Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Mechanics M1

Advanced/Advanced Subsidiary

Wednesday 6 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
6677/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Two particles, P and Q, have masses $3m$ and m respectively. They are moving in opposite directions towards each other along the same straight line on a smooth horizontal plane and collide directly. The speeds of P and Q immediately before the collision are $2u$ and $4u$ respectively. The magnitude of the impulse received by each particle in the collision is $\frac{21mu}{4}$.

(a) Find the speed of P after the collision.
(3)

(b) Find the speed of Q after the collision.
(3)

Question Number	Scheme	Marks
1(a)	For P: $-\frac{21mu}{4} = 3m(v_P - 2u)$	M1A1
	$v_P = \frac{u}{4}$	A1 (3)
(b)	For Q: $\frac{21mu}{4} = m(v_Q - -4u)$	M1A1
	$v_Q = \frac{5u}{4}$	A1 (3)
OR	CLM: $3m \times 2u - m \times 4u = 3m \times \frac{u}{4} + mv_Q$ M1 A1	
	$v_Q = \frac{5u}{4}$ A1	
		(6)
	Notes for Qu 1	
	1(a) M1 for using Impulse = Change in Momentum of P (must have 3m in both terms) (M0 if *clearly* adding momenta or if g is included) but condone sign errors. First A1 for a correct equation. (N.B. Could have $-v_P$ in place of v_P) Second A1 for $\frac{u}{4}$ oe (must be positive) **N.B.** If they try to find v_Q first and then use CLM to find v_P, M1 for a complete method to find v_P, A1 for correct equations, A1 for the answer for v_P. If an incorrect v_Q is then just stated in (b), award relevant marks if seen in working for (a). If no attempt at (b), then no marks for (b).	
	1(b) M1 for using Impulse = Change in Momentum of Q (must have m in both terms) (M0 if *clearly* adding momenta or if g is included) but condone sign errors. First A1 for a correct equation. (N.B. Could have $-v_Q$ in place of v_Q) Second A1 for $\frac{5u}{4}$ oe (must be positive) **OR**: M1 for CLM with correct no. of terms, condone missing m's or extra g's and sign errors First A1 for a correct equation Second A1 for $\frac{5u}{4}$ oe (must be positive)	

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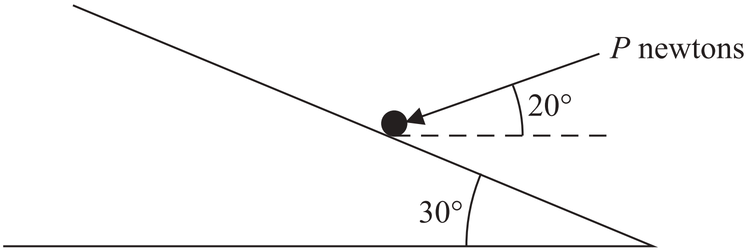

Figure 1

A particle of mass 2 kg lies on a rough plane. The plane is inclined to the horizontal at 30°. The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle is held in equilibrium by a force of magnitude P newtons. The force makes an angle of 20° with the horizontal and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 1. Find the least possible value of P.

(10)

Question Number	Scheme	Marks
2.	(Parallel to plane): $P\cos 50 + F = 2g\cos 60$	M1 A2
	(Perp to plane): $R - P\sin 50 = 2g\cos 30$	M1 A2
	Other possible equations:	
	$(\rightarrow)$: $R\cos 60 - F\cos 30 = P\cos 20$ M1 A2	
	$(\uparrow)$: $R\cos 30 + F\cos 60 = P\cos 70 + 2g$ M1 A2	
	$F = \frac{1}{4}R$	B1
	Attempt to eliminate *F* and *R* to give an equation in *P* only	M1
	Solve for *P*	**DM**1
	P = 6.7 (2 SF) or 6.66 (3SF)	A1
		(10)
	Notes for Qu 2	
	2. First M1 for resolving parallel to the plane with usual rules. 2*g* term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving perpendicular to the plane with usual rules. 2*g* term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error. B1 for *F* = ¼ *R* seen or implied Third M1, independent but must have two 3 (or 4) term equations, for attempt to eliminate *F* and *R* to give an equation in *P* only. Fourth DM1, dependent on third M1, for solving for *P*. Fifth A1 for 6.7 or 6.66 **Other possible equations**: First M1 for resolving horizontally with usual rules. *R* term must be using 30° or 60° angle and *F* term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving vertically with usual rules. *R* term must be using 30° or 60° angle and *F* term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error.	

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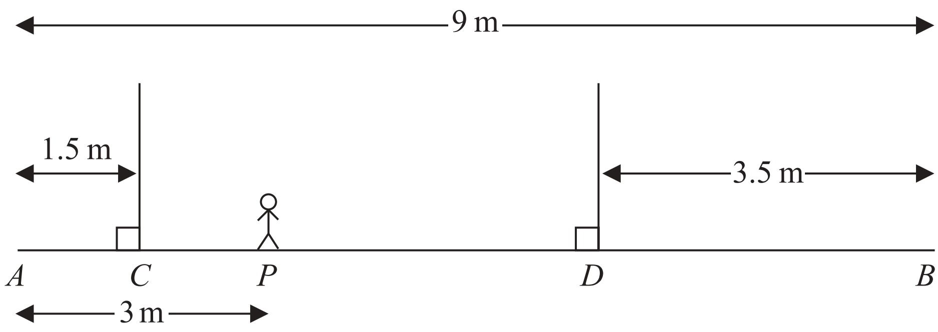

Figure 2

A wooden beam AB, of mass 150 kg and length 9 m, rests in a horizontal position supported by two vertical ropes. The ropes are attached to the beam at C and D, where $AC = 1.5$ m and $BD = 3.5$ m. A gymnast of mass 60 kg stands on the beam at the point P, where $AP = 3$ m, as shown in Figure 2. The beam remains horizontal and in equilibrium.

By modelling the gymnast as a particle, the beam as a uniform rod and the ropes as light inextensible strings,

(a) find the tension in the rope attached to the beam at C.

(3)

The gymnast at P remains on the beam at P and another gymnast, who is also modelled as a particle, stands on the beam at B. The beam remains horizontal and in equilibrium. The mass of the gymnast at B is the largest possible for which the beam remains horizontal and in equilibrium.

(b) Find the tension in the rope attached to the beam at D.

(4)

Question Number	Scheme	Marks
3.(a)	$M(D),\ (150g\times1)+(60g\times2.5)=Tc\times4$	M1 A1
	$Tc=75g$ or 735 N or 740 N Allow omission of N	A1 (3)
(b)	$M(B),\ (150g\times4.5)+(60g\times6)=T_D\times3.5$	M1 A2
	$T_D=2900\text{ N}$ or $\dfrac{2070g}{7}$ Allow omission of N	A1 (4)
		(7)
	Notes for Qu 3	
	3(a) M1 for a complete method to find Tc (M0 if they assume $T_C=T_D$) i.e. for producing an equation in Tc only. Each equation used must have correct no. of terms and be dimensionally correct. First A1 for correct equation. Second A1 for any of the 3 possible answers Other possible equations: $(\uparrow), Tc+T_D=60g+150g$ $M(A),\ (150g\times4.5)+(60g\times3)=(Tc\times1.5)+(T_D\times5.5)$ $M(C),\ (150g\times3)+(60g\times1.5)=T_D\times4$ $M(B),\ (150g\times4.5)+(60g\times6)=(Tc\times7.5)+(T_D\times3.5)$ $M(G),\ (T_D\times1)+(60g\times1.5)=T_C\times3$	
	3(b) **N.B.** (M0 if T_C is never equated to 0) M1 for a complete method to obtain an equation in T_D only. If they use more than one equation, each equation used must have correct no. of terms and be dimensionally correct. First and second A1 for a correct equation in T_D only. A1A0 if one error.Consistent omission of g is one error except in *M(D)* where it's not an error. Third A1 for either answer Other possible equations: $(\uparrow), T_D=60g+150g+Mg$ $M(A),\ (150g\times4.5)+(60g\times3)+9Mg=T_D\times5.5$ $M(C),\ (150g\times3)+(60g\times1.5)+7.5Mg=T_D\times4$ $M(D),\ (150g\times1)+(60g\times2.5)=3.5Mg$ $M(G),\ (T_D\times1)+(60g\times1.5)=4.5Mg$	

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4. A ball of mass 0.2 kg is projected vertically downwards with speed U m s^{-1} from a point A which is 2.5 m above horizontal ground. The ball hits the ground. Immediately after hitting the ground, the ball rebounds vertically with a speed of 10 m s^{-1}. The ball receives an impulse of magnitude 7 N s in its impact with the ground. By modelling the ball as a particle and ignoring air resistance, find

(a) the value of U.

(6)

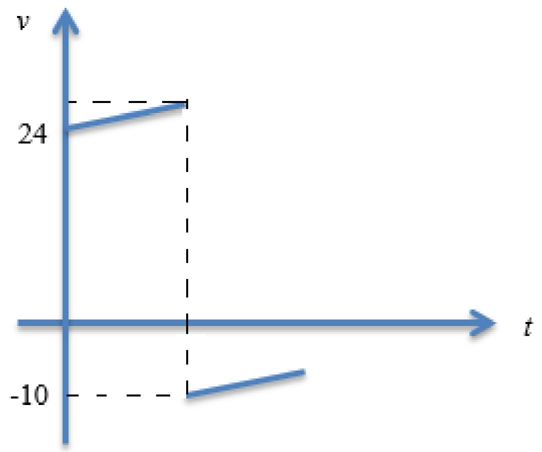
After hitting the ground, the ball moves vertically upwards and passes through a point B which is 1 m above the ground.

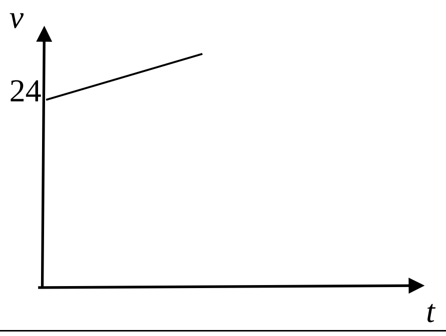
(b) Find the time between the instant when the ball hits the ground and the instant when the ball first passes through B.

(4)

(c) Sketch a velocity-time graph for the motion of the ball from when it was projected from A to when it first passes through B. (You need not make any further calculations to draw this sketch.)

(3)

Question Number	Scheme	Marks
4.(a)	$V^2 = U^2 + 2g \times 2.5$	M1A1
	Eliminate *V and* solve for *U*	A1 **(DM1)**
	$7 = 0.2(10 - -V)$	M1A1
	$U = 24$	A1 (6)
4.(b)	$1 = 10t - 4.9t^2$ **OR** e.g. $v^2 = 10^2 - 2\times 9.8 \times 1$ and $v = 10 - 9.8t$	
	$1 = 10t - 4.9t^2$ to give $\sqrt{80.4} = 10 - 9.8t$	M1 A1
	$t = \frac{10 \pm \sqrt{100 - 19.6}}{9.8}$ so $t = \frac{10 - \sqrt{10^2 - 2\times 9.8 \times 1}}{9.8}$	**DM**1
	$t = 0.11$ s or 0.105 s	A1 (4)
4(c)		B1**ft**1st line B1 2nd line B1 ,-10 (3)
		(13)
	Notes for Qu 4	
	4(a) First M1 for complete method, using *suvat*, to find equation in *U* and *V* only First A1 for a correct equation Second A1 – **treat as third DM1**, dependent on the other two M's, for eliminating *V* and solving for *U* Second M1 for using Impulse = Change in Momentum of ball (must have 0.2 in both terms and be using 10 as one of the velocities) (M0 if *clearly* adding momenta or if *g* is included) but condone sign errors. Third A1 for a correct equation, 7 and 10 must have the same sign but equation may have *V* instead of -*V* Fourth A1 for *U* = 24 (must appear here) **N.B.** If they use *U* instead of *V* in the impulse-momentum equation, can score max M1A0/6 for part (a). **N.B.** If they go from $V^2 = U^2 + 49$ to $V = U + 7$, can score max 5/6	

	4(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only using $s = 1$ or -1 First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 for either 0.105 (s) or 0.11 (s) (must be only ONE answer)	
	4(c) First B1**ft** for a straight line, with positive gradient, starting at their U value (or just at U) on the positive v-axis. Second B1 for a parallel (approx.) line placed correctly (B0 if a continuous vertical line is included) i.e. starting at a point where the t coordinate is equal to the t coordinate of the point where the first line stopped, and the v coordinate is negative. Third B1 for second line, placed correctly, starting on $v = -10$ N.B. Whole graph could be reflected in the t-axis **SC: If second line is placed correctly but extends up to the *t*-axis, or beyond, lose second B1 but can score the third B1.**	
4(b)	**ALTERNATIVE :** "the instant when the ball first passes through B" is taken to be when the ball is on the way down from A.	
	$s = vt - \frac{1}{2}at^2$ **OR** $v_B^2 = 24^2 + 2\times9.8\times1.5$ and $25 = v_B + 9.8t$	
	$1 = 25t - 4.9t^2$ to give $25 = \sqrt{605.4} + 9.8t$	M1 A1
	$t = \frac{25 \pm \sqrt{625 - 19.6}}{9.8}$ so $t = \frac{25 - \sqrt{625 - 19.6}}{9.8}$	**DM1**
	$t = 0.040$ (s) or 0.0403 (s) or 0.04 (s) (must only be ONE answer)	A1 (4)
4(c)	**ALTERNATIVE :** again "when it first passes through B" is taken to be when the ball is on the way down from A.	
		B2 line B1**ft** 24 (3)

	Notes for Qu 4 continued	
	4(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only using s = 1 or -1 First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 $t = 0.040$ (s) or 0.0403 (s)	
	4(c) B2 for a straight line, with positive gradient, starting on the positive v-axis. B1**ft** starting at their U value (or just at U)	

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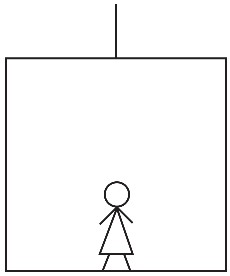

Figure 3

A lift of mass 250 kg is being raised by a vertical cable attached to the top of the lift. A woman of mass 60 kg stands on the horizontal floor inside the lift, as shown in Figure 3. The lift ascends vertically with constant acceleration $2\,\text{m}\,\text{s}^{-2}$. There is a constant downwards resistance of magnitude 100 N on the lift. By modelling the woman as a particle,

(a) find the magnitude of the normal reaction exerted by the floor of the lift on the woman.

(3)

The tension in the cable must not exceed 10 000 N for safety reasons, and the maximum upward acceleration of the lift is $3\,\text{m}\,\text{s}^{-2}$. A typical occupant of the lift is modelled as a particle of mass 75 kg and the cable is modelled as a light inextensible string. There is still a constant downwards resistance of magnitude 100 N on the lift.

(b) Find the maximum number of typical occupants that can be safely carried in the lift when it is ascending with an acceleration of $3\,\text{m}\,\text{s}^{-2}$.

(7)

Question Number	Scheme	Marks
5(a)	$R - 60g = 60 \times 2$	M1A1
	$R = 708$ N or 710 N (must be positive)	A1 (3)
5(b)	$75n$	B1
	$10000 - Mg - 100 = M \times 3$	M1A2
	using M = 250 +75n => $n = 6.9..$	**DM1**A1
	so 6 people	A1**ft** (7)
		(10)
	Notes for Qu 5	
	5(a) M1 for equation in R only, with usual rules First A1 for a correct equation Second A1 for 710 or 708 (N not needed)	
	5(b) B1 for 75n oe seen or implied First M1 for an equation in one unknown in the form $10000 - Mg - 100 = M \times a$ with usual rules (must be using 10000) where M can be any (relevant) number e.g. 250, 75, etc First A1 and second A1 for a correct equation with a = 3, A1A0 if one error (e.g. Use of a = 2 loses 1 A mark) Second **DM1**, dependent on first M1, for using M = 250 +75n and solving for n Third A1 for 6.9… (A0 for 7) Fourth A1**ft** for no. of people, ft on their n value (A0 for < 7) **N.B.** If no incorrect work seen, the third A mark can be implied by a correct answer (n = 6) **SC:** They may use <u>Trial and Error</u> to find the critical value of n, by writing down equations for the tension when n =1, 2, 3,…. until the tension exceeds 10000 oe This method can score the final DM1 A1 A1 if done fully correctly up to and including n = 7, with a correct answer given. It could also score some or all of the first 4 marks.	

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6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle P of mass 0.5 kg.

$\mathbf{F}_1 = (4\mathbf{i} - 6\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N.

Given that the resultant force of $\mathbf{F}_1$ and $\mathbf{F}_2$ is in the same direction as $-2\mathbf{i} - \mathbf{j}$,

(a) show that $p - 2q = -16$

(5)

Given that $q = 3$

(b) find the magnitude of the acceleration of P,

(5)

(c) find the direction of the acceleration of P, giving your answer as a bearing to the nearest degree.

(3)

Question Number	Scheme	Marks
6.(a)	$(4\mathbf{i}-6\mathbf{j})+(p\mathbf{i}+q\mathbf{j})=(4+p)\mathbf{i}+(q-6)\mathbf{j}$	M1
	$\frac{(4+p)}{(q-6)}=\frac{2}{1}$ or $-\frac{2}{1}$ **(or** $\frac{1}{2}$ or $-\frac{1}{2}$)	**DM**1 A1
	$2q-12=4+p$	
	$p-2q=-16$ GIVEN ANSWER	**DM**1 A1 (5)
(b)	$q=3 \Rightarrow p=-10$	B1
	EITHER $0.5\mathbf{a}=-6\mathbf{i}-3\mathbf{j}$ **OR** $\lvert\mathbf{R}\rvert=\sqrt{(-6)^2+(-3)^2}$	M1
	$\mathbf{a}=-12\mathbf{i}-6\mathbf{j}$ $=\sqrt{45}$ oe	A1
	$\lvert\mathbf{a}\rvert=\sqrt{(-12)^2+(-6)^2}$ $0.5a=\sqrt{45}$	M1
	$a=\sqrt{180}=13.4\text{ms}^{-2}$ $a=\sqrt{180}=13.4\text{ms}^{-2}$	A1 (5)
(c)	e.g. $\tan\theta=\frac{12}{6} \Rightarrow \theta=63.4^\circ$	M1A1
	Bearing $=180^\circ+63.4^\circ=243^\circ$ (nearest degree)	A1cao (3)
		(13)
	Notes for Qu 6	
	Allow column vectors throughout	
	6(a) First M1 for adding the two forces, with **i's** and **j**'s collected, seen or implied Second DM1, dependent on first M1, for an equation in *p* and *q* only. Allow $\frac{1}{2}$ or $-\frac{1}{2}$ or $-\frac{2}{1}$ instead of $\frac{2}{1}$ First A1 for a correct equation in any form Third DM1, dependent on the second M1, for (at least)one **correct** intermediate line of working Second A1 for correct **given answer**	
	6(b) B1 for $p=-10$ seen or implied **EITHER** First M1 for use of $\mathbf{F}=0.5\mathbf{a}$ with their <u>resultant force (must be a sum of the two forces)</u> First A1 for $\mathbf{a}=-12\mathbf{i}-6\mathbf{j}$ Second M1 (independent) for finding magnitude of their **a** Second A1 for $\sqrt{180}$ oe or 13.4 or better	

	OR First M1 for finding the magnitude of their resultant force **R** (must be a sum of the two forces) $R = \sqrt{(-6)^2 + (-3)^2}$ First A1 for $\sqrt{45}$ oe Second M1 for using $R = 0.5a$ to find a Second A1 for $a = 2\sqrt{45}$ oe 13.4 ms^{-2} or better	
	6(c) M1 for use of a relevant trig ratio from their **a** or their **R (may not be the sum of the two forces) or** $-2\mathbf{i} - \mathbf{j}$ First A1 for any relevant correct angle coming from a correct **a** or **R** or from $-2\mathbf{i} - \mathbf{j}$ Second A1 for 243	

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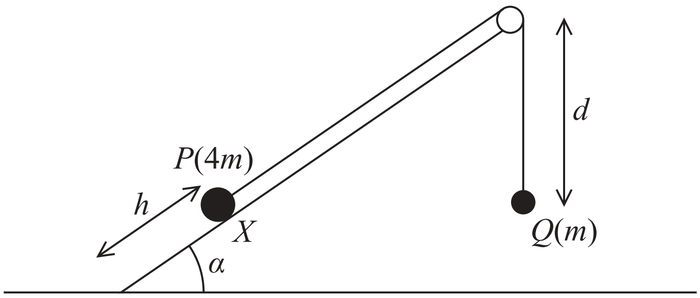

Figure 4

A particle P of mass $4m$ is held at rest at the point X on the surface of a rough inclined plane which is fixed to horizontal ground. The point X is a distance h from the bottom of the inclined plane. The plane is inclined to the horizontal at an angle α where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{4}$. The particle P is attached to one end of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which hangs freely at a distance d, where $d > h$, below the pulley, as shown in Figure 4.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The system is released from rest with the string taut and P moves down the plane.

For the motion of the particles before P hits the ground,

(a) state which of the information given above implies that the magnitudes of the accelerations of the two particles are the same,
(1)

(b) write down an equation of motion for each particle,
(5)

(c) find the acceleration of each particle.
(5)

When P hits the ground, it immediately comes to rest. Given that Q comes to instantaneous rest before reaching the pulley,

(d) show that $d > \frac{28h}{25}$.
(5)

Question Number	Scheme	Marks
7(a)	Inextensible string	B1 (1)
	MARK PARTS (b) and (c) together	
(b)	$4mg\sin\alpha - T - F = 4ma$	M1 A2
	$T - mg = ma$	M1 A1 (5)
(c)	$F = \frac{1}{4}R$	B1
	$R = 4mg\cos\alpha$	B1
	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1
	Eliminating R, F and T	M1
	$a = \frac{3}{25}g = 1.2$ or 1.18 (m s^{-2})	A1 (5)
(d)	$v^2 = 2\times\frac{3}{25}gh = \frac{6}{25}gh$	M1
	$0^2 = \frac{6}{25}gh - 2gs$	
	$s = \frac{3}{25}h$	M1 A1
	$d > \frac{3}{25}h + h = \frac{28}{25}h$ GIVEN ANSWER	**DM**1 A1 (5)
		(16)
	Notes for Qu 7	
	7(a) B1 for inextensible (and taut) string; B0 if any extras given or if an incorrect consequence of the inextensiblity of the string is given.	
	MARK PARTS (b) and (c) together **7(b)** **N.B.** Omission of m is a Method error i.e. M0 for that equation First M1 for equation of motion for P with usual rules (omission of 4 on RHS is M0) First A1 and second A1 for a correct equation, A1A0 if one error Second M1 for equation of motion for Q with usual rules Third A1 for a correct equation Use of e.g cos(4/5) instead of $\cos\alpha$ is an A error unless they recover correctly. N.B. Allow consistent use of $-a$	
	7(c) First B1 for $F = \frac{1}{4}R$ seen or implied Second B1 for $R = 4mg\cos\alpha$ seen or implied Third B1 for $\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$ seen or implied or an appropriate correct angle is used to give a correct trig ratio First M1 for eliminating R, F and T and finding an a value First A1 $a = \frac{3}{25}g = 1.2$ or 1.18 (m s^{-2}) (must be positive)	

	7(d) First M1 for finding *v* or v^2 for *P* using their a (M0 if *g* is used) Second M1 for a complete method to find *s*, independent but must have found *v* or v^2 (M0 if *g* not used) First A1 for $s = \frac{3}{25}h$ oe Third DM1, dependent on previous two M's, for adding *h* onto their *s* oe Second A1 for GIVEN ANSWER	