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Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Friday 20 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n+a)(n+b)(n+c)$$

where a, b and c are integers to be found.

(4)

Question Number	Scheme	Notes	Marks
1.	$\sum_{r=1}^{n} r(r^2-3) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r$		
	$= \frac{1}{4}n^2(n+1)^2 - 3\left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(r^2-3)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{4}n(n+1)\left[n(n+1)-6\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both the standard formulae	dM1
	$= \frac{1}{4}n(n+1)\left[n^2+n-6\right]$	{this step does not have to be written]	
	$= \frac{1}{4}n(n+1)(n+3)(n-2)$	Correct completion with no errors	A1 **cso**
			(4)
			4

		Question 1 Notes
1.	**Note**	Applying eg. $n=1$, $n=2$, $n=3$ to the printed equation without applying the standard formulae to give $a=1$, $b=3$, $c=-2$ or another combination of these numbers is M0A0M0A0.
	Alt	<u>**Alternative Method**</u>: Obtains $\sum_{r=1}^{n} r(r^2-3) \equiv \frac{1}{4}n(n+1)\left[n(n+1)-6\right] \equiv \frac{1}{4}n(n+a)(n+b)(n+c)$ So $a=1$. $n=1 \Rightarrow -2 = \frac{1}{4}(1)(2)(1+b)(1+c)$ and $n=2 \Rightarrow 0 = \frac{1}{4}(2)(3)(2+b)(2+c)$ leading to either $b=-2, c=3$ or $b=3, c=-2$
	dM1	**dependent on the previous M mark.** Substitutes in values of n and solves to find $b=...$ and $c=...$
	A1	Finds $a=1$, $b=3$, $c=-2$ or another combination of these numbers.
	Note	Using **only** a method of "proof by induction" scores 0 marks unless there is use of the standard formulae when the first M1 may be scored.
	Note	Allow final dM1A1 for $\frac{1}{4}n^4 + \frac{1}{2}n^3 - \frac{5}{4}n^2 - \frac{3}{2}n$ or $\frac{1}{4}n(n^3+2n^2-5n-6)$ or $\frac{1}{4}(n^4+2n^3-5n^2-6n) \rightarrow \frac{1}{4}n(n+1)(n+3)(n-2)$, from no incorrect working.
	Note	Give final A0 for eg. $\frac{1}{4}n(n+1)\left[n^2+n-6\right] \rightarrow = \frac{1}{4}n(n+1)(x+3)(x-2)$ unless recovered.

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2. A parabola P has cartesian equation $y^2 = 28x$. The point S is the focus of the parabola P.

(a) Write down the coordinates of the point S.

(1)

Points A and B lie on the parabola P. The line AB is parallel to the directrix of P and cuts the x-axis at the midpoint of OS, where O is the origin.

(b) Find the exact area of triangle ABS.

(4)

Question Number	Scheme	Notes	Marks
2.	$P: y^2 = 28x$ or $P(7t^2, 14t)$		
(a)	$(y^2 = 4ax \Rightarrow a = 7) \Rightarrow S(7,0)$	Accept $(7,0)$ or $x = 7, y = 0$ or 7 marked on the x-axis in a sketch	B1
			(1)
(b)	{A and B have x coordinate} $\frac{7}{2}$ So $y^2 = 28\left(\frac{7}{2}\right) \Rightarrow y^2 = 98 \Rightarrow y = ...$ **or** $y = \sqrt{(2(7)-3.5)^2 - (3.5)^2}$ $\left\{= \sqrt{(10.5)^2 - (3.5)^2}\right\}$ **or** $7t^2 = 3.5 \Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5}$	Divides their x coordinate from (a) by 2 **and** substitutes this into the parabola equation and takes the sqaure root to find $y = ...$ **or applies** $y = \sqrt{\left(2(\text{"7"}) - \left(\frac{\text{"7"}}{2}\right)\right)^2 - \left(\frac{\text{"7"}}{2}\right)^2}$ **or solves** $7t^2 = 3.5$ and finds $y = 2(7)\text{"their } t\text{"}$	M1
	$y = (\pm)7\sqrt{2}$	***At least one*** correct **exact** value of y. Can be un-simplified or simplified.	A1
	A, B have coordinates $\left(\frac{7}{2}, 7\sqrt{2}\right)$ and $\left(\frac{7}{2}, -7\sqrt{2}\right)$		
	Area triangle ABS = • $\frac{1}{2}\left(2(7\sqrt{2})\right)\left(\frac{7}{2}\right)$ • $\frac{1}{2}\begin{vmatrix} 7 & 3.5 & 3.5 & 7 \\ 0 & 7\sqrt{2} & -7\sqrt{2} & 0 \end{vmatrix}$	**dependent on the previous M mark** A **full** method for finding he area of triangle ABS.	dM1
	$= \frac{49}{2}\sqrt{2}$	Correct exact answer.	A1
			(4)
			5

		Question 2 Notes
2. (a)	**Note**	You can give B1 for part (a) for correct relevant work seen in either part (a) or part (b)
(b)	1st M1	Allow a slip when candidates find the x coordinate of their midpoint as long as $0 <$ their midpoint $<$ their a
	Note	Give 1st M0 if a candidate finds and uses $y = 98$
	1st A1	Allow any **exact value** of either $7\sqrt{2}$, $-7\sqrt{2}$, $\sqrt{98}$, $-\sqrt{98}$, $14\sqrt{0.5}$, awrt 9.9 or awrt -9.9
	2nd dM1	Either $\frac{1}{2}\left(2 \times \text{their } \text{"}7\sqrt{2}\text{"}\right)\left(\text{their } x_{\text{midpoint}}\right)$ or $\frac{1}{2}\left(2 \times \text{their } \text{"}7\sqrt{2}\text{"}\right)\left(\text{their "7"} - x_{\text{midpoint}}\right)$
	Note	Condone area triangle $ABS = \left(7\sqrt{2}\right)\left(\frac{7}{2}\right)$, i.e. $\left(\text{their } \text{"}7\sqrt{2}\text{"}\right)\left(\frac{\text{their "7"}}{2}\right)$
	2nd A1	Allow exact answers such as $\frac{49}{2}\sqrt{2}$, $\frac{49}{\sqrt{2}}$, $24.5\sqrt{2}$, $\frac{\sqrt{4802}}{2}$, $\sqrt{\frac{4802}{4}}$, $3.5\sqrt{2}$, $49\sqrt{\frac{1}{2}}$ or $\frac{7}{2}\sqrt{98}$ but do not allow $\frac{1}{2}(3.5)\left(2\sqrt{98}\right)$ seen by itself
	Note	Give final A0 for finding 34.64823228… without reference to a correct exact value.

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3.

$$f(x) = x^2 + \frac{3}{x} - 1, \qquad x < 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval $[-2, -1]$.

(a) Taking -1.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.
(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.
(2)

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Question Number	Scheme	Notes	Marks
3.	$\mathrm{f}(x)=x^2+\dfrac{3}{x}-1,\quad x<0$		
(a)	$\mathrm{f}'(x)=2x-3x^{-2}$	At one of either $x^2\to\pm Ax$ or $\dfrac{3}{x}\to\pm Bx^{-2}$ where A and B are non-zero constants.	M1
		Correct differentiation	A1
	$\mathrm{f}(-1.5)=-0.75,\ \mathrm{f}'(-1.5)=-\dfrac{13}{3}$	Either $\mathrm{f}(-1.5)=-0.75$ or $\mathrm{f}'(-1.5)=-\dfrac{13}{3}$ or awrt -4.33 **or a correct numerical expression** for either $\mathrm{f}(-1.5)$ or $\mathrm{f}'(-1.5)$ **Can be implied by later working**	B1
	$\left\{\alpha\simeq-1.5-\dfrac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}\right\}\Rightarrow\alpha\simeq-1.5-\dfrac{-0.75}{-4.333333...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $\mathrm{f}(-1.5)$ and $\mathrm{f}'(-1.5)$	dM1
	$\left\{\alpha=-1.67307692...\text{ or }-\dfrac{87}{52}\right\}\Rightarrow\alpha=-1.67$	**dependent on all 4 previous marks** -1.67 on their first iteration (Ignore any subsequent iterations)	A1 **cso cao**
	Correct differentiation followed by a correct answer scores full marks in (a) **Correct answer with <u>no</u> working scores no marks in (a)**		
			(5)
(b) **Way 1**	$\mathrm{f}(-1.675)=\ 0.01458022...$ $\mathrm{f}(-1.665)=-0.0295768...$	Chooses a suitable interval for x, which is within ±0.005 of their answer to (a) and at least one attempt to evaluate $\mathrm{f}(x)$.	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a root) $\alpha=-1.67$ (2 dp)	Both values correct awrt (or truncated) 1 sf, sign change and conclusion.	A1 **cso**
			(2)
(b) **Way 2**	**Alt 1: Applying Newton-Raphson again** Eg. Using $\alpha=-1.67,\ -1.673$ or $-\dfrac{87}{52}$		
	• $\alpha\simeq-1.67-\dfrac{-0.007507185629...}{-4.415692926...}\ \{=-1.671700115...\}$ • $\alpha\simeq-1.673-\dfrac{0.005743106396...}{-4.41783855...}\ \{=-1.671700019...\}$ • $\alpha\simeq-\dfrac{87}{52}-\dfrac{0.006082942257...}{-4.417893838...}\ \{=-1.67170036...\}$	Evidence of applying Newton-Raphson for a second time on their answer to part (a)	M1
	So $\alpha=-1.67$ (2 dp)	$\alpha=-1.67$	A1
			(2)
			7

		Question 3 Notes
3. (a)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	B1	B1 can be given for a correct numerical expression for either $\mathrm{f}(-1.5)$ or $\mathrm{f}'(-1.5)$ Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.
	Final dM1	This mark can be implied by applying at least one correct value of either $\mathrm{f}(-1.5)$ or $\mathrm{f}'(-1.5)$ in $-1.5 - \frac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}$. So just $-1.5 - \frac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	Give final dM0 for applying $1.5 - \frac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}$ without first quoting the correct N-R formula.
3. (b)	**A1**	**Way 1: correct solution only** Candidate needs to state **both** of their values for $\mathrm{f}(x)$ to awrt (or truncated) 1 sf along with **a reason and conclusion.** Reference to change of sign **or** eg. $\mathrm{f}(-1.675)\times\mathrm{f}(-1.665) < 0$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.67$, root (or α or part (a)) is correct, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion is "change of sign, hence root". No explicit reference to 2 decimal places is required.
	Note	Stating "root is in between -1.675 and -1.665" without some reference to $\alpha = -1.67$ is not sufficient for A1
	Note	Accept 0.015 as a correct evaluation of $\mathrm{f}(-1.675)$
	A1	**Way 2:** correct solution only Their conclusion in Way 2 needs to convey that they understand that $\alpha = -1.67$ to 2 decimal places. Eg. "therefore my answer to part (a) [which must be -1.67] is correct" is fine for A1.
	Note	$-1.67 - \frac{\mathrm{f}(-1.67)}{\mathrm{f}'(1.67)} = -1.67\,(2\text{ dp})$ is sufficient for M1A1 in part (b).
	Note	The root of $\mathrm{f}(x) = 0$ is $-1.67169988...$, so candidates can also choose x_1 which is less than $-1.67169988...$ and choose x_2 which is greater than $-1.67169988...$ with both x_1 and x_2 lying in the interval $\left[-1.675, -1.665\right]$ and evaluate $\mathrm{f}(x_1)$ and $\mathrm{f}(x_2)$.
3. (b)	**Note**	**Helpful Table** (see below)

x	$\mathrm{f}(x)$
-1.675	0.014580224
-1.674	0.010161305
-1.673	0.005743106
-1.672	0.001325627
-1.671	-0.003091136
-1.670	-0.007507186
-1.669	-0.011922523
-1.668	-0.016337151
-1.667	-0.020751072
-1.666	-0.025164288
-1.665	-0.029576802

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4. Given that

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) show that $\det(\mathbf{A}) > 0$ for all real values of k,

(3)

(b) find $\mathbf{A}^{-1}$ in terms of k.

(2)

Question Number	Scheme	Notes	Marks
4.	$\mathbf{A}=\begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}$, where k is a constant and let $\text{g}(k)=k^2+2k+3$		
(a) **Way 1**	$\{\det(\mathbf{A})=\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	$=(k+1)^2-1+3$	Attempts to complete the square [usual rules apply]	M1
	$=(k+1)^2+2>0$	$(k+1)^2+2$ and >0	A1 **cso**
			(3)
(a) **Way 2**	$\{\det(\mathbf{A})=\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	$\{b^2-4ac=\}\ 2^2-4(1)(3)$	Applies "b^2-4ac" to their $\det(\mathbf{A})$	M1
	All of • $b^2-4ac=-8<0$ • some reference to k^2+2k+3 being above the x-axis • so $\det(\mathbf{A})>0$	Complete solution	A1 **cso**
			(3)
(a) **Way 3**	$\{\text{g}(k)=\det(\mathbf{A})=\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	$\text{g}'(k)=2k+2=0\Rightarrow k=-1$ $g_{\min}=(-1)^2+2(-1)+3$	Finds the value of k for which $\text{g}'(k)=0$ **and** substitutes this value of k into $\text{g}(k)$	M1
	$\text{g}_{\min}=2$, so $\det(\mathbf{A})>0$	$\text{g}_{\min}=2$ and states $\det(\mathbf{A})>0$	A1 **cso**
			(3)
(b)	$\mathbf{A}^{-1}=\dfrac{1}{k^2+2k+3}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	$\dfrac{1}{\text{their }\det(\mathbf{A})}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	M1
		Correct answer in terms of k	A1
			(2)
			5

	Question 4 Notes	
4. (a)	**B1**	Also allow $k(k+2)--3$
	Note	**Way 2:** Proving $b^2-4ac=-8<0$ by itself could mean that $\det(\mathbf{A})>0$ or $\det(\mathbf{A})<0$.
	Note	To gain the final A1 mark for Way 2, candidates need to show $b^2-4ac=-8<0$ **and** make some reference to k^2+2k+3 being above the x-axis (eg. states that coefficient of k^2 is positive **or** evaluates $\det(\mathbf{A})$ for any value of k to give a positive result **or** sketches a quadratic curve that is above the x-axis) before then stating that $\det(\mathbf{A})>0$.
	Note	Attempting to solve $\det(\mathbf{A})=0$ by applying the quadratic formula or finding $-1\pm\sqrt{2}\text{i}$ is enough to score the M1 mark for Way 2. To gain A1 these candidates need to make some reference to k^2+2k+3 being above the x-axis (eg. states that coefficient of k^2 is positive **or** evaluates $\det(\mathbf{A})$ for any value of k to give a positive result **or** sketches a quadratic curve that is above the x-axis) before then stating that $\det(\mathbf{A})>0$.
(b)	**A1**	Allow either $\dfrac{1}{(k+1)^2+2}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$ or $\begin{pmatrix} \dfrac{k+2}{k^2+2k+3} & \dfrac{-3}{k^2+2k+3} \\ \dfrac{1}{k^2+2k+3} & \dfrac{k}{k^2+2k+3} \end{pmatrix}$ or equivalent.

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5.

$$2z + z^* = \frac{3 + 4\text{i}}{7 + \text{i}}$$

Find z, giving your answer in the form $a + b\text{i}$, where a and b are real constants. You must show all your working.

(5)

Question Number	Scheme	Notes	Marks
5.	$2z + z^* = \dfrac{3+4\text{i}}{7+\text{i}}$		
Way 1	$\{2z + z^* =\}\ 2(a+\text{i}b) + (a - \text{i}b)$	Left hand side $= 2(a+\text{i}b) + (a-\text{i}b)$ Can be implied by eg. $3a + \text{i}b$ **Note:** This can be seen anywhere in their solution	B1
	$.......... = \dfrac{(3+4\text{i})}{(7+\text{i})}\dfrac{(7-\text{i})}{(7-\text{i})}$	Multiplies numerator **and** denominator of the right hand side by $7 - \text{i}$ or $-7 + \text{i}$	M1
	$.......... = \dfrac{25+25\text{i}}{50}$	Applies $\text{i}^2 = -1$ to and collects like terms to give right hand side $= \dfrac{25+25\text{i}}{50}$ or equivalent	A1
	So, $3a + \text{i}b = \dfrac{1}{2} + \dfrac{1}{2}\text{i}$	**dependent on the previous B and M marks** Equates either real parts or imaginary parts to give at least one of $a = ...$ or $b = ...$	ddM1
	$\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	A1
			(5)
Way 2	$\{2z + z^* =\}\ 2(a+\text{i}b) + (a - \text{i}b)$	Left hand side $= 2(a+\text{i}b) + (a-\text{i}b)$ Can be implied by eg. $3a + \text{i}b$	B1
	$(3a + \text{i}b)(7+\text{i}) = ...$	Multiplies their $(3a + \text{i}b)$ by $(7+\text{i})$	M1
	$21a + 3a\text{i} + 7b\text{i} - b = ...$	Applies $\text{i}^2 = -1$ to give left hand side $= 21a + 3a\text{i} + 7b\text{i} - b$	A1
	So, $(21a - b) + (3a + 7b) = 3 + 4\text{i}$ gives $21a - b = 3$, $3a + 7b = 4$	**dependent on the previous B and M marks** Equates **both** real parts and imaginary parts to give at least one of $a = ...$ or $b = ...$	ddM1
	$\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	A1
			(5)
			5

	Question 5 Notes	
5.	**Note**	Some candidates may let $z = x + \text{i}y$ and $z^* = x - \text{i}y$. So apply the mark scheme with $x \equiv a$ and $y \equiv b$.
	Note	For the final A1 mark, you can accept exact equivalents for a, b.

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6. The rectangular hyperbola H has equation $xy = 25$

(a) Verify that, for $t \neq 0$, the point $P\left(5t, \frac{5}{t}\right)$ is a general point on H.

(1)

The point A on H has parameter $t = \frac{1}{2}$

(b) Show that the normal to H at the point A has equation

$$8y - 2x - 75 = 0$$

(5)

This normal at A meets H again at the point B.

(c) Find the coordinates of B.

(4)

Question Number	Scheme	Notes	Marks
6.	$H: xy = 25,\ \ P\left(5t, \frac{5}{t}\right)$ is a general point on H		
(a)	Either $5t\left(\frac{5}{t}\right) = 25$ **or** $y = \frac{25}{x} = \frac{25}{5t} = \frac{5}{t}$ **or** $x = \frac{25}{y} = \frac{25}{\frac{5}{t}} = 5t$ **or** states $c = 5$		B1
			(1)
(b)	$y = \frac{25}{x} = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2} = -\frac{25}{x^2}$	$\frac{dy}{dx} = \pm k\,x^{-2}$ where k is a numerical value	
	$xy = 25 \Rightarrow x\frac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct.	M1
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{5}{t^2}\left(\frac{1}{5}\right)$	$\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dx}{dt}}$	
	$\left\{\text{At } A,\ t = \frac{1}{2}, x = \frac{5}{2}, y = 10\right\} \Rightarrow \frac{dy}{dx} = -4$	Correct numerical gradient at A, which is found using calculus. **Can be implied by later working**	A1
	So, $m_N = \frac{1}{4}$	Applies $m_N = \frac{-1}{m_T}$, to find a numerical m_N, where m_T is found from using calculus. **Can be implied by later working**	M1
	• $y - 10 = \frac{1}{4}\left(x - \frac{5}{2}\right)$ • $10 = \frac{1}{4}\left(\frac{5}{2}\right) + c \Rightarrow c = \frac{75}{8} \Rightarrow y = \frac{1}{4}x + \frac{75}{8}$	Correct line method for a **normal** where a numerical $m_N\,(\neq m_T)$ is found from using calculus. **Can be implied by later working**	M1
	leading to $8y - 2x - 75 = 0$ (*)	Correct solution only	A1
			(5)
(c)	$y = \frac{25}{x} \Rightarrow 8\left(\frac{25}{x}\right) - 2x - 75 = 0$ **or** $x = \frac{25}{y} \Rightarrow 8y - 2\left(\frac{25}{y}\right) - 75 = 0$ **or** $x = 5t, y = \frac{5}{t} \Rightarrow 8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0$ Substitutes $y = \frac{25}{x}$ **or** $x = \frac{25}{y}$ **or** $x = 5t$ **and** $y = \frac{5}{t}$ into the printed equation or their normal equation to obtain an equation in either x only, y only or t only		M1
	$2x^2 + 75x - 200 = 0$ **or** $8y^2 - 75y - 50 = 0$ **or** $2t^2 + 15t - 8 = 0$ **or** $10t^2 + 75t - 40 = 0$		
	$(2x-5)(x+40) = 0 \Rightarrow x = ...$ **or** $(y-10)(8y+5) = 0 \Rightarrow y = ...$ **or** $(2t-1)(t+8) = 0 \Rightarrow t = ...$ **dependent on the previous M mark** Correct attempt of solving a 3TQ to find either $x = ..., y = ...$ or $t = ...$		dM1
	Finds at least one of either $x = -40$ or $y = -\frac{5}{8}$		A1
	$B\left(-40, -\frac{5}{8}\right)$	Both correct coordinates (If coordinates are not stated they can be paired together as $x = ..., y = ...$)	A1
			(4)
			10

		Question 6 Notes
6. (a)	**Note**	A conclusion is not required on this occasion in part (a).
	B1	Condone reference to $c = 5$ (as $xy = c^2$ and $\left(ct, \frac{c}{t}\right)$ are referred in the Formula book.)
(b)	**Note**	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{5}{t^2}\left(\frac{1}{5}\right) = -\frac{1}{t^2} \Rightarrow m_N = t^2 \Rightarrow y - 10 = t^2\left(x - \frac{5}{2}\right)$ **scores only the first M1.** When $t = \frac{1}{2}$ is substituted giving $y - 10 = \frac{1}{4}\left(x - \frac{5}{2}\right)$ the response then automatically gets A1(implied) M1(implied) M1
(c)	**Note**	You can imply the final three marks (dM1A1A1) for either • $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$ • $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$ • $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$ with no intermediate working.
		You can also imply the middle dM1A1 marks for either • $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow x = -40$ • $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow y = -\frac{5}{8}$ • $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow x = -40$ or $y = -\frac{5}{8}$ with no intermediate working.
	Note	**Writing** $x = -40,\ y = -\frac{5}{8}$ followed by $B\left(40, \frac{5}{8}\right)$ or $B\left(-\frac{5}{8}, -40\right)$ is final A0.
	Note	Ignore stating $B\left(\frac{5}{2}, 10\right)$ in addition to $B\left(-40, -\frac{5}{8}\right)$

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7.

$$\mathbf{P} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**. **(3)**

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$

(b) Write down the matrix **Q**. **(1)**

Given that the transformation V followed by the transformation U is the transformation T, which is represented by the matrix **R**,

(c) find the matrix **R**. **(2)**

(d) Show that there is a value of k for which the transformation T maps each point on the straight line $y = kx$ onto itself, and state the value of k. **(4)**

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Question Number	Scheme	Notes	Marks
7. (a)	Rotation	Rotation	B1
	67 degrees (anticlockwise)	Either $\arctan\left(\frac{12}{5}\right)$, $\tan^{-1}\left(\frac{12}{5}\right)$, $\sin^{-1}\left(\frac{12}{13}\right)$, $\cos^{-1}\left(\frac{5}{13}\right)$, awrt 67 degrees, awrt 1.2, truncated 1.1 (anticlockwise), awrt 293 degrees clockwise or awrt 5.1 clockwise	B1 o.e.
	about $(0, 0)$	**The mark is dependent on at least one of the previous B marks being awarded.** About $(0, 0)$ or about O or about the origin	dB1
	Note: Give 2nd B0 for 67 degrees clockwise o.e.		**(3)**
(b)	$\{\mathbf{Q} =\}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	Correct matrix	B1
			(1)
(c)	$\{\mathbf{R} = \mathbf{PQ} =\}\begin{pmatrix}\frac{5}{13} & -\frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}; = \begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}$	Multiplies **P** by their **Q** in the correct order and finds at least one element	M1
		Correct matrix	A1
			(2)
(d) **Way 1**	$\begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}\begin{pmatrix}x\\ kx\end{pmatrix} = \begin{pmatrix}x\\ kx\end{pmatrix}$	Forming the equation "their matrix **R**" $\begin{pmatrix}x\\ kx\end{pmatrix} = \begin{pmatrix}x\\ kx\end{pmatrix}$ **Allow x being replaced by any non-zero number eg. 1.** Can be implied by at least one correct ft equations below.	M1
	$-\frac{12}{13}x + \frac{5kx}{13} = x$ or $\frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = ...$	Uses their matrix equation to form an equation in k and progresses to give k = numerical value	M1
	So $k = 5$	**dependent on only the previous M mark** $k = 5$	A1 **cao**
	Dependent on all previous marks being scored in this part. Either • Solves **both** $-\frac{12}{13}x + \frac{5kx}{13} = x$ **and** $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ • Finds $k = 5$ and checks that it is true for the other component • Confirms that $\begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}\begin{pmatrix}x\\ 5x\end{pmatrix} = \begin{pmatrix}x\\ 5x\end{pmatrix}$		A1 **cso**
			(4)
(d) **Way 2**	Either $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$	Correct follow through equation in 2θ based on their matrix **R**	M1
	$\{k =\}\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$	Full method of finding 2θ, then θ and applying $\tan\theta$	M1
		$\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan(\text{awrt } 78.7^\circ)$ or $\tan(\text{awrt } 1.37)$. **Can be implied.**	A1
	So $k = 5$	$k = 5$ **by a correct solution only**	A1
			(4)
			10

		Question 7 Notes
7. (a)	**Note**	Condone "Turn" for the 1st B1 mark.
	Note	Penalise the first B1 mark for candidates giving a combination of transformations.
(c)	**Note**	Allow 1st M1 for eg. $\text{"their matrix }\mathbf{R}\text{"}\begin{pmatrix}1\\k\end{pmatrix}=\begin{pmatrix}1\\k\end{pmatrix}$ or $\text{"their matrix }\mathbf{R}\text{"}\begin{pmatrix}k\\k^2\end{pmatrix}=\begin{pmatrix}k\\k^2\end{pmatrix}$ or $\text{"their matrix }\mathbf{R}\text{"}\begin{pmatrix}\frac{1}{k}\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{k}\\1\end{pmatrix}$ or equivalent
	Note	$y=(\tan\theta)x:\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}=\begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}$

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8.

$$f(z) = z^4 + 6z^3 + 76z^2 + az + b$$

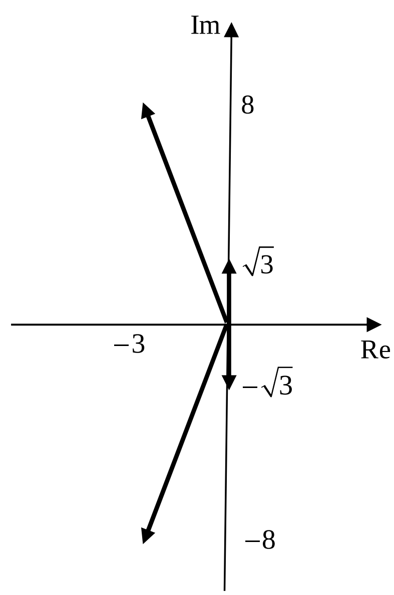
where a and b are real constants.

Given that $-3 + 8i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation. **(1)**

(b) Hence, or otherwise, find the other roots of the equation $f(z) = 0$ **(6)**

(c) Show on a single Argand diagram all four roots of the equation $f(z) = 0$ **(2)**

Question Number	Scheme	Notes	Marks
8.	$f(z)=z^4+6z^3+76z^2+az+b$, a, b are real constants. $z_1=-3+8i$ is given.		
(a)	$-3-8i$	$-3-8i$	B1
			(1)
(b)	$z^2+6z+73$	Attempt to expand $(z-(-3+8i))(z-(-3-8i))$ or any valid method ***to establish a quadratic factor*** eg. $z=-3\pm 8i \Rightarrow z+3=\pm 8i \Rightarrow z^2+6z+9=-64$ or sum of roots -6, product of roots 73 to give $z^2 \pm (\text{sum})z + \text{product}$	M1
		$z^2+6z+73$	A1
	$f(z)=(z^2+6z+73)(z^2+3)$	Attempts to find the other quadratic factor. eg. using long division to get as far as $z^2+...$ or eg. $f(z)=(z^2+6z+73)(z^2+...)$	M1
		z^2+3	A1
	$\{z^2+3=0 \Rightarrow z=\}$ $\pm\sqrt{3}i$	**dependent on only the previous M mark** Correct method of solving the 2nd quadratic factor	dM1
		$\sqrt{3}i$ **and** $-\sqrt{3}i$	A1
			(6)
(c)		**Criteria** • $-3\pm 8i$ plotted correctly in quadrants 2 and 3 with some evidence of symmetry • Their other two ***complex roots*** (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the *x*-axis	
		Satisfies at least one of the two criteria	B1 ft
		Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1 ft
			(2)
			9

Question 8 Notes

8. (b)	**Note**	Give 3rd M1 for $z^2+k=0,\ k>0 \Rightarrow$ **at least one of either** $z=\sqrt{k}\,i$ **or** $z=-\sqrt{k}\,i$
	Note	Give 3rd M0 for $z^2+k=0,\ k>0 \Rightarrow z=\pm ki$
	Note	Give 3rd M0 for $z^2+k=0,\ k>0 \Rightarrow z=\pm k$ or $z=\pm\sqrt{k}$
	Note	Candidates do not need to find $a=18,\ b=219$

Leave blank

9. The quadratic equation

$$2x^2 + 4x - 3 = 0$$

has roots α and β.

Without solving the quadratic equation,

(a) find the exact value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(5)

(b) Find a quadratic equation which has roots $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(4)

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Question Number	Scheme	Notes	Marks
9.	$2x^2+4x-3=0$ has roots α, β		
(a)	$\alpha+\beta=-\frac{4}{2}$ or -2, $\alpha\beta=-\frac{3}{2}$	**Both** $\alpha+\beta=-\frac{4}{2}$ **and** $\alpha\beta=-\frac{3}{2}$. This may be seen or implied anywhere in this question.	B1
(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=......$	**Use** of a **correct** identity for $\alpha^2+\beta^2$ (May be implied by their work)	M1
	$=(-2)^2-2\left(-\frac{3}{2}\right)=7$	7 **from correct working**	A1 **cso**
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$ or $=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=......$	**Use** of an **appropriate** and **correct** identity for $\alpha^3+\beta^3$ (May be implied by their work)	M1
	$=(-2)^3-3\left(-\frac{3}{2}\right)(-2)=-17$ or $=(-2)\left(7--\frac{3}{2}\right)=-17$	-17 **from correct working**	A1 **cso**
			(5)
(b)	Sum $=\alpha^2+\beta+\beta^2+\alpha$ $=\alpha^2+\beta^2+\alpha+\beta$ $=7+(-2)=5$	Uses at least one of their $\alpha^2+\beta^2$ or $\alpha+\beta$ in an attempt to find a **numerical value** for the sum of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$	M1
	Product $=(\alpha^2+\beta)(\beta^2+\alpha)$ $=(\alpha\beta)^2+\alpha^3+\beta^3+\alpha\beta$ $=\left(-\frac{3}{2}\right)^2-17-\frac{3}{2}=-\frac{65}{4}$	Expands $(\alpha^2+\beta)(\beta^2+\alpha)$ and uses at least one of their $\alpha\beta$ or $\alpha^3+\beta^3$ in an attempt to find a **numerical value** for the product of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$	M1
	$x^2-5x-\frac{65}{4}=0$	Applies $x^2-(\text{sum})x+\text{product}$ (Can be implied) ("$=0$" not required)	M1
	$4x^2-20x-65=0$	Any integer multiple of $4x^2-20x-65=0$, including the "$=0$"	A1
			(4)
	<u>Alternative:</u> Finding $\alpha^2+\beta$ and $\beta^2+\alpha$ explicitly		
(b)	Eg. Let $\alpha=\frac{-4+\sqrt{40}}{4}$, $\beta=\frac{-4+\sqrt{40}}{4}$ and so $\alpha^2+\beta=\frac{5-3\sqrt{10}}{2}$, $\beta^2+\alpha=\frac{5+3\sqrt{10}}{2}$		
	$\left(x-\left(\frac{5-3\sqrt{10}}{2}\right)\right)\left(x-\left(\frac{5+3\sqrt{10}}{2}\right)\right)$	Uses $\left(x-(\alpha^2+\beta)\right)\left(x-(\beta^2+\alpha)\right)$ with exact numerical values. (May expand first)	M1
	$=x^2-\left(\frac{5+3\sqrt{10}}{2}\right)x-\left(\frac{5-3\sqrt{10}}{2}\right)x+\left(\frac{5-3\sqrt{10}}{2}\right)\left(\frac{5+3\sqrt{10}}{2}\right)$	Attempts to expand using exact numerical values for $\alpha^2+\beta$ and $\beta^2+\alpha$	M1
	$\Rightarrow x^2-5x-\frac{65}{4}=0$	Collect terms to give a 3TQ. ("$=0$" not required)	M1
	$4x^2-20x-65=0$	Any integer multiple of $4x^2-20x-65=0$, including the "$=0$"	A1
			(4)
			9

		Question 9 Notes
9. (a)	**1st A1**	$\alpha+\beta=2,\ \alpha\beta=-\frac{3}{2} \Rightarrow \alpha^2+\beta^2=4-2\left(-\frac{3}{2}\right)=7$ is M1A0 cso
(a)	**Note**	Finding $\alpha+\beta=-2,\ \alpha\beta=-\frac{3}{2}$ by writing down or applying $\frac{-4+\sqrt{40}}{4}, \frac{-4+\sqrt{40}}{4}$ but then writing $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4+3=7$ and $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-8-9=-17$ scores B0M1A0M1A0 in part (a).
	Note	Applying $\frac{-4+\sqrt{40}}{4}, \frac{-4+\sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0 Eg: Give no credit for $\left(\frac{-4+\sqrt{40}}{4}\right)^2+\left(\frac{-4+\sqrt{40}}{4}\right)^2=7$ or for $\left(\frac{-4+\sqrt{40}}{4}\right)^3+\left(\frac{-4+\sqrt{40}}{4}\right)^3=-17$
(b)	**Note**	Candidates **are allowed** to apply $\frac{-4+\sqrt{40}}{4}, \frac{-4+\sqrt{40}}{4}$ explicitly in part (b).
	Note	A correct method leading to a candidate stating $a=4,\ b=-20, c=-65$ without writing a final answer of $4x^2-20x-65=0$ is **final** M1A0

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10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$
$$u_{n+1} = 3u_n + 2, \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}$$

(6)

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Question Number	Scheme	Notes	Marks
10.	$u_1 = 5,\ u_{n+1} = 3u_n + 2,\ n \geq 1$. Required to prove the result, $u_n = 2\times(3)^n - 1,\ n \in \mathbb{Z}^+$		
(i)	$n=1$: $u_1 = 2(3) - 1 = 5$	$u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$	B1
	(Assume the result is true for $n = k$)		
	$u_{k+1} = 3\left(2(3)^k - 1\right) + 2$	Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$	M1
	$= 2(3)^{k+1} - 1$	**dependent on the previous M mark** Expresses u_{k+1} in term of 3^{k+1}	dM1
		$u_{k+1} = 2(3)^{k+1} - 1$ **by correct solution only**	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n		A1 **cso**
			5
	Required to prove the result $\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n},\ n \in \mathbb{Z}^+$		
(ii)	$n=1$: LHS $= \frac{4}{3}$, RHS $= 3 - \frac{5}{3} = \frac{4}{3}$	Shows or states **both** LHS $= \frac{4}{3}$ **and** RHS $= \frac{4}{3}$ **or** states LHS = RHS $= \frac{4}{3}$	B1
	(Assume the result is true for $n = k$)		
	$\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{(3+2k)}{3^k} + \frac{4(k+1)}{3^{k+1}}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= 3 - \frac{3(3+2k)}{3^{k+1}} + \frac{4(k+1)}{3^{k+1}}$	**dependent on the previous M mark** Makes 3^{k+1} or $(3)3^k$ a common denominator for their fractions.	dM1
		Correct expression with common denominator 3^{k+1} or $(3)3^k$ for their fractions.	A1
	$= 3 - \left(\frac{3(3+2k) - 4(k+1)}{3^{k+1}}\right) = 3 - \left(\frac{5+2k}{3^{k+1}}\right)$		
	$= 3 - \frac{(3+2(k+1))}{3^{k+1}}$	$3 - \frac{(3+2(k+1))}{3^{k+1}}$ **by correct solution only**	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n		A1 **cso**
			6
			11

		Question 10 Notes
(i) & (ii)	**Note**	**Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
(i)	**Note**	$u_1 = 5$ by itself is not sufficient for the 1st B1 mark in part (i).
	Note	$u_1 = 3+2$ without stating $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$ is B0
(ii)	**Note**	LHS = RHS by itself is not sufficient for the 1st B1 mark in part (ii).