Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Tuesday 27 January 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. $$f(x) = x^4 - x^3 - 9x^2 + 29x - 60$$

Given that $x = 1 + 2i$ is a root of the equation $f(x) = 0$, use algebra to find the three other roots of the equation $f(x) = 0$

(7)

Question Number	Scheme	Notes	Marks
1.	$f(x) = x^4 - x^3 - 9x^2 + 29x - 60$		
	1 – 2i is also a root	Seen anywhere	B1
	$x^2 - 2x + 5$	M1: Attempt to expand $(x-(1+2i))(x-(1-2i))$ or any valid method to establish the quadratic factor A1: $x^2 - 2x + 5$	M1A1
	$f(x) = (x^2 - 2x + 5)(x^2 + x - 12)$	M1: Attempt other **quadratic** factor A1: $x^2 + x - 12$	M1A1
	$x^2 + x - 12 = (x+4)(x-3) \Rightarrow x = ...$	Attempt to solve their **other** quadratic factor.	M1
	$x = -4$ and $x = 3$	Both values correct	A1
			(7)
			Total 7
	Alternative using Factor Theorem		
	$f(3) = ...$ **or** $f(-4) = ...$	M1: Attempts f(3) **or** f(-4)	M1
	$f(3) = 0$ **or** $f(-4) = 0$	A1: Shows or states f(3) = 0 **or** f(-4) = 0	A1
	$f(3) = ...$ **and** $f(-4) = ...$	M1: Attempts f(3) **and** f(-4) or f(3) and g(-4) where g(x) = f(x)/(x - 3) or f(-4) and h(3) where h(x) = f(x)/(x + 4)	M1
	$f(3) = 0$ **and** $f(-4) = 0$	A1: Shows or states f(3) = 0 **and** f(-4) = 0 or shows or states f(3) = 0 and g(-4) = 0 where g(x) = f(x)/(x - 3) or shows or states f(-4) = 0 and h(3) = 0 where h(x) = f(x)/(x + 4)	A1
	NB $g(x) = x^3 + 2x^2 - 3x + 20$, $h(x) = x^3 - 5x^2 + 11x - 15$		
	$x = 3$ **or** $x = -4$	One of $x = 3$ **or** $x = -4$ clearly stated as a **root**	M1
	$x = 3$ **and** $x = -4$	Both $x = 3$ **and** $x = -4$ clearly stated as **roots**	A1
	$x = 1 - 2i$		B1

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2. $$f(x) = x^3 - 3x^2 + \frac{1}{2\sqrt{x^5}} + 2, \quad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α. Give your answer to 3 decimal places.

(5)

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x)=x^3-3x^2+\dfrac{1}{2\sqrt{x^5}}+2$		
(a)	$\mathrm{f}(2)=\ldots$ **and** $\mathrm{f}(3)=\ldots$	Attempts both f(2) and f(3)	M1
	$\mathrm{f}(2)=-1.9116..$, $\mathrm{f}(3)=2.032...$ **Sign change** (and $\mathrm{f}(x)$ is continuous) therefore a **root** α exists between $x=2$ and $x=3$	Both values correct : $\mathrm{f}(2)=-1.9116..$ (awrt -1.9), and $\mathrm{f}(3)=2.032...$(awrt 2.0 or e.g. $2+\dfrac{\sqrt{3}}{54}$) , **sign change** (or equivalent) and **conclusion**	A1
			(2)
(b)	$\mathrm{f}'(x)=3x^2-6x-\dfrac{5}{4}x^{-3.5}$	M1: $x^n\to x^{n-1}$ A1: $3x^2-6x$ A1: $-\dfrac{5}{4}x^{-3.5}$ or equivalent un-simplified and no other terms (+ c loses this mark)	M1A1A1
	$\alpha=3-\dfrac{2.032075015}{8.973270821}$	Correct attempt at Newton-Raphson using their values of $\mathrm{f}(3)$ and $\mathrm{f}'(3)$.	M1
	$\alpha=2.774$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores <u>no</u> marks in (b)**		
			(5)
	NB if the answer is incorrect it must be clear that both $\mathrm{f}(3)$ and $\mathrm{f}'(3)$ are being used in the Newton-Raphson process. So that just $3-\dfrac{f(3)}{f'(3)}$ with an incorrect answer and no other evidence scores M0.		
			Total 7

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3. Given that $z = x + \mathrm{i}y$, where x and y are real numbers, solve the equation

$$(z - 2\mathrm{i})(z^* - 2\mathrm{i}) = 21 - 12\mathrm{i}$$

where z^* is the complex conjugate of z.

(6)

Question Number	Scheme	Notes	Marks
3	$(z-2\text{i})(z^*-2\text{i})=21-12\text{i}$		
	$z^*=x-\text{i}y$		B1
	$(x+\text{i}y-2\text{i})(x-\text{i}y-2\text{i})=....$	Substitutes for z and their z^* and attempts to expand	M1
	$=x^2-x(y+2)\text{i}+x(y-2)\text{i}+y^2-4$		
	$=x^2+y^2-4-4x\text{i}$		
	$x^2+y^2-4=21$ and $4x=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$4x=12\Rightarrow x=....$	Solves real and imaginary parts to obtain at least one value of x or y	M1
	$x=3,\ \ y=\pm 4$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			(6)
			Total 6
Way 2	$(z-2\text{i})(z^*-2\text{i})=zz^*-2\text{i}(z+z^*)-4$	Attempt to expand	M1
	$=(x+\text{i}y)(x-\text{i}y)-2\text{i}(x+\text{i}y+x-\text{i}y)-4$	$z^*=x-\text{i}y$ (may be implied)	B1
	$=x^2+y^2-4x\text{i}-4$		
	$x^2+y^2-4=21$ and $4x=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$4x=12\Rightarrow x=....$	Solves real and imaginary parts to obtain at least one value of x or y	M1
	$x=3,\ \ y=\pm 4$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			Total 6
Way 3	$(z-2\text{i})(z^*-2\text{i})=zz^*-2\text{i}(z+z^*)-4$	Attempt to expand	M1
	$zz^*-2\text{i}(z+z^*)-4=21-12\text{i}$		
	$zz^*-4=21,\quad 2(z+z^*)=12$	Compares real and imaginary parts (allow sign errors only)	M1
	$z^2-6z+25=0\ \left(\text{or}\left(z^*\right)^2-6z^*+25=0\right)$	Correct quadratic	B1
	$z^2-6z+25=0\ \left(\text{or}\left(z^*\right)^2-6z^*+25=0\right)$ $\Rightarrow z=.....$ or $z^*=.....$	Solves to obtain at least one value of z or z^*	M1
	$z=3,\pm4\text{i}$	$x=3$ cso $y=\pm 4$ cso	A1, A1
			Total 6

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4. The parabola C has cartesian equation $y^2 = 12x$

The point $P(3p^2, 6p)$ lies on C, where $p \neq 0$

(a) Show that the equation of the normal to the curve C at the point P is

$$y + px = 6p + 3p^3$$

(5)

This normal crosses the curve C again at the point Q.

Given that $p = 2$ and that S is the focus of the parabola, find

(b) the coordinates of the point Q,

(5)

(c) the area of the triangle PQS.

(4)

Question Number	Scheme	Notes	Marks
4(a)	$y^2 = 12x \Rightarrow y = \sqrt{12}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{12}x^{-\frac{1}{2}}$	$\frac{dy}{dx} = kx^{-\frac{1}{2}}$	M1
	$y^2 = 12x \Rightarrow 2y\frac{dy}{dx} = 12$	$\alpha y\frac{dy}{dx} = \beta$	
	$\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = 6.\frac{1}{6p}$	their $\frac{dy}{dp} \times \left(\frac{1}{\text{their}\frac{dx}{dp}}\right)$	
	$\frac{dy}{dx} = \frac{1}{2}\sqrt{12}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 12$ or $\frac{dy}{dx} = 6.\frac{1}{6p}$ or equivalent expressions	Correct differentiation	A1
	$m_T = \frac{1}{p} \Rightarrow m_N = -p$	Correct perpendicular gradient rule	M1
	$y - 6p = -p(x - 3p^2)$	$y - 6p =$ their $m_N(x - 3p^2)$ or $y = mx + c$ with their m_N and $(3p^2, 6p)$ in an attempt to find 'c'. **Their m_N must have come from calculus and should be a function of p which is not their tangent gradient.**	M1
	$y + px = 6p + 3p^3$ *	Achieves printed answer with no errors	A1*
			(5)
(b)	$p = 2 \Rightarrow y + 2x = 12 + 24$	Substitutes the given value of p into the normal	M1
	$y + \frac{y^2}{6} = 36$	Substitutes to obtain an equation in one variable (x, y or "q")	M1
	$y^2 + 6y - 216 = 0$		
	$(y + 18)(y - 12) = 0 \Rightarrow y =$	Solves their 3TQ	M1
	$y = -18 \Rightarrow x = 27$	A1: One correct coordinate A1: Both coordinates correct	A1, A1
			(5)
(c)	Focus is (3, 0) or a = 3 or OS = 3	Must be seen or used in (c)	B1
	$y = 0 \Rightarrow x = 18$		
	$A = \frac{1}{2}(18-3)(12) + \frac{1}{2}(18-3)(18)$	M1: Correct attempt at area A1: Correct expression	M1A1
	A = 225	Correct area	A1
			(4)
			Total 14

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5. The quadratic equation

$$4x^2 + 3x + 1 = 0$$

has roots α and β.

(a) Write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$.

(2)

(b) Find the value of $(\alpha^2 + \beta^2)$.

(2)

(c) Find a quadratic equation which has roots

$$(4\alpha - \beta) \text{ and } (4\beta - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(4)

Question Number	Scheme	Notes	Marks
5(a)	$\alpha+\beta=-\frac{3}{4},\quad \alpha\beta=\frac{1}{4}$		B1, B1
			(2)
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{9}{16}-\frac{1}{2}=\frac{1}{16}$	M1:Use of $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ A1: $\frac{1}{16}$ cso (allow 0.0625)	M1 A1
			(2)
(c)	Sum $4\alpha-\beta+4\beta-\alpha=3(\alpha+\beta)=-\frac{9}{4}$	Attempt numerical sum	M1
	Product $(4\alpha-\beta)(4\beta-\alpha)=17\alpha\beta-4(\alpha^2+\beta^2)$ $=\frac{17}{4}-\frac{1}{4}=4$	Attempt numerical product	M1
	$x^2-\left(-\frac{9}{4}\right)x+4(=0)$	Uses x^2 – (sum)x + (prod) with sum, prod numerical (= 0 not reqd.)	M1
	$4x^2+9x+16=0$	Any multiple (including = 0)	A1
			(4)
			Total 8

	Alternative: Finds roots explicitly		
(a)	$x=-\frac{3}{8}\pm\frac{\sqrt{7}}{8}\mathrm{i}$		
	$\alpha+\beta=-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}=-\frac{3}{4}$		B1
	$\alpha\beta=\left(-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}\right)\left(-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}\right)=\frac{1}{4}$		B1
			(2)
(b)	$\alpha^2+\beta^2=\left(-\frac{3}{8}+\frac{\sqrt{7}}{8}\mathrm{i}\right)^2+\left(-\frac{3}{8}-\frac{\sqrt{7}}{8}\mathrm{i}\right)^2=\frac{1}{16}$	M1: Substitutes their α and β and attempt to square and add both brackets A1: $\frac{1}{16}$ cso (allow 0.0625)	M1 A1
			(2)
(c)	$4\alpha-\beta=-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i},\ 4\beta-\alpha=-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}$		
	$\mathrm{f}(x)=\left(x-\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)\right)\left(x-\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)\right)$	Uses $(x-(4\alpha-\beta))(x-(4\beta-\alpha))$ With numerical values (May expand first)	M1
	$\mathrm{f}(x)=x^2+x\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)-x\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)+\left(-\frac{9}{8}+\frac{5\sqrt{7}}{8}\mathrm{i}\right)\left(-\frac{9}{8}-\frac{5\sqrt{7}}{8}\mathrm{i}\right)$ Attempt to expand (may occur in terms of α and β but must be numerical for both M's)		M1
	$=x^2+\frac{9}{4}x+4(=0)$	Collects terms (= 0 not reqd.)	M1
	$4x^2+9x+16=0$	Any multiple (including = 0)	A1
			(4)
			Total 8

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6.

(i) $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

(b) Describe fully the single transformation represented by the matrix **B**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by the matrix **C**.

(c) Find **C**.

(2)

(ii) $\mathbf{M} = \begin{pmatrix} 2k+5 & -4 \\ 1 & k \end{pmatrix}$, where k is a real number.

Show that det $\mathbf{M} \neq 0$ for all values of k.

(4)

Question Number	Scheme	Notes	Marks
6(i)(a)	**A**: Stretch scale factor 3 parallel to the x-axis	B1: **Stretch** B1: SF 3 parallel to (or along) x-axis Allow e.g. horizontal stretch SF 3 (Ignore any reference to the origin)	B1B1
			(2)
(b)	**B**: Rotation 210 degrees (anticlockwise) about (0, 0) or about O	B1: Rotation about (0, 0) B1: 210 degrees (anticlockwise) (or equivalent e.g. -150º or 150º clockwise). Allow equivalents in radians.	B1B1
			(2)
(c)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	Attempts **BA** (This statement is sufficient)	M1
	$= \begin{pmatrix} -\frac{3\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{3}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	det**M** = $(2k+5).k - 1\times(-4)\left(= 2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$b^2 - 4ac = 25 - 32$	Attempts discriminant or uses quadratic formula	M1
	$b^2 - 4ac < 0$ So **no real roots** so **det$\mathbf{M} \neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1
			(4)
			Total 10
(ii) Way 2	$(2k+5).k - 1\times(-4)\left(= 2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$= 2\left(k+\frac{5}{4}\right)^2 + \frac{7}{8}$	Attempts to complete the square:	M1
	det$\mathbf{M} > 0\ \forall\ k$ Therefore **det$\mathbf{M} \neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1
(ii) Way 3	$(2k+5).k - 1\times(-4)\left(= 2k^2+5k+4\right)$	M1: Correct attempt at determinant A1: Correct determinant (allow un-simplified)	M1A1
	$\frac{\mathrm{d}(det\mathbf{M})}{\mathrm{d}k} = 4k+5 = 0 \Rightarrow k = -\frac{5}{4}$		
	$k = -\frac{5}{4} \Rightarrow det\mathbf{M} = \frac{7}{8}$	Attempts coordinates of turning point	M1
	Minimum $det\mathbf{M}$ is $\frac{7}{8}$ therefore **det$\mathbf{M} \neq 0$**	Convincing explanation **and** conclusion with no previous errors	A1

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7. Given that, for all positive integers n,

$$\sum_{r=1}^{n}(r+a)(r+b) = \frac{1}{6}n(2n+11)(n-1)$$

where a and b are constants and $a > b$,

(a) find the value of a and the value of b.

(8)

(b) Find the value of

$$\sum_{r=9}^{20}(r+a)(r+b)$$

(3)

Question Number	Scheme	Notes	Marks
7	$\sum_{r=1}^{n}(r+a)(r+b)=\frac{1}{6}n(2n+11)(n-1)$		
(a)	$(r+a)(r+b)=r^2+ra+rb+ab$		B1
	$\sum_{r=1}^{n}(r+a)(r+b)=\frac{1}{6}n(n+1)(2n+1)+(a+b)\frac{1}{2}n(n+1)+abn$		M1A1B1
	M1: Attempt to use one of the standard formulae correctly A1: $\frac{1}{6}n(n+1)(2n+1)+(a+b)\frac{1}{2}n(n+1)$ B1: abn		
	$\frac{1}{6}n\left[(n+1)(2n+1)+3(a+b)(n+1)+6ab\right]=\frac{1}{6}n(2n+11)(n-1)$		
	$(n+1)(2n+1)+3(a+b)(n+1)+6ab=2n^2+9n-11$		
	$2n^2+3n+1+3(a+b)(n+1)+6ab=2n^2+9n-11$		
	$3+3a+3b=9,\ 3a+3b+1+6ab=-11$ $(a+b=2,\ ab=-3)$	M1: Compares coefficients to obtain at least one equation in a and b M1: One correct equation M1: Both equations correct	M1M1M1
	$b=-1,\ a=3$	Both values correct. This can be withheld if $b=3,\ a=-1$ is not rejected.	A1
			(8)
(b)	$\sum_{r=9}^{20}(r+a)(r+b)$		
	$\sum_{r=9}^{20}(r+a)(r+b)=\text{f}(20)-\text{f}(8\text{ or }9)$	**<u>Use</u>** of $\text{f}(20)-\text{f}(8\text{ or }9)$	M1
	$=\frac{1}{6}(20)(51)(19)-\frac{1}{6}(8)(27)(7)$	Correct (possibly un-simplified) numerical expression	A1
	=3230 – 252 = 2978	cao	A1
			(3)
			Total 11

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8. (i) A sequence of numbers is defined by

$$u_1 = 5 \qquad u_2 = 13$$

$$u_{n+2} = 5u_{n+1} - 6u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2^n + 3^n$$

(6)

(ii) Prove by induction that for $n \geqslant 2$, where $n \in \mathbb{Z}$,

$$f(n) = 7^{2n} - 48n - 1$$

is divisible by 2304

(6)

Question Number	Scheme	Notes	Marks
8(i)	When n = 1 $u_1 = 2^1 + 3^1 = 5$ When n = 2 $u_2 = 2^2 + 3^2 = 13$	**Both**	B1
	True for n = 1 and n = 2		
	Assume $u_k = 2^k + 3^k$ and $u_{k+1} = 2^{k+1} + 3^{k+1}$		
	$u_{k+2} = 5u_{k+1} - 6u_k = 5\left(2^{k+1} + 3^{k+1}\right) - 6\left(2^k + 3^k\right)$	M1: Attempts u_{k+2} in terms of u_{k+1} and u_k A1: Correct expression	M1A1
	$= 5.2^{k+1} + 5.3^{k+1} - 6.2^k - 6.3^k$		
	$= 5.2^{k+1} - 3.2^{k+1} + 5.3^{k+1} - 2.3^{k+1}$	Attempt u_{k+2} in terms of $2^{f(k)}$ and $3^{f(k)}$ only	M1
	So $u_{k+2} = 2.2^{k+1} + 3.3^{k+1}$		
	$= 2^{(k+1)+1} + 3^{(k+1)+1}$ or $2^{k+2} + 3^{k+2}$	Correct expression with **no errors**	A1
	If true for k and k + 1 then shown true for k + 2 and as true for n = 1 and n = 2, true for $n \in \mathbb{Z}^+$	**Full conclusion** with all previous marks scored	A1
			(6)
(ii)	$f(2) = 7^4 - 48(2) - 1 = 2304$ So true for n = 2	Shows true for n = 2	B1
	Assume $f(k) = 7^{2k} - 48k - 1 = 2304p$ for some integer p		
	$f(k+1) - f(k) = 7^{2k+2} - 48(k+1) - 1 - \left(7^{2k} - 48k - 1\right)$	Attempt f(k+1) – f(k)	M1
	$= 7^{2k+2} - 7^{2k} - 48$		
	$= 7^{2k}(49-1) - 48$		
	$= 48f(k) + 48^2 k$	M1: Attempt rhs in terms of f(k) or $7^{2k} - 48k - 1$ A1: Correct expression which is a multiple of 2304	M1A1
	$= 48 \times 2304p + 2304k$		
	$f(k+1) = 49 \times 2304p + 2304k$	Obtains f(k+1) as a correct multiple of 2304 with **no errors**	A1
	If true for k then shown true for k + 1 and as true for n = 2, true for $n \geq 2$ ($n \in \mathbb{Z}$)	**Full conclusion** with all previous marks scored	A1
			(6)
			Total 12