Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Tuesday 15 January 2019

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Core Mathematics C34
Advanced

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. (a) Express $7\sin 2\theta - 2\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 90°$, the equation

$$7\sin 2\theta - 2\cos 2\theta = 4$$

giving your answers in degrees to one decimal place.

(4)

(c) Express $28\sin\theta\cos\theta + 8\sin^2\theta$ in the form $a\sin 2\theta + b\cos 2\theta + c$, where a, b and c are constants to be found.

(3)

(d) Use your answers to part (a) and part (c) to deduce the exact maximum value of $28\sin\theta\cos\theta + 8\sin^2\theta$

(2)

Question Number	Scheme	Notes	Marks
1(a)	$R=\sqrt{53}$	cao	B1
	$\tan\alpha=\frac{2}{7}\Rightarrow\alpha=...$ $\tan\alpha=\pm\frac{2}{7}$ or $\tan\alpha=\pm\frac{7}{2}$ or $\sin\alpha=\pm\frac{2}{'\sqrt{53}'}$ or $\sin\alpha=\pm\frac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\frac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\frac{2}{'\sqrt{53}'}$ $\Rightarrow\alpha=...$ Uses one of these equations to find a value for α		M1
	$\alpha=15.95^{\circ}$	Awrt 15.95° (Allow awrt 0.28 (rad))	A1
			(3)
(b)	$\sqrt{53}\sin(2\theta-15.95^{\circ})=4\Rightarrow\sin(2\theta-15.95^{\circ})=\frac{4}{\sqrt{53}}(0.549)$ Attempts to use part (a) $"\sqrt{53}"\sin(2\theta-"15.95^{\circ}")=4$ and proceeds to $\sin(2\theta\pm"15.95^{\circ}")=K,\quad \lvert K\rvert<1$ Allow the letter α for "15.95"		M1
	$2\theta-15.95^{\circ}=33.3287...\Rightarrow\theta=24.6^{\circ}$	Awrt 24.6° (Allow awrt 0.43 (rad))	A1
	$2\theta-15.95^{\circ}=180^{\circ}-33.287...\Rightarrow\theta=...$ Correct attempt at a second solution in the range. E.g. $2\theta_2\mp'15.95^{\circ}'=180^{\circ}-'33.3287...^{\circ}'\Rightarrow\theta_2=\frac{180^{\circ}-'33.3287...^{\circ}'\pm'15.95^{\circ}'}{2}$ (May be implied by their θ_2) It is **dependent** upon having scored the previous M. **Do not allow mixing of radians and degrees so if working in radians must be using π not 180**		dM1
	$\theta=81.3^{\circ}$	Awrt 81.3° **only**	A1
	Ignore extra answers outside range but deduct the final A for extra answers in range.		
			(4)
(c)	$28\sin\theta\cos\theta=a\sin2\theta\Rightarrow a=14$	$a=14$	B1
	$8\sin^2\theta=b\left(\pm1\pm2\sin^2\theta\right)+c$ or $8\sin^2\theta=8\left(\frac{1}{2}\left(\pm1\pm\cos2\theta\right)\right)$ or $8\sin^2\theta=4\sin^2\theta+4\sin^2\theta=4\sin^2\theta+4\left(1-\cos^2\theta\right)=\pm4\cos2\theta\pm4$ Attempts to use a $\cos2\theta$ identity e.g. $\cos2\theta=\pm1\pm2\sin^2\theta$ or $\sin^2\theta=\frac{1}{2}\left(\pm1\pm\cos2\theta\right)$ at some point in their working and applies it to the given expression.		M1
	$b=-4,\ c=4$ or $...-4\cos2\theta+4$	Correct values or correct expression	A1
			(3)
(d)	$\left(28\sin\theta\cos\theta+8\sin^2\theta\right)_{\max}=2'\sqrt{53}'+'4'$	Maximum $=2\times$ their $\sqrt{53}+$ their c May be implied e.g. by their decimal answer.	M1
	$2\sqrt{53}+4$	Cao (must be exact not decimals)	A1
	Attempts to use calculus for the maximum should reach $2R+c$ as above for M1 .		
			(2)
			Total 12

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2. Given that

$$\frac{3x^2 + 4x - 7}{(x+1)(x-3)} \equiv A + \frac{B}{x+1} + \frac{C}{x-3}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, find the series expansion of

$$\frac{3x^2 + 4x - 7}{(x+1)(x-3)} \qquad |x| < 1$$

in ascending powers of x, up to and including the term in x^2

Give each coefficient as a simplified fraction.

(6)

Question Number	Scheme	Notes	Marks
2	$\dfrac{3x^2+4x-7}{(x+1)(x-3)} \equiv A+\dfrac{B}{x+1}+\dfrac{C}{x-3}$		
(a)	$A=3$	Must be clearly identified as the value of A. (May be implied by their partial fractions)	B1
	$3x^2+4x-7=A(x+1)(x-3)+B(x-3)+C(x+1)$ And then expands and compares coefficients or substitutes values of x leading to a value for B or C **Or** $3x^2+4x-7\div(x+1)(x-3)=3+\dfrac{"10x+2"}{(x+1)(x-3)}$ $\Rightarrow 10x+2=B(x-3)+C(x+1)$ And then expands and compares coefficients or substitutes values of x leading to a value for B or C A correct method may be implied by correct values provided no incorrect work is seen		M1
	$B=2$ or $C=8$	One of B or C correct	A1
	$B=2$ and $C=8$	Both B and C correct	A1
			(4)

(b) Way 1	**If correct values for *A*, *B* and *C* are obtained by an incorrect method in part (a), allow a full recovery in (b)**		
	$\frac{1}{x+1}=(1+x)^{-1}=(1-x+x^2...)$	Attempts to expand $(1+x)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	$\frac{1}{x-3}=-(3-x)^{-1}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$	$\frac{1}{x-3}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$. Takes out a correct factor including the minus sign **and** a correct bracket.	B1
	$\left(1-\frac{1}{3}x\right)^{-1}=1+\frac{1}{3}x+\frac{1}{9}x^2...$	Attempts to expand $\left(1\pm\frac{1}{3}x\right)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	Note $-(3-x)^{-1}$ can be expanded as $-\left(3^{-1}+(-1)3^{-2}(-x)+\frac{(-1)(-2)}{2!}3^{-3}(-x)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term **or** $\frac{1}{x-3}$ can be expanded as $(x-3)^{-1}=3^{-1}\left(\frac{x}{3}-1\right)^{-1}\left(=3^{-1}\left(-1+\frac{x}{3}\right)^{-1}\right)$ $=3^{-1}\left(-1-(-1)^{-2}\left(\frac{x}{3}\right)+\frac{-1(-2)}{2}(-1)^{-3}\left(\frac{x}{3}\right)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term		
	$\frac{3x^2+4x-7}{(x+1)(x-3)}\approx(3+)2(1-x+x^2)-\frac{8}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2\right)$ Combines using their expansions and at least their *B* and *C* (so allow if they forget/don't add their *A*)		M1
	$=\frac{7}{3}-\frac{26}{9}x+\frac{46}{27}x^2$	Any 2 correct terms	A1
		All terms correct	A1
	Allow $2\frac{1}{3}$ for $\frac{7}{3}$, $-2\frac{8}{9}$ for $-\frac{26}{9}$, $1\frac{19}{27}$ for $\frac{46}{27}$		
			(6)
			Total 10

2(b)	**(b) Way 2 not requiring part (a) using** $\frac{3x^2+4x-7}{(x+1)(x-3)}=(3x^2+4x-7)(x+1)^{-1}(x-3)^{-1}$		
	$(1+x)^{-1}=(1-x+x^2...)$	Attempts to expand $(1+x)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	$\frac{1}{x-3}=-(3-x)^{-1}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$	$\frac{1}{x-3}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$ or $-3^{-1}\left(1-\frac{1}{3}x\right)^{-1}$ Takes out a correct factor including the minus sign.	B1
	$\left(1-\frac{1}{3}x\right)^{-1}=1+\frac{1}{3}x+\frac{1}{9}x^2...$	Attempts to expand $\left(1\pm\frac{1}{3}x\right)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	Note $-(3-x)^{-1}$ can be expanded as $-\left(3^{-1}+(-1)3^{-2}(-x)+\frac{(-1)(-2)}{2!}3^{-3}(-x)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term **or** $\frac{1}{x-3}$ can be expanded as $(x-3)^{-1}=3^{-1}\left(\frac{x}{3}-1\right)^{-1}\left(=3^{-1}\left(-1+\frac{x}{3}\right)^{-1}\right)$ $=3^{-1}\left(-1-(-1)^{-2}\left(\frac{x}{3}\right)+\frac{-1(-2)}{2}(-1)^{-3}\left(\frac{x}{3}\right)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term		
	$\frac{3x^2+4x-7}{(x+1)(x-3)}\approx(3x^2+4x-7)\left(-\frac{1}{3}\right)\left(1+\frac{1}{3}x+\frac{1}{9}x^2\right)(1-x+x^2)=...$ Attempts to multiply out all 3 brackets		M1
	$=\frac{7}{3}-\frac{26}{9}x+\frac{46}{27}x^2$	Any 2 correct terms	A1
		All terms correct	A1
			(6)

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3. The function f is defined by

$$f : x \mapsto 2x^2 + 3kx + k^2 \qquad x \in \mathbb{R}, -4k \leqslant x \leqslant 0$$

where k is a positive constant.

(a) Find, in terms of k, the range of f.

(4)

The function g is defined by

$$g : x \mapsto 2k - 3x \qquad x \in \mathbb{R}$$

Given that $gf(-2) = -12$

(b) find the possible values of k.

(4)

Question Number	Scheme	Notes	Marks
3(a)	$\mathrm{f}\left(-\frac{3k}{4}\right)=\ldots\text{or } \mathrm{f}(-4k)=\ldots$	Attempts $\mathrm{f}\left(-\frac{3k}{4}\right)$ **or** $\mathrm{f}(-4k)$	M1
	Note: Candidates who use completion of the square to obtain e.g. $a\left(x+\frac{3k}{4}\right)^2+b$ must then identify the "b" as an "end point" if they do not explicitly find $\mathrm{f}\left(-\frac{3k}{4}\right)$		
	$y_{\min}=-\frac{k^2}{8}$ or $y>-\frac{k^2}{8}$ or $y\geq-\frac{k^2}{8}$ or $y_{\max}=21k^2$ or $y<21k^2$ or $y\leq 21k^2$	One correct "end" of the range. May be implied by their final answer. Allow strict and non-strict inequality symbols or other indications that values are max or min.	A1
	$\mathrm{f}\left(-\frac{3k}{4}\right)=\ldots\text{and } \mathrm{f}(-4k)=\ldots$	Attempts $\mathrm{f}\left(-\frac{3k}{4}\right)$ **and** $\mathrm{f}(-4k)$	M1
	Note: Candidates who use completion of the square to obtain e.g. $a\left(x+\frac{3k}{4}\right)^2+b$ must then identify the "b" as an "end point" if they do not explicitly find $\mathrm{f}\left(-\frac{3k}{4}\right)$		
	$-\frac{k^2}{8}\leq \mathrm{f}(x)\leq 21k^2$ $\left[-\frac{k^2}{8},21k^2\right]$ $\mathrm{f}(x)\geq-\frac{k^2}{8}$ **and** $\mathrm{f}(x)\leq 21k^2$ $\mathrm{f}(x)\geq-\frac{k^2}{8}\cap \mathrm{f}(x)\leq 21k^2$	Correct range. Allow alternative notation as shown and allow y or "range" for f(x) but do not allow x for f(x).	A1
			(4)
(b)	$\mathrm{gf}(-2)=2k-3\left(2(-2)^2+3k(-2)+k^2\right)$ or $\mathrm{gf}(x)=2k-3\left(2x^2+3kx+k^2\right)$	Correct expression for gf(−2) or gf(x). Award this mark as soon as a correct expression is seen.	B1
	$2k-3\left(2(-2)^2+3k(-2)+k^2\right)=-12$	Puts their gf(−2) = ±12 to obtain an equation in k only. Must be using $x=-2$.	M1
	$3k^2-20k+12=0$ $\Rightarrow(3k-2)(k-6)=0\Rightarrow k=\frac{2}{3},\ 6$	Solves a 3TQ – see general guidance. **Dependent on the previous M.**	dM1
		Correct values. Allow equivalent fractions for $\frac{2}{3}$ or 0.6 with a clear dot over the 6.	A1
			(4)
			Total 8

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4. The curve C has equation

$$81y^3 + 64x^2y + 256x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(6)

Question Number	Scheme	Notes	Marks
4	$81y^3+64x^2y+256x=0$		
(a)	$\frac{\mathrm{d}(81y^3)}{\mathrm{d}x}=243y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	$\frac{\mathrm{d}(81y^3)}{\mathrm{d}x}=ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}(64x^2y)}{\mathrm{d}x}=128xy+64x^2\frac{\mathrm{d}y}{\mathrm{d}x}$	$\frac{\mathrm{d}(64x^2y)}{\mathrm{d}x}=\alpha xy+\beta x^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$243y^2\frac{\mathrm{d}y}{\mathrm{d}x}+128xy+64x^2\frac{\mathrm{d}y}{\mathrm{d}x}+256(=0)$	Correct differentiation. The "= 0" is not required but there should be no extra terms.	A1
	For the first 3 marks you can ignore any spurious "$\frac{\mathrm{d}y}{\mathrm{d}x}=$" at the start.		
	$243y^2\frac{\mathrm{d}y}{\mathrm{d}x}+64x^2\frac{\mathrm{d}y}{\mathrm{d}x}=-128xy-256\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}(243y^2+64x^2)=-128xy-256$ $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ Makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject allowing sign errors only with "= 0" seen or implied. This depends on there being **exactly two** $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms. One coming from the differentiation of $81y^3$ and one coming from the differentiation of $64x^2y$		M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-128xy-256}{243y^2+64x^2}$	Correct expression (oe)	A1
	Note that the final M1A1 in (a) <u>can</u> be recovered in part (b)		
			(5)

(b)	**Note that full marks are available in (b) following an incorrect denominator in (a)**		
	$-128xy-256=0$	Sets their numerator = 0. Note that this may appear from putting $\frac{dy}{dx}=0$ into their differentiation in part (a) before making $\frac{dy}{dx}$ the subject.	M1
	$81y^3+64y\left(-\frac{2}{y}\right)^2+256\left(-\frac{2}{y}\right)=0$ or $81\left(-\frac{2}{x}\right)^3+64x^2\left(-\frac{2}{x}\right)+256x=0$	Substitutes to obtain an equation in one variable. **Dependent on the first M.**	**d**M1
	$y^4=\frac{256}{81}\Rightarrow y=...$ or $x^4=\frac{81}{16}\Rightarrow x=...$	Solves an equation of the form $y^4=p$ or $x^4=q$ $(p,q>0)$ **Depends on the previous M.**	**d**M1
	$y=\pm\frac{4}{3}$ or $x=\pm\frac{3}{2}$	2 Correct values for x or 2 correct values for y. Allow unsimplified for this mark.	A1
	$y=(\pm)"\frac{4}{3}"\Rightarrow x=...$ or $x=(\pm)"\frac{3}{2}"\Rightarrow y=...$	Attempts at least one value of the other variable having previously found and solved an equation in one variable.	M1
	Examples: $\left(\pm\frac{3}{2},\mp\frac{4}{3}\right)$ or $x=\pm\frac{3}{2},\ y=\mp\frac{4}{3}$ or $x=\frac{3}{2},\ y=-\frac{4}{3}$, and $x=-\frac{3}{2},\ y=\frac{4}{3}$ or $\left(\frac{3}{2},-\frac{4}{3}\right),\left(-\frac{3}{2},\frac{4}{3}\right)$	Correct values which must now be simplified and paired correctly. **Do not isw and mark their final answer.**	A1
			(6)
			Total 11

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5. The angle x and the angle y are such that

$$\tan x = m \quad \text{and} \quad 4\tan y = 8m + 5$$

where m is a constant.

Given that $16\sec^2 x + 16\sec^2 y = 537$

(a) find the two possible values of m.

(4)

Given that the angle x and the angle y are acute, find the exact value of

(b) $\sin x$

(2)

(c) $\cot y$

(2)

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Question Number	Scheme	Notes	Marks
5	$\tan x = m$ and $4\tan y = 8m+5$		
(a)	Examples: $\sec^2 x = 1+m^2$ or $\sec^2 y = 1+\left(\frac{8m+5}{4}\right)^2$ or $16\sec^2 y = 16+16(8m+5)^2$	Attempts to express $\sec^2 x$ or $\sec^2 y$ in terms of m using a **correct** identity.	M1
	$16\left(\sec^2 x+\sec^2 y\right)=16\left(1+m^2+1+\left(\frac{8m+5}{4}\right)^2\right)=537$ Uses their expressions in m and 537 to obtain a quadratic equation in terms of m which may be unsimplified.		M1
	$m^2+m-6=0 \Rightarrow m=2,\ -3$	Solves their 3TQ as far as $m = \ldots$	M1
		Correct values	A1
			(4)
(b)	$\tan x = 2 \Rightarrow \sin x = \frac{2}{\sqrt{1^2+2^2}} = \ldots$ Correct method for the value of sin x. Must be from an appropriate identity or exact work but m does not need to be exact. So e.g. $\sin\left(\tan^{-1} 2\right) = 0.8944..$ scores M0 Can be for using either of their values of m.		M1
	$=\frac{2}{\sqrt{5}}$	cao (oe) **and no other values**	A1
			(2)
(c)	$\tan y = \frac{21}{4} \Rightarrow \cot y = \frac{4}{21}$	Correct method to obtain a value for cot y. So uses $4\tan y = 8m+5$ and their m to find a value for tan y and finds reciprocal. Can be for using either of their values of m.	M1
		cao (oe) **and no other values**	A1
			(2)
			Total 8

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6. Relative to a fixed origin O, the points A, B and C have coordinates (2, 1, 9), (5, 2, 7) and (4, −3, 3) respectively.

The line l passes through the points A and B.

(a) Find a vector equation for the line l.

(2)

(b) Find, in degrees, the acute angle between the line l and the line AC.

(3)

The point D lies on the line l such that angle ACD is 90°

(c) Find the coordinates of D.

(4)

(d) Find the exact area of triangle ADC, giving your answer as a fully simplified surd.

(2)

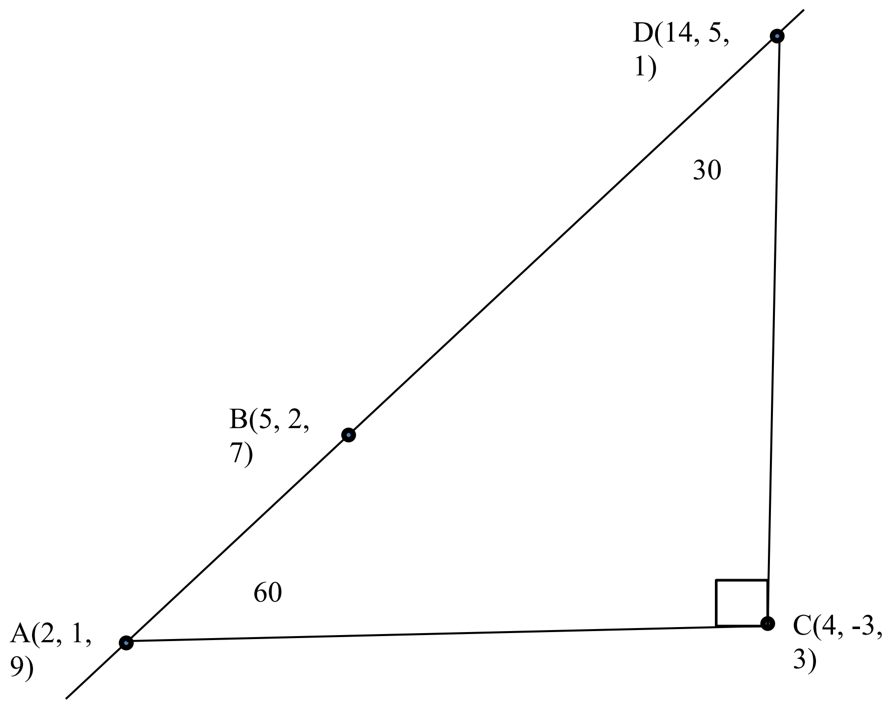
Question Number	Scheme	Notes	Marks
6(a)	$\pm\overrightarrow{AB} = \pm\left(\begin{pmatrix}5\\2\\7\end{pmatrix} - \begin{pmatrix}2\\1\\9\end{pmatrix}\right)$	Correct attempt at direction. May be implied by at least 2 correct components if no method seen.	M1
	$\mathbf{r} = \begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$ or $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 9\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	Accept equivalents but it must be an equation and it must be "r =" or $\begin{pmatrix}x\\y\\z\end{pmatrix} = \ldots$	A1
	Equivalent correct answers include: $\mathbf{r} = \begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}-3\\-1\\2\end{pmatrix}$ $\mathbf{r} = \begin{pmatrix}5\\2\\7\end{pmatrix} + \lambda\begin{pmatrix}-3\\-1\\2\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}5\\2\\7\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$		
	Do not allow e.g. $\mathbf{r} = \begin{pmatrix}2\mathbf{i}\\\mathbf{j}\\9\mathbf{k}\end{pmatrix} + \lambda\begin{pmatrix}3\mathbf{i}\\\mathbf{j}\\-2k\end{pmatrix}$ unless a correct form is seen earlier then isw		
			(2)

(b) Way 1	$\overrightarrow{AC} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$	Attempts $\pm\overrightarrow{AC}$. May be implied by at least 2 correct components if no method seen.	M1
	$\pm\overrightarrow{AB}.\pm\overrightarrow{AC} = \lvert AB\rvert\lvert AC\rvert\cos\theta \Rightarrow 6-4+12 = \sqrt{14}\sqrt{56}\cos\theta$ $\Rightarrow \cos\theta \Rightarrow \dfrac{14}{\sqrt{14}\sqrt{56}} \Rightarrow \theta = ...$ Attempt the scalar product of $\pm\overrightarrow{AB}$ or their direction vector from part (a) and their $\pm\overrightarrow{AC}$ and proceeds to $\theta = \ldots$		dM1
	$\theta = 60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)
	(b) Way 2 (cosine rule on triangle *ABC*)		
	$AB = \sqrt{14},\ AC = 2\sqrt{14},\ BC = \sqrt{42}$	Attempts the lengths of all 3 sides	M1
	$42 = 14 + 56 - 2\sqrt{14}\sqrt{56}\cos\theta$ $\Rightarrow \cos\theta = \dfrac{28}{2\sqrt{14}\sqrt{56}} \Rightarrow \theta = ...$	Attempt cosine rule and proceeds to $\theta = \ldots$	dM1
	$\theta = 60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)
	(b) Way 3 using vector product		
	$\overrightarrow{AC} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$	Attempts $\pm\overrightarrow{AC}$	M1
	$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -6 \\ 3 & 1 & -2 \end{vmatrix} = \begin{pmatrix} 14 \\ -14 \\ 14 \end{pmatrix} \Rightarrow \sin\theta = \dfrac{\sqrt{14^2+14^2+14^2}}{\sqrt{2^2+4^2+6^2}\sqrt{3^2+1^2+2^2}}$ Attempt the vector product of $\pm\overrightarrow{AB}$ or their direction vector from part (a) and their $\pm\overrightarrow{AC}$ and proceeds to $\theta = \ldots$		M1
	$\theta = 60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)

(c) Way 1	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on *AB*) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$\begin{pmatrix}2\\-4\\-6\end{pmatrix}\bullet\left\{\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}\right\}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}\bullet\begin{pmatrix}3\lambda-2\\\lambda+4\\-2\lambda+6\end{pmatrix}$ $6\lambda-4-4\lambda-16-36+12\lambda=0\Rightarrow\lambda=...$ Attempts $\overrightarrow{AC}\bullet\overrightarrow{CD}=0$ and solves for λ. It must be a correct AC or their attempt at AC and a correct attempt at CD or what they think is CD as long as it is clearly identified as CD.		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses **their** value of λ to find *D*. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1
			(4)
	(c) Way 2:		
	$AC=2\sqrt{14}\Rightarrow AD=\dfrac{2\sqrt{14}}{\cos 60°}\left(=4\sqrt{14}\right)$	Correct attempt at the length of *AD*	M1
	$AB=\sqrt{14}\Rightarrow AD=4AB$ or $(3\lambda)^2+\lambda^2+(2\lambda)^2=\left(4\sqrt{14}\right)^2\Rightarrow\lambda=...$	Uses ratio of *AB* to *AD* to find a value for "λ" or uses the length of *AD* and applies Pythagoras to "λ"×their direction of *l* to find a value for "λ"	M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of "λ" to find *D*. **Dependent on both previous M's**	**dd**M1
	D(14, 5, 1) or 14**i** + 5**j** + **k** etc.	Correct coordinates or vector and **no other points or vectors**	A1
	(c) Way 3		
	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on *AB*) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$(3\lambda-2)^2+(\lambda+4)^2+(6-2\lambda)^2=(AC\tan 60)^2$ $\lambda^2-2\lambda-8=0\Rightarrow\lambda=...$ Attempts $(CD)^2=(AC\tan 60)^2$ and solves for λ		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of λ to find *D*. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1

	(c) Way 4		
	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on AB) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$(3\lambda-2)^2+(\lambda+4)^2+(6-2\lambda)^2+AC^2=(3\lambda)^2+\lambda^2+(2\lambda)^2$ $28\lambda-112=0\Rightarrow\lambda=...$ Attempts $AC^2+CD^2=AD^2$ and solves for λ		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of λ to find D. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1

(d)	Area ADC = $\frac{1}{2}AC\times CD=\frac{1}{2}\sqrt{56}\sqrt{168}$	Correct triangle area method	M1
	$=28\sqrt{3}$	cao	A1
			(2)
	Alternatives for (d)		
	$\frac{1}{2}AC\times AD\sin 60^\circ=\frac{1}{2}\sqrt{56}\sqrt{224}\frac{\sqrt{3}}{2}$, $\frac{1}{2}AD\times DC\sin 30^\circ=\frac{1}{2}\sqrt{168}\sqrt{224}\frac{1}{2}$ $\frac{1}{2}AC\times AC\tan 60^\circ=\frac{1}{2}\sqrt{56}\sqrt{56}\sqrt{3}$		
			Total 11



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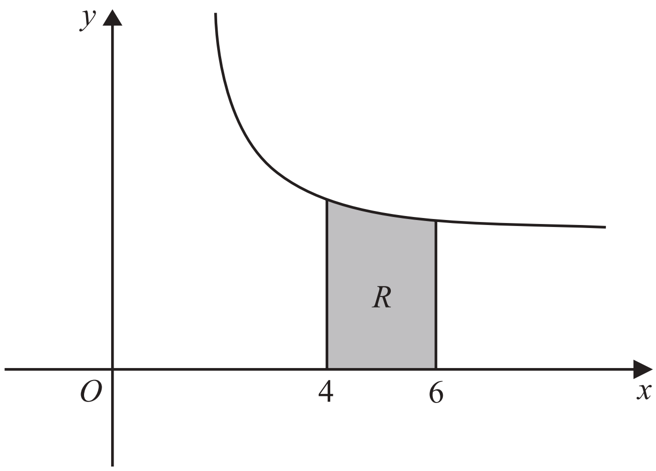

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{x + 7}{\sqrt{2x - 3}} \qquad x > \frac{3}{2}$$

The region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 6$

(a) Use the trapezium rule with 4 strips of equal width to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Using the substitution $u = 2x - 3$, or otherwise, use calculus to find the exact area of R, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants to be found.

(7)

Question Number	Scheme	Notes	Marks
7(a)	Strip width = 0.5	Correct value stated or used within the formula.	B1
	$\frac{11\sqrt{5}}{5}+\frac{13}{3}+2\left(\frac{23\sqrt{6}}{12}+\frac{12\sqrt{7}}{7}+\frac{25\sqrt{2}}{8}\right)$ or $(4.91...+4.33...+2(4.69...+4.53...+4.41...))$ Correct structure for their y values (if their values need checking, look for 2sf) Must have y values starting at $x = 4$ and ending at $x = 6$		M1
	Area $\approx \frac{1}{2}\times\frac{1}{2}\left(\frac{11\sqrt{5}}{5}+\frac{13}{3}+2\left(\frac{23\sqrt{6}}{12}+\frac{12\sqrt{7}}{7}+\frac{25\sqrt{2}}{8}\right)\right)$ or Area $\approx \frac{1}{2}\times\frac{1}{2}(4.91...+4.33...+2(4.69...+4.53...+4.41...))$ Correct numerical expression for the area (allow decimal values to 2sf), may be implied by their area value		A1
	9.14	9.14 **only**	A1
			(4)
(b)	$u=2x-3\Rightarrow\frac{du}{dx}=2$	Correct derivative. Accept any correct equivalents e.g. du = 2dx	B1
	$\int\frac{x+7}{\sqrt{2x-3}}\,dx=\int\frac{\frac{u+3}{2}+7}{\sqrt{u}}\,\frac{1}{2}\,du$	M1: Fully substitutes. Just replacing "dx" with "du" with no evidence of where the "du" has come from is M0 but allow slips e.g. omission of " + 7" A1: Fully correct expression.	M1A1
	$\frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}+34u^{\frac{1}{2}}\right)(+c)$	Fully correct integration in any form (+ c not required)	A1
	Note: Integration by parts gives $\frac{1}{4}\int(u+17)u^{-\frac{1}{2}}\,du=\frac{1}{4}\left(2u^{\frac{1}{2}}\right)(u+17)-\frac{1}{4}\int 2u^{-\frac{1}{2}}\,du=\frac{1}{2}u^{\frac{3}{2}}+\frac{17}{2}u^{\frac{1}{2}}-\frac{1}{3}u^{\frac{3}{2}}(+c)$		
	$x=4, u=5\quad x=6, u=9$	Correct u limits seen anywhere.	B1
	If they return to x then this B1 is for replacing u with $2x-3$		
	$\frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}}+34u^{\frac{1}{2}}\right]_5^9=\frac{1}{4}\left\{\left(\frac{2}{3}(9)^{\frac{3}{2}}+34(9)^{\frac{1}{2}}\right)-\left(\frac{2}{3}(5)^{\frac{3}{2}}+34(5)^{\frac{1}{2}}\right)\right\}$ Substitutes their (changed) u limits into a changed function and subtracts either way round or substitutes x limits if they undo the substitution and subtracts either way round		M1
	$=30-\frac{28}{3}\sqrt{5}$	cao	A1
			(7)
			Total 11

	Note that 7(b) is hence or otherwise so other substitutions will be seen. The mark scheme will follow the same structure as in the example below:		
7(b)	$u^2 = 2x-3 \Rightarrow 2u\frac{du}{dx} = 2$	Correct derivative. Accept correct equivalents e.g. $2u = 2\frac{dx}{du}$, $dx = u du$	B1
	$\int \frac{x+7}{\sqrt{2x-3}}dx = \int \frac{\frac{u^2+3}{2}+7}{u} u\, du$	M1: Fully substitutes. Allow slips e.g. omission of " + 7" A1: Fully correct expression.	M1A1
	$\frac{u^3}{6}+\frac{17}{2}u(+c)$	Fully correct integration in any form (+ c not required)	A1
	$x=4, u=\sqrt{5} \quad x=6, u=3$	Correct u limits seen anywhere.	B1
	If they return to x then this B1 is for replacing u with $\sqrt{2x-3}$		
	$\left[\frac{1}{6}u^3+\frac{17}{2}u\right]_{\sqrt{5}}^{3} = \left\{\left(\frac{27}{6}+\frac{17}{2}(3)\right)-\left(\frac{1}{6}(\sqrt{5})^3+\frac{17}{2}(\sqrt{5})\right)\right\}$ Substitutes their (changed) u limits into a changed function and subtracts either way round or substitutes x limits if they undo the substitution and subtracts either way round		M1
	$= 30 - \frac{28}{3}\sqrt{5}$	cao	A1
			(7)

	Note that 7(b) can also be done by parts:		
7(b)	$\int \frac{x+7}{\sqrt{2x-3}}dx = \int (x+7)(2x-3)^{-\frac{1}{2}}\ dx$	Uses $\frac{x+7}{\sqrt{2x-3}}$ as $(x+7)(2x-3)^{-\frac{1}{2}}$ and makes some progress with attempting to integrate even if it is incorrect.	B1
	$\int (x+7)(2x-3)^{-\frac{1}{2}} dx = (x+7)(2x-3)^{\frac{1}{2}} - \int (2x-3)^{\frac{1}{2}} dx$ M1: Integration by parts in the correct direction A1: Correct expression		M1A1
	$\int (2x-3)^{\frac{1}{2}} dx = \frac{1}{3}(2x-3)^{\frac{3}{2}}$	$\int (2x-3)^{\frac{1}{2}} dx = k(2x-3)^{\frac{3}{2}}$	M1
		$\int (2x-3)^{\frac{1}{2}} dx = \frac{1}{3}(2x-3)^{\frac{3}{2}}$	A1
	$\left[(x+7)(2x-3)^{\frac{1}{2}} - \frac{1}{3}(2x-3)^{\frac{3}{2}}\right]_4^6 = \left\{\left(11(9)^{\frac{1}{2}}-\frac{1}{3}(9)^{\frac{3}{2}}\right)-\left(11(5)^{\frac{1}{2}}-\frac{1}{3}(5)^{\frac{3}{2}}\right)\right\}$ Substitutes the limits 4 and 6 into a changed function and subtracts either way round		M1
	$= 30 - \frac{28}{3}\sqrt{5}$	cao	A1
			(7)

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8. A curve has parametric equations

$$x = t^2 - t, \qquad y = \frac{4t}{1-t} \qquad t \neq 1$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, giving your answer as a simplified fraction.

(4)

(b) Find an equation for the tangent to the curve at the point P where $t = -1$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The tangent to the curve at P cuts the curve at the point Q.

(c) Use algebra to find the coordinates of Q.

(5)

Question Number	Scheme	Notes	Marks
8(a)	$\frac{dx}{dt}=2t-1$	Correct derivative	B1
	Quotient rule $\frac{dy}{dt}=\frac{(1-t)\times 4-4t\times(-1)}{(1-t)^2}$ Obtains $\frac{dy}{dt}=\frac{\alpha(1-t)\pm\beta t}{(1-t)^2},\ \alpha>0,\ \beta>0$ **or product rule** $\frac{dy}{dt}=4t(1-t)^{-2}+4(1-t)^{-1}$ Obtains $\frac{dy}{dt}=p(1-t)^{-1}\pm qt(1-t)^{-2},\ \ p>0,\ q>0$ **If an incorrect formula is quoted this scores M0**		M1
	NB: May see $\frac{4t}{1-t}=-4+\frac{4}{1-t}\Rightarrow\frac{dy}{dt}=-4(1-t)^{-2}\times -1$ Allow M1 for $\frac{4t}{1-t}=A+\frac{B}{1-t}\Rightarrow\frac{dy}{dt}=k(1-t)^{-2}$		
	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{(1-t)\times 4-4t\times(-1)}{(1-t)^2}\times\frac{1}{2t-1}$ **Correct** application of the chain rule using their derivatives. This is an independent method mark. Their $\mathbf{\frac{dy}{dt}}$ divided by their $\mathbf{\frac{dx}{dt}}$ **or** their $\mathbf{\frac{dy}{dt}}$ multiplied by their $\mathbf{\frac{dt}{dx}}$		M1
	$\frac{dy}{dx}=\frac{4}{(2t-1)(1-t)^2}$ Allow e.g. $\frac{4}{(2t-1)(1-2t+t^2)},\ \frac{4}{2t^3-5t^2+4t-1}$ but not $\frac{1}{(2t-1)}\times\frac{4}{(1-t)^2}$ **But isw once a correct answer is seen**		A1
			(4)
(b)	$t=-1\rightarrow(2,\ -2)$ or $x=2,\ y=-2$	Correct coordinates for P	B1
	$t=-1\Rightarrow\frac{dy}{dx}=\frac{4}{(2(-1)-1)(1-(-1))^2}\left(=-\frac{1}{3}\right)$	Attempts gradient. May be implied by their value for the gradient.	M1
	$y+2=-\frac{1}{3}(x-2)$	Correct straight line method for the tangent **not the normal**. If using $y=mx+c$ must reach as far as finding a value for c.	M1
	$x+3y+4=0$	Any integer multiple.	A1
			(4)

(c) Way 1	$t^2 - t + 3\left(\frac{4t}{1-t}\right) + 4 = 0$	Substitutes to obtain an equation in t only.	M1
	$t^3 - 2t^2 - 7t - 4 = 0$	Correct cubic	A1
	$(t+1)(t^2-3t-4)=0$ or $(t+1)^2(t-4)=0$ Attempt to factorise using $(t \pm 1)$ or $(t \pm 1)^2$ as a factor. Look for $(t \pm 1)(at^2+...)$ or $(t \pm 1)^2(at+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $at^2 + \ldots$ or $at + \ldots$ **This mark is dependent on having obtained a cubic equation that has a constant term** **This mark is not for just solving their cubic e.g. using a calculator. However, if they have a correct cubic equation and the root $t = 4$ is seen, this method can be implied.**		M1
	$t = 4$	Correct value of t	A1
	$\left(12, -\frac{16}{3}\right)$	Correct coordinates	A1
			(5)
			Total 13

(c) Way 2	$y = \frac{4t}{1-t} \Rightarrow t = \frac{y}{4+y} \Rightarrow x = \left(\frac{y}{4+y}\right)^2 - \frac{y}{4+y}$ $\Rightarrow \left(\frac{y}{4+y}\right)^2 - \frac{y}{4+y} + 3y + 4 = 0$	Finds x in terms of y by eliminating t and substitutes to obtain an equation in y only. When eliminating t using y, the algebra must be correct so allow sign errors only for making t the subject from y.	M1
	$3y^3 + 28y^2 + 76y + 64 = 0$	Correct cubic	A1
	$(y+2)(3y^2+22y+32)=0$ or $(y+2)^2(3y+16)=0$ Attempt to factorise using $(y \pm 2)$ or $(y \pm 2)^2$ as a factor. Look for $(y \pm \text{"}-2\text{"})(ay^2+...)$ or $(y \pm \text{"}-2\text{"})^2(ay+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $ay^2 + \ldots$ or $ay + \ldots$ **This mark is dependent on having obtained a cubic equation that has a constant term** **This mark is not for just solving their cubic e.g. using a calculator. However, if they have a correct cubic equation and the root y = -16/3 is seen, this method can be implied.**		M1
	$y = -\frac{16}{3}$	Correct value of y	A1
	$\left(12, -\frac{16}{3}\right)$	Correct coordinates	A1
			(5)

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9.

(a) Find $\int x \sin 2x \, dx$

(3)

(b) Find $\int (x + \sin 2x)^2 \, dx$

(4)

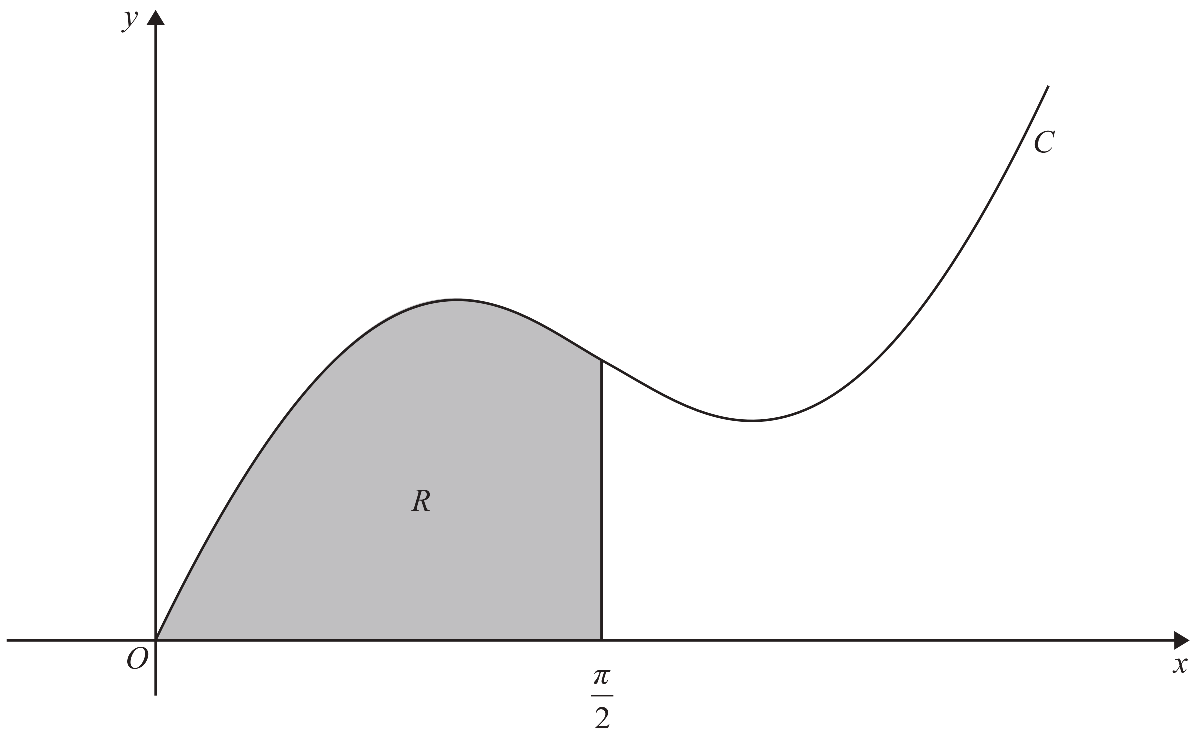

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x + \sin 2x$.

The region R, shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = \frac{\pi}{2}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the exact value for the volume of this solid, giving your answer as a single, simplified fraction.

(3)

Question Number	Scheme	Notes	Marks
9(a)	$\int x\sin 2x\,\mathrm{d}x = -x.\frac{1}{2}\cos 2x + \frac{1}{2}\int \cos 2x(\,\mathrm{d}x)$	$\int x\sin 2x\,\mathrm{d}x = \pm px.\cos 2x \pm q\int \cos 2x(\,\mathrm{d}x)$	M1
		Correct expression (dx not required)	A1
	$\int x\sin 2x\,\mathrm{d}x = \frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x(+c)$	Correct integration in any form – does not need to be simplified but is cso so e.g. any double sign errors should be penalised here. Condone poor notation e.g. cos2x.x rather than $x$cos2x. **The constant of integration is not required.**	A1 **cso**
			(3)
(b)	$(x+\sin 2x)^2 = x^2 + 2x\sin 2x + \sin^2 2x$	Correct (possibly unsimplified) expansion. Condone poor notation so allow e.g. 2sin2x.x for 2$x$sin2x and sin2x^2 for sin^{2}2x	B1
	$\int \sin^2 2x\,\mathrm{d}x = \frac{1}{2}\int(1-\cos 4x)\,\mathrm{d}x$	Uses $\cos 4x = \pm 1 \pm 2\sin^2 2x$	M1
	$\int \sin^2 2x\,\mathrm{d}x = \frac{1}{2}x - \frac{1}{8}\sin 4x(+c)$	Correct integration	A1
	$\int (x+\sin 2x)^2\,\mathrm{d}x = \frac{x^3}{3} + \frac{1}{2}\sin 2x - x\cos 2x + \frac{1}{2}x - \frac{1}{8}\sin 4x(+c)$ Allow in any correct possibly unsimplified form. Follow through their answer to part (a) so allow for: $\int (x+\sin 2x)^2\,\mathrm{d}x = \frac{x^3}{3} + 2\times\text{their part (a)} + \frac{1}{2}x - \frac{1}{8}\sin 4x(+c)$ **The constant of integration not required.**		A1ft
	In part (b) allow mixed variables for the first 3 marks but for the final mark the expression must be in terms of *x* only.		
			(4)
(c)	$(\text{Volume} =)\pi\int (x+\sin 2x)^2\,\mathrm{d}x$	States or implies that the volume required is $\pi\int (x+\sin 2x)^2$ **Note that the π <u>is</u> required but may appear later in their working.**	M1
	$=(\pi)\left(\frac{\pi^3}{24} + 0 + \frac{\pi}{2} + \frac{\pi}{4} - 0 - (0)\right)$ Applies at least the limit $\frac{\pi}{2}$ to an expression of the form: $\alpha x^3 + \beta x + (\text{at least one trig function})$. The substitution of x = 0 does not need to be seen. Must be exact work and not just decimals.		M1
	$= \frac{\pi^4 + 18\pi^2}{24}$ or $\frac{\pi^2(\pi^2+18)}{24}$	Cso. Allow any equivalent exact single fraction but come from correct integration. Note that incorrect coefficients of sin… will fortuitously give the correct answer.	A1 **cso**
	Note: Condone mixing *x* with θ		
			(3)
			Total 10

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10.

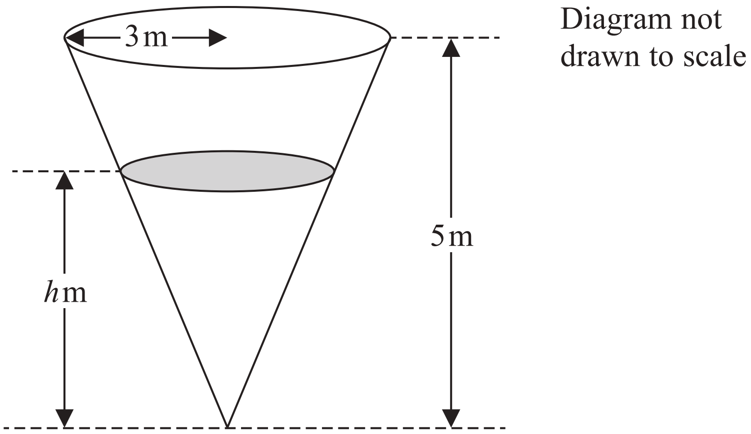


Figure 3

Figure 3 shows a container in the shape of an inverted right circular cone which contains some water.

The cone has an internal radius of 3 m and a vertical height of 5 m as shown in Figure 3.

At time t seconds, the height of the water is h metres, the volume of the water is V m^3 and water is leaking from a hole in the bottom of the container at a constant rate of 0.02 m^3 s^{-1}

[*The volume of a cone of radius r and height h is* $\frac{1}{3}\pi r^2 h$.]

(a) Show that, while the water is leaking,

$$h^2 \frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{1}{k\pi}$$

where k is a constant to be found.

(5)

Given that the container is initially full of water,

(b) express h in terms of t.

(3)

(c) Find the time taken for the container to empty, giving your answer to the nearest minute.

(2)

Question Number	Scheme	Notes	Marks
10(a)	E.g. $\frac{r}{h}=\frac{3}{5},\frac{3}{r}=\frac{5}{h},5r=3h,r=\frac{3}{5}h,h=\frac{5}{3}r$	Any correct equation connecting r and h	B1
	$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ $\left(V=\frac{9}{75}\pi h^3\right)$	Obtains $V=kh^3$ or equivalent using their equation connecting h and r. $V=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ is sufficient.	M1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{27}{75}\pi h^2$	Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$. Allow for $\frac{\mathrm{d}V}{\mathrm{d}h}=\alpha h^2$. **Dependent on the first M.**	**d**M1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}h}\Rightarrow\frac{27}{75}\pi h^2=-0.02\frac{\mathrm{d}t}{\mathrm{d}h}$	Uses e.g. $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}h}$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm0.02$ May be implied by their work.	M1
	$h^2\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{18\pi}$ *	Correct equation or states $k=18$	A1 **cso**
			(5)
(a) Way 2	**Avoids the need to find** $\frac{\mathrm{d}V}{\mathrm{d}h}$		
	$\frac{r}{h}=\frac{3}{5},\frac{3}{r}=\frac{5}{h},5r=3h,r=\frac{3}{5}h,h=\frac{5}{3}r$	Any correct equation connecting r and h	B1
	$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ $\left(V=\frac{9}{75}\pi h^3\right)$	Obtains $V=kh^3$ (or equivalent) using their equation connecting h and r.	M1
	Examples: $V=\frac{9}{75}\pi h^3\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=3\times\frac{9}{75}\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ or $h^3=\frac{75V}{9\pi}\Rightarrow3h^2\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75V}{9\pi}\frac{\mathrm{d}V}{\mathrm{d}t}$ The M1 is for differentiating both sides with respect to t to obtain $\alpha\frac{\mathrm{d}V}{\mathrm{d}t}=\beta h^2\frac{\mathrm{d}h}{\mathrm{d}t}$		dM1
	$-0.02=3\times\frac{9}{75}\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ or $3h^2\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75V}{9\pi}\times-0.02$	Replaces $\frac{\mathrm{d}V}{\mathrm{d}t}$ with ±0.02	M1
	$h^2\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{18\pi}$ *	Correct equation or states $k=18$	A1
			(5)

(b) Way 1	$\frac{h^3}{3} = -\frac{1}{18\pi}t(+c)$	$ph^3 = qt(+c)$. **Note that "+ c" is not required for this mark.**	M1
	$t = 0, h = 5 \Rightarrow c = \frac{125}{3}$	Uses h = 5 and t = 0 to find c. **There must be a constant of integration for this mark.**	M1
	Note that both M marks are available if the letter k is used i.e. if they haven't obtained a value for k in part (a)		
	$h = \sqrt[3]{125 - \frac{t}{6\pi}}$	Correct equation (oe)	A1
			(3)
(b) Way 2	$\frac{dV}{dt} = -0.02 \Rightarrow V = -0.02t + c$ $15\pi = c \Rightarrow V = 15\pi - 0.02t$	Uses $\frac{dV}{dt} = \pm 0.02$ to obtain $V = \pm 0.02t + c$ and then uses r = 3 and h = 5 when t = 0 to find c. This may be implied by sight of $V = 15\pi - 0.02t$ (**but must be V = …**)	M1
	$\frac{9}{75}\pi h^3 = 15\pi - 0.02t$ $h^3 = 125 - \frac{t}{6\pi} \Rightarrow h = ...$	Replaces V with V in terms of h and rearranges to find h	M1
	$h = \sqrt[3]{125 - \frac{t}{6\pi}}$	Correct equation (oe)	A1
			(3)
(c) Way 1	$h = 0 \Rightarrow 125 - \frac{t}{6\pi} = 0 \Rightarrow t = ...$	Puts h = 0 and solves for t	M1
	$t = 750\pi$ seconds		
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
(c) Way 2	$\left[\frac{h^3}{3}\right]_5^0 = \left[-\frac{1}{18\pi}t\right]_0^T \Rightarrow 0 - \frac{125}{3} = -\frac{1}{18\pi}T \Rightarrow T = ...$ Uses the limits 0 and 5 with their ph^3 and 0 and T or t with their qt and solves for T (or t). The limits can be either way round and the substitution of 0 does not need to be seen. A minimum could be $\frac{125}{3} = \frac{1}{18\pi}t \Rightarrow t = ...$(as in the main scheme)		M1
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
(c) Way 3	$\frac{1}{3}\pi(3)^2 \times 5 \div 0.02 = ...$ or e.g. solves $15\pi - 0.02t = 0$	Calculates the volume of the cone and divides by 0.02	M1
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
			Total 10

Leave blank

11. (a) Given that $0 \leqslant f(x) \leqslant \pi$, sketch the graph of $y = f(x)$ where

$$f(x) = \arccos(x - 1), \qquad 0 \leqslant x \leqslant 2$$

(2)

The equation $\arccos(x - 1) - \tan x = 0$ has a single root α.

(b) Show that $0.9 < \alpha < 1.1$

(2)

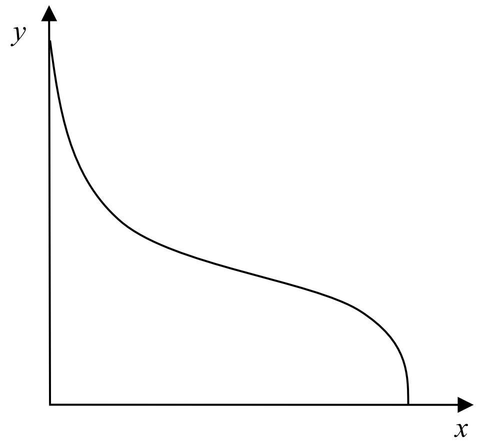
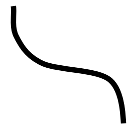
The iteration formula

$$x_{n+1} = \arctan(\arccos(x_n - 1))$$

can be used to find an approximation for α.

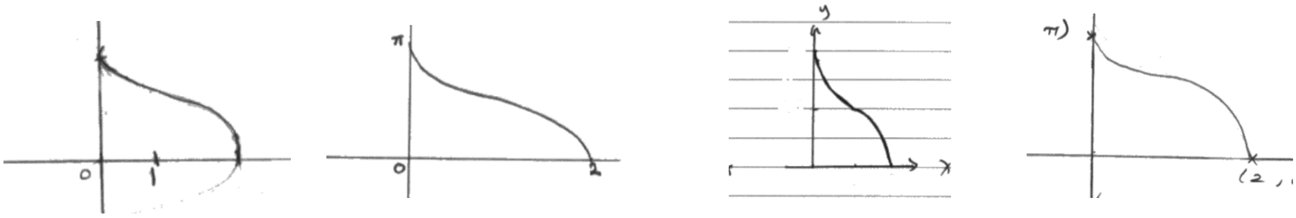
(c) Taking $x_0 = 1.1$ find, to 3 decimal places, the values of x_1 and x_2

(2)

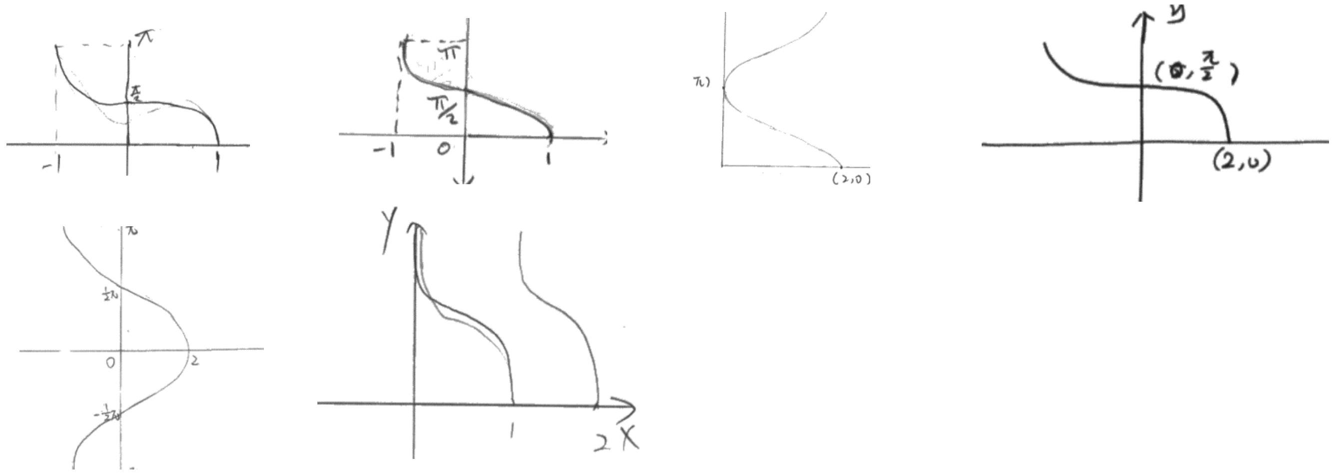
Question Number	Scheme	Notes	Marks
11(a)		 Correct shape anywhere. Ignore any extra "cycles" or other parts of graphs. The curve should become steeper at each end.	M1
		Correct shape in the correct position with no "extra cycles" or other parts of graphs. Ignore any labels on axes, correct or otherwise.	A1
	See next page for example marking		
			(2)
(b)	$f(0.9)=0.4108...,\ f(1.1)=-0.4941...$	Substitutes both $x = 0.9$ and $x = 1.1$ and obtains at least one answer correct to 1sf or truncated so allow 0.4 and – 0.4 or – 0.5.	M1
	Change of sign therefore $0.9 < \alpha < 1.1$ Both values correct (to one sig fig or truncated), change of sign + conclusion Allow equivalent statements e.g. positive, negative therefore root etc. but this mark may be withheld if there are any contradictory statements e.g. therefore root lies between f(0.9) and f(1.1)		A1
			(2)
(c)	$\arctan\left(\arccos(1.1-1)\right)$	Attempt the given formula with $x = 1.1$ Score for $\arctan\left(\arccos(1.1-1)\right)$ This may be implied by awrt 0.97 (using radians) or awrt 89 (using degrees) for x_1	M1
	$(x_1 =)0.974,\ \ (x_2 =)1.011$	$(x_1 =)$awrt 0.974, $(x_2 =)$awrt 1.011. Ignore any subsequent iterations and ignore labelling if answers are clearly the second and third terms.	A1
			(2)
			Total 6

Example marking of Q11 part (a)

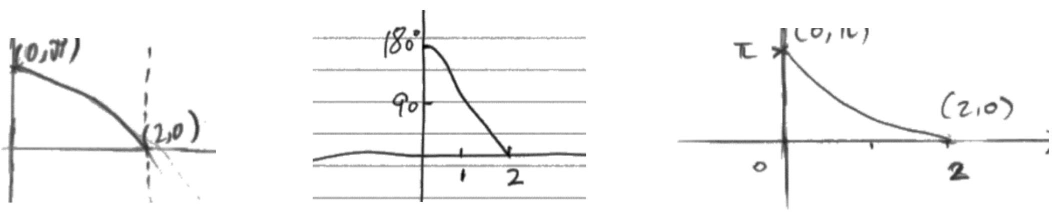
These sketches score both marks:



These sketches score M1A0:



These sketches score M0A0:



Leave blank

12. Given that k is a positive constant,

(a) sketch the graph with equation

$$y = 2|x| - k$$

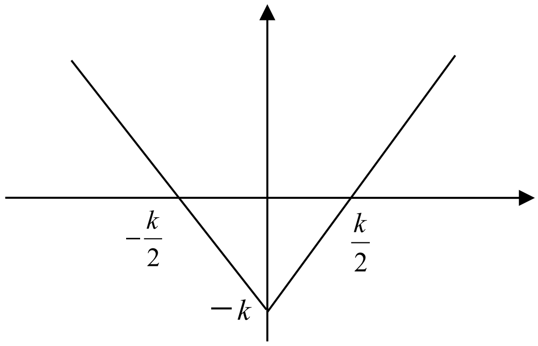
Show on your sketch the coordinates of each point at which the graph crosses the x-axis and the y-axis.

(2)

(b) Find, in terms of k, the values of x for which

$$2|x| - k = \frac{1}{2}x + \frac{1}{4}k$$

(3)

Question Number	Scheme	Notes	Marks
12(a)			
	V Shape with the vertex anywhere on the y-axis with the branches approximately symmetrical about the y-axis. Ignore any dashed or dotted lines.		B1
	There must be a sketch for this mark. Intercepts (must be **crossing**) at $\left(-\frac{k}{2},0\right)$, $\left(\frac{k}{2},\ 0\right)$ and $(0, -k)$ and no others. Allow if the coordinates are the wrong way round provided the positioning is correct. The zeros are not needed as long as the expressions are correct (as above). Allow if the correct **coordinates** are seen away from the sketch but they must be the right way round in this case and must correspond with the sketch. If there is any ambiguity, the sketch has precedence.		B1
			(2)
(b)	$2x-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow x=...$ or $-2x-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow x=...$ Attempt to solve either equation to make x or k the subject		M1
	$x=\frac{5k}{6}$ or $x=-\frac{k}{2}$	One correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	$x=\frac{5k}{6}$ and $x=-\frac{k}{2}$	Both x values correct for. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	Note that the $x=-\frac{k}{2}$ must clearly be from work in (b) and not from work in (a) when attempting the sketch unless it is clearly stated as an answer to (b).		
			(3)

	(b) Alternative by squaring:		
	$2\lvert x\rvert - k = \frac{1}{2}x + \frac{k}{4} \Rightarrow 2\lvert x\rvert = \frac{1}{2}x + \frac{5k}{4} \Rightarrow 4x^2 = \frac{1}{4}x^2 + \frac{5k}{4}x + \frac{25}{16}k^2$ $\Rightarrow 60x^2 - 20kx - 25k^2 = 0 \Rightarrow x = ...$ Adds k to both sides, squares and solves to obtain a 3TQ and solves for x		M1
	$x = \frac{5k}{6}$ or $x = -\frac{k}{2}$	One correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	$x = \frac{5k}{6}$ and $x = -\frac{k}{2}$	Both x values correct for. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
			(3)
			Total 5

	(b) Special case		
	$2x - k = \frac{1}{2}x + \frac{k}{4} \Rightarrow 4x^2 - 4kx + k^2 = \frac{1}{4}x^2 + \frac{k}{4}x + \frac{1}{16}k^2$ $\Rightarrow 60x^2 - 68kx + 15k^2 = 0 \Rightarrow x = ...$ Squares both sides to obtain 3 terms each time and solves the resulting 3TQ solves for x		M1
	$x = \frac{5k}{6}$	Correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$	A1
	If this is all they do, 2 marks will be the maximum		

Leave blank

13. A scientist is studying a population of insects. The number of insects, N, in the population, t days after the start of the study is modelled by the equation

$$N = \frac{240}{1 + ke^{-\frac{t}{16}}}$$

where k is a constant.

Given that there were 50 insects at the start of the study,

(a) find the value of k

(2)

(b) use the model to find the value of t when $N = 100$

(3)

(c) Show that

$$\frac{dN}{dt} = \frac{1}{p}N - \frac{1}{q}N^2$$

where p and q are integers to be found.

(5)

Question Number	Scheme	Notes	Marks
13	$N = \dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}$		
(a)	$\dfrac{240}{1+k\mathrm{e}^{(0)}} = 50 \Rightarrow k = ...$	Substitutes $t = 0$ and $N = 50$ and solves for k	M1
	$k = 3.8\left(=\dfrac{19}{5}\right)$	cao	A1
			(2)
(b)	$100 = \dfrac{240}{1+3.8\mathrm{e}^{-\frac{t}{16}}} \Rightarrow 380\mathrm{e}^{-\frac{t}{16}} = 140$	Puts $N = 100$ and solves as far as $p\mathrm{e}^{-\frac{t}{16}} = q$ using correct processing (allow sign/copying/arithmetic slips)	M1
	$\mathrm{e}^{-\frac{t}{16}} = \dfrac{7}{19} \Rightarrow -\dfrac{t}{16} = \ln\left(\dfrac{7}{19}\right)$	Takes ln's **correctly** to reach $\pm\dfrac{t}{16} = \ln(\alpha), \alpha > 0$ **Dependent on the previous M**	**d**M1
	$t = 16\ln\left(\dfrac{19}{7}\right)$ or $-16\ln\left(\dfrac{7}{19}\right)$ or $8\ln\left(\dfrac{361}{49}\right)$ or $4\ln\left(\dfrac{130321}{2401}\right)$ etc	Cao (accept equivalents) or awrt 16	A1
			(3)
	(b) For mis-read $N = \dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 2/3)**		
	$100 = \dfrac{240}{1+3.8\mathrm{e}^{\frac{t}{16}}} \Rightarrow 380\mathrm{e}^{\frac{t}{16}} = 140$	Puts $N = 100$ and solves as far as $p\mathrm{e}^{\frac{t}{16}} = q$ using correct processing (allow sign/copying/arithmetic slips)	M1
	$\mathrm{e}^{\frac{t}{16}} = \dfrac{7}{19} \Rightarrow \dfrac{t}{16} = \ln\left(\dfrac{7}{19}\right)$	Takes ln's **correctly** to reach $\pm\dfrac{t}{16} = \ln(\alpha), \alpha > 0$ **Dependent on the previous M**	**d**M1
	$t = 16\ln\left(\dfrac{7}{19}\right)$ etc.		A0

Part (c) General Guidance for Marking:

M1 is for their attempt at differentiating
A1 is for correct differentiation (in terms of k or follow through their k)
M1 is for $\mathrm{e}^{...}$ or $k\mathrm{e}^{...}$ or $1+k\mathrm{e}^{...}$ in terms of N
M1 is for obtaining $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N
A1 fully correct

Note that a value of k is not necessary to do part (c)

(c) Way 1	$N=240\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{-1}$	$\frac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{-\frac{t}{16}}\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$	M1
	$\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}t}=-240\left(1+3.8\mathrm{e}^{-\frac{t}{16}}\right)^{-2}\times-\frac{3.8}{16}\mathrm{e}^{-\frac{t}{16}}$	Correct derivative. Follow through their k or the letter k	A1ft
	May see **quotient rule**: $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{(0)-240\times-\frac{k}{16}\mathrm{e}^{-\frac{t}{16}}}{\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{2}}$ But this must satisfy the conditions above i.e. they need to obtain $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{A\mathrm{e}^{-\frac{t}{16}}}{\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{2}}$ May see **product rule**: $\frac{\mathrm{d}N}{\mathrm{d}t}=0+\frac{240k\mathrm{e}^{-\frac{t}{16}}}{16}\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$ But this must satisfy the conditions above i.e. they need to obtain $\frac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{-\frac{t}{16}}\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$ **If an incorrect rule is quoted this scores M0**		
	$N=\frac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow 1+k\mathrm{e}^{-\frac{t}{16}}=\frac{240}{N}$	Attempt to find $\mathrm{e}^{-\frac{t}{16}}$ or $k\mathrm{e}^{-\frac{t}{16}}$ or $1+k\mathrm{e}^{-\frac{t}{16}}$ in terms of N	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{-\frac{t}{16}}$ by $\frac{240}{N}$ in their solution		
	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{57\left(\frac{240-N}{3.8N}\right)}{\left(\frac{240}{N}\right)^{2}}$	Obtains $\frac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{1}{16}N-\frac{1}{3840}N^{2}$	Cao (Allow p = 16, q = 3840)	A1
			(5)

	(c) Way 1 mis-read $N=\dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$N=240\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^{-1}$ $\Rightarrow\dfrac{\mathrm{d}N}{\mathrm{d}t}=-240\left(1+3.8\mathrm{e}^{\frac{t}{16}}\right)^{-2}\times\dfrac{3.8}{16}\mathrm{e}^{\frac{t}{16}}$	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{\frac{t}{16}}\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ Correct derivative. Follow through their k or the letter k	M1 A1ft
	May see **quotient rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{(0)-240\times\dfrac{k}{16}\mathrm{e}^{\frac{t}{16}}}{\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^2}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{A\mathrm{e}^{\frac{t}{16}}}{\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^2}$ May see **product rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=0-\dfrac{240k\mathrm{e}^{\frac{t}{16}}}{16}\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{\frac{t}{16}}\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ **If an incorrect formula is quoted this scores M0**		
	$N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow 1+k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}$	Attempt to find $\mathrm{e}^{\frac{t}{16}}$ or $k\mathrm{e}^{\frac{t}{16}}$ or $1+k\mathrm{e}^{\frac{t}{16}}$ in terms of N	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{-57\left(\dfrac{240-N}{3.8N}\right)}{\left(\dfrac{240}{N}\right)^2}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

	(c) Way 2		
	$\left(N=\dfrac{240}{1+ke^{-\frac{t}{16}}}\Rightarrow ke^{-\frac{t}{16}}=\dfrac{240}{N}-1\right)$		
	$\Rightarrow -\dfrac{k}{16}e^{-\frac{t}{16}}\dfrac{dt}{dN}=-\dfrac{240}{N^2}$ Or $\Rightarrow -\dfrac{k}{16}e^{-\frac{t}{16}}=-\dfrac{240}{N^2}\dfrac{dN}{dt}$	Differentiates to obtain $Ae^{-\frac{t}{16}}\dfrac{dt}{dN}=\dfrac{B}{N^2}$ or $Ae^{-\frac{t}{16}}=\dfrac{B}{N^2}\dfrac{dN}{dt}$	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+ke^{-\frac{t}{16}}}\Rightarrow ke^{-\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $e^{-\frac{t}{16}}$ or $ke^{-\frac{t}{16}}$ or $1+ke^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{-\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{\frac{1}{16}\left(\frac{240}{N}-1\right)}{\frac{240}{N^2}}$	Obtains $\dfrac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\dfrac{dN}{dt}=\dfrac{1}{16}N-\dfrac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
			(5)
	(c) Way 2 mis-read $N=\dfrac{240}{1+ke^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$\left(N=\dfrac{240}{1+ke^{\frac{t}{16}}}\Rightarrow ke^{\frac{t}{16}}=\dfrac{240}{N}-1\right)$		
	$\Rightarrow \dfrac{k}{16}e^{\frac{t}{16}}\dfrac{dt}{dN}=-\dfrac{240}{N^2}$ or $\Rightarrow \dfrac{k}{16}e^{\frac{t}{16}}=-\dfrac{240}{N^2}\dfrac{dN}{dt}$	Differentiates to obtain $Ae^{\frac{t}{16}}\dfrac{dt}{dN}=\dfrac{B}{N^2}$ or $Ae^{\frac{t}{16}}=\dfrac{B}{N^2}\dfrac{dN}{dt}$	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+ke^{\frac{t}{16}}}\Rightarrow ke^{\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $e^{\frac{t}{16}}$ or $ke^{\frac{t}{16}}$ or $1+ke^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{\frac{1}{16}\left(1-\frac{240}{N}\right)}{\frac{240}{N^2}}$	Obtains $\dfrac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\dfrac{dN}{dt}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

	(c) Way 3		
	$\left(N=\frac{240}{1+ke^{-\frac{t}{16}}}\Rightarrow ke^{-\frac{t}{16}}=\frac{240}{N}-1\right)$		
	$\Rightarrow -\frac{t}{16}=\ln\frac{1}{k}\left(\frac{240}{N}-1\right)$ $\Rightarrow t=-16\ln\frac{1}{k}-16\ln\left(\frac{240}{N}-1\right)$ $\Rightarrow \frac{dt}{dN}=-16\left(\frac{N}{240-N}\right)\left(-\frac{240}{N^2}\right)$	Makes t the subject, takes ln's and differentiates using the chain rule.	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\frac{240}{1+ke^{-\frac{t}{16}}}\Rightarrow ke^{-\frac{t}{16}}=\frac{240}{N}-1$	Attempt to find $e^{-\frac{t}{16}}$ or $ke^{-\frac{t}{16}}$ or $1+ke^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{-\frac{t}{16}}$ by $\frac{240}{N}$ in their solution		
	$=\frac{3840}{N(240-N)}$ $\Rightarrow \frac{dN}{dt}=\frac{N(240-N)}{3840}$	Obtains $\frac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\frac{dN}{dt}=\frac{1}{16}N-\frac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
	(c) Way 3 mis-read $N=\frac{240}{1+ke^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$N=\frac{240}{1+ke^{\frac{t}{16}}}\Rightarrow ke^{\frac{t}{16}}=\frac{240}{N}-1$		
	$\frac{t}{16}=\ln\frac{1}{k}\left(\frac{240}{N}-1\right)$ $\Rightarrow t=16\ln\frac{1}{k}+16\ln\left(\frac{240}{N}-1\right)$ $\Rightarrow \frac{dt}{dN}=16\left(\frac{N}{240-N}\right)\left(-\frac{240}{N^2}\right)$	Makes t the subject, takes ln's and differentiates using the chain rule.	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\frac{240}{1+ke^{\frac{t}{16}}}\Rightarrow ke^{\frac{t}{16}}=\frac{240}{N}-1$	Attempt to find $e^{\frac{t}{16}}$ or $ke^{\frac{t}{16}}$ or $1+ke^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{\frac{t}{16}}$ by $\frac{240}{N}$ in their solution		
	$=\frac{3840}{N(N-240)}$	Obtains $\frac{dN}{dt}$ in terms of N only (may include k's)	M1

	$\Rightarrow \frac{dN}{dt} = \frac{N(N-240)}{3840}$		
	$\frac{dN}{dt} = -\frac{1}{16}N + \frac{1}{3840}N^2$		A0

	(c) Way 4		
	$\left(1+ke^{-\frac{t}{16}}\right)N = 240$ $\Rightarrow N \times -\frac{k}{16}e^{-\frac{t}{16}} + \left(1+ke^{-\frac{t}{16}}\right)\frac{dN}{dt} = 0$ or $\left(1+ke^{-\frac{t}{16}}\right) + N \times -\frac{k}{16}e^{-\frac{t}{16}}\frac{dt}{dN} = 0$	Multiplies by $\left(1+ke^{-\frac{t}{16}}\right)$ and differentiates with respect to t or N using the product rule	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N = \frac{240}{1+ke^{-\frac{t}{16}}} \Rightarrow ke^{-\frac{t}{16}} = \frac{240}{N} - 1$	Attempt to find $e^{-\frac{t}{16}}$ or $ke^{-\frac{t}{16}}$ or $1+ke^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{-\frac{t}{16}}$ by $\frac{240}{N}$ in their solution		
	$\Rightarrow \frac{dN}{dt} = \frac{N(240-N)}{3840}$	Obtains $\frac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\frac{dN}{dt} = \frac{1}{16}N - \frac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
	(c) Way 4 mis-read $N = \frac{240}{1+ke^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$\left(1+ke^{\frac{t}{16}}\right)N = 240$ $\Rightarrow N \times \frac{k}{16}e^{\frac{t}{16}} + \left(1+ke^{\frac{t}{16}}\right)\frac{dN}{dt} = 0$ or $\left(1+ke^{\frac{t}{16}}\right) + N \times \frac{k}{16}e^{\frac{t}{16}}\frac{dt}{dN} = 0$	Multiplies by $\left(1+ke^{\frac{t}{16}}\right)$ and differentiates with respect to t or N using the product rule	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N = \frac{240}{1+ke^{\frac{t}{16}}} \Rightarrow ke^{\frac{t}{16}} = \frac{240}{N} - 1$	Attempt to find $e^{\frac{t}{16}}$ or $ke^{\frac{t}{16}}$ or $1+ke^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{\frac{t}{16}}$ by $\frac{240}{N}$ in their solution		
	$\Rightarrow \frac{dN}{dt} = \frac{N(N-240)}{3840}$	Obtains $\frac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\frac{dN}{dt} = -\frac{1}{16}N + \frac{1}{3840}N^2$		A0

There may be other methods not covered in the MS but the marking should follow the same pattern.