Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Find the value of

$$\sum_{r=1}^{200} (r+1)(r-1)$$

(4)

P 4 4 9 6 5 A 0 2 3 2

Question Number	Scheme		Marks
1.	$(r+1)(r-1)=r^2-1$	Correct expansion. Allow $r^2-r+r-1$	B1
	$\sum_{r=1}^{200} r^2 = \frac{1}{6}200(201)(401)$	Use of $\frac{1}{6}n(n+1)(2n+1)$ with $n=200$	M1
	$\sum_{r=1}^{200} -1 = -200$	Cao (May be implied by their work)	B1
	$\sum_{r=1}^{200}(r^2-1) = 2686700 - 200 = 2686500$	2686500	A1
	Note use of $\sum_{r=1}^{200} -1 = -1$ gives a sum of 2686699 and usually scores B1M1B0A0		
			Total 4

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2. Given that $-2 + 3\text{i}$ is a root of the equation

$$z^2 + pz + q = 0$$

where p and q are real constants,

(a) write down the other root of the equation.

(1)

(b) Find the value of p and the value of q.

(3)

Question Number	Scheme		Marks
	Mark (a) and (b) together		
2.(a)	$-2-3\text{i}$	cao	B1
			(1)
	Way 1		
(b)	$p = -\text{sum of roots} = -(-2+3\text{i}-2-3\text{i})$ **or** $q = \text{product of roots} = (-2+3\text{i})(-2-3\text{i})$	A correct approach for **either** p **or** q	M1
	$p=4,\ q=13$	1st A1: One value correct 2nd A1: Both values correct **Can be implied by a correct equation or expression e.g** $z^2+4z+13$	A1A1
			(3)
			Total 4
	(b) Way 2		
	$(z-(-2+3\text{i}))(z-(-2-3\text{i}))$	$z-(-2+3\text{i})$ **and** $z-(-2-3\text{i})$ and attempt to expand (condone invisible brackets)	M1
	Equation is $z^2+4z+13(=0)$ **or** $p=4,\ q=13$	1st A1: One value correct 2nd A1: Both values correct Condone use of x instead of z	A1 A1
	(b) Way 3		
	$(-2+3\text{i})^2+p(-2+3\text{i})+q=0 \Rightarrow (3p-12)\text{i}+q-2p-5=0$		
	$(3p-12)\text{i}+q-2p-5=0$ $\Rightarrow 3p-12=0,\ q-2p-5$ $\Rightarrow p=\ldots$ or $q=\ldots$	Substitutes – 2 + 3i or – 2 – 3i into the given equation, compares real and imaginary parts and obtains a real value for p or a real value for q	M1
	$p=4,\ q=13$	1st A1: One value correct 2nd A1: Both values correct	A1A1
	(b) Way 4		
	$z^2+pz+q=0 \Rightarrow z=\frac{-p\pm\sqrt{p^2-4q}}{2}$		
	$\frac{-p\pm\sqrt{p^2-4q}}{2}=-2\pm3\text{i} \Rightarrow -\frac{p}{2}=-2 \Rightarrow p=\ldots$		M1
	Correct method to find a value for p		
	$p=4$		A1
	$p^2-4q=-36 \Rightarrow q=13$	Correct value for q	A1
	(b) Way 5		
	$(-2+3\text{i})^2+p(-2+3\text{i})+q=0$ and $(-2-3\text{i})^2+p(-2-3\text{i})+q=0$		
	$\Rightarrow 24i-6pi=0 \Rightarrow p=\ldots$ M1: Substitutes both roots into the given equation and attempts to solve simultaneously to obtain a **real** value for p or a **real** value q		M1
	$p=4,\ q=13$	1st A1: One value correct 2nd A1: Both values correct	A1A1

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3.

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$$

where a is a real constant and $a \neq 6$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(3)

Given that $\mathbf{A} + 2\mathbf{A}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a.

(3)

Question Number	Scheme		Marks
3.(a)	$\det\mathbf{A} = 4\times -3 - a\times -2 (= 2a-12)$	Any correct form (possibly un-simplified) of the determinant	B1
	$\text{adj}\mathbf{A} = \begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix}$	Correct attempt at swapping elements in the major diagonal and changing signs in the minor diagonal. Three or four of the numbers in the matrix should be correct e.g. allow one slip	M1
	$\mathbf{A}^{-1} = \frac{1}{2a-12}\begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix}$	Correct inverse	A1
			(3)
(b)	$\begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix} + \frac{2}{2a-12}\begin{pmatrix} -3 & 2 \\ -a & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Correct statement for **A**, **"**their" inverse and use of the correct identity matrix	M1
	$\Rightarrow \begin{pmatrix} 4-\frac{6}{2a-12} & -2+\frac{4}{2a-12} \\ a-\frac{2a}{2a-12} & -3+\frac{8}{2a-12} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
	So e.g. $4-\frac{6}{2a-12} = 1 \Rightarrow a = ...$	**Adds** their **A** and 2 x their$\mathbf{A}^{-1}$ and compares corresponding elements to form an equation in *a* and attempts to solve as far as *a* = … **or adds** an element of their **A** and the corresponding element of 2 x their$\mathbf{A}^{-1}$ to form an equation in *a* and attempts to solve as far as *a* = …	M1
	$a = 7$ only	Cao (from a **correct** equation i.e. their $\mathbf{A}^{-1}$ might be incorrect) **If they solve a second equation and get a different value for *a*, this mark can be withheld.**	A1
			(3)
			Total 6
	(b) Way 2 (does not use the inverse)		
	$\mathbf{A}+2\mathbf{A}^{-1}=\mathbf{I}\Rightarrow\mathbf{A}^2+2\mathbf{I}=\mathbf{A}$ $\begin{pmatrix} 16-2a & -2 \\ a & 9-2a \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$	Correct statement for $\mathbf{A}^2+2\mathbf{I}=\mathbf{A}$ using **A**, **"**their" $\mathbf{A}^2$ and use of the correct identity matrix	M1
	$\Rightarrow \begin{pmatrix} 16-2a+2 & -2 \\ a & 9-2a+2 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$		
	So $16-2a+2=4$ or $11-2a=-3$	**Adds** their $\mathbf{A}^2$ and $2\mathbf{I}$, compares elements, forms an equation in *a* and attempts to solve as far as *a* = …	M1
	$a = 7$	cao	A1

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4. $$f(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}} - 3, \qquad x > 0$$

Given that α is the only real root of the equation $f(x) = 0$,

(a) show that $4 < \alpha < 5$

(2)

(b) Taking 4.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval [4, 5] to find another approximation to α, giving your answer to 3 decimal places.

(3)

Question Number	Scheme		Marks
4.(a)	$\mathrm{f}(4) = \ldots$ **and** $\mathrm{f}(5) = \ldots$	Attempt to evaluate both f(4) and f(5) NB f(5) = $2\sqrt{5} - 3$ but this must be evaluated to score the A1	M1
	$\mathrm{f}(4) = -1$, $\mathrm{f}(5) = 1.472\ldots$ Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α exists between $x = 4$ and $x = 5$	Both values correct $\mathrm{f}(4) = -1$, and $\mathrm{f}(5) = 1.472\ldots$ (awrt 1.5) , sign change (or equivalent) and conclusion E.g. $\mathrm{f}(4) = -1 < 0$ and $\mathrm{f}(5) = 1.472 > 0$ so $4 < \alpha < 5$ scores M1A1	A1
			(2)
(b)	$\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$	M1: $x^n \to x^{n-1}$ A1: Either $\frac{3}{2}x^{\frac{1}{2}}$ or $-\frac{3}{2}x^{-\frac{1}{2}}$ A1: Correct derivative	M1A1A1
	$x_1 = 4.5 - \frac{\mathrm{f}(4.5)}{\mathrm{f}'(4.5)} = 4.5 - \frac{0.1819805153..}{2.474873734...}$	Correct attempt at Newton-Raphson Can be implied by a correct answer or their working provided a correct derivative is seen or implied.	M1
	$= 4.426$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores no marks in (b)**		
			(5)
(c)	$\frac{5 - \alpha}{1.472} = \frac{\alpha - 4}{1}$ or $\frac{\alpha - 4}{1} = \frac{5 - 4}{1.472 + 1}$	A **correct** statement for α or 5 - α or α - 4	M1
	$\alpha(1.472 + 1) = 5 + 4 \times 1.472$ so $\alpha = \ldots$	Attempt to make "α" the subject (allow poor manipulation). **Dependent** on the previous M1.	**d**M1
	$\alpha = 4.405$	cao	A1
	There are no marks for interval bisection		
			(3)
			Total 10

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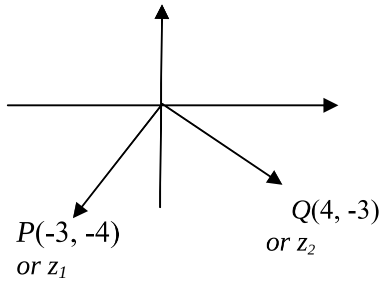
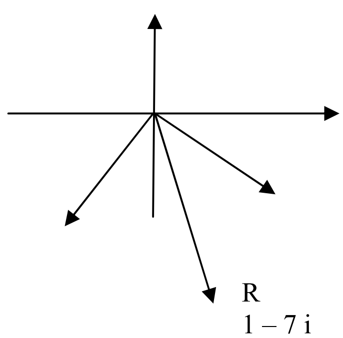
5. Given that $z_1 = -3 - 4\text{i}$ and $z_2 = 4 - 3\text{i}$

(a) show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2
(2)

(b) Given that O is the origin, show that OP is perpendicular to OQ.
(2)

(c) Show the point R on your diagram, where R represents $z_1 + z_2$
(1)

(d) Prove that $OPRQ$ is a square.
(2)

Question	Scheme		Marks
5. (a)	 	M1: One point in third quadrant and one in the fourth quadrant. Can be vectors, points or even lines. A1: The points representing the complex numbers plotted correctly. The points must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as complex numbers.	M1A1
			(2)
(b)	M1 requires a correct strategy e.g.		
	1. Gradient OP = $\frac{4}{3}$, Gradient OQ = $\frac{-3}{4}$ $\quad \frac{4}{3}\times-\frac{3}{4}=$...........		M1
	2. Angles with Im axis are $\tan^{-1}\frac{3}{4}$ and $\tan^{-1}\frac{4}{3}$. $\quad \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=$.....		
	3. Angles with Re axis are $\tan^{-1}\frac{4}{3}$ and $\tan^{-1}\frac{3}{4}$. $180-\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=\right)$.....		
	4. $OP^2=3^2+4^2,\ OQ^2=3^2+4^2,\ PQ^2=1^2+7^2\ OP^2+OQ^2=$.....		
	5. $\overrightarrow{OP}=\begin{pmatrix}-3\\-4\end{pmatrix}, \overrightarrow{OQ}=\begin{pmatrix}4\\-3\end{pmatrix}\Rightarrow \overrightarrow{OP}.\overrightarrow{OQ}=$.....		
	$\frac{4}{3}\times-\frac{3}{4}=-1$ so right angle or 53.1 + 26.9 = 90 (accept 53 + 27) or radians or $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3}=\frac{\pi}{2}$ $OP^2+OQ^2=PQ^2=50$ so right angle or $\overrightarrow{OP}.\overrightarrow{OQ}=0$ so right angle **Correct work with no slips and conclusion**		A1
			(2)
(c)	(c) $z_1 + z_2 = 1-7\text{ i}$  	New point as shown. It must be the point 1 – 7 i and it must be correctly plotted. The point must be indicated by a scale (could be ticks on axes) **or** labelled with coordinates or as a complex number. **May be on its own axes.**	B1
			(1)
(d)	Writes down another fact about OPQR other than OP being perpendicular to OQ: e.g. OP = OQ, OP is parallel to QR, QR = PR, QR is perpendicular to PR		B1
	Sufficient justification that OPQR is a square and conclusion If their explanation could relate to something other than a square score B0		B1
			(2)
			Total 7

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6. It is given that α and β are roots of the equation $3x^2 + 5x - 1 = 0$

(a) Find the exact value of $\alpha^3 + \beta^3$

(3)

(b) Find a quadratic equation which has roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(5)

Question Number	Scheme		Marks
6.(a)	$\alpha+\beta=-\frac{5}{3}$ **and** $\alpha\beta=-\frac{1}{3}$	**Both** correct statements	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$	**Use** of a **correct** identity for $\alpha^3+\beta^3$ (may be implied by their work)	M1
	$\alpha^3+\beta^3=\left(-\frac{5}{3}\right)^3+\left(-\frac{5}{3}\right)=-\frac{170}{27}$	Correct value (allow exact equivalent – even the correct recurring decimal -6.296296......)	A1
	Special Case – but must be a complete method – generally there are no marks for finding the roots explicitly $\alpha=\frac{-5+\sqrt{37}}{6},\ \beta=\frac{-5-\sqrt{37}}{6}\Rightarrow\alpha^3+\beta^3=\left(\frac{-5+\sqrt{37}}{6}\right)^3+\left(\frac{-5-\sqrt{37}}{6}\right)^3=-\frac{170}{27}$ Could score 3/3 in (a) B1: Both correct roots M1: Cube and add A1: Correct value		
			(3)
(b)	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-\frac{170}{27}}{-\frac{1}{3}}=\frac{170}{9}$	M1: Uses the identity $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ A1: Correct sum (or equivalent)	M1 A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=-\frac{1}{3}$	Correct product	B1
	$x^2-\frac{170}{9}x-\frac{1}{3}$	Uses $x^2-(\textit{their sum})\,x+(\textit{their product})$ (= 0 not needed here)	M1
	$9x^2-170x-3=0$	This equation or any integer multiple including = 0. **Follow through their sum and product.**	A1ft
			(5)
			Total 8
	(b) Alternative using explicit roots		
	$\alpha=\frac{-5+\sqrt{37}}{6},\ \beta=\frac{-5-\sqrt{37}}{6}$		
	$\frac{\alpha^2}{\beta}=\frac{85-14\sqrt{37}}{9},\ \frac{\beta^2}{\alpha}=\frac{85+14\sqrt{37}}{9}$		
	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{170}{9}$	M1: Adds their $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ A1: Correct sum	M1A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=-\frac{1}{3}$	Correct product	B1
	$x^2-\frac{170}{9}x-\frac{1}{3}$	Uses $x^2-(\textit{their sum})\,x+(\textit{their product})$ (= 0 not needed here)	M1
	$9x^2-170x-3=0$	This equation or any integer multiple including = 0. **Follow through their sum and product.**	A1ft

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7.

$$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**.
(3)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the x-axis.

(b) Write down the matrix **Q**.
(1)

Given that V followed by U is the transformation T, which is represented by the matrix **R**,

(c) find the matrix **R**.
(2)

(d) Show that there is a real number k for which the transformation T maps the point $(1, k)$ onto itself. Give the exact value of k in its simplest form.
(5)

Question Number	Scheme		Marks
7. (a)	**Rotation**, **30 degrees** (anticlockwise), **about *O*** Allow $\frac{\pi}{6}$ (radians) for 30 degrees. Anticlockwise may be omitted but do not allow -30 degrees or 30 degrees clockwise **B1: Rotation B1: 30 degrees B1: About O**		B1, B1, B1
			(3)
(b)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Correct matrix	B1
			(1)
(c)	**R** = **PQ**= $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Multiplies **P** by their **Q** This statement is sufficient in correct order	M1
	$= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
(d)	$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$	$\mathbf{R}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ A correct statement but allow poor notation provided there is an indication that the candidate understands that the point (1, *k*) is mapped onto itself. This Method mark could be implied by a correct equation or correct follow through equation below.	M1
	$\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ or $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k$	One correct equation (not a matrix equation)	A1
	$\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ or $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k \Rightarrow k = ...$	Attempts to solve their equation for *k*. **Dependent on the first M.**	**d**M1
	$k = 2 - \sqrt{3}$	cao	A1
	Solves both $\frac{\sqrt{3}}{2} + \frac{k}{2} = 1$ and $\frac{1}{2} - \frac{k\sqrt{3}}{2} = k$ or checks other component	Solves both equations **explicitly** to obtain the same correct value for *k* or clearly verifies that $k = 2 - \sqrt{3}$ is valid for the other equation	B1
			(5)
			Total 11

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8. The hyperbola H has cartesian equation $xy = 16$
The parabola P has parametric equations $x = 8t^2$, $y = 16t$.

(a) Find, using algebra, the coordinates of the point A where H meets P.
(3)

Another point B (8, 2) lies on the hyperbola H.

(b) Find the equation of the normal to H at the point (8, 2), giving your answer in the form $y = mx + c$, where m and c are constants.
(5)

(c) Find the coordinates of the points where this normal at B meets the parabola P.
(6)

Question	Scheme		Marks
8.(a)	$8t^2 \times 16t = 16$ **or** $\left(\frac{16}{x}\right)^2 = 32x$ **or** $y^2 = 32 \times \left(\frac{16}{y}\right)$	Attempts to obtains an equation in one variable *x*, *y* or *t*	M1
	$t = \frac{1}{2}$ or $x = 2$ or $y = 8$	A correct value for *t*, *x* or *y*	A1
	(2, 8)	Correct coordinates following correct work with no other points	B1
			(3)
(b)	$\left(y = \frac{16}{x} \Rightarrow\right)\frac{dy}{dx} = -16x^{-2}$ **or** $\left(y + x\frac{dy}{dx} = 0 \Rightarrow\right)\frac{dy}{dx} = -\frac{y}{x}$ **or** $\left(\dot{x} = 4,\ \dot{y} = -\frac{4}{t^2} \Rightarrow\right)\frac{dy}{dx} = -\frac{1}{t^2}$	Correct derivative in terms of *x*, *y* and *x*, or *t*	B1
	at (8, 2) $\frac{dy}{dx} = -\frac{16}{(8)^2} = -\frac{1}{4}$	Uses $x = 8$, $x = 8$ and $y = 2$, or $t = 2$	M1
	gradient of normal is 4	Correct normal gradient	A1
	$y - 2 = 4(x - 8)$ **or** $y = 4x + c$ and uses $x = 8$ and $y = 2$ to find *c*	Correct straight line method using the point (8, 2) and a numerical gradient from their $\frac{dy}{dx}$ which is not the tangent gradient.	M1
	$y = 4x - 30$	Correct equation	A1
			(5)
(c)	$16t = 32t^2 - 30$ **or** $y = \frac{y^2}{8} - 30$ **or** $\frac{16\sqrt{x}}{\sqrt{8}} = 4x - 30$	Uses their straight line from part (b) and the parabola to obtain an equation in one variable (*x*, *y* or *t*)	M1
	$(4t+3)(4t-5) = 0 \Rightarrow t = ...$ $(y-20)(y+12) = 0 \Rightarrow y = ...$ $(2x-25)(2x-9) = 0 \Rightarrow x = ...$	Attempts to solve three term quadratic (see general guidance) to obtain *t* = ... or *y* = ... or *x* = ... **Dependent on the previous M**	**d**M1
	Note if they solve the **tangent** with the parabola this gives $x^2 - 544x + 256 = 0$ which has roots *x* = 543.53... and 0.47...... (Seeing these values would imply a correct attempt to solve their 3TQ)		
	$t = -\frac{3}{4}$ and $\frac{5}{4}$ **or** $y = 20$ and -12 **or** $x = \frac{25}{2}$ and $\frac{9}{2}$	Correct values for *t* or *y* or *x*	A1
	$t = -\frac{3}{4} \Rightarrow (x, y) =$ or $t = \frac{5}{4} \Rightarrow (x, y) =$ $y = 20 \Rightarrow x =$ or $y = -12 \Rightarrow x =$ $x = \frac{25}{2} \Rightarrow y =$ or $x = \frac{9}{2} \Rightarrow y =$	Uses their values of *t* to find at least one point **or** uses their values of *y* to find at least one *x* **or** uses their values of *x* to find at least one *y*. **Not dependent on previous method marks.**	M1
	$\left(\frac{25}{2}, 20\right)$, $\left(\frac{9}{2}, -12\right)$	A1: One correct pair of coordinates A1: Both pairs correct	A1 A1
			(6)
			Total 14

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9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

(5)

(ii) Prove by induction that,

$$4^n + 6n + 8 \text{ is divisible by } 18$$

for all positive integers n.

(6)

Question	Scheme		Marks
9.(i)	$\sum_{r=1}^{n} r(r+1)(r+2)=6$ **and** $\frac{n(n+1)(n+2)(n+3)}{4}=\frac{1\times2\times3\times4}{4}=6$ **Minimum: lhs = rhs = 6**		B1
	Assume result true for n = "k" so $\sum_{r=1}^{k} r(r+1)(r+2)=\frac{k(k+1)(k+2)(k+3)}{4}$ Minimum **Assume result true**		M1
	$\sum_{r=1}^{k+1} r(r+1)(r+2)=\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$ Adds the (k + 1)th term to the given result		M1
	$=\frac{1}{4}(k+1)(k+2)(k+3)(k+4)$	Achieves this result with no errors **Note this may be written down directly from the line above.**	A1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso
			(5)
(ii)	$f(1)=4^1+6\times1+8=18$	f(1) = 18 is the minimum	B1
	$f(k+1)-f(k)=4^{k+1}+6(k+1)+8-(4^k+6k+8)$	M1: Attempts f(k + 1) – f(k)	M1
	$=3\times4^k+6=3(4^k+6k+8)-18k-18$	A1: $3(4^k+6k+8)$ or $3f(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$f(k+1)=3(4^k+6k+8)-18(k+1)+f(k)$	Makes f(k + 1) the subject	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.		A1cso
			(6)
			Total 11
(ii) ALT 1	$f(1)=4^1+6\times1+8=18$		B1
	$f(k+1)-4f(k)=4^{k+1}+6(k+1)+8-4(4^k+6k+8)$	M1: Attempts f(k + 1) – 4f(k)	M1
	$=-18k-18$	A1: – 18k A1: – 18	A1A1
	$f(k+1)=4f(k)-18(k+1)$	Makes f(k + 1) the subject	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso
(ii) ALT 2	$f(1)=4^1+6\times1+8=18$		B1
	$f(k+1)=4^{k+1}+6(k+1)+8$	M1: Attempts f(k + 1)	M1
	$=4(4^k+6k+8)-18k-18$	A1: $4(4^k+6k+8)$ or $4f(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$f(k+1)=4f(k)-18(k+1)$	Makes f(k + 1) the subject (implicit with first M)	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n.</u>		A1cso

	See general case below for f(k) – mf(k)	

	f(k) – mf(k)		
(ii)	$\text{f}(1) = 4^1 + 6\times1 + 8 = 18$		B1
	$\text{f}(k+1) - m\text{f}(k) = 4^{k+1} + 6(k+1) + 8 - m(4^k + 6k + 8)$	M1: Attempts f(k + 1) – mf(k)	M1
	$= (4-m)(4^k + 6k + 8) - 18k - 18$	A1: $(4-m)(4^k + 6k + 8)$ or $(4-m)\text{f}(k)$ A1: $-18-18k$ or $-18(k+1)$	A1A1
	$\text{f}(k+1) = 4\text{f}(k) - 18(k+1)$	Makes f(k + 1) the subject	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso