Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Wednesday 13 January 2016 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. $z = 3 + 2\text{i}, \qquad w = 1 - \text{i}$

Find in the form $a + b\text{i}$, where a and b are real constants,

(a) zw **(2)**

(b) $\dfrac{z}{w^*}$, showing clearly how you obtained your answer. **(3)**

Given that

$$|z + k| = \sqrt{53}, \text{ where } k \text{ is a real constant}$$

(c) find the possible values of k. **(4)**

January 2016
WFM01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1. (a)	$\{(3+2i)(1-i)\} = 3-3i+2i+2$	At least 3 correct terms	M1
	$= 5-i$	**cao** (Correct answer **only** scores both marks)	A1
			(2)
(b)	$w^* = 1+i$	Understanding that $w^* = 1+i$	B1
	$\left\{\frac{z}{w^*} = \right\}\frac{3+2i}{1+i} \times \frac{1-i}{1-i}$	Multiplies top and bottom by the conjugate of the denominator	M1
	$\left\{= \frac{3-3i+2i+2}{1+1}\right\} = \frac{5}{2} - \frac{1}{2}i$	$\frac{5}{2} - \frac{1}{2}i$ or $2.5-0.5i$	A1
			(3)
(c)	$\left\{\lvert 3+2i+k\rvert = \sqrt{53} \Rightarrow\right\} (3+k)^2 + 4; = 53$	Substitutes for z and uses Pythagoras correctly.	M1;
		Correct equation in any form	A1
	$(3+k)^2 + 4 = 53 \Rightarrow (3+k)^2 = 49 \Rightarrow k =$ or $(3+k)^2 + 4 = 53 \Rightarrow k^2 + 6k - 40 = 0$ $\Rightarrow (k-4)(k+10) = 0 \Rightarrow k =$	**dependent on the previous M mark** Attempt to solve for k	dM1
	$\{k = \}\ 4,\ -10$	Both $\{k =\}\ 4, -10$	A1
			(4)
			9

		Question 1 Notes
1. (b)	**Note**	**<u>Alternative acceptable method:</u>** $\left(\frac{z}{w^*}\right)\left(\frac{w}{w}\right) = \frac{zw}{\lvert w\rvert^2} = \frac{5-i}{2} = \frac{5}{2} - \frac{1}{2}i$
(b)	**Note**	Give A0 for writing down $\frac{5-i}{2}$ without reference to $\frac{5}{2} - \frac{1}{2}i$ or $2.5-0.5i$
	Note	Give B0M0A0 for writing down $\frac{5}{2} - \frac{1}{2}i$ from no working in part (b).
	Note	Give B0M1A0 for $\frac{3+2i}{1-i} \times \frac{1+i}{1+i}$
	Note	Simplifying a correct $\frac{5}{2} - \frac{1}{2}i$ in part (b) to a final answer of $5-i$ is A0
(c)	**Note**	Give final A0 if a candidate rejects one of $k = 4$ or $k = -10$
(b)	**ALT**	$\frac{3+2i}{1+i} = a+bi$ **B1**; $\Rightarrow 3+2i = (a+bi)(1+i) \Rightarrow 3 = a-b,\ 2 = a+b \Rightarrow a = ...,\ b = ...$ for **M1** and $\frac{5}{2} - \frac{1}{2}i$ for **A1**

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2.

$$\mathrm{f}(x) = x^2 - \frac{3}{\sqrt{x}} - \frac{4}{3x^2}, \qquad x > 0$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval [1.6, 1.7]

(2)

(b) Taking 1.6 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to find a second approximation to α. Give your answer to 3 decimal places.

(5)

Question Number	Scheme	Notes	Marks
2.	$f(x) = x^2 - \frac{3}{\sqrt{x}} - \frac{4}{3x^2}$		
(a)	$f(1.6) = -0.3325....$ $f(1.7) = 0.1277....$	Attempts to evaluate both $f(1.6)$ and $f(1.7)$ and either f(1.6) = awrt –0.3 or f(1.7) = awrt 0.1	M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a root) α is between $x = 1.6$ and $x = 1.7$	Both f(1.6) = awrt –0.3 and f(1.7) = awrt 0.1, sign change and conclusion.	A1 **cso**
			(2)
(b)	$f'(x) = 2x + \frac{3}{2}x^{-\frac{3}{2}} + \frac{8}{3}x^{-3}$	At least one of either $x^2 \to \pm Ax$ or $-\frac{3}{\sqrt{x}} \to \pm Bx^{-\frac{3}{2}}$ or $-\frac{4}{3x^2} \to \pm Cx^{-3}$ where *A*, *B* and *C* are non-zero constants.	M1
		At least 2 differentiated terms are correct	A1
		Correct differentiation	A1
	$\left\{\alpha \simeq 1.6 - \frac{f(1.6)}{f'(1.6)}\right\} \Rightarrow \alpha \simeq 1.6 - \frac{-0.332541....}{4.592200....}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $f(1.6)$ and $f'(1.6)$	dM1
	$\{\alpha = 1.672414... \Rightarrow\}\ \alpha = 1.672$	**dependent on all 4 previous marks** 1.672 ***on their first iteration*** (Ignore any subsequent applications)	A1 **cso cao**
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores no marks in (b)**		
			(5)
			7

		Question 2 Notes
2. (a)	A1	**correct solution only.** Candidate needs to state both f(1.6) = awrt –0.3 **and** f(1.7) = awrt 0.1 along with **a reason and conclusion.** Reference to change of sign **or** $f(1.6) \times f(1.7) < 0$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. root is in between 1.6 and 1.7, hence root is in interval, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion is "change of sign, hence root".
(b)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	Note	If the answer is incorrect it must be clear that we must see evidence of both $f(1.6)$ and $f'(1.6)$ being used in the Newton-Raphson process. So that just $1.6 - \frac{f(1.6)}{f'(1.6)}$ with an incorrect answer and no other evidence scores M0.

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3. The quadratic equation

$$x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) (i) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(ii) show that $\alpha^2 + \beta^2 = -2$

(iii) find the value of $\alpha^3 + \beta^3$

(5)

(b) (i) show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

(ii) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(6)

Question Number	Scheme	Notes	Marks
3.	$x^2 - 2x + 3 = 0$		
(a) (i)	$\alpha + \beta = 2, \quad \alpha\beta = 3$	Both $\alpha + \beta = 2, \quad \alpha\beta = 3$	B1
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = ...$	**Use** of a **correct** identity for $\alpha^2 + \beta^2$ (May be implied by their work)	M1
	$= 2^2 - 6 = -2$ *	-2 **from a correct solution only**	A1 *
(iii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = ...$ or $= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = ...$	**Use** of a **correct** identity for $\alpha^3 + \beta^3$ (May be implied by their work)	M1
	$= 8 - 3(3)(2) = -10$ or $= 2(-2-3) = -10$	-10 **from a correct solution only**	A1
			(5)
(b)(i)	$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + 2(\alpha\beta)^2 + \beta^4 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$	Correct algebraic proof	B1 *
(ii)	Sum $= \alpha^3 + \beta^3 - (\alpha + \beta) = -10 - 2 = -12$	Correct working without using explicit roots leading to a correct sum.	B1
	Product $= (\alpha^3 - \beta)(\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta$	Attempts to expand giving at least one term	M1
	$= (\alpha\beta)^3 - ((\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2) + \alpha\beta$		
	$= 27 - (4 - 18) + 3 = 44$	Correct product	A1
	$\{x^2 - \text{sum}\, x + \text{product} = 0 \Rightarrow\}\ x^2 + 12x + 44 = 0$	Applying $x^2 - (\text{sum})x + \text{product}$	M1
		$x^2 + 12x + 44 = 0$	A1
			(6)
			11

Question 3 Notes

(a) (i)	**1st A1**	$\alpha + \beta = -2, \quad \alpha\beta = 3 \Rightarrow \alpha^2 + \beta^2 = 4 - 6 = -2$ is M1A0 cso
(b) (ii)	**1st A1**	$\alpha + \beta = -2, \quad \alpha\beta = 3 \Rightarrow (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta = 44$ is first M1A1
(a)	**Note**	Applying $1 + \sqrt{2}\,\mathrm{i}, 1 - \sqrt{2}\,\mathrm{i}$ explicitly in part (a) will score B0M0A0M0A0
(b)	**Note**	Applying $1 + \sqrt{2}\,\mathrm{i}, 1 - \sqrt{2}\,\mathrm{i}$ explicitly in part (b) will score a maximum of B1B0M0A0M1A0
(a)	**Note**	Finding $\alpha + \beta = 2, \alpha\beta = 3$ by writing down or applying $1 + \sqrt{2}\,\mathrm{i}, 1 - \sqrt{2}\,\mathrm{i}$ but then writing $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 6 = -2$ and $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 8 - 3(3)(2) = -10$ scores B0M1A0M1A0 in part (a). Such candidates will be able to score all marks in part (b) if they use the method as detailed on the scheme in part (b).
(b)(ii)	**Note**	A correct method leading to a candidate stating $p = 1, q = 12, r = 44$ without writing a final answer of $x^2 + 12x + 44 = 0$ is **final** M1A0

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4.

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation represented by the matrix **A**.
(3)

(b) Hence find the smallest positive integer value of n for which

$$\mathbf{A}^n = \mathbf{I}$$

where **I** is the 2×2 identity matrix.
(1)

The transformation represented by the matrix **A** followed by the transformation represented by the matrix **B** is equivalent to the transformation represented by the matrix **C**.

Given that $\mathbf{C} = \begin{pmatrix} 2 & 4 \\ -3 & -5 \end{pmatrix}$,

(c) find the matrix **B**.
(4)

Question Number	Scheme	Notes	Marks
4. (a)	Rotation	Rotation	B1
	225 degrees (anticlockwise)	225 degrees or $\frac{5\pi}{4}$ (anticlockwise) or 135 degrees clockwise	B1 o.e.
	about (0, 0)	**This mark is dependent on at least one of the previous B marks being awarded.** About (0, 0) or about O or about the origin	dB1
	Note: Give 2nd B0 for 225 degrees clockwise		**(3)**
(b)	$\{n=\}\ 8$	8	B1 **cao**
			(1)
(c) **Way 1**	$\mathbf{A}^{-1}=\begin{pmatrix}-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$ or $\begin{pmatrix}-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\end{pmatrix}$	Correct matrix	B1
	$\{\mathbf{B}=\mathbf{CA}^{-1}\}=\begin{pmatrix}2 & 4\\ -3 & -5\end{pmatrix}\begin{pmatrix}-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}=\ldots$	Attempts $\mathbf{CA}^{-1}$ and finds at least one element of the matrix **B**	M1
	$=\begin{pmatrix}\sqrt{2} & -3\sqrt{2}\\ -\sqrt{2} & 4\sqrt{2}\end{pmatrix}$	**dependent on the previous B1M1 marks** At least 2 correct elements	A1
		All elements are correct	A1
			(4)
(c) **Way 2**	$\{\mathbf{BA}=\}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}=\begin{pmatrix}2 & 4\\ -3 & -5\end{pmatrix}$	Correct statement using 2×2 matrices. All 3 matrices must contain four elements. **(Can be implied).** (Allow one slip in copying down **C**)	B1
	$-\frac{a}{\sqrt{2}}-\frac{b}{\sqrt{2}}=2,\ \frac{a}{\sqrt{2}}-\frac{b}{\sqrt{2}}=4$ or $-\frac{c}{\sqrt{2}}-\frac{d}{\sqrt{2}}=-3,\ \frac{c}{\sqrt{2}}-\frac{d}{\sqrt{2}}=-5$ and finds at least one of either a or b or c or d	Applies $\mathbf{BA}=\mathbf{C}$ and attempts simultaneous equations in a and b **or** c and d **and** finds at least one of either a or b or c or d	M1
	$=\begin{pmatrix}\sqrt{2} & -3\sqrt{2}\\ -\sqrt{2} & 4\sqrt{2}\end{pmatrix}$ or $a=\sqrt{2},\ b=-3\sqrt{2},\ c=-\sqrt{2},\ d=4\sqrt{2}$	**dependent on the previous B1M1 marks** At least 2 correct elements	A1
		All elements are correct	A1
			(4)
			8

		Question 4 Notes
4. (a)	**Note**	Condone "Turn" for the 1st B1 mark.
(c)	**Note**	You can ignore previous working prior to a candidate finding $\mathbf{CA}^{-1}$ (i.e. you can ignore the statements $\mathbf{C}=\mathbf{BA}$ or $\mathbf{C}=\mathbf{AB}$).
	A1 A1	You can allow equivalent matrices/values, e.g. $\begin{pmatrix}\frac{2}{\sqrt{2}} & -\frac{6}{\sqrt{2}}\\ -\frac{2}{\sqrt{2}} & \frac{8}{\sqrt{2}}\end{pmatrix}$

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(8r^3 - 3r) = \frac{1}{2}n(n+1)(2n+3)(an+b)$$

where a and b are integers to be found.

(4)

Given that

$$\sum_{r=5}^{10}(8r^3 - 3r + kr^2) = 22\,768$$

(b) find the exact value of the constant k.

(4)

Question Number	Scheme	Notes	Marks
5. (a)	$\left\{\sum_{r=1}^{n} 8r^3 - 3r\right\} = 8\left(\frac{1}{4}n^2(n+1)^2\right) - 3\left(\frac{1}{2}n(n+1)\right)$	Attempt to substitute at least one of the standard formulae correctly into the given expression	M1
		Correct expression	A1
	$= \frac{1}{2}n(n+1)\left[4n(n+1)-3\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n + 1)$ having used both standard formulae correctly	dM1
	$= \frac{1}{2}n(n+1)\left[4n^2+4n-3\right]$	{*this step does not have to be written*}	
	$= \frac{1}{2}n(n+1)(2n+3)(2n-1)$	Correct completion with no errors	A1 **cso**
			(4)
(b)	Let $f(n) = \frac{1}{2}n(n+1)(2n+3)(2n-1)$, $g(n) = \frac{8}{4}n^2(n+1)^2$ & $h(n) = \pm\frac{3}{2}n(n+1)$		
	$\left\{\sum_{r=5}^{10} 8r^3 - 3r\right\} = \frac{1}{2}(10)(11)(23)(19) - \frac{1}{2}(4)(5)(11)(7)$ $\{= 24035 - 770 = 23265\}$	Attempts to find either • $f(10)$ **and** $f(4)$ or $f(5)$ • $g(10)$ **and** $g(4)$ or $g(5)$ **and** $h(10)$ **and** $h(4)$ or $h(5)$	M1
	$\sum_{r=5}^{10} kr^2 = k\left(\frac{1}{6}(10)(11)(21) - \frac{1}{6}(4)(5)(9)\right)\{= k(385-30) = 355k\}$ or $= k(5^2+6^2+7^2+8^2+9^2+10^2)\{= 355k\}$	Correct attempt at $\sum_{r=5}^{10} kr^2$	M1
	$23265 + 355k = 22768 \Rightarrow k = -\frac{497}{355}$ or $-\frac{7}{5}$	**dependent on both previous M marks.** Uses both previous method mark results to form a linear equation in k using 22768 and solves to give $k = ...$	ddM1
		$k = -\frac{497}{355}$ or $-\frac{7}{5}$ or -1.4 or equivalent	A1 o.e.
			(4)
			8

	Question 5 Notes	
5. (a)	**Note**	Applying eg. $n = 1$, $n = 2$ to the printed equation without applying the standard formula to give $a = 2$, $b = -1$ is M0A0M0A0
	Alt **dM1** **A1 cso**	**Alternative Method:** Using $2n^4 + 4n^3 + \frac{1}{2}n^2 - \frac{3}{2}n \equiv an^4 + (b + \frac{5}{2}a)n^3 + (\frac{5}{2}b + \frac{3}{2}a)n^2 + \frac{3}{2}bn$ o.e. Equating coefficients to give both $a = 2$, $b = -1$ Demonstrates that the identity works for **all** of its terms
(b)	**Note**	$f(10) - f(5) = \frac{1}{2}(10)(11)(23)(19) - \frac{1}{2}(5)(6)(13)(9)$ $\{= 24035 - 1755 = 22280\}$
	Note	Applying $\sum_{r=5}^{10} 8r^3 - \sum_{r=5}^{10} 3r + k\sum_{r=5}^{10} r^2$ gives either • $(24200 - 165 + 385k) - (800 - 30 + 30k) = 22768$ • $23400 - 135 + 355k = 22768$
	Note	$985 + 25k + 1710 + 36k + 2723 + 49k + 4072 + 64k + 5805 + 81k + 7970 + 100k = 23265 + 355k$ is fine for the first two M1M1 marks with the final ddM1A1 leading to $k = -1.4$

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6. The rectangular hyperbola H has equation $xy = c^2$, where c is a non-zero constant.

The point $P\left(cp, \frac{c}{p}\right)$, where $p \neq 0$, lies on H.

(a) Show that the normal to H at P has equation

$$yp - p^3x = c(1 - p^4)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Find, in terms of c and p, the coordinates of Q.

(4)

Question Number	Scheme	Notes	Marks
6. (a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{dy}{dx} = k\,x^{-2}$	M1
	$xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct.	
	$\frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = -\frac{c}{p^2}.\frac{1}{c}$	their $\frac{dy}{dp} \times \frac{1}{\text{their}\frac{dx}{dp}}$	
	$\frac{dy}{dx} = -c^2x^{-2}$ or $x\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = -\frac{c}{p^2}.\frac{1}{c}$	Correct differentiation	A1
	So, $m_N = p^2$	Perpendicular gradient rule where $m_N (\neq m_T)$ is found from using calculus.	M1
	$y - \frac{c}{p} = p^2(x - cp)$ or $y = p^2x + \frac{c}{p} - cp^3$	Correct line method where m_N is found from using calculus.	M1
	$py - p^3x = c(1 - p^4)$ *		A1*
			(5)
(b)	$y = \frac{c^2}{x} \Rightarrow p\frac{c^2}{x} - p^3x = c(1-p^4)$ or $x = \frac{c^2}{y} \Rightarrow py - p^3\frac{c^2}{y} = c(1-p^4)$ Substitutes $y = \frac{c^2}{p}$ or $x = \frac{c^2}{y}$ into the printed equation to obtain an equation in either *x*, c and *p* only or in *y*, *c* and *p* only.		M1
	$p^3x^2 + c(1-p^4)x - c^2p = 0$ or $py^2 - c(1-p^4)y - c^2p^3 = 0$		
	$(x - cp)(p^3x + c) = 0 \Rightarrow x = \ldots$ or $\left(y - \frac{c}{p}\right)(yp + cp^4) = 0 \Rightarrow y = \ldots$ Correct attempt of solving a 3TQ to find the *x* or *y* coordinate of *Q*		M1
	$Q\left(-\frac{c}{p^3}, -cp^3\right)$ — Can be simplified or un-simplified.	At least one correct coordinate.	A1
		Both correct coordinates	A1
	Note: If *Q* is stated as coordinates then they must be correct for the final A1 mark.		**(4)**
(b) **ALT**	Let Q be $\left(cq, \frac{c}{q}\right)$ so $\frac{c}{q}p - p^3cq = c(1-p^4)$ Substitutes $x = cq$ or $y = \frac{c}{q}$ into the printed equation to obtain an equation in only *p*, *c* and *q*.		M1
	$cp - p^3cq^2 = cq - cqp^4 \Rightarrow p - q - p^3q^2 + qp^4 = 0$		
	$(p - q)(1 + p^3q) = 0 \Rightarrow q = \ldots$ Correct attempt to find *q* in terms of *p*		M1
	$Q\left(-\frac{c}{p^3}, -cp^3\right)$ — Can be simplified or un-simplified.	At least one correct coordinate	A1
		Both correct coordinates	A1
			(4)
			9

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7.

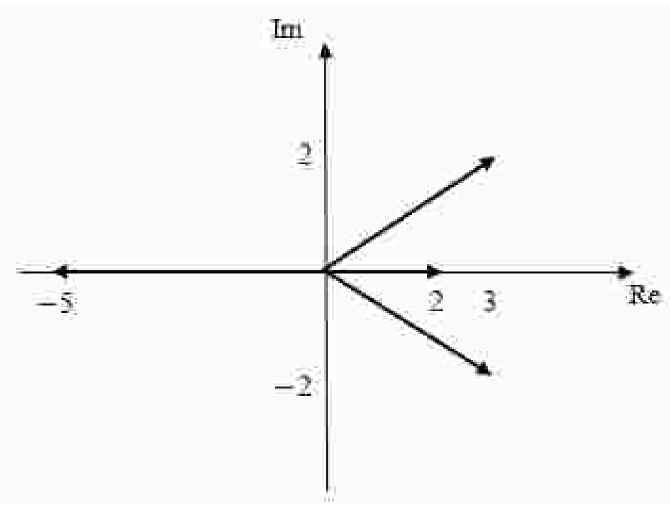
$$f(x) = x^4 - 3x^3 - 15x^2 + 99x - 130$$

(a) Given that $x = 3 + 2i$ is a root of the equation $f(x) = 0$, use algebra to find the three other roots of the equation $f(x) = 0$

(7)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

Question Number	Scheme	Notes	Marks
7.	$f(x) = x^4 - 3x^3 - 15x^2 + 99x - 130$		
(a)	3 – 2i is also a root	3 – 2i	B1
	$x^2 - 6x + 13$	Attempt to expand $(x-(3+2i))(x-(3-2i))$ or any valid method to establish the quadratic factor e.g. $x = 3 \pm 2i \Rightarrow x - 3 = \pm 2i \Rightarrow x^2 - 6x + 9 = -4$ or sum of roots 6, product of roots 13	M1
		$x^2 - 6x + 13$	A1
	$f(x) = \left(x^2 - 6x + 13\right)\left(x^2 + 3x - 10\right)$	Attempt other quadratic factor. **Note:** Using long division to get as far as $x^2 \pm kx$ is fine for this mark.	M1
		$x^2 + 3x - 10$	A1
	$\{x^2 + 3x - 10 =\}\ (x+5)(x-2) \Rightarrow x = ...$	Correct method for solving a 3TQ on their 2nd quadratic factor	M1
	$x = -5,\ x = 2$	Both values correct	A1
			(7)
	Note: Writing down $2,\ -5,\ 3+2i,\ 3-2i$ with **no** working is B1M0A0M0A0M0A0		
(a)	**<u>Alternative using Factor Theorem</u>**		
	3 – 2i	3 – 2i	B1
	$\{f(2) =\}\ 2^4 - 3\times2^3 - 15\times2^2 + 99\times2 - 130 = 0$	Attempts to find $f(2)$	M1
		Shows that $f(2) = 0$	A1
	$\{f(-5) =\}\ (-5)^4 - 3(-5)^3 - 15(-5)^2 + 99\times(-5) - 130 = 0$	Attempts to find $f(-5)$	M1
		Shows that $f(-5) = 0$	A1
	$x = 2,\ x = -5$	**Either** shows that $f(2) = 0$ and states $x = 2$ **or** shows that $f(-5) = 0$ and states $x = -5$	M1
		Shows both $f(2) = 0$ & $f(-5) = 0$ **and** states both $x = -5,\ x = 2$	A1
			(7)
(b)		• $3 \pm 2i$ plotted correctly in quadrants 1 and 4 with some evidence of symmetry • **dependent on the final M mark being awarded in part (a).** Their other two roots plotted correctly.	
		Satisfies at least one of the criteria.	B1ft
		Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1ft
			(2)
			9

Leave blank

8. The parabola P has equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of P.

The point B, which does not lie on the parabola, has coordinates (q, r) where q and r are positive constants and $q > a$. The line l passes through B and S.

(a) Show that an equation of the line l is

$$(q - a)\,y = r(x - a)$$

(3)

The line l intersects the directrix of P at the point C.

Given that the area of triangle OCS is three times the area of triangle OBS, where O is the origin,

(b) show that the area of triangle OBC is $\frac{6}{5}qr$

(5)

Question Number	Scheme	Notes	Marks
8.	$S(a,0),\ B(q,r),\ C\left(-a, -\frac{2ar}{q-a}\right)$ or $C(-a, -3ar)$		
(a)	$m = \frac{r-0}{q-a}$	Correct gradient using $(a, 0)$ and (q, r) (Can be implied)	B1
	• $y = \frac{r}{q-a}(x-a)$ or • $y - r = \frac{r}{q-a}(x-q)$ • $0 = \frac{ra}{q-a} + "c" \Rightarrow "c" = -\frac{ra}{q-a}$ and $y = \frac{r}{q-a}x - \frac{ra}{q-a}$	Correct straight line method	M1
	leading to $(q-a)y = r(x-a)$*	cso	A1*
			(3)
(b)	$C\left(\{-a\}, -\frac{2ar}{q-a}\right)$ or height $OCS = \frac{2ar}{q-a}$	$-\frac{2ar}{q-a}$ or $\frac{2ar}{q-a}$	B1
	$\frac{2ar}{q-a} = 3r$ or $\frac{1}{2}(a)\left(\frac{2ar}{q-a}\right) = 3\left(\frac{1}{2}\right)(a)(r) \Rightarrow ...$	Applies height OCS $= 3r$ or applies Area(OSC) = 3Area(OSB) and rearranges to give $\lambda a = \mu q$ where λ, μ are numerical values.	M1
	$\Rightarrow 5a = 3q$	$5a = 3q$ or $a = \frac{3}{5}q$	A1
	Area$(OBC) = 4\left(\frac{1}{2}\right)\left(\frac{3q}{5}\right)r$ or $= \left(\frac{1}{2}\right)\left(\frac{3q}{5}\right)r + \left(\frac{3}{2}\right)\left(\frac{3q}{5}\right)r$	**dependent on the previous M mark** Uses their $a = \frac{3}{5}q$ and applies a correct method to find Area(OBC) in terms of only q and r	dM1
	$= \frac{6}{5}qr$ (*)	$\frac{6}{5}qr$	A1* **cso**
			(5)
			8
	Alternative Method (Similar Triangles)		
(b)	$\frac{3r}{2a} = \frac{r}{q-a}$	$\frac{3r}{2a} = \frac{r}{q-a}$ or equivalent	B1
	$\frac{3r}{2a} = \frac{r}{q-a} \Rightarrow ...$	$\frac{3r}{2a} = \frac{r}{q-a}$ or equivalent and rearranges to give $\lambda a = \mu q$ where λ, μ are numerical values.	M1
	… then apply the original mark scheme.		

Question 8 Notes

8. (a)	**Note**	The first two marks B1M1 can be gained together by applying the formula $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
		to give $\frac{y-0}{r-0} = \frac{x-a}{q-a}$
(b)	**Note**	If a candidate uses either $-\frac{2ar}{q-a}$ or $-3r$ they can get 1st M1 but not 2nd M1 in (b).

Leave blank

9. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)

Question Number	Scheme	Notes	Marks
9.	$\text{f}(n) = 4^{n+1} + 5^{2n-1}$		
	$\text{f}(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$\text{f}(k+1) - \text{f}(k) = 4^{k+2} + 5^{2(k+1)-1} - (4^{k+1} + 5^{2k-1})$	Attempts $\text{f}(k+1) - \text{f}(k)$	M1
	$\text{f}(k+1) - \text{f}(k) = 3(4^{k+1}) + 24(5^{2k-1})$		
	$= 3(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= 24(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$ — Either	$3(4^{k+1} + 5^{2k-1})$ or $3\text{f}(k)$; $21(5^{2k-1})$ $24(4^{k+1} + 5^{2k-1})$ or $24\text{f}(k)$; $-21(4^{k+1})$	A1; A1
	$\text{f}(k+1) = 3\text{f}(k) + 21(5^{2k-1}) + \text{f}(k)$ or $\text{f}(k+1) = 24\text{f}(k) - 21(4^{k+1}) + \text{f}(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $\text{f}(k+1)$ the subject	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>is true for all $n \in \mathbb{Z}^+$</u>.		A1 **cso**
			(6)
			6
WAY 2	**General Method:** Using $\text{f}(k+1) - m\text{f}(k)$		
	$\text{f}(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$\text{f}(k+1) - m\text{f}(k) = 4^{k+2} + 5^{2(k+1)-1} - m(4^{k+1} + 5^{2k-1})$	Attempts $\text{f}(k+1) - \text{f}(k)$	M1
	$\text{f}(k+1) - m\text{f}(k) = (4-m)(4^{k+1}) + (25-m)(5^{2k-1})$		
	$= (4-m)(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= (25-m)(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$	$(4-m)(4^{k+1} + 5^{2k-1})$ or $(4-m)\text{f}(k)$; $21(5^{2k-1})$ $(25-m)(4^{k+1} + 5^{2k-1})$ or $(25-m)\text{f}(k)$; $-21(4^{k+1})$	A1; A1
	$\text{f}(k+1) = (4-m)\text{f}(k) + 21(5^{2k-1}) + m\text{f}(k)$ or $\text{f}(k+1) = (25-m)\text{f}(k) - 21(4^{k+1}) + m\text{f}(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $\text{f}(k+1)$ the subject	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>is true for all $n \in \mathbb{Z}^+$</u>.		A1 **cso**
WAY 3	$\text{f}(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$\text{f}(k+1) = 4^{k+2} + 5^{2(k+1)-1}$	Attempts $\text{f}(k+1)$	M1
	$\text{f}(k+1) = 4(4^{k+1}) + 25(5^{2k-1})$		
	$= 4(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= 25(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$ — Either	$4(4^{k+1} + 5^{2k-1})$ or $4\text{f}(k)$; $21(5^{2k-1})$ $25(4^{k+1} + 5^{2k-1})$ or $25\text{f}(k)$; $-21(4^{k+1})$	A1; A1
	$\text{f}(k+1) = 4\text{f}(k) + 21(5^{2k-1})$ or $\text{f}(k+1) = 25\text{f}(k) - 21(4^{k+1})$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $\text{f}(k+1)$ the subject	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>is true for all $n \in \mathbb{Z}^+$</u>.		A1 **cso**
	Note Some candidates may set $\text{f}(k) = 21M$ and so may prove the following general results		

		• $\left\{f(k+1) = 4f(k) + 21(5^{2k-1})\right\} \Rightarrow f(k+1) = 84M + 21(5^{2k-1})$ • $\left\{f(k+1) = 25f(k) - 21(4^{k+1})\right\} \Rightarrow f(k+1) = 525M - 21(4^{k+1})$