Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Wednesday 29 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference **WFM01/01**

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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P 4 4 9 6 7 A 0 1 3 2

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1. $$f(x) = 6\sqrt{x} - x^2 - \frac{1}{2x}, \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [3, 4].

(2)

(b) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval [3, 4] to find another approximation to α. Give your answer to 3 decimal places.

(3)

P 4 4 9 6 7 A 0 2 3 2

Question Number	Scheme		Marks
1. (a)	$\text{f}(x) = 6\sqrt{x} - x^2 - \dfrac{1}{2x}$		
	$\text{f}(3) = 1.225638179...$ $\text{f}(4) = -4.125\left(-\dfrac{33}{8}\right)$	Either any one of $\text{f}(3) =$ awrt 1.2 or $\text{f}(4) =$ awrt -4.1	M1
	Sign change (and $\text{f}(x)$ is continuous) therefore a root α exists between $x = 3$ and $x = 4$	both values correct, sign change (or equivalent) and conclusion	A1
			[2]
(b)	$\text{f}'(x) = 3x^{-\frac{1}{2}} - 2x + \dfrac{1}{2x^2}$	$x^n \to x^{n-1}$ on at least one term At least two terms differentiated correctly (May be un-simplified) Correct differentiation (May be un-simplified)	M1 A1 A1
	$\{\text{f}'(3) = -4.212393637...\}$		
	$\alpha = 3 - \dfrac{\text{f}(3)}{\text{f}'(3)} = 3 - \left(\dfrac{"1.225638179..."}{"-4.212393637"}\right)$	Correct application of Newton-Raphson using their values of $\text{f}(3)$ and $\text{f}'(3)$. May be implied by a correct answer.	M1
	$= 3.29096003...\ \{= 3.291\ (3\text{dp})\}$	awrt 3.291	A1
	Ignore any further applications of N-R		
			[5]
(c)	$\dfrac{\alpha - 3}{"1.225638179..."} = \dfrac{4 - \alpha}{"4.125"}$ or $\dfrac{\alpha - 3}{"1.225638179..."} = \dfrac{1}{"1.225638179..." - "-4.125"}$	This mark can be implied. Do not allow if any 'negative lengths' are used or if either fraction is the wrong way up	M1
	$\alpha = 3 + \left(\dfrac{"1.225638179..."}{"1.225638179..." + "4.125"}\right)1$	Attempt to make α the subject	M1
	$\alpha = \dfrac{3 \times "4.125" + 4 \times "1.225638179..."}{"1.225638179..." + "4.125"}$ would score both method marks		
	$= 3.229063924...$ $= 3.229\ (3\text{dp})$	awrt 3.229	A1
			[3]
			10
	NB if -4.125 is used this gives 2.577273119….		

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2. The quadratic equation

$$5x^2 - 4x + 2 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

(2)

(b) Find the value of $\alpha^2 + \beta^2$.

(2)

(c) Find a quadratic equation which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$

giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers.

(4)

Question Number	Scheme		Marks
2. (a)	$5x^2 - 4x + 2 = 0$ has roots α and β		
	$\alpha + \beta = \frac{4}{5}$, $\alpha\beta = \frac{2}{5}$	At least one of $\alpha + \beta$ or $\alpha\beta$ correct Both $\alpha + \beta$ and $\alpha\beta$ correct	B1 B1
			[2]
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\left\{= \left(\frac{4}{5}\right)^2 - 2\left(\frac{2}{5}\right)\right\}$	Writes down or applies the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$= -\frac{4}{25}(-0.16)$	$-\frac{4}{25}$	A1**cso**
			[2]
Note 1	**cso** so: $\alpha + \beta = -\frac{4}{5}$, $\alpha\beta = \frac{2}{5}$ scores B1B0 in (a) and $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\left\{= \left(-\frac{4}{5}\right)^2 - 2\left(\frac{2}{5}\right)\right\}$ $= -\frac{4}{25}$ M1A0 in (b) **But allow recovery of marks in (c)**		
Note 2	$\alpha + \beta = 4$, $\alpha\beta = 2$ is quite common and gives $\alpha^2 + \beta^2 = 12$, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 3$, $\frac{1}{\alpha^2\beta^2} = \frac{1}{4}$, and $4x^2 - 12x + 1 = 0$. This scores a maximum of 4/8		
(c)	A quadratic equation with roots of $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$		
	Sum of roots $\left\{= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} =\right\} = \frac{-\frac{4}{25}}{\frac{4}{25}}$ $\{= -1\}$	Applies $\frac{\text{their } (\alpha^2 + \beta^2)}{\text{their } (\alpha\beta)^2}$	M1
	Product of roots $\left\{= \frac{1}{\alpha^2\beta^2} =\right\} = \frac{1}{\left(\frac{4}{25}\right)}$ $\left\{= \frac{25}{4}\right\}$	Applies $\frac{1}{\text{their } (\alpha\beta)^2}$	M1
	So, $x^2 - (-1)x + \frac{25}{4} (= 0)$	Applies $x^2 - (\text{their sum})x + (\text{their product}) (= 0)$ **Dependent on at least one of the previous M's having been scored.**	dM1
	$4x^2 + 4x + 25 = 0$	$4x^2 + 4x + 25 = 0$ or any integer multiple	A1
			[4]
			8
	Alternative to part (c) 1st M1: $\left(x - \frac{1}{\alpha^2}\right)\left(x - \frac{1}{\beta^2}\right) = 0$ 2nd M1: $(\alpha^2\beta^2)x^2 - (\alpha^2 + \beta^2)x + 1 = 0$ 3rd M1: $\frac{4x^2}{25} + \frac{4x}{25} + 1 = 0$ 4th A1: $4x^2 + 4x + 25 = 0$		

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- ft denotes "follow through"
- cao denotes "correct answer only"
- oe denotes "or equivalent"

Other Possible Solutions

Question Number	Scheme		Marks
2.	$5x^2 - 4x + 2 = 0$ has roots α and β		
***Aliter* Way 2**	$x = \dfrac{4 \pm \sqrt{-24}}{10} = \dfrac{2}{5} \pm \dfrac{\sqrt{6}}{5}\text{i}$. Hence let, say $\alpha = \dfrac{2}{5} + \dfrac{\sqrt{6}}{5}\text{i}$ and $\beta = \dfrac{2}{5} - \dfrac{\sqrt{6}}{5}\text{i}$		
(a)	$\alpha + \beta = \dfrac{4}{5}$, $\alpha\beta = \dfrac{2}{5}$	At least one of $\alpha + \beta$ or $\alpha\beta$ correct Both $\alpha + \beta$ and $\alpha\beta$ correct	B1 B1
			[2]
(b)	$\alpha^2 = -\dfrac{2}{25} + \dfrac{4\sqrt{6}}{25}\text{i}$, $\beta^2 = -\dfrac{2}{25} - \dfrac{4\sqrt{6}}{25}\text{i}$	Uses their α and their β to find both α^2 and β^2	M1
	So, $\alpha^2 + \beta^2 = -\dfrac{4}{25}$	$-\dfrac{4}{25}$	A1
			[2]
(c)	A quadratic equation with roots of $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$		
	$\dfrac{1}{\alpha^2} = 25\left(\dfrac{1}{-2 + 4\sqrt{6}\,\text{i}}\right) = 25\left(\dfrac{-2 + 4\sqrt{6}\,\text{i}}{4 + 96}\right) = \dfrac{1}{2}\left(-1 - 2\sqrt{6}\,\text{i}\right) = -\dfrac{1}{2} - \sqrt{6}\,\text{i}$ Hence, $\dfrac{1}{\beta^2} = -\dfrac{1}{2} + \sqrt{6}\,\text{i}$	A valid attempt to find either $\dfrac{1}{\alpha^2}$ or $\dfrac{1}{\beta^2}$.	M1
	So, $\left(x - \left(-\dfrac{1}{2} - \sqrt{6}\,\text{i}\right)\right)\left(x - \left(-\dfrac{1}{2} + \sqrt{6}\,\text{i}\right)\right) = 0$	An attempt to form a quadratic equation using their $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$.	M1
	So, $x^2 - (-1)x + \dfrac{25}{4}\ (= 0)$	... leading to a quadratic expression with integer coefficients.	M1
	leading to, $4x^2 + 4x + 25 = 0$	$4x^2 + 4x + 25 = 0$ or any integer multiple	A1
			[4]
			8

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3.

$$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$$

(a) Show that **A** is non-singular.
(2)

The triangle R is transformed to the triangle S by the matrix **A**.

Given that the area of triangle R is 10 square units,

(b) find the area of triangle S.
(2)

Given that

$$\mathbf{B} = \mathbf{A}^4$$

and that the triangle R is transformed to the triangle T by the matrix **B**,

(c) find, without evaluating **B**, the area of triangle T.
(2)

Question Number	Scheme		Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$ **,** $\text{Area}(R) = 10$**,** $\mathbf{B} = \mathbf{A}^4$		
	$\det(\mathbf{A}) = 6(1) - 4(1)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non-singular)	$\det(\mathbf{A}) = 2$ or $6-4$ **and** some reference to zero e.g. $2 \neq 0$ is sufficient	A1
			[2]
(b)	$\text{Area}(S) = 2(10); = 20$	(their $\det(\mathbf{A})) \times (10)$ 20	M1; A1
	$(10) \div$ (their $\det(\mathbf{A})$) is M0		
			[2]
(c)	$\text{Area}(T) = 2^4(10); = 160$	(their $\det(\mathbf{A}))^4 \times (10)$ 160	M1 ; A1
	$(10) \div$ (their $\det(\mathbf{A}))^4$ is M0		
	$\mathbf{A}^2 = \begin{pmatrix} 40 & 28 \\ 7 & 5 \end{pmatrix} \Rightarrow \left\|\mathbf{A}^2\right\| = 4 \Rightarrow \text{Area}(T) = 4^2(10); = 160$ Is acceptable (their $\det(\mathbf{A}^2))^2 \times (10)$; M1 160; A1		
	BUT there must be no attempt to evaluate $\mathbf{A}^4$ to give $\det(\mathbf{A}) = 16$		
			[2]
			6
Note 1	If they think $\det(\mathbf{A}) = \dfrac{1}{\det(\mathbf{A})}$ then no marks in (a) but allow M's in (b) and (c).		
	NB $\mathbf{A}^4 = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 1796 & 1260 \\ 315 & 221 \end{pmatrix}$		

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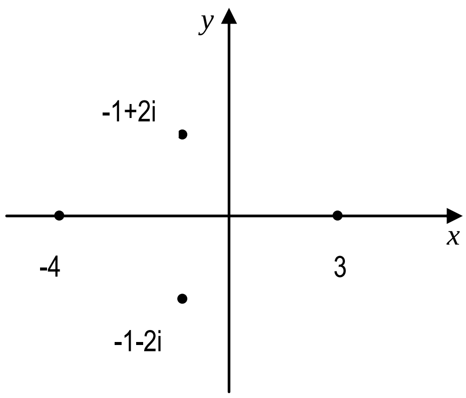
4. $$\mathrm{f}(x) = x^4 + 3x^3 - 5x^2 - 19x - 60$$

(a) Given that $x = -4$ and $x = 3$ are roots of the equation $\mathrm{f}(x) = 0$, use algebra to solve $\mathrm{f}(x) = 0$ completely.

(7)

(b) Show the four roots of $\mathrm{f}(x) = 0$ on a single Argand diagram.

(2)

Question Number	Scheme		Marks
4. (a)	$\mathrm{f}(x) = x^4 + 3x^3 - 5x^2 - 19x - 60$		
	Quadratic factor: $(x+4)(x-3)\ \{= x^2 + x - 12\}$	$(x \pm 4)(x \pm 3)$ or $x^2 \pm x \pm 12$ $(x+4)(x-3)$ or $x^2 + x - 12$	M1 A1
	$\mathrm{f}(x) = \{x^2 + x - 12\}(x^2 + 2x + 5)$	Attempt to find the other quadratic factor of the form $(x^2 + bx + c)$ $(x^2 + 2x + 5)$	M1 A1
	$x = \dfrac{-2 \pm \sqrt{4-20}}{2}$ or $(x+1)^2 - 1 + 5 = 0$, $x = ...$	Solving a 3-term quadratic by formula or completion of the square	M1
	$= -1 + 2\mathrm{i}$ and $-1 - 2\mathrm{i}$	Allow $-1 \pm 2\mathrm{i}$ (-4 and 3 are not needed for this mark)	A1 A1ft
			[7]
(b)	 	Note that the points are $(-4, 0)$, $(3, 0)$, $(-1, 2)$ and $(-1, -2)$. The points $(-4, 0)$ and $(3, 0)$ plotted on the Argand diagram with -4 and 3 indicated. They could be labelled as e.g. x_1 and x_2 and referred to elsewhere. The distinct points representing the other two **complex** roots plotted correctly and symmetrically about the x-axis. The points must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as complex numbers. They could be labelled as e.g. x_3 and x_4 and referred to elsewhere. If there is any contradiction in position in an otherwise correct diagram (e.g. -1 + 2i further to the left than -4, deduct one mark.	B1 B1ft
			[2]
			9
	Alternative by long division		
	1st M1: for attempting to divide $\mathrm{f}(x)$ by $(x \pm 3)$ or $(x \pm 4)$. 1st A1: $\dfrac{\mathrm{f}(x)}{(x-3)} = x^3 + 6x^2 + 13x + 20$ or $\dfrac{\mathrm{f}(x)}{(x+4)} = x^3 - x^2 - x - 15$ 2nd M1: Attempt quadratic factor $\dfrac{x^3 + 6x^2 + 13x + 20}{(x+4)}$ or $\dfrac{x^3 - x^2 - x - 15}{(x-3)}$ 2nd A1: $(x^2 + 2x + 5)$		
	Alternative by comparing coefficients		
	$\mathrm{f}(x) = (x^2 + x - 12)(ax^2 + bx + c) = x^4 + 3x^3 - 5x^2 - 19x - 60$ $\Rightarrow a = 1,\ c = 5,\ b + a = 3$ or $c + b - 12a = -5 \Rightarrow b = 2$ M1: Compares coefficients to obtain values for a, b and c A1: $a = 1$, $b = 2$ and $c = 5$		

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5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(9r^2 - 4r) = \frac{1}{2}n(n+1)(6n-1)$$

for all positive integers n.

(4)

Given that

$$\sum_{r=1}^{12}\left(9r^2 - 4r + k(2^r)\right) = 6630$$

(b) find the exact value of the constant k.

(4)

Question Number	Scheme		Marks
5. (a)	$\sum_{r=1}^{n}\left(9r^2-4r\right)$		
	$=\frac{9}{6}n(n+1)(2n+1)-\frac{4}{2}n(n+1)$	An attempt to use at least one of the standard formulae correctly. Correct expression.	M1 A1
	$=\frac{3}{2}n(n+1)(2n+1)-2n(n+1)$		
	$=\frac{1}{2}n(n+1)\left(3(2n+1)-4\right)$	An attempt to factorise out at least $n(n+1)$. May not come until their last line.	M1
	$=\frac{1}{2}n(n+1)\left(6n+3-4\right)$		
	$=\frac{1}{2}n(n+1)(6n-1)$ (*)	Achieves the correct answer with no errors	A1 *
	There are no marks for proof by induction		
			[4]
	$\sum_{r=1}^{12}\left(9r^2-4r+k(2^r)\right)=6630$		
	$\sum_{r=1}^{12}\left(9r^2-4r\right)=\frac{1}{2}(12)(13)(71)\ \{=5538\}$	Attempt to evaluate $\sum_{r=1}^{12}\left(9r^2-4r\right)$ May be implied by 5538	M1
	$\sum_{r=1}^{12}(2^r)=\frac{2(1-2^{12})}{1-2}\ \{=8190\}$	Attempt to apply the sum to n terms of a GP $\frac{2(1-2^{12})}{1-2}$	M1 A1
	So, $5538+8190k=6630 \Rightarrow 8190k=1092$ giving, $k=\frac{2}{15}$ oe		A1
			[4]
			8
(b)	2^{nd} M1 1^{st} A1: These two marks can be implied by seeing 8190 or 8190k $\sum_{r=1}^{12}(2^r)=2^{12}=4096$ is common and gives $k=\frac{273}{1024}$ (0.2666...) (Usually scores M1M0A0A0)		

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6.

(i) $$\mathbf{B} = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} 4 & -2 \\ 1 & 0 \end{pmatrix}$$

(a) Find $\mathbf{B}^{-1}$.

(2)

The transformation represented by **Y** is equivalent to the transformation represented by **B** followed by the transformation represented by the matrix **A**.

(b) Find **A**.

(2)

(ii) $$\mathbf{M} = \begin{pmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{pmatrix}$$

The matrix **M** represents an enlargement scale factor k, centre (0, 0), where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(a) Find the value of k.

(2)

(b) Find the value of θ.

(2)

Question Number	Scheme		Marks
6. (i) (a)	$\mathbf{B}^{-1} = -\frac{1}{2}\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}\left(=\frac{1}{2}\begin{pmatrix}4 & 2\\3 & 1\end{pmatrix}\right)\left(=\begin{pmatrix}2 & 1\\\frac{3}{2} & \frac{1}{2}\end{pmatrix}\right)$	Either $-\frac{1}{2}$ or $\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}$	M1
		Correct matrix	A1
			[2]
(b)	$\mathbf{Y}=\mathbf{AB} \Rightarrow \mathbf{YB}^{-1}=\mathbf{ABB}^{-1} \Rightarrow \mathbf{YB}^{-1}=\mathbf{A}$		
	$\mathbf{A} = \begin{pmatrix}4 & -2\\1 & 0\end{pmatrix}.-\frac{1}{2}\begin{pmatrix}-4 & -2\\-3 & -1\end{pmatrix}$	Multiplies their **Y** by $\mathbf{B}^{-1}$ This statement is sufficient	M1
	$= -\frac{1}{2}\begin{pmatrix}-10 & -6\\-4 & -2\end{pmatrix}$ or $\begin{pmatrix}5 & 3\\2 & 1\end{pmatrix}$	Correct matrix	A1
	NB $\mathbf{B}^{-1}\mathbf{Y} = \begin{pmatrix}9 & -4\\\frac{13}{2} & -3\end{pmatrix}$		
			[2]
(ii) (a)	$k = \sqrt{3-(-1)}\,;=2$	Applies $\sqrt{(\text{their } \det\mathbf{M})}$ 2 (Accept correct answer only)	M1 A1
			[2]
(b)	$\cos\theta = -\frac{\sqrt{3}}{2},\ \sin\theta=\frac{1}{2},\ \tan\theta=-\frac{1}{\sqrt{3}}$	Writes down a correct trigonometric ratio Or a correct expression for the required angle e.g. $180-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (This mark can be implied by a correct answer)	M1
	$\theta = 150^\circ$ or $\frac{5\pi}{6}$	150° or $\frac{5\pi}{6}$ (Accept correct answer only)	A1
			[2]
			8
(i)(b)	***Alternative method for (i)(b)*** $\mathbf{AB}=\mathbf{Y} \Rightarrow \begin{pmatrix}p & q\\r & s\end{pmatrix}\begin{pmatrix}-1 & 2\\3 & -4\end{pmatrix}=\begin{pmatrix}4 & -2\\1 & 0\end{pmatrix}$ $\begin{Bmatrix}-p+3q=4 & -r+3s=1\\2p-4q=-2 & 2r-4s=0\end{Bmatrix}$ leading to $\mathbf{A}=\begin{pmatrix}5 & 3\\2 & 1\end{pmatrix}$	Applies the matrix equation **AB** = **Y** for an unknown **A.** This statement is sufficient Correct matrix	M1 A1 **[2]**
	Alternative method for (ii)(b)- Marks likely to come in the order (b), (a) $\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}k & 0\\0 & k\end{pmatrix} \Rightarrow k\cos\theta=-\sqrt{3},\ k\sin\theta=1,\ \tan\theta=-\frac{1}{\sqrt{3}} \Rightarrow \theta=150^\circ$ or $\frac{5\pi}{6}$ M1: Writes down a correct trigonometric ratio. A1: 150° or $\frac{5\pi}{6}$ $k\sin\theta=1 \Rightarrow \frac{1}{2}k=1 \Rightarrow k=2$ **(from correct** θ) M1: Uses their value of θ to obtain an equation in k. A1: $k=2$		

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7. (i) Given that

$$\frac{2w - 3}{10} = \frac{4 + 7\mathrm{i}}{4 - 3\mathrm{i}}$$

find w, giving your answer in the form $a + b\mathrm{i}$, where a and b are real constants. You must show your working.

(4)

(ii) Given that

$$z = (2 + \lambda\mathrm{i})(5 + \mathrm{i})$$

where λ is a real constant, and that

$$\arg z = \frac{\pi}{4}$$

find the value of λ.

(4)

Question Number	Scheme		Marks
7. (i) **Way 1**	$\dfrac{2w-3}{10} = \dfrac{4+7\text{i}}{4-3\text{i}}$		
	$\dfrac{2w-3}{10} = \dfrac{(4+7\text{i})}{(4-3\text{i})} \times \dfrac{(4+3\text{i})}{(4+3\text{i})}$	Multiplies by $\dfrac{(4+3\text{i})}{(4+3\text{i})}$	M1
	$= \dfrac{(16+12\text{i}+28\text{i}-21)}{16+9}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator	M1
	$\left\{= \dfrac{1}{25}(-5+40\text{i})\right\}$		
	So $w = \dfrac{\frac{10}{25}(-5+40\text{i})+3}{2} = \dfrac{-2+16\text{i}+3}{2}$	Rearranges to $w = ...$	ddM1
	and $w = \dfrac{1}{2} + 8\text{i}$	$\dfrac{1}{2} + 8\text{i}$ Do not allow $\dfrac{1+16\text{i}}{2}$	A1
			[4]
(ii)	$(2+\lambda\text{i})(5+\text{i}) = 10+2\text{i}+5\lambda\text{i}-\lambda$	Multiplies out to give a four term expression and applies $\text{i}^2 = -1$ Correct expression	M1 A1
	$= (10-\lambda)+(2+5\lambda)\text{i}$ $\left\{\arg z = \dfrac{\pi}{4} \Rightarrow\right\}$ $\dfrac{2+5\lambda}{10-\lambda} = \tan\left(\dfrac{\pi}{4}\right)$	$\dfrac{\text{their combined imaginary part}}{\text{their combined real part}} = \tan\left(\dfrac{\pi}{4}\right)$ or sets real part = imaginary part	M1 oe
	$\{10-\lambda = 2+5\lambda \Rightarrow 8 = 6\lambda \Rightarrow\}$ $\lambda = \dfrac{4}{3}$	$\dfrac{4}{3}$ oe or awrt 1.33	A1
			[4]
			8
Way 2	***Alternative method for part (i)*** $2w = \dfrac{10(4+7\text{i})}{(4-3\text{i})} + 3 = \dfrac{40+70\text{i}+12-9\text{i}}{(4-3\text{i})}$		
	$2w = \dfrac{(52+61\text{i})}{(4-3\text{i})} \times \dfrac{(4+3\text{i})}{(4+3\text{i})}$	Multiplies by $\dfrac{\text{their}(4-3\text{i})^*}{\text{their}(4-3\text{i})^*}$	M1
	$= \dfrac{(208+156\text{i}+244\text{i}-183)}{16+9}$ $= \dfrac{1}{25}(25+400\text{i}) = 1+16\text{i}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator.	M1
	So, $w = \dfrac{1+16\text{i}}{2}$	Rearranges to $w = ...$ If w is made the subject as a first step only award this mark if the previous two M's are scored.	ddM1
	and $w = \dfrac{1}{2} + 8\text{i}$	$\dfrac{1}{2} + 8\text{i}$	A1

Question Number	Scheme		Marks
7(i) **Way 3**	$\frac{2(u+\text{i}v)-3}{10}=\frac{4+7\text{i}}{4-3\text{i}}$		
	$\Rightarrow (2(u+\text{i}v)-3)(4-3\text{i})=40+70\text{i}$	Replaces *w* with *u* + i*v* and eliminates fractions	M1
	$\therefore 8u+6v-12=40$ and $8v-6u+9=70$	Correct equations	A1
	$u=\frac{1}{2},\ \ v=8$	Solves simultaneously to at least *u* = or *v* = Correct values	M1 A1
			[4]

7(i) **Way 4**	$\frac{2w-3}{10}=\frac{4+7\text{i}}{4-3\text{i}}\Rightarrow\frac{2w-3}{10}-\frac{4+7\text{i}}{4-3\text{i}}=0$		
	$\Rightarrow\frac{(2w-3)(4-3\text{i})-10(4+7\text{i})}{10(4-3\text{i})}=0$		
	$8w-6\text{i}w=52+61\text{i}$		
	$w=\frac{52+61\text{i}}{8-6\text{i}}$		
	$w=\frac{52+61\text{i}}{8-6\text{i}}\times\frac{8+6\text{i}}{8+6\text{i}}$	Multiplies by $\frac{\text{their}(8-6\text{i})^*}{\text{their}(8-6\text{i})^*}$	M1
	$w=\frac{416+800\text{i-}366}{100}$	Simplifies realising that a real number is needed in the denominator and applies $\text{i}^2=-1$ on their numerator expression and denominator	M1
	$w=\frac{1}{2}+8\text{i}$	The ddM1 can be awarded now	ddM1 A1
	Cross multiplication essentially follows the same scheme		
			[4]

7(ii)	$z=(2+\lambda\text{i})(5+\text{i})\Rightarrow\arg z=\arg(2+\lambda\text{i})(5+\text{i})$		
	$\arg(2+\lambda\text{i})(5+\text{i})=\arg(2+\lambda\text{i})+\arg(5+\text{i})$	Use of $\arg z_1z_2=\arg z_1+\arg z_2$ $\arg z=\arg(2+\lambda\text{i})+\arg(5+\text{i})$	M1 A1
	$\frac{\pi}{4}=\arctan\left(\frac{\lambda}{2}\right)+\arctan\left(\frac{1}{5}\right)$		
	$1=\frac{\frac{\lambda}{2}+\frac{1}{5}}{1-\frac{\lambda}{2}\frac{1}{5}}$	Use of the **correct** addition formula $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	M1
	$10-\lambda=5\lambda+2\Rightarrow\lambda=\frac{4}{3}$	$\frac{4}{3}$ oe	A1
			[4]

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8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(ap^2, 2ap)$ lies on the parabola C.

(a) Show that an equation of the tangent to C at P is

$$py = x + ap^2$$

(4)

The tangent to C at the point P intersects the directrix of C at the point B and intersects the x-axis at the point D.

Given that the y-coordinate of B is $\frac{5}{6}a$ and $p > 0$,

(b) find, in terms of a, the x-coordinate of D.

(6)

Given that O is the origin,

(c) find, in terms of a, the area of the triangle OPD, giving your answer in its simplest form.

(2)

Question Number	Scheme		Marks
8.(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$	$\frac{dy}{dx} = \pm k\,x^{-\frac{1}{2}}$	
	or (implicitly) $2y\frac{dy}{dx} = 4a$	or $k\,y\frac{dy}{dx} = c$	M1
	or (chain rule) $\frac{dy}{dx} = 2a \times \frac{1}{2ap}$	or $\frac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	
	$x = a\,p^2,\ m_T = \frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{a\,p^2}} = \frac{\sqrt{a}}{\sqrt{a}\,p} = \frac{1}{p}$ or $m_T = \frac{dy}{dx} = \frac{4a}{2(2ap)} = \frac{1}{p}$	$\frac{dy}{dx} = \frac{1}{p}$	A1
	T: $y - 2a\,p = \frac{1}{p}\left(x - a\,p^2\right)$	Applies $y - 2a\,p = \left(\text{their } m_T\right)\left(x - a\,p^2\right)$ Where $\left(\text{their } m_T\right)$ is a function of p and has come from calculus.	M1
	T: $py - 2a\,p^2 = x - a\,p^2$		
	T: $py = x + a\,p^2$	Correct solution.	A1 **cso** *
			[4]
(b)	$B\left(-a, \frac{5}{6}a\right) \Rightarrow p\left(\frac{5}{6}a\right) = -a + a\,p^2$ or $p\left(\frac{5}{6}a\right) = x + a\,p^2$ or $py = -a + a\,p^2$	Substitutes $x = -a$ or $y = \frac{5}{6}a$ or both into **T** (or their rearranged **T)**	M1
	$p\left(\frac{5}{6}a\right) = -a + a\,p^2 \quad (6p^2 - 5p - 6 = 0)$	Correct equation **in any form** with $x = -a$ and $y = \frac{5}{6}a$	A1
	$\Rightarrow (3p+2)(2p-3) = 0$ leading to $p = ...$	Attempts to solve their 3TQ in p having substituted both $x = -a$ and $y = \frac{5}{6}a$ into **T**	M1
	$\Rightarrow \left\{p = -\frac{2}{3}\text{ (reject)}\right\}\ p = \frac{3}{2}$	$p = \frac{3}{2}$ (Can just be stated from a correct quadratic)	A1
	So, $0 = x + a\left(\frac{3}{2}\right)^2$	Substitutes "$p = \frac{3}{2}$" and $y = 0$ in **T**	M1
	giving, $x = -\frac{9a}{4}$	$x = -\frac{9a}{4}$	A1
			[6]
(c)	When $p = \frac{3}{2},\ y_P = 2a\left(\frac{3}{2}\right) = 3a$		
	$\text{Area}(OAD) = \frac{1}{2}\left(\frac{9a}{4}\right)(3a) = \frac{27a^2}{8}$ Or $\text{Area}(OAD) = \frac{1}{2}\begin{vmatrix} 0 & \frac{9a}{4} & -\frac{9a}{4} & 0 \\ 0 & 3a & 0 & 0 \end{vmatrix} = \frac{1}{2} \times 3a \times \frac{9a}{4}$	Applies $\frac{1}{2}(\text{their } \lvert OD \rvert)(\text{their } y_P)$ Allow if $OD < 0$ and a correct method in terms of a and p e.g. $\frac{1}{2} \times -ap^2 \times 2ap$	M1
		$\frac{27a^2}{8}$	A1
	Do not allow $\frac{1}{2} \times 2ap \times \left(\frac{5ap}{6} - ap^2\right)$ as this implies that $y = 0$ has not been used for D		
			[2]
			12

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9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 7^n - 2^n \text{ is divisible by 5}$$

(6)

Question Number	Scheme		Marks
	$\mathrm{f}(n) = 7^n - 2^n$ is divisible by 5		
9.	$\mathrm{f}(1) = 7^1 - 2^1 = 5$	Shows or states that $\mathrm{f}(1) = 5$	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 7^{k+1} - 2^{k+1} - (7^k - 2^k)$	Applies $\mathrm{f}(k+1) - \mathrm{f}(k)$	M1
	$= 7(7^k) - 2(2^k) - (7^k - 2^k)$	Achieves an expression in 7^k and 2^k. Correct expression in 7^k and 2^k	M1 A1
	$= 6(7^k) - 2^k$ $= 6(7^k - 2^k) + 5(2^k)$ $= 6\mathrm{f}(k) + 5(2^k)$	Or $(7^k - 2^k) + 5(7^k)$ Or $\mathrm{f}(k) + 5(7^k)$	
	$\therefore \mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $2\mathrm{f}(k) + 5(7^k)$	$\mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $\mathrm{f}(k+1) = 2\mathrm{f}(k) + 5(7^k)$ or e.g. $\mathrm{f}(k+1) = \mathrm{f}(k) + 5(7^k) + 7^k - 2^k$ Correctly achieves f(k + 1) that is clearly a multiple of 5	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion with all previous marks scored.	A1 cso
			[6]
			6

Question Number	Scheme		Marks
Aliter	$\mathrm{f}(n) = 7^n - 2^n$ is divisible by 5		
9. Way 2	$\mathrm{f}(1) = 7^1 - 2^1 = 5$	Shows or states $\mathrm{f}(1) = 5$	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$\mathrm{f}(k+1) = 7^{k+1} - 2^{k+1}$	Applies $\mathrm{f}(k+1)$	M1
	$= 7(7^k) - 2(2^k)$	Achieves an expression in 7^k and 2^k Correct expression in 7^k and 2^k	M1 A1
	$= 7(7^k - 2^k) + 5(2^k)$ or $5(7^k) + 2(7^k - 2^k)$ $\therefore \mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $5(7^k) + 2\mathrm{f}(k)$	$\mathrm{f}(k+1) = 7\mathrm{f}(k) + 5(2^k)$ or $5(7^k) + 2\mathrm{f}(k)$ Correctly achieves f(k + 1) that is clearly a multiple of 5	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion with all previous marks scored.	A1 cso
			[6]

Question Number	Scheme		Marks
Aliter	$\mathrm{f}(n) = 7^n - 2^n$ is divisible by 5		
9. Way 3	$\mathrm{f}(1) = 7^1 - 2^1 = 5$	Shows or states $\mathrm{f}(1) = 5$	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 7^k - 2^k$ is divisible by 5 for $k \in \mathbb{Z}^+$.		
	$\mathrm{f}(k+1) - 2\mathrm{f}(k) = 7^{k+1} - 2^{k+1} - 2\left(7^k - 2^k\right)$	Applies $\mathrm{f}(k+1) - 2\mathrm{f}(k)$	M1
	$= 5(7^k)$	Achieves an expression in 7^k Correct expression in 7^k	M1 A1
	$\therefore \mathrm{f}(k+1) = 5(7^k) + 2\mathrm{f}(k)$	$5(7^k) + 2\mathrm{f}(k)$	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	Correct conclusion with all previous marks scored.	A1 cso
			[6]