finding $\sin\theta$ as an exact value is M0 M1 A0 |
| | **Note** | Anything that rounds to 26.8 without reference to $12\sqrt{5}$ is A0 |
| | **Note** | If they use $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{AB}=(5\mathbf{i}+7\mathbf{j}+6\mathbf{k})$ in part (b), then this can be followed through in part (c) for the 2nd M mark as e.g. $\dfrac{1}{2}\left(\sqrt{110}\right)\left(\sqrt{56}\right)\sin\theta$ |
| | **Note** | Finding $12\sqrt{5}$ in part (c) is M1 dM1 A1, even if there is little or no evidence of finding an exact value for $\sin\theta$. So $\dfrac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{56}\right)\sin(29.2°)=12\sqrt{5}$ is M1 dM1 A1 |

| | | Question 7 Notes Continued |
|---|---|---|
| **7.** (d) | **Note** | Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is not required for the M mark |
| | **A1** | Writing $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ a multiple of $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ |
| | **Note** | Writing $\mathbf{r} = ...$ or $l_2 = ...$ or $l = ...$ or Line 2 = … is required for the A mark |
| | **Note** | Other valid $\mathbf{p} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix}$ are e.g. $\mathbf{p} = \begin{pmatrix} 13 \\ -5 \\ 10 \end{pmatrix}$ or $\mathbf{p} = \begin{pmatrix} 5 \\ 7 \\ 6 \end{pmatrix}$. So $\mathbf{r} = \begin{pmatrix} 13 \\ -5 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ is M1 A1 |
| | **Note** | Give A0 for writing $l_2 : \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ or ans $= \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ unless recovered |
| | **Note** | Using scalar parameter $\lambda$ or other scalar parameters (e.g. $\mu$ or *s* or *t*) is fine for M1 and/or A1 |
| (e) | **ddM1** | Substitutes their value of $\mu$ into $\overrightarrow{OQ}$, where $\overrightarrow{OQ}$ = their equation for $l_2$ |
| | **Note** | If they use $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$ in part (b), then this can be followed through in part (e) for the 2nd M mark and the 3rd M mark |
| | **Note** | You imply the final M mark in part (e) for at least 2 correctly followed through components for *Q* from their $\mu$ |
| | | |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** (c) **Alt 1** | **<u>Vector Cross Product:</u>** Use this scheme if a vector cross product method is being applied | | |
| | $\overrightarrow{AP} \times \overrightarrow{AB} = \underline{\begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -6 & 6 \\ 4 & -6 & 2 \end{vmatrix} = 24\mathbf{i} + 0\mathbf{j} - 48\mathbf{k} \right\}$ | | |
| | Area $PAB = \frac{1}{2}\sqrt{(24)^2 + (-48)^2}$ | Uses a vector product and $\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$ | M1 |
| | | Uses a vector product and $\frac{1}{2}\sqrt{("24")^2 + ("0")^2 + ("-48")^2}$ | M1 |
| | $= 12\sqrt{5}$ | $12\sqrt{5}$ | A1 **cao** |
| | | | **[3]** |
| 7. (c) **Alt 2** | **Note:** $\cos APB = \frac{5}{\sqrt{30}}$ or $\frac{1}{6}\sqrt{30}$ **Note:** $\lvert\overrightarrow{PA}\rvert = \sqrt{216}$ and $\lvert\overrightarrow{PB}\rvert = \sqrt{80}$ | | |
| | $\sin\theta = \frac{\sqrt{30-25}}{\sqrt{30}} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{6}}{6}$ | A correct method for converting an exact value for $\cos q$ to an exact value for $\sin q$ | M1 |
| | Area $PAB = \frac{1}{2}\left(\sqrt{216}\right)\left(\sqrt{80}\right)\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\left\{= 12\sqrt{30}\left(\frac{\sqrt{5}}{\sqrt{30}}\right)\right\} = 12\sqrt{5}$ | $\frac{1}{2}$(their *PA*)(their *PB*)$\sin\theta$ | M1 |
| | | $12\sqrt{5}$ | A1 **cao** |
| | | | **[3]** |

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**8.**

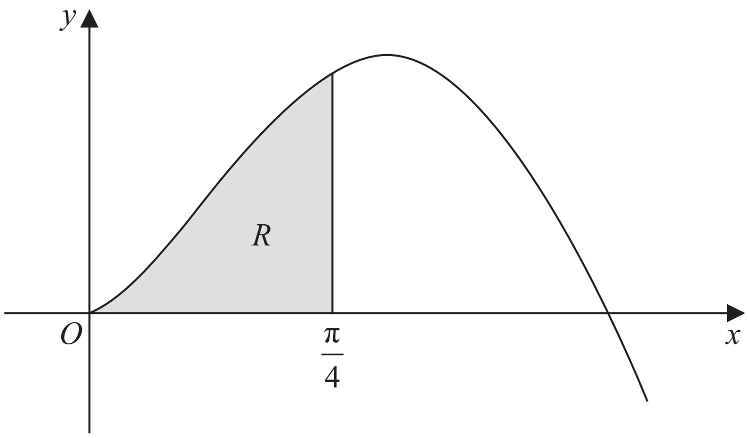


**Figure 3**

(a) Find $\int x \cos 4x \, dx$

**(3)**

Figure 3 shows part of the curve with equation $y = \sqrt{x} \sin 2x, \quad x \geqslant 0$

The finite region $R$, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and the line with equation $x = \frac{\pi}{4}$

The region $R$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Find the exact value of the volume of this solid of revolution, giving your answer in its simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(6)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8.** (a) | $\left\{\int x\cos 4x\,\mathrm{d}x\right\}$ $= \frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$ | $\pm\alpha x\sin 4x \pm \beta\int \sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$; $\alpha,\ \beta \neq 0$ | M1 |
| | | $\frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$ **Can be simplified or un-simplified** | A1 |
| | $= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\ \{+c\}$ | $\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x$ o.e. with or without $+c$ **Can be simplified or un-simplified** | A1 |
| | **Note:** You can ignore subsequent working following on from a correct solution | | **[3]** |
| (b) **Way 1** | $\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ | $\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied | B1 |
| | $\left\{\int x\sin^2 2x\,\mathrm{d}x = \right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ | For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1 - 2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral Can be implied. | M1 |
| | | Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ | A1 |
| | $\left\{\int\left(\frac{1}{2}x - \frac{1}{2}x\cos 4x\right)\mathrm{d}x\right\}$ $= \frac{1}{4}x^2 - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\{+c\}$ | Integrates to give $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x$; $A, B, C \neq 0$ which can be simplified or un-simplified. **Note:** Allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b) | M1 |
| | $\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x = \left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$ | | |
| | $= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(0 - 0 - \frac{1}{32}\cos 0\right)$ | **dependent on the previous M mark** see notes | dM1 |
| | $= \left(\frac{\pi^2}{64} + \frac{1}{32}\right) - \left(-\frac{1}{32}\right) = \frac{\pi^2}{64} + \frac{1}{16}$ | | |
| | So, $V = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e. | two term exact answer | A1 o.e. |
| | | | **[6]** |
| | | | **9** |

**Question 8 Notes**

| | | |
|---|---|---|
| | **SC** | **<u>Special Case for the 2nd M and 3rd M mark for those who use their answer from part (a)</u>** |

You can apply the 2nd M and 3rd M marks for integration of the form
$\pm Ax^2 \pm$ (their answer to part (*a*))
where their answer to part (a) is in the form

- $\pm Bx\sin kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\cos px$
- $\pm Bx\sin kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\sin kx \pm C\sin px$
- $\pm Bx\cos kx \pm C\sin px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\sin px$
- $\pm Bx\cos kx \pm C\cos px$ to give $\pm Ax^2 \pm Bx\cos kx \pm C\cos px$

$k, p \neq 0$, $k, p$ can be 1

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8.** (b) **Way 2** | $\{V=\}\ \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ | $\pi\int\left(\sqrt{x}\sin 2x\right)^2\{\mathrm{d}x\}$ Ignore limits and $\mathrm{d}x$. Can be implied | B1 |
| | $\left\{\int x\sin^2 2x\,\mathrm{d}x=\right\}$ $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ | For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1-2\sin^2 2x$) **and** some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral. Can be implied | M1 |
| | | Simplifies $\int x\sin^2 2x\,\{\mathrm{d}x\}$ to $\int x\left(\frac{1-\cos 4x}{2}\right)\{\mathrm{d}x\}$ **Note:** This mark can be implied for stating $u=x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1-\cos 4x}{2}$ **or** $u=\frac{1}{2}x$ **and** $\frac{\mathrm{d}v}{\mathrm{d}x}=1-\cos 4x$ | A1 |
| | $= x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\int\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)\mathrm{d}x$ | | |
| | $= x\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)-\left(\frac{1}{4}x^2+\frac{1}{32}\cos 4x\right)\{+c\}$ | Integrates to give $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x;\ A, B, C\neq 0$ or an expression that can be simplified to this form | M1 **(B1 on ePEN)** |
| | $\left\{\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2\mathrm{d}x=\left[\frac{1}{4}x^2-\frac{1}{8}x\sin 4x-\frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}}\right\}$ | | |
| | $=\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2-\frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right)-\frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right)-\left(0-0-\frac{1}{32}\cos 0\right)$ | **dependent on the previous M mark** see notes | dM1 |
| | $=\left(\frac{\pi^2}{64}+\frac{1}{32}\right)-\left(-\frac{1}{32}\right)=\frac{\pi^2}{64}+\frac{1}{16}$ | | |
| | So, $V=\pi\left(\frac{\pi^2}{64}+\frac{1}{16}\right)$ or $\frac{1}{64}\pi^3+\frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32}+\frac{1}{8}\right)$ o.e. | | A1 o.e. |
| | | | **[6]** |

| | | Question 8 Notes Continued |
|---|---|---|
| **8.** (a) | **SC** | Give ***Special Case*** M1A0A0 for writing down the correct "by parts" formula and using $u=x,\ \frac{\mathrm{d}v}{\mathrm{d}x}=\cos 4x,$ but making only one error in the application of the correct formula |
| (b) | **Note** | You can imply B1 for seeing $\pi\int y^2\{\mathrm{d}x\}$, followed by $y^2=\left(\sqrt{x}\sin 2x\right)^2$ or $y^2=x\sin^2 2x$ |
| | **Note** | If the form $\cos 4x=\cos^2 2x-\sin^2 2x$ or $\cos 4x = 2\cos^2 2x-1$ is used, the 1st M cannot be gained until $\cos^2 2x$ has been replaced by $\cos^2 2x = 1-\sin^2 2x$ and the result is applied to their integral |
| | **Note** | Mixing $x's$ and e.g. $\theta's$: Condone $\cos 4\theta = 1-2\sin^2 2\theta,\ \sin^2 2\theta=\frac{1-\cos 4\theta}{2}$ or $\lambda\sin^2 2\theta=\lambda\left(\frac{1-\cos 4\theta}{2}\right)$ if recovered in their integration |
| | **Final M1** | **Complete** method of applying limits of $\frac{\pi}{4}$ and 0 to all terms of an expression of the form $\pm Ax^2\pm Bx\sin 4x\pm C\cos 4x;\ A, B, C\neq 0$ and subtracting the correct way round. |
| | **Note** | For the final M1 mark in Way 1, allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b) |

| | | **Question 8 Notes Continued** |
|---|---|---|
| **8.** (b) | **Note** | Evidence of a proper consideration of the limit of 0 on $\cos 4x$ **where applicable** is needed for the<br>final M mark<br>E.g. $\left[\frac{1}{4}x^2 - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\right]_0^{\frac{\pi}{4}} =$<br>• $= \left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) + \frac{1}{32}$ is final M1<br>• $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - 0$ is final M0<br>• $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \frac{1}{32}$ is final M0 (adding)<br>• $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - \left(\frac{1}{32}\right)$ is final M1 (condone)<br>• $\left(\frac{1}{4}\left(\frac{\pi}{4}\right)^2 - \frac{1}{8}\left(\frac{\pi}{4}\right)\sin\left(4\left(\frac{\pi}{4}\right)\right) - \frac{1}{32}\cos\left(4\left(\frac{\pi}{4}\right)\right)\right) - (0+0+0)$ is final M0 |
| **8.** (b) | **Note** | <u>**Alternative Method**</u>:<br>$\left\{\begin{matrix} u = \sin^2 2x & \frac{dv}{dx} = x \\ \frac{du}{dx} = 2\sin 4x & v = \frac{1}{2}x^2 \end{matrix}\right\}, \left\{\begin{matrix} u = x^2 & \frac{dv}{dx} = \sin 4x \\ \frac{du}{dx} = 2x & v = -\frac{1}{4}\cos 4x \end{matrix}\right\}$<br>$\int x\sin^2 2x\,dx$<br>$= \frac{1}{2}x^2\sin^2 2x - \int \frac{1}{2}x^2(2\sin 4x)\,dx$<br>$= \frac{1}{2}x^2\sin^2 2x - \int x^2\sin 4x\,dx$<br>$= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x - \int 2x.\left(-\frac{1}{4}\cos 4x\right)dx\right)$<br>$= \frac{1}{2}x^2\sin^2 2x - \left(-\frac{1}{4}x^2\cos 4x + \frac{1}{2}\int x\cos 4x\,dx\right)$<br>$= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\int x\cos 4x\,dx$<br>$= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{2}\left(\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right)\ \{+c\}$<br>$= \frac{1}{2}x^2\sin^2 2x + \frac{1}{4}x^2\cos 4x - \frac{1}{8}x\sin 4x - \frac{1}{32}\cos 4x\ \{+c\}$<br>$V = \pi\int_0^{\frac{\pi}{4}}\left(\sqrt{x}\sin 2x\right)^2 dx = \pi\left(\frac{\pi^2}{64} + \frac{1}{16}\right)$ or $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$ or $\frac{\pi}{2}\left(\frac{\pi^2}{32} + \frac{1}{8}\right)$ o.e. |