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- A railway truck P , of mass m kg, is moving along a straight horizontal track with speed 15 m s^{-1} . Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 m s^{-1} and the speed of Q is 9 m s^{-1} . The direction of motion of P is reversed by the collision.

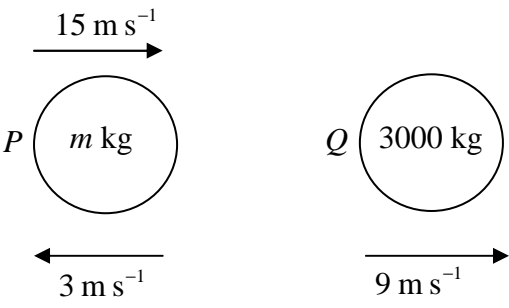
Modelling the trucks as particles, find

- the magnitude of the impulse exerted by P on Q , (2)

- the value of m . (3)



January 2012
6677 Mechanics M1
Mark Scheme

Question Number	Scheme	Marks
<p>1 (a)</p>	 <p>For Q $I = 3000 \times 9 = 27\,000$ (N s)</p>	<p>M1 A1 (2)</p>
<p>(b)</p>	<p>Conservation of linear momentum $15m = -3m + 3000 \times 9$ Leading to $m = 1500$</p>	<p>M1 A1 A1 (3) 5</p>
	<p><i>Alternative to (b)</i> For P $27\,000 = m(15 - (-3))$ Leading to $m = 1500$</p>	<p>M1 A1 A1 (3)</p>

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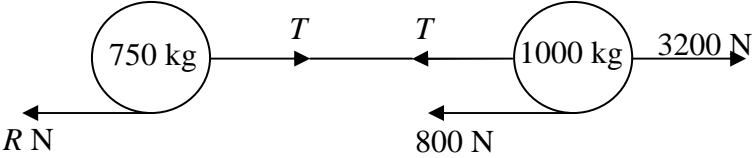
2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is 0.88 ms^{-2} ,

(a) show that $R = 860$, (3)

(b) find the tension in the tow-bar. (3)



Question Number	Scheme	Marks
<p>2 (a)</p>	 <p>For the whole system $R (\rightarrow) \quad 3200 - 800 - R = 1750 \times 0.88$ Leading to $R = 860 \text{ *}$</p> <p>(b) For the caravan $R (\rightarrow) \quad T - 860 = 750 \times 0.88$ Leading to $T = 1520 \text{ (N)}$</p>	<p>M1 A1 A1 (3)</p> <p>M1 A1 A1 (3)</p> <p>6</p>
	<p><i>Alternative for (b)</i> For the car $R (\rightarrow) \quad 3200 - 800 - T = 1000 \times 0.88$ Leading to $T = 1520 \text{ (N)}$</p>	<p>M1 A1 A1 (3)</p>

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3. Three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$

$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j}) \text{ N}$$

$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q .

(3)

The force \mathbf{F}_3 is now removed. The resultant of \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} .
Find

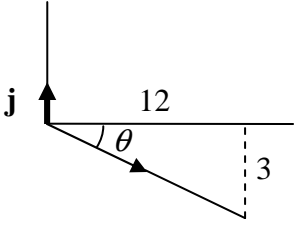
(b) the magnitude of \mathbf{R} ,

(2)

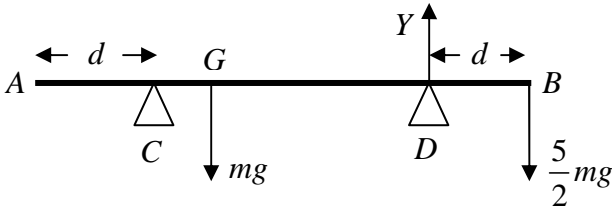
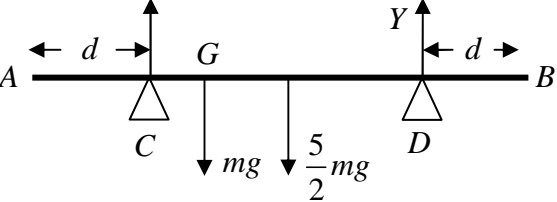
(c) the angle, to the nearest degree, that the direction of \mathbf{R} makes with \mathbf{j} .

(3)

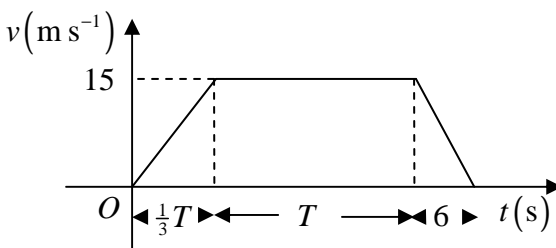
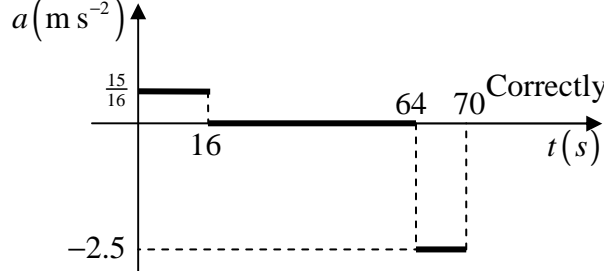


Question Number	Scheme	Marks
3 (a)	$7 + 5 + p = 0 \quad \text{or} \quad -9 + 6 + q = 0$ $p = -12$ $q = 3$	M1 A1 A1 (3)
3 (b)	$\mathbf{R} = 12\mathbf{i} - 3\mathbf{j}$ $ \mathbf{R} = \sqrt{(12^2 + (-3)^2)} = \sqrt{153} \text{ or } 3\sqrt{17} \text{ or } 12.4 \text{ or better (N)}$	M1 A1 (2)
3 (c)	$\tan \theta = \frac{3}{12}$ $\theta = 14.03^\circ \dots$ <p>Angle with \mathbf{j} is 104°, to the nearest degree cao</p>	 M1 A1 A1 (3)

8

Question Number	Scheme	Marks
<p>4 (a)</p>	 <p>M(D) $mg \times GD = \frac{5}{2}mg \times d$</p> <p>$GD = \frac{5}{2}d$ *</p>	<p>M1 A1</p> <p>DM1 A1</p> <p>(4)</p>
<p>(b)</p>	 <p>M(C) $mg \times \frac{d}{2} + \frac{5}{2}mg \times \frac{3}{2}d = Y \times 3d$</p> <p>Leading to $Y = \frac{17}{12}mg$</p>	<p>M1 A2(1, 0)</p> <p>DM1 A1</p> <p>(5)</p> <p>9</p>

Question Number	Scheme	Marks
5 (a)	$v = u + at(\uparrow) \Rightarrow 0 = u - g\left(\frac{25}{14}\right)$ $u = 17 \frac{1}{2} *$	M1 M(A)1 A1 (3)
5 (b)	$v^2 = u^2 + 2as(\uparrow) \Rightarrow 0^2 = 17.5^2 - 2gs$ $s = 15.6 \text{ (m) or } 16 \text{ (m)}$	M1 A1 (2)
5 (c)	$s = ut + \frac{1}{2}at^2(\uparrow) \Rightarrow 6.6 = 17.5t - \frac{1}{2}gt^2$ $4.9t^2 - 17.5t + 6.6 = 0$ $t = \frac{17.5 \pm \sqrt{(17.5^2 - 129.36)}}{9.8} = \frac{17.5 \pm 13.3}{9.8}$ $t = 3.142.. (22/7) \text{ or } 0.428...(3/7)$ $T = t_2 - t_1 = 2.71 \text{ (2.7)}$ <p>OR</p> $v^2 = u^2 + 2as(\uparrow) \Rightarrow v^2 = 17.5^2 - 2gx6.6$ $v = \pm 13.3$ $v = u + at(\uparrow) \Rightarrow \pm 13.3 = 17.5 - gt$ $t = \frac{17.5 \pm 13.3}{9.8}$ $= 3.14.. (22/7) \text{ or } 0.428..(3/7)$ $T = 3.14.. - 0.428.. = 2.71 \text{ or } 2.7$ <p>OR</p> $v^2 = u^2 + 2as(\uparrow) \Rightarrow v^2 = 17.5^2 - 2gx6.6 \text{ or } 0^2 = u^2 - 2gx(15.625 - 6.6)$ $v = 13.3 \qquad u = 13.3$ $v = u + at(\uparrow) \Rightarrow 0 = 13.3 - gt$ $t = \frac{13.3}{g}$ $T = 2 \times \frac{13.3}{g} = 2.7 \text{ or } 2.71$	M1 A1 DM1 A1 DM1 A1 (6)
		M1A1 DM1 A1 DM1 A1 (6)
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Question Number	Scheme	Marks
6 (a)	$v = u + at \Rightarrow 0 = 15 - 2.5t$ $t = 6 \text{ (s)}$	M1 A1 (2)
(b)		Shape 15, T B1 B1 (2)
(c)	$\frac{1}{2} 15 \left(\frac{4}{3} T + 6 + T \right) = 885$ $\frac{7}{3} T = 118 - 6$ $T = 112 \times \frac{3}{7} = 48$	ft their 6 M1 A1ft M1 A1 (4)
(d)	$a = \frac{15}{\frac{1}{3} T} = \frac{15}{16}, 0.9375, 0.938, 0.94$	M1 A1 (2)
(e)		3 horizontal lines B1 B1 B1 Correctly placed; no cts vert line -2.5, ft their $\frac{15}{16}$ (3) 13

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7. [In this question, the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively. Position vectors are relative to a fixed origin O .]

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$.

- (a) Calculate the speed of P . (2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j}) \text{ km}$. At time t hours, the position vector of P is $\mathbf{p} \text{ km}$.

- (b) Write down \mathbf{p} in terms of t . (1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q} \text{ km}$, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j})$$

Find

- (c) the value of t when P is due west of Q , (3)

- (d) the distance between P and Q when P is due west of Q . (3)



Question Number	Scheme	Marks
7 (a)	$\sqrt{((-4)^2 + 8^2)} = \sqrt{80}$ (km h ⁻¹) accept exact equivalents or 8.9 or better	M1 A1 (2)
(b)	$\mathbf{p} = (2\mathbf{i} - 8\mathbf{j}) + t(-4\mathbf{i} + 8\mathbf{j})$	B1 (1)
(c)	Equating j components $-8 + 8t = 12 - 8t$ $t = \frac{5}{4} \text{ oe}$	M1 A1 A1 (3)
(d)	Using their t from (c) to find the i -cpts of p and q and subtract them $10\frac{1}{2} - (-3) = 13\frac{1}{2} \text{ (km)}$	M1 A1 ft A1 (3) 9

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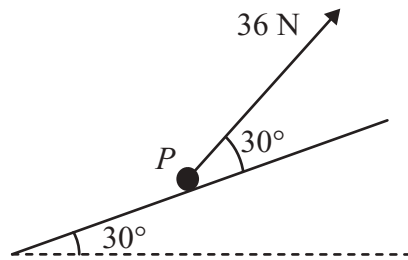


Figure 2

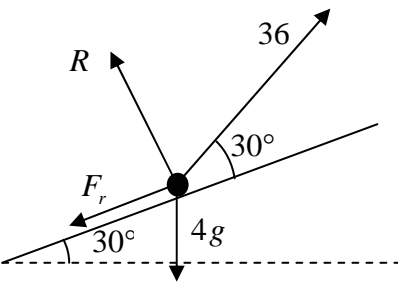
A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of 16 m s^{-1} under the action of a force of magnitude 36 N . The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P , and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ . Find

- (a) the magnitude of the normal reaction between P and the plane, (4)
- (b) the value of μ . (5)

The force of magnitude 36 N is removed.

- (c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest. (5)



Question Number	Scheme	Marks
<p>8 (a)</p>	 $R + 36 \sin 30^\circ = 4g \cos 30^\circ$ $R \approx 15.9, 16$	<p>M1 A1 M1 A1 (4)</p>
	<p>(b) Use of $F_r = \mu R$</p> $36 \cos 30^\circ = F + 4g \sin 30^\circ$ $\mu = \frac{36 \cos 30^\circ - 4g \sin 30^\circ}{R} \approx 0.726$ <p style="text-align: right;">0.73</p>	<p>B1 M1 A1 M1 A1 (5)</p>
	<p>(c) After force is removed</p> $R = 4g \cos 30^\circ$ $-\mu 4g \cos 30^\circ - 4g \sin 30^\circ = 4a$ $a = (-)11.06 \dots$ $v^2 = u^2 + 2as \Rightarrow 0^2 = 16^2 - 2 \times 11.06 \dots \times s$ $s = \frac{16^2}{2 \times 11.06 \dots} \approx 11.6 \text{ (m)}$ <p style="text-align: right;">12</p>	<p>B1 M1 A1 M1 A1 (5) 14</p>