Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Thursday 14 May 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Given that

$$2z^3 - 5z^2 + 7z - 6 \equiv (2z - 3)(z^2 + az + b)$$

where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Given that z is a complex number, find the three exact roots of the equation

$$2z^3 - 5z^2 + 7z - 6 = 0$$

(3)

June 2015
Further Pure Mathematics F1 WFM01
Mark Scheme

Question Number	Scheme		Marks
1.(a)	$2z^3-5z^2+7z-6=(2z-3)(z^2+az+b)$		
	$a = -1$ and $b = 2$	B1: One of $a = -1$ or $b = 2$ B1: Both $a = -1$ and $b = 2$	B1 B1
	Values may be implied by a correct quadratic e.g. sight of z^2-z+2		
			(2)
(b)	$z = 1\frac{1}{2}$	z = 1.5 or equivalent	B1
	$z=\frac{1}{2}\pm\left(\frac{1}{2}\sqrt{7}\right)\text{i}$	M1: Solves their 3 term quadratic (usual rules) as far as z = ... A1: Allow $z=\frac{1\pm\text{i}\sqrt{7}}{2}$ or equivalent e.g. $z=\frac{1}{2}\pm\left(\sqrt{\frac{7}{4}}\right)\text{i}$	M1A1
	Answers must be exact and accept correct answers only for both marks. **Answers that are not exact with no working score M0A0**		
			(3)
			[5 marks]

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2. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(3r-2)^2 = \frac{n}{2}\left(an^2 + bn + c\right)$$

where a, b and c are integers to be found.

(5)

Question Number	Scheme		Marks
2.	$(3r-2)^2 = 9r^2 - 12r + 4$	Correct expansion	B1
	$\sum_{r=1}^{n}(3r-2)^2 = \sum_{r=1}^{n}9r^2 - \sum_{r=1}^{n}12r + \sum_{r=1}^{n}4$ $= 9\frac{n}{6}(n+1)(2n+1) - 12\frac{n}{2}(n+1) + 4n$	B1ft: "4" = "4"n M1: Uses valid formulae for sum of squares **and** sum of integers (their 9 or 12 may be followed through from their coefficients)	B1ft M1
	$= \frac{n}{2}\left(3(n+1)(2n+1) - 12(n+1) + 8\right)$ or $\frac{n}{6}\left(9(n+1)(2n+1) - 36(n+1) + 24\right)$	Takes out factor $\frac{n}{2}$ or $\frac{n}{6}$. Dependent on the B1ft having been scored.	dM1
	$= \frac{n}{2}\left(6n^2 - 3n - 1\right)$	Correct result or states $a = 6$, $b = -3$, $c = -1$	A1
			(5)
	You should always award marks as in the scheme but generally there are no marks for proof by induction		
			[5 marks]

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3. It is given that α and β are roots of the equation

$$2x^2 - 7x + 4 = 0$$

(a) Find the exact value of $\alpha^2 + \beta^2$

(3)

(b) Find a quadratic equation which has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(3)

Question Number	Scheme		Marks
3.(a)	$\alpha+\beta=\frac{7}{2}$ **and** $\alpha\beta=2$	Allow $\frac{4}{2}$ for 2	B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{7}{2}\right)^2-2(2)=\frac{33}{4}$	M1: Uses $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ A1: $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25	M1 A1
			(3)
(b)	Sum of roots is $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{33/4}{2}=\frac{33}{8}$ and product of roots is $\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$	M1: Attempts sum **or** product of new roots correctly (may be implied) A1: Sum = $\frac{33}{8}$ and product = 1	M1 A1
	$x^2-\frac{33}{8}x+1=0 \therefore 8x^2-33x+8=0$	$8x^2-33x+8=0$ or any integer multiple including the "= 0"	A1
			(3)
			[6 marks]
	Alternative – finds roots explicitly:		
(a)	$\alpha,\beta=\frac{1}{4}\left(7\pm\sqrt{17}\right)$	Correct exact roots including $\sqrt{17}$	B1
	$\alpha^2+\beta^2=2\left(\frac{49}{16}\right)+2\frac{17}{16}=2\times\frac{66}{16}=\frac{33}{4}$	M1: Squares and adds their roots A1: cao $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25	M1 A1
			(3)
(b)	$\left(x-\frac{7+\sqrt{17}}{7-\sqrt{17}}\right)\left(x-\frac{7-\sqrt{17}}{7+\sqrt{17}}\right)=\ldots$	Uses $\left(x-\frac{\alpha}{\beta}\right)\left(x-\frac{\beta}{\alpha}\right)$ with numerical $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and attempts to expand. There are no marks until numerical values are used.	M1
	$=x^2-\frac{33}{8}x+1$		A1
	$8x^2-33x+8=0$	This answer with no errors or any integer multiple including the "= 0"	A1cso
			(3)
	Note: **Roots of the form** $\frac{1}{k}\left(7\pm\sqrt{17}\right)$, $k\neq 4$ **will give a correct answer – in this case lose the final mark as not cso.**		

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4.

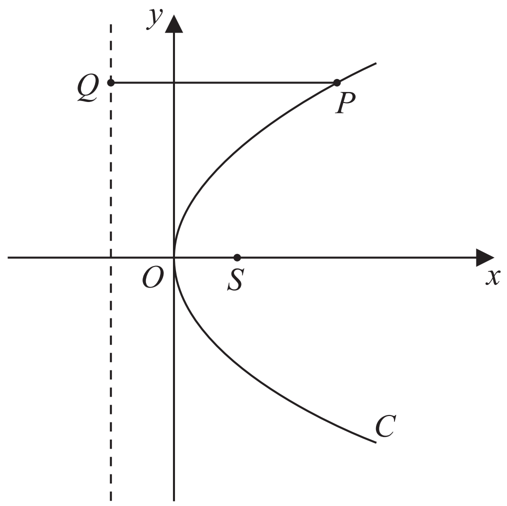

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C and the point Q lies on the directrix of C.
The point P lies on C where $y > 0$ and the line segment QP is parallel to the x-axis.

Given that the length of PS is 13

(a) write down the length of PQ.

(1)

Given that the point P has x coordinate 9

find

(b) the value of a,

(2)

(c) the area of triangle PSQ.

(3)

Question Number	Scheme		Marks
4.(a)	$(PQ =)\ 13$	Sight of 13 (Must be seen in (a))	B1
			(1)
(b)	$9 + a = 13 \Rightarrow a = ...$ or $(9 \pm a)^2 + 36a = 169 \Rightarrow a = ...$	M1: Uses $9 \pm a = 13$ or $(9 \pm a)^2 + 36a = 169$ to obtain a value for a	M1 A1
		A1: $a = 4$ only	
			(2)
(c)	$y = 12$	Correct y coordinate of P.	M1
	Uses Area of triangle = $\frac{1}{2} \times 13 \times "y"$ or $\frac{1}{2} \times \begin{vmatrix} -4 & 9 & 4 & -4 \\ y & y & 0 & y \end{vmatrix}$	A correct triangle area method	M1
	= 78	cao	A1
			(3)
	Alternative method for area of triangle using midpoint of QS (M) **Area =** $\frac{1}{2} \times QS \times MP = \frac{1}{2} \times \sqrt{208} \times \sqrt{117}$ **The method for QS and MP must be correct for their values** **There are other methods for the area and the method should be correct for their values to score the M1 e.g. Box – Triangles = 156 – 48 – 30 = 78**		
			[6 marks]

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5. In the interval $2 < x < 3$, the equation

$$6 - x^2 \cos\left(\frac{x}{5}\right) = 0, \text{ where } x \text{ is measured in radians}$$

has exactly one root α.

(a) Starting with the interval [2, 3], use interval bisection twice to find an interval of width 0.25 which contains α.

(4)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α. Give your answer to 2 decimal places.

(3)

Question Number	Scheme		Marks
5.(a)	$f(2) = ...$ and $f(3) = ...$	Attempts to evaluate both $f(2)$ and $f(3)$ (ignore use of degrees for this mark) **NB degrees usually scores M1A0M0A0** **NB** $f(2) \approx 2$ and $f(3) \approx -3$ for degrees	M1
	$f(2) = 2.3...$, $f(3) = -1.4...$	Needs accuracy to 1 figure truncated or rounded	A1
	f(2.5) = 0.5….and f(2.75)= -0.4….	Evaluates both f(2.5) and f(2.75) (and not f(2.25))	M1
	(2.5, 2.75)	$2.5 \le x \le 2.75$ or $2.5 < x < 2.75$ or $2.5 \le \alpha \le 2.75$ or $2.5 < \alpha < 2.75$ or $[2.5, 2.75]$ or $(2.5, 2.75)$ or equivalent in words. Allow a mixture of 'ends' but not incorrect statements such as $2.75 < x < 2.5$. Needs accuracy to 1 figure truncated or rounded for f(2.5) and f(2.75) and conclusion	A1
			(4)
	Note that some candidates only indicate the sign of f not its value. In this case the M's can still score as defined but not the A's. However if f(2) and f(3) are correctly evaluated in (b) then the first A1 can be given retrospectively.		

Common Approach in the form of a table:

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	2.31576...	3	-1.428...	2.5	0.5151...
2.5	0.5151...	3	-1.428...	2.75	-0.4472...

$2.5 < \alpha < 2.75$

Would score full marks in (a)

Question Number	Scheme		Marks
(b)	$\frac{\alpha-2}{2.3158} = \frac{3-\alpha}{1.4280}$ or $\frac{\alpha-2}{2.3158} = \frac{3-2}{3.7438}$	Correct equation involving α or x and **their** values even in degrees. Use of negative lengths scores M0	M1
	$\alpha(1.4280 + 2.3158) = 3 \times 2.3158 + 2 \times 1.4280$ so $\alpha = ...$	Makes α or x the subject. Dependent on the previous M but condone poor algebra.	dM1
	$(\alpha =) 2.62$	cao and cso (Allow x =)	A1
	A correct statement followed by 2.62 scores 3/3		
			(3)
	Using $y = mx + c$:		
	$m = f(2) - f(3) = -3.74.....$ $c = f(2) - 2m = 9.80...$	Correct method to find equation of straight line	M1
	$y = 0 \Rightarrow x = ...$	Substitutes y = 0 and makes x or α the subject. Dependent on the previous M	dM1
	$(\alpha =) 2.62$	cao and cso (Allow x =)	A1
	Also allow candidates to find the value of e.g $3-\alpha$ or $\alpha - 2$ and then add to 2 or subtract from 3: M1 for a correct method for $3-\alpha$ or $\alpha-2$, dM1 for adding to 2 or subtracting from 3 and A1 for 2.62 cao and cso.		
			[7 marks]

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6. The rectangular hyperbola, H, has cartesian equation

$$xy = 36$$

The three points $P\left(6p, \frac{6}{p}\right)$, $Q\left(6q, \frac{6}{q}\right)$ and $R\left(6r, \frac{6}{r}\right)$, where p, q and r are distinct, non-zero values, lie on the hyperbola H.

(a) Show that an equation of the line PQ is

$$pqy + x = 6(p + q)$$

(4)

Given that PR is perpendicular to QR,

(b) show that the normal to the curve H at the point R is parallel to the line PQ.

(6)

Question Number	Scheme		Marks
6(a)	Gradient of PQ is $\dfrac{\frac{6}{p}-\frac{6}{q}}{6p-6q}\left(=-\dfrac{1}{pq}\right)$	Correct gradient in any form	B1
	Equation of PQ is $y-\dfrac{6}{q}=\dfrac{\frac{6}{p}-\frac{6}{q}}{6p-6q}(x-6q)$	M1: Uses straight line equation in any form correctly for their gradient or uses $y = mx + c$ and attempts to find c in terms of p and q A1: Correct line in any form	M1 A1
	$y-\dfrac{6}{q}=\dfrac{-1}{pq}(x-6q)$		
	$pq(y-\frac{6}{q})=-(x-6q)\Rightarrow pqy+x=6(p+q)$*	Cso. Reaches the given answer with at least one intermediate step.	A1*
			(4)
	Alternative simultaneous equations		
	$\dfrac{6}{p}=m(6p)+c,\quad \dfrac{6}{q}=m(6q)+c$	Correct equations	B1
	$m=\dfrac{q-p}{pq(p-q)}$, $c=\dfrac{6}{p}+\dfrac{6}{q}$	M1: Solves simultaneously to obtain **either** "m" **or** "c" in terms of p and q A1: $m=\dfrac{q-p}{pq(p-q)}$ **and** $c=\dfrac{6}{p}+\dfrac{6}{q}$	M1A1
	$pqy+x=6(p+q)$*	Cso. Reaches the given answer with at least one intermediate step.	A1*
(b)	$y=\dfrac{36}{x}\Rightarrow\dfrac{dy}{dx}=-36x^{-2}$ and uses $x = 6r$ $x\dfrac{dy}{dx}+y=0\Rightarrow\dfrac{dy}{dx}=-\dfrac{y}{x}$ and use $x = 6r$, $y = 6/r$ $\dfrac{dx}{dr}=6\Rightarrow\dfrac{dy}{dr}=-\dfrac{6}{r^2}$ and uses $\dfrac{dy}{dx}=\dfrac{dy}{dr}\div\dfrac{dx}{dr}$	$\dfrac{dy}{dx}=kx^{-2}$ and uses $x = 6r$ $\dfrac{dy}{dx}=k\dfrac{y}{x}$ and uses $x = 6r$, $y = 6/r$ $\dfrac{dx}{dr}=k\Rightarrow\dfrac{dy}{dr}=\dfrac{k}{r^2}$ and uses $\dfrac{dy}{dx}=\dfrac{dy}{dr}\div\dfrac{dx}{dr}$	M1
	So at R gradient of curve = $-\dfrac{1}{r^2}$	Allow any un-simplified correct form e.g. $-36(6r)^{-2},-\dfrac{\frac{6}{r}}{6r},-\dfrac{6}{r^2}\div 6$	A1
	So gradient of normal = r^2	Correct use of perpendicular gradient rule	M1
	$-\dfrac{1}{pr}\times-\dfrac{1}{qr}=-1$	M1: Uses gradient PR perpendicular to gradient QR A1: Correct equation connecting p, q and r	M1 A1
	So $r^2=\dfrac{-1}{pq}$ which is the gradient of PQ so the normal at R is **parallel** to PQ	Conclusion with all previous marks scored. **Must see the word 'parallel' used.**	A1cso
			(6)
			[10 marks]

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7.

$z = -3k - 2k\text{i}$, where k is a real, positive constant.

(a) Find the modulus and the argument of z, giving the argument in radians to 2 decimal places and giving the modulus as an exact answer in terms of k.

(3)

(b) Express in the form $a + \text{i}b$, where a and b are real and are given in terms of k where necessary,

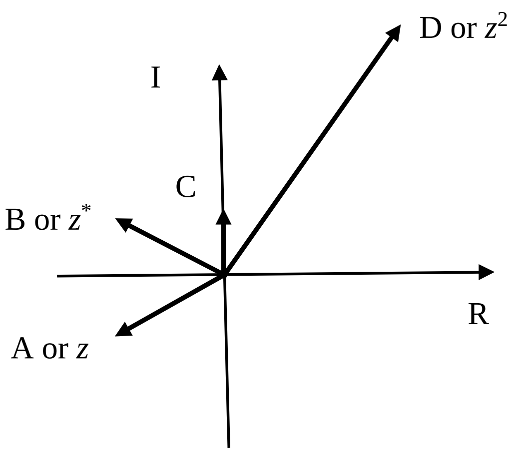
(i) $\dfrac{4}{z + 3k}$

(ii) z^2

(5)

(c) Given that $k = 1$, plot the points A, B, C and D representing z, z^*, $\dfrac{4}{z + 3k}$ and z^2 respectively on a single Argand diagram.

(3)

Question Number	Scheme		Marks
7.(a)	$\lvert z\rvert = k\sqrt{13}$	Accept $\sqrt{13k^2}$ but not $\sqrt{9k^2+4k^2}$	B1
	$\arg z = \pi + \arctan\left(\frac{2}{3}\right) = \pi + 0.588$ $= 3.73$ or -2.55	M1: Uses $\arctan\left(\pm\frac{2}{3}\right)\ (\pm 0.588^c.../\pm 33.6^o...)$ or $\arctan\left(\pm\frac{3}{2}\right)(\pm 0.98^c.../\pm 56.3^o...)$ A1: 3.73 or -2.55 only	M1 A1
			(3)
(b)(i)	$\frac{4}{z+3k} = \frac{4}{-2k\mathrm{i}} = \frac{2}{k}\mathrm{i}$	M1: Substitutes z and multiplies numerator and denominator by conjugate of denominator or equivalent A1: $\frac{2}{k}\mathrm{i}$ oe (Allow un-simplified e.g. $\frac{8k}{4k^2}\mathrm{i}$). Allow $0+\frac{2}{k}\mathrm{i}$	M1 A1
(ii)	$z^2 = (-3k-2k\mathrm{i})(-3k-2k\mathrm{i}) = 9k^2 + 12\mathrm{i}k^2 + 4\mathrm{i}^2k^2$	Multiplies out obtaining 3 term quadratic in i	M1
	$= 5k^2 + 12k^2\mathrm{i}$	M1: Uses $\mathrm{i}^2 = -1$ (may be implied) A1: cao	M1A1
			(5)
(c)		Plots z in 3rd quadrant and z^* as mirror image in 2nd quadrant and both correctly labelled	B1
		Plots a complex number on positive imaginary axis and correctly labelled	B1
		Plots and labels D in the first quadrant, positioned correctly relative to the other points and further from the origin than all the other points.	B1
		<u>Notes:</u> 1. Penalise the omission of labels once and penalise it the first time it occurs. 2. For labels allow letters, in terms of z, coordinates or labels on axes. 3. If there are separate Argand Diagrams, imagine them superimposed. 4. Accept points, lines or arrows.	(3)
			(3)
			[11 marks]

Leave blank

8.

$$\mathbf{P} = \begin{pmatrix} 3a & -4a \\ 4a & 3a \end{pmatrix}, \text{ where } a \text{ is a constant and } a > 0$$

(a) Find the matrix $\mathbf{P}^{-1}$ in terms of a.

(3)

The matrix **P** represents the transformation U which transforms a triangle T_1 onto the triangle T_2.
The triangle T_2 has vertices at the points $(-3a, -4a)$, $(6a, 8a)$, and $(-20a, 15a)$.

(b) Find the coordinates of the vertices of T_1

(3)

(c) Hence, or otherwise, find the area of triangle T_2 in terms of a.

(3)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through an angle α **clockwise** about the origin, where $\tan \alpha = \frac{4}{3}$ and $0 < \alpha < \frac{\pi}{2}$

(d) Write down the matrix **Q**, giving each element as an exact value.

(2)

The transformation U followed by the transformation V is the transformation W.
The matrix **R** represents the transformation W.

(e) Find the matrix **R**.

(2)

Question Number	Scheme		Marks
8.(a)	$\mathbf{P}^{-1} = \frac{1}{25a^2}\begin{pmatrix} 3a & 4a \\ -4a & 3a \end{pmatrix}$ or $\frac{1}{25a}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$	M1: Switches signs on minor diagonal B1: Correct determinant. Allow simplified or un-simplified e.g. 3*a*(3*a*)-(-4*a*)(4*a*), score when first seen. A1: Completely correct inverse with determinant simplified.	M1 B1 A1
			(3)
(b)	$\frac{1}{25a}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} -3a & 6a & -20a \\ -4a & 8a & 15a \end{pmatrix}$	Sets up correct multiplication including $\frac{1}{25a}$ or equivalent	M1
	$= \begin{pmatrix} -1 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$	Correct matrix	A1
	$(-1,0)$, $(2,0)$ and $(0,5)$	Follow through their matrix but must be written as coordinates	A1ft
			(3)
(c)	Area of triangle $T_1 = \frac{1}{2}\times 3\times 5$ o.e. $\left(\frac{15}{2}\right)$	Correct area for triangle T_1.	M1
	Area scale factor is $25a^2$ so Area of triangle $T_2 = \frac{15}{2}\times 25a^2 = 187.5a^2$ oe	M1: Multiplies their area of T_1 by their det**P** to find required area A1: cao	M1A1
			(3)
	Alternative 1: Shoelace method		
	area $T_2 = \frac{1}{2}\times\begin{vmatrix} -3a & 6a & -20a & -3a \\ -4a & 8a & 15a & -4a \end{vmatrix}$	Correct statement.	M1
	$\frac{1}{2}\times\left\lvert \begin{matrix} -3a\times 8a+(6a\times 15a)+(-20a\times -4a) \\ -\{(-4a\times 6a)+(-8a\times 20a)+(15a\times -3a)\} \end{matrix}\right\rvert$	Correct calculation	M1
	$=187.5a^2$ oe	cao	A1
	Alternative 2: Encloses T_2 by a rectangle and subtracts triangles:		
	Rectangle area = $494a^2$ and **one** triangle area of $161.5a^2$, $91a^2$ or $54a^2$	Correct values	M1
	$494a^2 - 161.5a^2 - 91a^2 - 54a^2$	Complete method for area	M1
	$=187.5a^2$ oe	cao	A1
(d)	$\mathbf{Q} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$	M1: $\begin{pmatrix} \frac{3}{5} & \alpha \\ \beta & \frac{3}{5} \end{pmatrix}$ $\frac{3}{5}$ in both entries of main diagonal and $\alpha \neq 0$ and $\beta \neq 0$ A1: Correct matrix	M1A1
			(2)
(e)	$\mathbf{R} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} 3a & -4a \\ 4a & 3a \end{pmatrix} = \begin{pmatrix} 5a & 0 \\ 0 & 5a \end{pmatrix}$ oe	M1: Sets up correct multiplication in correct order. "Their **Q**" × **P** A1: cao	M1 A1
			(2)
			[13 marks]

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9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r^2(2r-1) = \frac{1}{6}n(n+1)(3n^2+n-1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^n = \begin{pmatrix} 6n+1 & -12n \\ 3n & 1-6n \end{pmatrix}$$

(6)

Question Number	Scheme		Marks
9.(i)	If $n=1$, $\sum_{r=1}^{n} r^2(2r-1)=1$ and $\frac{1}{6}n(n+1)(3n^2+n-1)=1$, **LHS=RHS** so true for n = 1.		B1
	$\sum_{r=1}^{k+1} r^2(2r-1)=\frac{1}{6}k(k+1)(3k^2+k-1)+(k+1)^2(2(k+1)-1)$ (Adds the $(k+1)^{th}$ term to the sum of the first k terms)		M1
	$=\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$	dM1: Attempt factor of $\frac{1}{6}(k+1)$ A1: $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$	dM1A1
	$=\frac{1}{6}(k+1)(k+2)(3k^2+7k+3)=\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$		A1
	Achieves this result with no errors and $3k^2+7k+3$ seen **Allow work that shows equivalence between** e.g. $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ and $\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$		
	True for *n* = *k* + 1 if true for *n* = *k*, and as true for *n* = 1 true by induction for all *n*.		A1cso
	Full conclusion and all previous marks scored		
			(6)
(ii)	n = 1: $\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^1 = \begin{pmatrix} 6+1 & -12 \\ 3 & 1-6 \end{pmatrix}$ so true for n = 1	Shows true for n = 1	B1
	$\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^{k+1} = \begin{pmatrix} 6k+1 & -12k \\ 3k & 1-6k \end{pmatrix}\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}$ or $\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 6k+1 & -12k \\ 3k & 1-6k \end{pmatrix}$		M1
	Either statement scores M1		
	$\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ or e.g. $\begin{pmatrix} 7(6k+1)+3(-12k) & -12(6k+1)+(-12k)(-5) \\ 3k(7)+3(1-6k) & -12(3k)+(1-6k)(-5) \end{pmatrix}$	M1: Correct attempt at multiplication (if unclear, at least 2 terms must be correct) A1: Correct matrix possibly un-simplified	M1A1
	If the previous A1 was awarded for $\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ then allow the next A mark for the matrix as shown. If the previous A1 was awarded for e.g. $\begin{pmatrix} 7(6k+1)+3(-12k) & -12(6k+1)+(-12k)(-5) \\ 3k(7)+3(1-6k) & -12(3k)+(1-6k)(-5) \end{pmatrix}$ then this must be simplified to $\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ before the next A mark can be awarded.		
	$\begin{pmatrix} 6(k+1)+1 & -12(k+1) \\ 3(k+1) & 1-6(k+1) \end{pmatrix}$	States **or shows by equivalence** that the result is true for $n = k + 1$	A1
	True for *n* = *k* + 1 if true for *n* = *k*, and as true for *n* = 1 true by induction for all *n*.		A1
	Full conclusion and all previous marks scored		(6)
			[12 marks]