

June 2012
6677 Mechanics M1
Mark Scheme

Question Number	Scheme	Marks
1.	<p style="text-align: center;"> Before $\xrightarrow{3 \text{ m s}^{-1}}$ $A(5m)$ $\xleftarrow{4 \text{ m s}^{-1}}$ $B(2m)$ $\xrightarrow{3.3 \text{ N s}}$ After $\xrightarrow{0.8 \text{ m s}^{-1}}$ $\xrightarrow{v \text{ m s}^{-1}}$ </p> <p>(a) CLM $5m \times 3 - 2m \times 4 = 5m \times 0.8 + 2mv$ Leading to $v = 1.5$ (Speed is 1.5 m s^{-1})</p> <p>(b) Impulse for A $5m(0.8 - 3) = -3.3$ Leading to $m = 0.3$</p>	<p>M1 A1 A1 (3)</p> <p>M1 A1 A1 (3) [6]</p>
	<p><i>Alternative for (b)</i> Impulse for B $2m(1.5 - -4) = 3.3$ Leading to $m = 0.3$</p>	<p>M1 A1 A1 (3)</p>

Question 1(a)

M1 for attempt at CLM equation, with correct no. of terms, correct masses and dimensionally consistent. Allow consistent extra g's, consistent missing m's and sign errors. However, M0 if masses are not paired with the correct speeds.

First A1 for a correct equation.

Second A1 for $v = 1.5$. (-1.5 A0)

N.B. Allow M1 for an attempt to equate the impulses on the particles but must have $5m(0.8 - 3)$ or $5m(3 - 0.8)$ on one side of the equation and $2m(\pm v \pm 4)$ on the other.

Question 1(b)

M1 for attempt at impulse = difference in momenta, for either particle, (must be considering one particle) (M0 if g's are included or if mass omitted or if just m used)
 Allow Initial Momentum – Final Momentum.

A1 cao (i.e. no ft on their v) for a correct equation in m only.

A1 for $m = 0.3$

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2.

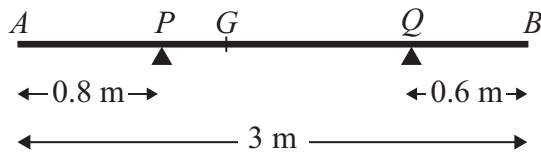


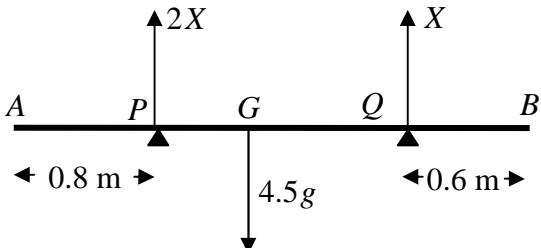
Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q , where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G . Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod, (3)

(b) the distance AG . (4)



Question Number	Scheme	Marks
2.	<div style="text-align: center;">  </div> <p>(a) $\uparrow \quad 2X + X = 4.5g$ Leading to $X = \frac{3g}{2}$ or 14.7 or 15 (N)</p> <p>(b) $M(A) \quad 4.5g \times AG = (2X) \times 0.8 + X \times 2.4$ $AG = \frac{4}{3}$ (m), 1.3, 1.33,...</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A2 ft (1,0)</p> <p>A1 (4)</p> <p>[7]</p>

Question 2(a)

First M1 for a complete method for finding R_Q , either by resolving vertically, or taking moments twice, with usual criteria (allow M1 even if $R_P = 2R_Q$ not substituted)

First A1 for a correct equation in either R_Q or R_P ONLY.

Second A1 for 1.5g or 14.7 or 15 (A0 for a negative answer)

Question 2(b)

First M1 for taking moments about any point, with usual criteria.

A2 ft for a correct equation (A1A0 one error, A0A0 for two or more errors, ignoring consistent omission of g's) in terms of X and their x (which may not be AG at this stage)

Third A1 for $AG = 4/3, 1.3, 1.33, \dots$ (any number of decimal places, since g cancels) need 'AG =' or x marked on diagram

N.B. if $R_Q = 2R_P$ throughout, mark as a misread as follows:

(a) M1A1A0 (resolution method) (b) M1A0A1A1, assuming all work follows through correctly..

3.

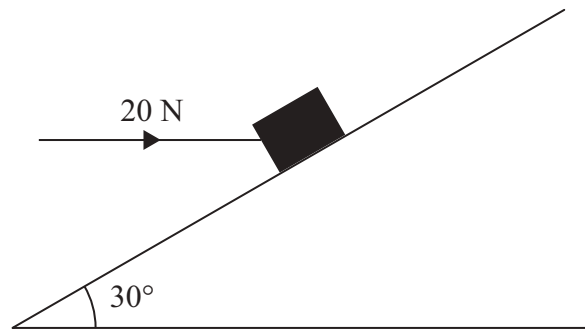


Figure 2

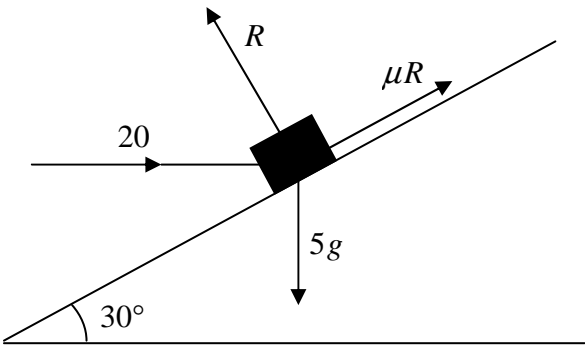
A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

(a) the magnitude of the normal reaction of the plane on the box, (4)

(b) the coefficient of friction between the box and the plane. (5)



Question Number	Scheme	Marks
3.	<div style="text-align: center;">  </div> <p>(a) \perp plane $R = 20 \cos 60^\circ + 5g \cos 30^\circ$ $= 52.4 \text{ (N)}$ or 52</p> <p>(b) P plane $F_r = \mu R$ $F + 20 \cos 30^\circ = 5g \cos 60^\circ$ Leading to $\mu = 0.137$ or 0.14</p>	<p>M1 A2(1,0) A1 (4)</p> <p>B1 M1 A2(1, 0) A1 (5) [9]</p>

Question 3(a)

First M1 for resolving perpendicular to plane with usual criteria

First A2 for a correct equation (A1A0 one error, A0A0 for two or more errors)

Second A1 for either 52 or 52.4

N.B. In part (a), the M1 is for a complete method, so they must have sufficient equations to be able to solve for R . The A2 marks are then for *all* the equations.

Question 3(b)

B1 for use of $F = \mu R$ (could just be on diagram)

First M1 (allow if F is used rather than μR) for resolving parallel to the plane with usual criteria

First A2 for a correct equation (A1A0 one error, A0A0 for two or more errors)

Second A1 for either 0.14 or 0.137

N.B. If they resolve vertically AND horizontally, there are max 6 marks available (M1A2, M1A2) for the TWO equations, but if they only have one equation, there are no marks available for that equation.

The marks for the horizontal resolution should be entered first on ePen.

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4. A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A . The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2} , reducing its speed from 20 m s^{-1} to 8 m s^{-1} . The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B .

(a) Sketch a speed-time graph to represent the motion of the car from A to B .

(3)

(b) Find the time for which the car is decelerating.

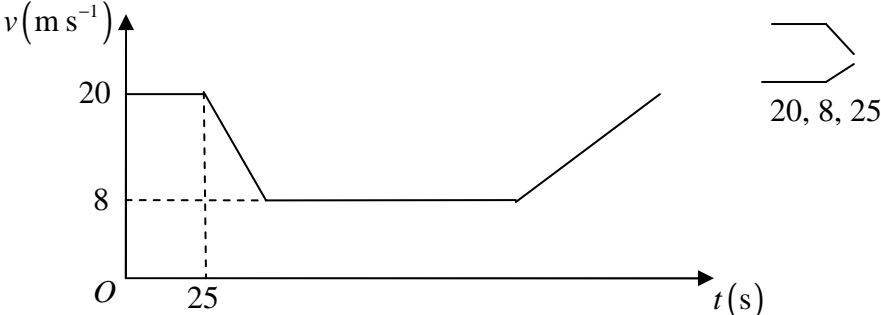
(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B .

(8)



Question Number	Scheme	Marks
4.	<p>(a) </p> <p>(b) $v = u + at \Rightarrow 8 = 20 - 0.4t$ $t = 30 \text{ (s)}$</p> <p>(c) $1960 = (25 \times 20) + (30 \times 8) + (\frac{1}{2} \times 30 \times 12) + (60 \times 8) + 8 \times t + \frac{1}{2} \times t \times 12$</p> $1960 = 500 + 240 + 180 + 480 + 14t$ $T = 115 + 40$ $= 155$ <p>N.B. SEE ALTERNATIVES</p>	<p>B1 B1 B1 (3)</p> <p>M1 A1 (2)</p> <p>M1A3 ft (2,1,0)</p> <p>DM1 A1</p> <p>DM1 A1</p> <p>(8) [13]</p>

Question 4(a)

First B1 for 1st section of graph

Second B1 for 2nd section

Third B1 for the figures 20, 8 and 25

Question 4(b)

M1 for a complete method to produce an equation in t only; allow $(20 - 8)/0.4$

A1 for 30 N.B.

Give A0 for $t = -30$, even if changed to 30, but then allow use of 30 in part (c), where full marks could then be scored.

Question 4(c)

First M1 (generous) for clear attempt to find whole area under *their* graph (must include at least one “1/2”), in terms of *a single unknown time (t say), and equate it to 1960.*

First A3, ft on their (b), for a correct equation.

Deduct 1 mark for each numerical error, or omission, in each of the 4 *sections of the area* corresponding to each stage of the motion. (they may ‘slice’ it, horizontally into 3 sections, or a combination of the two)

Second DM1, dependent on first M1, for simplifying to produce an equation with all their *t* terms collected.

Fourth A1 for a correct equation for *t* or *T*

Third DM1, dependent on second M1. for solving for *T*

Fifth A1 155

Please note that any incorrect answer to (b) will lead to an answer of 155 in (c) and can score max 6/8;

Solutions with the correct answer of 155 will need to be checked carefully.

Solutions to 4 (c) **N.B.** $t = T - 115$

- | | |
|---|--|
| <p>A. $1960 = (25 \times 20) + (30 \times 8) + (\frac{1}{2} \times 30 \times 12) + (60 \times 8) + 8 \times t + \frac{1}{2} \times t \times 12$
 $1960 = 500 + 240 + 180 + 480 + 14t$
 $T = 115 + 40$
 $= 155$</p> | <p>M1 A3 ft
M1 A1
M1
A1</p> |
| <p>B. $1960 = (25 \times 20) + \frac{1}{2} \times 30 \times (20 + 8) + (60 \times 8) + \frac{1}{2} \times t \times (20 + 8)$
 $1960 = 500 + 420 + 480 + 14t$
 $T = 115 + 40$
 $= 155$</p> | <p>M1 A3 ft
M1 A1
M1
A1</p> |
| <p>C. $1960 = 8T + \frac{1}{2} \times 12 \times (55 + 25) + \frac{1}{2} \times 12 \times (T - 115)$
 $1960 = 8T + 480 + 6T - 690$
 $1960 = 14T - 210$
 $155 = T$</p> | <p>M1 A3 ft

M1 A1
M1 A1</p> |
| <p>D. $1960 = 20T - \frac{1}{2} \times 12 \times (60 + T - 25)$
 $1960 = 20T - 6T - 210$
 $1960 = 14T - 210$
 $155 = T$</p> | <p>M1 A3 ft

M1 A1
M1 A1</p> |
| <p>E. $1960 = (55 \times 20) - \frac{1}{2} \times 30 \times 12 + (60 \times 8) + \frac{1}{2} \times t \times (20 + 8)$
 $1960 = 1100 - 180 + 480 + 14t$
 $T = 115 + 40$
 $= 155$</p> | <p>M1 A3 ft
M1 A1
M1
A1</p> |
| <p>F. $1960 = (8 \times 115) + \frac{1}{2} \times 12 \times (55 + 25) + \frac{1}{2} \times 28 \times (T - 115)$
 $1960 = 920 + 480 + 14T - 1610$
 $1960 = 14T - 210$
 $155 = T$</p> | <p>M1 A3 ft

M1 A1
M1 A1</p> |

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5. A particle P is projected vertically upwards from a point A with speed $u \text{ m s}^{-1}$. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1} .

(a) Show that $u = 21$

(3)

At time t seconds after projection, P is 19 m above A .

(b) Find the possible values of t .

(5)

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P .

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)



Question Number	Scheme	Marks
5.	(a) $v^2 = u^2 + 2as \Rightarrow 28^2 = u^2 + 2 \times 9.8 \times 17.5$ Leading to $u = 21$ *	M1 A1 A1 (3) cso
	(b) $s = ut + \frac{1}{2}at^2 \Rightarrow 19 = 21t - 4.9t^2$ $4.9t^2 - 21t + 19 = 0$ $t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 19}}{9.8}$ $t = 2.99$ or 3.0 $t = 1.30$ or 1.3	M1 A1 DM1 A1 A1 (5)
	(c) N2L $4g - 5000 = 4a$ $(a = -1240.2)$ $v^2 = u^2 + 2as \Rightarrow 0^2 = 28^2 - 2 \times 1240.2 \times s$ Leading to $s = 0.316$ (m)	M1 A1 or 0.32 M1 A1 (4) [12]
	OR $\frac{1}{2} \times 4 \times 28^2 + 4gs = 5000s$ Work-Energy: $s = 0.316$ or 0.32	M1 A1 M1 A1

Question 5(a)

First M1 for a complete method for finding u e.g.

$$28^2 = u^2 + 2gx17.5$$

or $28^2 = u^2 + 2(-g)x(-17.5)$

or $28^2 = 2gs \Rightarrow s = 40$ then $0^2 = u^2 + 2(-g)x(22.5)$

condone sign errors

First A1 for a correct equation(s) with $g = 9.8$

Second A1 for “ $u = 21$ ” PRINTED ANSWER

N.B. Allow a verification method, but they must state, as a conclusion, that “ $u = 21$ ”, to score the final A1.

Question 5(b)

First M1 for a complete method for finding at least one t value i.e. for producing an equation in t only.
(condone sign errors but not missing terms)

First A1 for a correct quadratic equation in t only or TWO correct linear equations in t only.

Second DM1, dependent on first M1, for attempt to solve the quadratic or one of the linear equations.

Second A1 for 3.0 or 3 or 2.99

Third A1 for 1.3 or 1.30

Question 5(c)

First M1 for resolving vertically with usual rules.

First A1 for a correct equation

Second M1 for use of $v^2 = u^2 + 2as$, with $v = 0$, $u = 28$ or $u = 0$ and $v = 28$ and their a , (or any other complete method which produces an equation in s , which could be negative)

M0 if they haven't *calculated* a value of a .

Second A1 for 0.32 or 0.316. (must be positive since it's a distance)

6. [*In this question i and j are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship *S* is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j}) \text{ km h}^{-1}$.

- (a) Find the direction in which *S* is moving, giving your answer as a bearing. (3)

At time *t* hours after noon, the position vector of *S* is *s* km. When $t = 0$, $s = 40\mathbf{i} - 6\mathbf{j}$.

- (b) Write down *s* in terms of *t*. (2)

A fixed beacon *B* is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j}) \text{ km}$.

- (c) Find the distance of *S* from *B* when $t = 3$ (4)

- (d) Find the distance of *S* from *B* when *S* is due north of *B*. (4)

Question Number	Scheme	Marks
<p>6.</p>	<p>(a) $\arctan \frac{7.5}{12} = 32^\circ$ Bearing is 302 (allow more accuracy)</p>	<p>M1 A1 A1 (3)</p>
	<p>(b) $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j} + t(-12\mathbf{i} + 7.5\mathbf{j})$</p>	<p>M1 A1 (2)</p>
	<p>(c) $t = 3,$ $\mathbf{s} = 4\mathbf{i} + 16.5\mathbf{j}$ $\mathbf{s} - \mathbf{b} = -3\mathbf{i} + 4\mathbf{j}$ $SB = \sqrt{((-3)^2 + 4^2)} = 5 \text{ (km)}$</p>	<p>M1 M1 DM1 A1 (4)</p>
	<p>(d) Equating i components $40 - 12t = 7$ or $-33 + 12t = 0$ $t = 2\frac{3}{4}$</p>	<p>M1 A1</p>
	<p>When $t = 2\frac{3}{4},$ $\mathbf{s} = (7\mathbf{i}) + 14\frac{5}{8}\mathbf{j}$ $SB = 2\frac{1}{8} \text{ (km)}$ 2.125, 2.13</p>	<p>M1 A1 (4)</p>
<p>OR When $t = 2\frac{3}{4},$ $7.5t - 18.5 = 2.125, 2.13$</p>	<p>[13] M1 A1</p>	

Question 6(a)

First M1 for $\arctan\left(\frac{\pm 7.5}{\pm 12}\right)$ either way up
First A1 for a correct value from their expression, usually 32° or 58°
Second A1 for 302 (allow more accurate answers)

Question 6(b)

M1 for a clear attempt at $(40\mathbf{i} - 6\mathbf{j}) + t(-12\mathbf{i} + 7.5\mathbf{j})$
A1 for any correct expression

Question 6(c)

First M1 is really B1 for $4\mathbf{i} + 16.5\mathbf{j}$ (seen or implied but can be in unsimplified form)
Second M1 is for a subtraction, $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$.
Third DM1, dependent on second M1, for finding magnitude of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$
A1 for 5

Question 6(d)

First M1 for equating \mathbf{i} -component of their answer in part (b) to 7 or
the \mathbf{i} -component of their $\mathbf{s} - \mathbf{b}$ or $\mathbf{b} - \mathbf{s}$ to zero

First A1 for 2.75 cao
Second M1 (independent) for attempt to find \mathbf{j} -component of their \mathbf{s} at their
 $t = 2.75$
Second A1 2.125 or 2.13 cao

Leave blank

7.



Figure 3

Two particles P and Q , of mass 0.3 kg and 0.5 kg respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time $t = 0$, a constant force F of magnitude 4 N is applied to Q in the direction PQ , as shown in Figure 3. The system moves under the action of this force until $t = 6$ s. During the motion, the resistance to the motion of P has constant magnitude 1 N and the resistance to the motion of Q has constant magnitude 2 N.

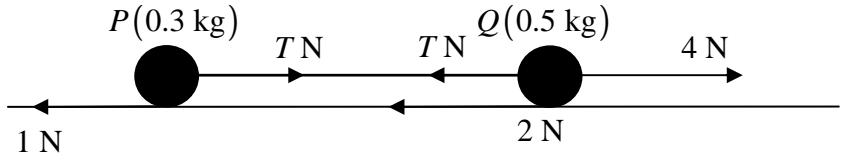
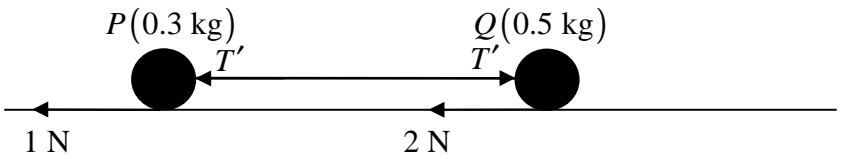
Find

- (a) the acceleration of the particles as the system moves under the action of F , (3)
- (b) the speed of the particles at $t = 6$ s, (2)
- (c) the tension in the rod as the system moves under the action of F . (3)

At $t = 6$ s, F is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

- (d) the distance moved by P as the system decelerates, (4)
- (e) the thrust in the rod as the system decelerates. (3)



Question Number	Scheme	Marks
7.	<div style="text-align: center;">  </div> <p>(a) For system N2L $4 - 3 = 0.8a$ $a = 1.25 \text{ (m s}^{-2}\text{)}, 1.3$</p> <p>(b) $v = u + at \Rightarrow v = 0 + 1.25 \times 6 = 7.5 \text{ (m s}^{-1}\text{)}$</p> <p>(c) For P N2L $T - 1 = 0.3 \times 1.25$ ft their a $T = 1.375 \text{ (N)} 1.38, 1.4$</p> <p>OR For Q N2L $4 - 2 - T = 0.5 \times 1.25$</p> <div style="text-align: center;">  </div> <p>(d) For system N2L $-3 = 0.8a \Rightarrow a = -3.75$ $v^2 = u^2 + 2as \Rightarrow 0^2 = 7.5^2 - 2 \times 3.75s$ $s = 7.5 \text{ (m)}$</p> <p>(e) For P N2L $T' + 1 = 0.3 \times 3.75$ $T' = 0.125 \text{ (N)}, 0.13$</p>	<p>M1 A1 A1 (3)</p> <p>M1 A1 (2)</p> <p>M1 A1ft A1 (3)</p> <p>M1 A1 M1 A1 (4)</p> <p>M1 A1 A1 (3)</p> <p>[15]</p>
	<p>Alternative for (e) For Q N2L $2 - T' = 0.5 \times 3.75$ $T' = 0.125 \text{ (N)}, 0.13$</p>	<p>M1 A1 A1 (3)</p>

Question 7(a) (In parts (a), (c), (d) and (e) use the value of the mass being used to guide you as to which part of the system is being considered, and mark equation(s) accordingly)

M1 for resolving horizontally to produce an equation in a ONLY.

First A1 for a correct equation

Second A1 for 1.25

Question 7(b)

M1 for a complete method to find the speed

A1 cao 7.5

Question 7(c)

M1 for resolving horizontally, for either P or Q , to produce an equation in T only.

First A1ft for a correct equation, ft on their a

Second A1 cao for 1.38 (N) or 1.375 (N)

Question 7(d)

First M1 for resolving horizontally to produce an equation in a ONLY.

First A1cao for -3.75 (or 3.75)

Second M1 for use of $v^2 = u^2 + 2as$, with $v = 0$, $u =$ their (b) and their a , (or any other complete method which produces an equation in s only)

M0 if they haven't *calculated* a value of a .

Second A1 for 7.5 m

Question 7(e)

M1 for resolving horizontally, for either P or Q , to produce an equation in T only.

M0 if they haven't *calculated* a value of a

First A1cao for a correct equation

Second A1 cao for 0.125 or 0.13 (N) (must be positive)