Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 16 January 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. $f(x) = 2^x - 10 \sin x - 2$, where x is measured in radians

(a) Show that $f(x) = 0$ has a root, α, between 2 and 3

(2)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α.
Give your answer to 3 decimal places.

(3)

January 2017
WFM01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = 2^x - 10\sin x - 2$, x measured in radians		
(a)	$\mathrm{f}(2) = -7.092974268...$ $\mathrm{f}(3) = 4.588799919...$	Attempts to find values for both $\mathrm{f}(2)$ and $\mathrm{f}(3)$	M1
	Sign change {negative, positive} {and $\mathrm{f}(x)$ is continuous} therefore {a root} α is between $x = 2$ and $x = 3$	**Both** $\mathrm{f}(2) =$ awrt -7 **and** $\mathrm{f}(3) =$ awrt 5 or truncated 4 or truncated 4.5, sign change and conclusion.	A1 **cso**
			(2)
(b)	$\dfrac{\alpha - 2}{\text{"}7.092974268...\text{"}} = \dfrac{3 - \alpha}{\text{"}4.588799919...\text{"}}$ **or** $\dfrac{\alpha - 2}{3 - \alpha} = \dfrac{\text{"}7.092974268...\text{"}}{\text{"}4.588799919...\text{"}}$ **or** $\dfrac{\alpha - 2}{\text{"}7.092974268...\text{"}} = \dfrac{3 - 2}{\text{"}4.588799919...\text{"} + \text{"}7.092974268...\text{"}}$	A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.	M1
	Either $\alpha = \left(\dfrac{(3)(\text{"}7.092974268...\text{"}) + (2)(\text{"}4.588799919...\text{"})}{\text{"}4.588799919...\text{"} + \text{"}7.092974268...\text{"}}\right)$ **or** $\alpha = 2 + \left(\dfrac{\text{"}7.092974268...\text{"}}{\text{"}4.588799919...\text{"} + \text{"}7.092974268...\text{"}}\right)(1)$ **or** $\alpha = 2 + \left(\dfrac{\text{"}-7.092974268...\text{"}}{\text{"}-4.588799919...\text{"} + \text{"}-7.092974268...\text{"}}\right)(1)$	**dependent on the previous M mark.** Rearranges to make $\alpha = ...$	dM1
	$\{\alpha = 2.607182963...\} \Rightarrow \alpha = 2.607$ (3 dp)	2.607	A1 **cao**
			(3)
(b) **Way 2**	$\dfrac{x}{\text{"}7.092974268...\text{"}} = \dfrac{1 - x}{\text{"}4.588799919...\text{"}} \Rightarrow x = \dfrac{\text{"}7.092974268...\text{"}}{11.68177419...} = 0.6071829632...$		
	$\alpha = 2 + 0.6071829632...$	Finds x using a correct method of similar triangles and applies "2 + their x"	M1 dM1
	$\{\alpha = 2.607182963...\} \Rightarrow \alpha = 2.607$ (3 dp)	2.607	A1 **cao**
(b) **Way 3**	$\dfrac{1 - x}{\text{"}7.092974268...\text{"}} = \dfrac{x}{\text{"}4.588799919...\text{"}} \Rightarrow x = \dfrac{\text{"}4.588799919...\text{"}}{11.68177419...} = 0.3928170366...$		
	$\alpha = 3 - 0.3928170366...$	Finds x using a correct method of similar triangles and applies "3 − their x"	M1 dM1
	$\{\alpha = 2.607182963...\} \Rightarrow \alpha = 2.607$ (3 dp)	2.607	A1 **cao**
			5

		Question 1 Notes
1. (a)	**A1**	**correct solution only** Candidate needs to state **both** $\text{f}(2) = \text{awrt} - 7$ **and** $\text{f}(3) = \text{awrt } 5$ or truncated 4 or truncated 4.5 along with **a reason and conclusion.** Reference to change of sign **or** e.g. $\text{f}(2) \times \text{f}(3) < 0$ **or** a diagram **or** < 0 and > 0 **or** one negative, one positive are sufficient reasons. There must be a (minimal, not incorrect) conclusion, e.g. root is between 2 and 3, hence root is in the interval, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion is “change of sign, hence root”.
(a)	**Note** **Note**	In degrees, $\text{f}(2) = 1.651005033...$, $\text{f}(3) = 5.476640438...$ Some candidates will write $\text{f}(2) = 4$, $\text{f}(3) = -0.4147...$

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2. The quadratic equation

$$2x^2 - x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$ **(1)**

(b) find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ **(2)**

(c) find a quadratic equation which has roots

$$\left(2\alpha - \frac{1}{\beta}\right) \text{ and } \left(2\beta - \frac{1}{\alpha}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers. **(4)**

Question Number	Scheme	Notes	Marks
2.	$2x^2 - x + 3 = 0$ has roots α, β		
	Note: Parts (a) and (b) can be marked together.		
(a)	$\alpha + \beta = \frac{1}{2}, \ \alpha\beta = \frac{3}{2}$	**Both** $\alpha + \beta = \frac{1}{2}$ **and** $\alpha\beta = \frac{3}{2}$	B1
			(1)
(b)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{1}{2}}{\frac{3}{2}}$	Attempts to substitute at least one of their $(\alpha + \beta)$ or their $\alpha\beta$ into $\frac{\beta+\alpha}{\alpha\beta}$	M1
	$= \frac{1}{3}$	$\frac{1}{3}$ **from correct working**	A1 **cso**
			(2)
(c)	Sum $= \left(2\alpha - \frac{1}{\beta}\right) + \left(2\beta - \frac{1}{\alpha}\right)$ $= 2(\alpha + \beta) - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ $= 2\left(\frac{1}{2}\right) - \left(\frac{1}{3}\right) = \frac{2}{3}$	Uses at least one of $2(\text{their } (\alpha + \beta))$ or their $\frac{1}{\alpha} + \frac{1}{\beta}$ in an attempt to find a **numerical value** for the sum of $\left(2\alpha - \frac{1}{\beta}\right)$ and $\left(2\beta - \frac{1}{\alpha}\right)$.	M1
	Product $= \left(2\alpha - \frac{1}{\beta}\right)\left(2\beta - \frac{1}{\alpha}\right)$ $= 4\alpha\beta - 2 - 2 + \frac{1}{\alpha\beta}$ $= 4\left(\frac{3}{2}\right) - 4 + \frac{1}{\left(\frac{3}{2}\right)}$ $= 6 - 4 + \frac{2}{3} = \frac{8}{3}$	Expands $\left(2\alpha - \frac{1}{\beta}\right)\left(2\beta - \frac{1}{\alpha}\right)$ and uses their $\alpha\beta$ at least once in an attempt to find a **numerical value** for the product of $\left(2\alpha - \frac{1}{\beta}\right)$ and $\left(2\beta - \frac{1}{\alpha}\right)$.	M1
	$x^2 - \frac{2}{3}x + \frac{8}{3} = 0$	Applies $x^2 - (\text{their sum})x + \text{their product}$ (Can be implied) **Note:** (" = 0" not required for this mark.)	M1
	$3x^2 - 2x + 8 = 0$	Any integer multiple of $3x^2 - 2x + 8 = 0$ including the "= 0"	A1
			(4)
			7

		Question 2 Notes
2. (a)	**Note**	Finding $a+b=\frac{1}{2}, ab=\frac{3}{2}$ by writing down $a, b=\frac{1+\sqrt{23}\text{i}}{4}, \frac{1-\sqrt{23}\text{i}}{4}$ or by applying $a+b=\left(\frac{1+\sqrt{23}\text{i}}{4}\right)+\left(\frac{1-\sqrt{23}\text{i}}{4}\right)=\frac{1}{2}$ and $ab=\left(\frac{1+\sqrt{23}\text{i}}{4}\right)\left(\frac{1-\sqrt{23}\text{i}}{4}\right)=\frac{3}{2}$ scores B0 in part (a).
(b), (c)	**Note**	Those candidates who apply $a+b=\frac{1}{2}, ab=\frac{3}{2}$ in part (b) and/or part (c) having written down/applied $a, b=\frac{1+\sqrt{23}\text{i}}{4}, \frac{1-\sqrt{23}\text{i}}{4}$ in part (a) will be penalised the final A mark in part (b) **and** penalised the final A mark in part (c).
(b)	**Note**	Applying $a, b=\frac{1+\sqrt{23}\text{i}}{4}, \frac{1-\sqrt{23}\text{i}}{4}$ explicitly in part (b) will score M0A0. E.g.: Give no credit for $\frac{1}{\frac{1+\sqrt{23}\text{i}}{4}}+\frac{1}{\frac{1-\sqrt{23}\text{i}}{4}}=\frac{1}{3}$ or for $\frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}=\left(\left(\frac{1+\sqrt{23}\text{i}}{4}\right)+\left(\frac{1-\sqrt{23}\text{i}}{4}\right)\right) \div \left(\left(\frac{1+\sqrt{23}\text{i}}{4}\right)\left(\frac{1-\sqrt{23}\text{i}}{4}\right)\right)=\frac{1}{3}$
(c)	**Note**	Candidates **are not allowed** to apply $a, b=\frac{1+\sqrt{23}\text{i}}{4}, \frac{1-\sqrt{23}\text{i}}{4}$ explicitly in part (c).
	Note	A correct method leading to a candidate stating $p=3, q=-2, r=8$ without writing a final answer of $3x^2-2x+8=0$ is **final** A0

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3. $$f(x) = x^4 + 2x^3 + 26x^2 + 32x + 160$$

Given that $x = -1 + 3i$ is a root of the equation $f(x) = 0$, use algebra to find the three other roots of $f(x) = 0$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(7)

Question Number	Scheme	Notes	Marks
3.	$\mathrm{f}(x) = x^4 + 2x^3 + 26x^2 + 32x + 160,\ x_1 = -1 + 3\mathrm{i}$ is given.		
	$x_2 = -1 - 3\mathrm{i}$	Writes down the root $-1-3\mathrm{i}$ **Note:** $-1-3\mathrm{i}$ needs to be stated explicitly somewhere in the candidate's working for B1	B1
	$x^2 + 2x + 10$	Attempt to expand $(x-(-1+3\mathrm{i}))(x-(-1-3\mathrm{i}))$ or $(x-(-1+3\mathrm{i}))(x-(\text{their complex } x_2))$ or any valid method ***to establish a quadratic factor*** e.g. $x = -1 \pm 3\mathrm{i} \Rightarrow x+1 = \pm 3\mathrm{i} \Rightarrow x^2+2x+1 = -9$ or sum of roots -2, product of roots 10 to give $x^2 \pm (\text{their sum})x + (\text{their product})$	M1
		$x^2 + 2x + 10$	A1
	$\mathrm{f}(x) = (x^2+2x+10)(x^2+16)$	Attempts to find the other quadratic factor. e.g. using long division to get as far as $x^2 + \ldots$ or e.g. $\mathrm{f}(x) = (x^2+2x+10)(x^2+\ldots)$	M1
		$x^2 + 16$	A1
	$\left\{x^2 + 16 = 0 \Rightarrow x =\right\} = \pm\sqrt{16}\,\mathrm{i};\ = \pm 4\mathrm{i}$	**dependent on only the previous M mark** Correct method of solving ***their*** 2nd quadratic factor to give $x = \ldots$.	dM1
		$4\mathrm{i}$ **and** $-4\mathrm{i}$	A1
			(7)
			7
	Question 3 Notes		
3.	**Note**	$x_1 = -1+3\mathrm{i}$, $x_2 = -1-3\mathrm{i}$ leading to $(x-1+3\mathrm{i})(x-1-3\mathrm{i})$ is 1st M0 1st A0	
	Note	Give 3rd M1 for $x^2 + k = 0,\ k > 0 \Rightarrow$ **at least one of either** $x = \sqrt{k}\,\mathrm{i}$ **or** $x = -\sqrt{k}\,\mathrm{i}$ Therefore $x^2 + 16 = 0$ leading to a final answer of $x = \sqrt{16}\,\mathrm{i}$ only is 3rd M1.	
	Note	$x^2 + 16 = 0$ leading to $x = \pm\sqrt{(16\mathrm{i})}$ unless recovered is 3rd M0 3rd A0.	
	Note	Give 3rd M0 for $x^2 + k = 0,\ k > 0 \Rightarrow x = \pm k\mathrm{i}$	
	Note	Give 3rd M0 for $x^2 + k = 0,\ k > 0 \Rightarrow x = \pm k$ or $x = \pm\sqrt{k}$ Therefore $x^2 + 16 = 0$ leading to $x = \pm 4$ is 3rd M0. Therefore $x^2 + 16 = 0$ leading to $(x+4)(x-4) = 0 \Rightarrow x = \pm 4$ is 3rd M0.	
	Note	No working leading to $x = -1-3\mathrm{i}, 4\mathrm{i}, -4\mathrm{i}$ is B1M0A0M0A0M0A0.	
	Note	Candidates can go from $x^2 + 16 = 0$ to $x = \pm 4\mathrm{i}$ for the final dM1A1 marks.	
	3rd dM1	You can give this mark for a correct method for solving ***their*** quadratic $\neq x^2 + k,\ k > 0$ which can be a 3TQ.	
	Note	e.g. their 2nd quadratic is $x^2 - 16 = 0$ leading to $(x+4)(x-4) = 0 \Rightarrow x = \pm 4$ gets 3rd M1.	

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4. (a) Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(2r+1)(3r+1) = \frac{1}{6}n(n+1)\left(an^2 + bn + c\right)$$

where a, b and c are integers to be determined.

(5)

(b) Hence find the value of

$$\sum_{r=10}^{20} r(2r+1)(3r+1)$$

(2)

Question Number	Scheme	Notes	Marks
4. (a)	$\left\{\sum_{r=1}^{n} r(2r+1)(3r+1) = \right\} \sum_{r=1}^{n} (\underline{6r^3 + 5r^2 + r})$	$6r^3 + 5r^2 + r$	B1
	$= 6\left(\frac{1}{4}n^2(n+1)^2\right) + 5\left(\frac{1}{6}n(n+1)(2n+1)\right) + \left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(2r+1)(3r+1)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{6}n(n+1)\big(9n(n+1) + 5(2n+1) + 3\big)$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute all three standard formulae.	dM1
	$= \frac{1}{6}n(n+1)(9n^2 + 19n + 8)$	Correct completion with no errors. **Note:** $a = 9,\ b = 19,\ c = 8$	A1 **cso**
			(5)
(b)	Let $f(n) = \frac{1}{6}n(n+1)(9n^2 + 19n + 8)$. So $\sum_{r=10}^{20} r(2r+1)(3r+1) = f(20) - f(9)$		
	$= \left(\frac{1}{6}(20)(20+1)(9(20)^2 + 19(20) + 8)\right) - \left(\frac{1}{6}(9)(9+1)(9(9)^2 + 19(9) + 8)\right)$	Attempts to find either $f(20) - f(9)$ or $f(20) - f(10)$	M1
	$\left\{= \left(\frac{1}{6}(20)(21)(3988)\right) - \left(\frac{1}{6}(9)(10)(908)\right) = 279160 - 13620\right\} = 265540$	265540	A1
			(2)
			7

		Question 4 Notes
4. (a)	**Note**	Applying e.g. $n = 1,\ n = 2,\ n = 3$ to the printed equation without applying the standard formulae to give $a = 9,\ b = 19,\ c = 8$ is B0M0A0M0A0.
	Alt 1 **dM1** **A1 cso**	**Alt Method 1:** Using $\frac{3}{2}n^4 + \frac{14}{3}n^3 + \frac{9}{2}n^2 + \frac{4}{3}n \equiv \frac{1}{6}an^4 + \frac{1}{6}(a+b)n^3 + \frac{1}{6}(b+c)n^2 + \frac{1}{6}cn$ o.e. Equating coefficients and finds at least two of $a = 9,\ b = 19,\ c = 8$ Finds $a = 9,\ b = 19,\ c = 8$ and demonstrates the identity works for all of its terms.
	Alt 2 **dM1** **A1**	**Alt Method 2:** $6\left(\frac{1}{4}n^2(n+1)^2\right) + 5\left(\frac{1}{6}n(n+1)(2n+1)\right) + \left(\frac{1}{2}n(n+1)\right) \equiv \frac{1}{6}n(n+1)(an^2 + bn + c)$ Substitutes $n = 1,\ n = 2,\ n = 3$ into this identity o.e. and finds at least two of $a = 9,\ b = 19,\ c = 8$ Finds $a = 9,\ b = 19,\ c = 8$.
	Note	Allow final dM1A1 for $\frac{3}{2}n^4 + \frac{14}{3}n^3 + \frac{9}{2}n^2 + \frac{4}{3}n$ or $\frac{1}{6}n(9n^3 + 28n^2 + 27n + 8)$ or $\frac{1}{6}(9n^4 + 28n^3 + 27n^2 + 8n) \rightarrow \frac{1}{6}n(n+1)(9n^2 + 19n + 8)$, from no incorrect working.
(b)	**Note**	Give M1A0 for applying $f(20) - f(10)$. i.e. $279160 - 20130\ \{= 259030\}$
	Note	Give M0A0 for applying $20(41)(61) - 9(19)(28) = 50020 - 4788 = 45232$
	Note	Give M0A0 for applying $20(41)(61) - 10(21)(31) = 50020 - 6510 = 43510$
	Note	Give M0A0 for listing individual terms. e.g. $6510 + 8602 + \ldots + 42978 + 50020 = 265540$

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5. The complex number z is given by

$$z = -7 + 3\text{i}$$

Find

(a) $|z|$

(1)

(b) arg z, giving your answer in radians to 2 decimal places.

(2)

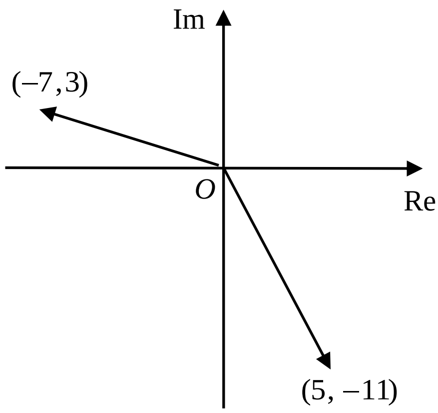
Given that $\dfrac{z}{1 + \text{i}} + w = 3 - 6\text{i}$

(c) find the complex number w, giving your answer in the form $a + b\text{i}$, where a and b are real numbers. You must show all your working.

(3)

(d) Show the points representing z and w on a single Argand diagram.

(2)

Question Number	Scheme	Notes	Marks
5.	$z = -7 + 3\mathrm{i}$; $\dfrac{z}{1+\mathrm{i}} + w = 3 - 6\mathrm{i}$		
(a)	$\left\{\lvert z\rvert = \sqrt{(-7)^2 + (3)^2}\right\} = \sqrt{58}$ or $7.61577...$	$\sqrt{58}$ or awrt 7.62	B1
			(1)
(b)	$\arg z = \pi - \arctan\left(\dfrac{3}{7}\right)$ or $= \dfrac{\pi}{2} + \arctan\left(\dfrac{7}{3}\right)$ or $= -\pi - \arctan\left(\dfrac{3}{7}\right)$	Uses trigonometry in order to find an angle in the 2nd quadrant. i.e. in the range of either $(1.57..., 3.14...)$ or $(-3.14, -4.71...)$ or $(90°, 180°)$ or $(-180°, -270°)$. **Note:** $\arctan\left(-\dfrac{3}{7}\right)$ by itself is not sufficient for M1.	M1
	$\{= \pi - 0.40489...\} = 2.7367...$ $\{= 2.74\ (2\text{ dp})\}$ **or** $\{= -\pi - 0.40489...\} = -3.5464...$ $\{= -3.55\ (2\text{ dp})\}$	either awrt 2.74 or awrt -3.55	A1 o.e.
	{**Note:** $\arg z = 156.8014...°$ or $-203.1985...°$}		**(2)**
(c) **Way 1**	$\dfrac{(-7+3\mathrm{i})}{(1+\mathrm{i})}\dfrac{(1-\mathrm{i})}{(1-\mathrm{i})} + w = 3 - 6\mathrm{i}$	$\dfrac{(-7+3\mathrm{i})}{(1+\mathrm{i})}\dfrac{(1-\mathrm{i})}{(1-\mathrm{i})} + w = 3 - 6\mathrm{i}$ or $\dfrac{z}{(1+\mathrm{i})}\dfrac{(1-\mathrm{i})}{(1-\mathrm{i})} + w = 3 - 6\mathrm{i}$ or can be implied by $-2 + 5\mathrm{i} + w = 3 - 6\mathrm{i}$	M1
	$-2 + 5\mathrm{i} + w = 3 - 6\mathrm{i}$		
	$w = 5 - 11\mathrm{i}$	**dependent on the previous M mark** Rearranges to make $w = \ldots$	dM1
		$5 - 11\mathrm{i}$	A1
			(3)
(c) **Way 2**	$z + w(1+\mathrm{i}) = (3 - 6\mathrm{i})(1+\mathrm{i})$	Fully correct method of multiplying each term by $(1+\mathrm{i})$	M1
	$w(1+\mathrm{i}) = (9 - 3\mathrm{i}) - (-7 + 3\mathrm{i})$		
	$w = \dfrac{(16 - 6\mathrm{i})}{(1+\mathrm{i})}\dfrac{(1-\mathrm{i})}{(1-\mathrm{i})}$	**dependent on the previous M mark** Rearranges to make $w = ...$ and multiplies by $\dfrac{(1-\mathrm{i})}{(1-\mathrm{i})}$	dM1
	$w = 5 - 11\mathrm{i}$	$5 - 11\mathrm{i}$	A1
			(3)
(d)	 	Plotting $-7 + 3\mathrm{i}$ correctly. The point must be indicated by a scale (could be ticks on the axes) **or** labelled with coordinates or a complex number *z*.	B1
		Plotting their *w* correctly. The point must be indicated by a scale (could be ticks on the axes) **or** labelled with coordinates or a complex number *w*.	B1ft
		<u>**Special Case**</u> Award SC B1B0 if both $-7 + 3\mathrm{i}$ and their *w* are plotted correctly relative to each other without any scale or labelled coordinates.	
			8

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6. $$\mathrm{f}(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \qquad x > 0$$

The root α of the equation $\mathrm{f}(x) = 0$ lies in the interval [0.6, 0.7].

(a) Taking 0.6 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.
(5)

(b) Show that your answer to part (a) is correct to 3 decimal places.
(2)

Question Number	Scheme	Notes	Marks
6.	$f(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \quad x > 0$		
(a)	$f'(x) = 3x^2 + \frac{1}{2}x^{-2} + \frac{3}{2}x^{\frac{1}{2}}$ **or** $f'(x) = 3x^2 + (2x)^{-2}(2) + \frac{3}{2}x^{\frac{1}{2}}$	At least one of either $x^3 \to \pm Ax^2$ or $-\frac{1}{2x} \to \pm Bx^{-2}$ or $x^{\frac{3}{2}} \to \pm Cx^{\frac{1}{2}}$ where A, B and C are non-zero constants.	M1
		At least 2 differentiated terms are correct	A1
		Correct differentiation.	A1
	$\left\{\alpha \simeq 0.6 - \frac{f(0.6)}{f'(0.6)}\right\} \Rightarrow \alpha \simeq 0.6 - \frac{-0.1525753318...}{3.630783893...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $f(0.6)$ and $f'(0.6)$	dM1
	$\{\alpha = 0.6420226971...\} \Rightarrow \alpha = 0.642$ (3 dp)	**dependent on all 4 previous marks** 0.642 on their first iteration (Ignore any subsequent iterations)	A1 **cso cao**
	Correct differentiation followed by a correct answer scores full marks in (a) **Correct answer with <u>no</u> working scores no marks in (a)**		
			(5)
(b) **Way 1**	$f(0.6415) = -0.001630649...$ $f(0.6425) = 0.002020826...$	Chooses a suitable interval for x, which is within ± 0.0005 of their answer to (a) and at least one attempt to evaluate $f(x)$.	M1
	Sign change {negative, positive} {and $f(x)$ is continuous} therefore {a root} $\alpha = 0.642$ (3 dp)	Both values correct awrt (or truncated) to 1 sf, sign change and conclusion.	A1 **cso**
			(2)
(b) **Way 2**	**Applying Newton-Raphson again** Using $\alpha = 0.642$ or better e.g. $\alpha = 0.64200226971...$		
	• $\alpha \simeq 0.642 - \frac{0.0001949626...}{3.651474882...}$ $\{= 0.641946607...\}$ • $\alpha \simeq 0.642022697 - \frac{0.0002778408...}{3.651497787...}$ $\{= 0.641946608...\}$	Evidence of applying Newton-Raphson for a second time on their answer to part (a)	M1
	$\alpha = 0.642$ (3 dp)	$\alpha = 0.642$ (3 dp)	A1 **cso**
	Note: You can recover work for Way 2 in part (a)		**(2)**
			7

		Question 6 Notes
6. (a)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	Final dM1	This mark can be implied by applying at least one correct ***value*** of either $f(0.6)$ or $f'(0.6)$ in $0.6 - \frac{f(0.6)}{f'(0.6)}$. So just $0.6 - \frac{f(0.6)}{f'(0.6)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	If a candidate writes $0.6 - \frac{f(0.6)}{f'(0.6)} = 0.642$ with no differentiation, send the response to review.

		Question 6 Notes
6. (b)	**A1**	**Way 1: correct solution only** Candidate needs to state **both** of their values for $f(x)$ to awrt (or truncated) 1 sf along with **a reason and conclusion.** Reference to change of sign **or** e.g. $f(0.6415) \times f(0.6425) < 0$ **or** a diagram **or** < 0 and > 0 **or** one negative, one positive are sufficient reasons. There must be a correct conclusion, e.g. $\alpha = 0.642$ (3 dp). Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion is "change of sign, so $\alpha = 0.642$ (3 dp)."
	Note	Stating "root is in between 0.6415 and 0.6425" without some reference to $\alpha = 0.642$ (3 dp) is not sufficient for A1.
	Note	The root of $f(x) = 0$ is $0.6419466...$, so candidates can also choose x_1 which is less than $0.6419466...$ and choose x_2 which is greater than $0.6419466...$ with both x_1 and x_2 lying in the interval $[0.6415, 0.6425]$ and evaluate $f(x_1)$ and $f(x_2)$.
	Note	**<u>Conclusions to part (b)</u>** Their conclusion needs to convey that they understand that $\alpha = 0.642$ to 3 decimal places. Therefore acceptable conclusions are: e.g. 1: $\alpha = 0.642$ (3 dp) e.g. 2: (a) is correct to 3 dp {Note: their answer to part (a) must be 0.642} e.g. 3: my answer to part (a) is correct to 3 dp {Note: their answer to part (a) must be 0.642} e.g. 4: the answer is correct to 3 d.p. {Note: their answer to part (a) must be 0.642} **Note** that saying "α is correct to 3 dp" or "0.642 is correct" or "$\alpha = 0.642$" are not acceptable conclusions.
	Note	$0.642 - \dfrac{f(0.642)}{f'(0.642)} = 0.642$ (3 dp) is sufficient for M1A1 in part (b).
6. (b)	**Note**	**<u>Helpful Table</u>**

x	$f(x)$
0.6415	-0.001630649
0.6416	-0.001265547
0.6417	-0.000900435
0.6418	-0.000535312
0.6419	-0.000170180
0.6420	0.000194963
0.6421	0.000560115
0.6422	0.000925278
0.6423	0.001290451
0.6424	0.001655634
0.6425	0.002020827

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7. (i)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.
(2)

The matrix **B** represents a stretch, scale factor 3, parallel to the x-axis.

(b) Find the matrix **B**.
(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} -4 & 3 \\ -3 & -4 \end{pmatrix}$$

The matrix **M** represents an enlargement with scale factor k and centre (0, 0), where $k > 0$, followed by a rotation anticlockwise through an angle θ about (0, 0).

(a) Find the value of k.
(2)

(b) Find the value of θ, giving your answer in radians to 2 decimal places.
(2)

(c) Find $\mathbf{M}^{-1}$
(2)

Question Number	Scheme	Notes	Marks
7. (i)(a)	Reflection	Reflection	B1
	in the y-axis.	**dependent on the previous B mark** Allow y-axis **or** $x = 0$	dB1
			(2)
(i)(a) **Way 2**	Stretch scale factor -1	Stretch scale factor -1	B1
	parallel to the x-axis	**dependent on the previous B mark** parallel to the x-axis	dB1
			(2)
(b)	$\{\mathbf{B} =\}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & ... \\ ... & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & ... \\ ... & 3 \end{pmatrix}$	M1
		Correct matrix	A1
			(2)
	Note: Parts (ii)(a) and (ii)(b) can be marked together.		
(ii)(a)	$\{k =\}\ \sqrt{(-4)^2 - (3)(-3)};\ = 5$ **or** $k\cos\theta = -4,\ k\sin\theta = -3$ to give $\theta = ...$ and then $k = ...$	Attempts $\sqrt{\pm 16 \pm 9}$ **or** uses full method of trigonometry to find $k = ...$	M1;
		5 only	A1 **cao**
			(2)
(b)	$5\cos\theta = -4,\ 5\sin\theta = -3,\ \tan\theta = \frac{3}{4}$ or $\tan^{-1}\left(\frac{3}{4}\right)$ and e.g. $\theta = \pi + \tan^{-1}\left(\frac{3}{4}\right)$	Uses trigonometry to find an expression in the range $(3.14..., 4.71...)$ or $(-3.14..., -1.57...)$ or $(180°, 270°)$ or $(-180°, -90°)$	M1
	$\{\theta = \pi + 0.64350...\} = 3.78509...\{= 3.79\ (2\text{ dp})\}$	awrt 3.79 or awrt -2.50	A1
			(2)
(c)	$\{\mathbf{M}^{-1} =\}\ \frac{1}{25}\begin{pmatrix} -4 & -3 \\ 3 & -4 \end{pmatrix}$	$\frac{1}{25}$ or $\begin{pmatrix} -4 & -3 \\ 3 & -4 \end{pmatrix}$	M1
		$\frac{1}{25}\begin{pmatrix} -4 & -3 \\ 3 & -4 \end{pmatrix}$ or $\begin{pmatrix} -0.16 & -0.12 \\ 0.12 & -0.16 \end{pmatrix}$ o.e.	A1 o.e.
			(2)
			10

Question 7 Notes

7. (i)	**Note**	Give B1B0 for "Reflection in the y-axis about $(0, 0)$".
(i)	**Note**	Send to review a response which states, e.g. "enlargement parallel to the x-axis"
(ii)(b)	**Note**	Allow M1 (implied) for awrt $217°$ or awrt $-143°$
(ii)(b)	**Note**	$\begin{pmatrix} k\cos\theta & -k\sin\theta \\ k\sin\theta & k\cos\theta \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ -3 & -4 \end{pmatrix}$
(ii) (c)	**Note**	Allow M1 for $\begin{pmatrix} -0.8 & -0.6 \\ 0.6 & -0.8 \end{pmatrix}$

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8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ lies on C.

(a) Using calculus, show that the normal to C at P has equation

$$y + tx = at^3 + 2at$$

(5)

The point S is the focus of the parabola C.

The point B lies on the positive x-axis and $OB = 5OS$, where O is the origin.

(b) Write down, in terms of a, the coordinates of the point B.

(1)

A circle has centre B and touches the parabola C at two distinct points Q and R.

Given that $t \neq 0$,

(c) find the coordinates of the points Q and R.

(4)

(d) Hence find, in terms of a, the area of triangle BQR.

(2)

Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 4ax$, a is a positive constant. $P(at^2, 2at)$ lies on C; k, p, q are constants.		
(a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}(2)a^{\frac{1}{2}}x^{-\frac{1}{2}} = \dfrac{\sqrt{a}}{\sqrt{x}}$	$\dfrac{dy}{dx} = \pm k x^{-\frac{1}{2}}$	
	$y^2 = 4ax \Rightarrow 2y\dfrac{dy}{dx} = 4a$	$py\dfrac{dy}{dx} = q$	M1
	$\dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = 2a\left(\dfrac{1}{2at}\right)$	their $\dfrac{dy}{dt} \times \dfrac{1}{\text{their } \frac{dx}{dt}}$	
	$\dfrac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\dfrac{dy}{dx} = 4a$ or $\dfrac{dy}{dx} = 2a\left(\dfrac{1}{2at}\right)$	Correct differentiation	A1
	So, $m_N = -t$	Applies $m_N = \dfrac{-1}{m_T}$, where m_T is found from using calculus. **Can be implied by later working**	M1
	$y - 2at = -t(x - at^2)$ **or** $y = -tx + 2at + at^3$	Correct straight line method for an equation of a **normal** where $m_N (\neq m_T)$ is found from using calculus.	M1
	leading to $y + tx = at^3 + 2at$ (*)	Correct solution only	A1
	Note: m_N must be a function of t for the 2nd M1 and the 3rd M1 mark.		**(5)**
(b)	Coordinates of B are $(5a, 0)$	$(5a, 0)$. Condone $x = 5a$ if coordinates are not stated.	B1
			(1)
(c)	$\left\{\text{their } (5a, 0) \text{ into } y + tx = at^3 + 2at \Rightarrow\right\}\ 5at = at^3 + 2at$		M1
	$\{m_{BP} =\}\ \dfrac{2at - 0}{at^2 - 5a} = -t$		
	$PB^2 = (at^2 - 5a)^2 + (2at)^2 \Rightarrow \dfrac{d(PB^2)}{dt} = 2(at^2 - 5a)2at + 2(2at)2a = 0$		
	$PB^2 = a^2t^4 - 10a^2t^2 + 25a^2 + 4a^2t^2 = a^2t^4 - 6a^2t^2 + 25a^2 \Rightarrow \dfrac{d(PB^2)}{dt} = 4a^2t^3 - 12a^2t = 0$		
	Substitutes their coordinates of B into the normal equation **or** finds m_{BP} and sets this equal to their m_N **or** minimises PB or PB^2 **to obtain an equation in *a* and *t* only**. **Note:** $t \equiv q$ or p.		
	$t^3 - 3t = 0$ or $t^2 - 3 = 0 \Rightarrow t = ...$	**dependent on the previous M mark** Solves to find $t = ...$	dM1
	$\{Q, R$ are$\}$ $(3a, 2\sqrt{3}a)$ and $(3a, -2\sqrt{3}a)$	At least one set of coordinates is correct.	A1
		Both sets of coordinates are correct.	A1
			(4)
(d)	Area $BQR = \dfrac{1}{2}(2(2a\sqrt{3}))(5a - 3a)$ or $= \dfrac{1}{2}\begin{vmatrix} 5a & 3a & 3a & 5a \\ 0 & 2\sqrt{3}a & -2\sqrt{3}a & 0 \end{vmatrix}$	Points are in the form $B(ka, 0)$, $Q(\alpha, \beta)$ and $R(\alpha, -\beta)$, $k \neq 0$ **and** applies either $\dfrac{1}{2}\left(\lvert(ka - \alpha)\rvert\right)(2\beta)$ **or** writes down a correct ft determinant statement.	M1
	$= 4a^2\sqrt{3}$	$4a^2\sqrt{3}$	A1
			(2)
			12

Question Number	Scheme	Notes	Marks
8. (c) **Way 2**	$y^2 = 4ax$ into $(x - 5a)^2 + y^2 = r^2$ $(x - 5a)^2 + 4ax = r^2$ $x^2 - 10ax + 25a^2 + 4ax = r^2$ $x^2 - 6ax + 25a^2 - r^2 = 0$ $\left\{"b^2 - 4ac = 0" \Rightarrow\right\}\ 36a^2 - 4(1)(25a^2 - r^2) = 0$	Substitutes $y^2 = 4ax$ into $(x - \text{their } x_A)^2 + y^2 = r^2$ and applies $"b^2 - 4ac = 0"$ to the resulting quadratic equation.	M1
	$36a^2 - 100a^2 + 4r^2 = 0$ $4r^2 = 64a^2 \Rightarrow r^2 = 16a^2 \Rightarrow r = 4a$ So $r = 4a$ gives $x^2 - 6ax + 25a^2 - 16a^2 = 0$ $x^2 - 6ax + 9a^2 = 0 \Rightarrow (x - 3a)(x - 3a) = 0$ $\Rightarrow x = 3a$	**dependent on the previous M mark** Obtains $r = ka,\ k > 0$, where k is a constant and uses this result to form and solve a quadratic to find x which is in terms of a.	dM1
	$\left\{y^2 = 4ax \Rightarrow\right\}\ y^2 = 4a(3a) = 12a^2 \Rightarrow y = \pm 2\sqrt{3}a$		
	$\{Q, R \text{ are}\}\ (3a, 2\sqrt{3}a)$ and $(3a, -2\sqrt{3}a)$	At least one set of coordinates is correct.	A1
		Both sets of coordinates are correct.	A1
			(4)

	Question 8 Notes	
8. (c)	**A marks**	Allow $(3a, \sqrt{12}\,a)$ and $(3a, -\sqrt{12}\,a)$ as exact alternatives to $(3a, 2\sqrt{3}a)$ and $(3a, -2\sqrt{3}a)$ respectively.

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9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n}\left(4r^3 - 3r^2 + r\right) = n^3(n+1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^{2n} + 3n - 1$$

is divisible by 9

(6)

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Question Number	Scheme	Notes	Marks
9.	(i) $\sum_{r=1}^{n}(4r^3 - 3r^2 + r) = n^3(n+1)$; (ii) $f(n) = 5^{2n} + 3n - 1$ is divisible by 9		
(i)	$n=1$: LHS $= 4 - 3 + 1 = 2$, RHS $= 1^3(1+1) = 2$	Shows or states **both** LHS = 2 **and** RHS = 2 **or** states LHS = RHS = 2	B1
	(Assume the result is true for $n = k$)		
	$\sum_{r=1}^{k+1}(4r^3 - 3r^2 + r) = k^3(k+1) + 4(k+1)^3 - 3(k+1)^2 + (k+1)$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= (k+1)\left[k^3 + 4(k+1)^2 - 3(k+1) + 1\right]$ or $(k+1)\left[k^3 + 4k^2 + 5k + 2\right]$ or $(k+2)\left[k^3 + 3k^2 + 3k + 1\right]$	**dependent on the previous M mark**. Takes out a factor of either $(k+1)$ or $(k+2)$	dM1
	$= (k+1)(k+1)(k+1)(k+2)$	**dependent on both the previous M marks**. Factorises out and obtains either $(k+1)(k+1)(....)$ or $(k+1)(k+2)(....)$	ddM1
	$= (k+1)^3(k+1+1)$ or $= (k+1)^3(k+2)$	Achieves this result with no errors.	A1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
	Note: Expanded quartic is $k^4 + 5k^3 + 9k^2 + 7k + 2$		6
(ii) **Way 1**	$f(1) = 5^2 + 3 - 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) - f(k) = (5^{2(k+1)} + 3(k+1) - 1) - (5^{2k} + 3k - 1)$	Attempts $f(k+1) - f(k)$	M1
	$f(k+1) - f(k) = 24(5^{2k}) + 3$		
	$= 24(5^{2k} + 3k - 1) - 9(8k - 3)$	$24(5^{2k} + 3k - 1)$ or $24f(k)$	A1
	or $= 24(5^{2k} + 3k - 1) - 72k + 27$	$-9(8k - 3)$ or $-72k + 27$	A1
	$f(k+1) = 24f(k) - 9(8k - 3) + f(k)$ or $f(k+1) = 24f(k) - 72k + 27 + f(k)$ or $f(k+1) = 25(5^{2k} + 3k - 1) - 72k + 27$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k} + 3k - 1)$	dM1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$,</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
(ii) **Way 2**	$f(1) = 5^2 + 3 - 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 5^{2(k+1)} + 3(k+1) - 1$	Attempts $f(k+1)$	M1
	$f(k+1) = 25(5^{2k}) + 3k + 2$		
	$= 25(5^{2k} + 3k - 1) - 9(8k - 3)$	$25(5^{2k} + 3k - 1)$ or $25f(k)$	A1
	or $= 25(5^{2k} + 3k - 1) - 72k + 27$	$-9(8k - 3)$ or $-72k + 27$	A1
	$f(k+1) = 25f(k) - 9(8k - 3)$ or $f(k+1) = 25f(k) - 72k + 27$ or $f(k+1) = 25(5^{2k} + 3k - 1) - 72k + 27$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k} + 3k - 1)$	dM1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$,</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			12

Question Number	Scheme	Notes	Marks
	(ii) $\mathrm{f}(n) = 5^{2n} + 3n - 1$ is divisible by 9		
(ii) **Way 3**	**General Method:** Using $\mathrm{f}(k+1) - m\mathrm{f}(k)$; where m is an integer		
	$\mathrm{f}(1) = 5^2 + 3 - 1 = 27$	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (5^{2(k+1)} + 3(k+1) - 1) - m(5^{2k} + 3k - 1)$	Attempts $\mathrm{f}(k+1) - m\mathrm{f}(k)$	M1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (25 - m)(5^{2k}) + 3k(1 - m) + (2 + m)$		
	$= (25 - m)(5^{2k} + 3k - 1) - 9(8k - 3)$	$(25 - m)(5^{2k} + 3k - 1)$ or $(25 - m)\mathrm{f}(k)$	A1
	or $= (25 - m)(5^{2k} + 3k - 1) - 72k + 27$	$-9(8k - 3)$ or $-72k + 27$	A1
	$\mathrm{f}(k+1) = (25 - m)\mathrm{f}(k) - 9(8k - 3) + m\mathrm{f}(k)$ or $\mathrm{f}(k+1) = (25 - m)\mathrm{f}(k) - 72k + 27 + m\mathrm{f}(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ or $(5^{2k} + 3k - 1)$	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>, As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>is true for all n $(\in \mathbb{Z}^+)$</u>		A1 **cso**
(ii) **Way 4**	**General Method:** Using $\mathrm{f}(k+1) - m\mathrm{f}(k)$		
	$\mathrm{f}(1) = 5^2 + 3 - 1 = 27$	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (5^{2(k+1)} + 3(k+1) - 1) - m(5^{2k} + 3k - 1)$	Attempts $\mathrm{f}(k+1) - m\mathrm{f}(k)$	M1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (25 - m)(5^{2k}) + 3k(1 - m) + (2 + m)$		
	e.g. $m = -2 \Rightarrow \mathrm{f}(k+1) + 2\mathrm{f}(k) = 27(5^{2k}) + 9k$	$m = -2$ and $27(5^{2k})$	A1
		$m = -2$ and $9k$	A1
	$\mathrm{f}(k+1) = 27(5^{2k}) + 9k - 2\mathrm{f}(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ or $(5^{2k} + 3k - 1)$	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>, As the result has been shown to be <u>true for $n = 1$</u>, then the result <u>is true for all n $(\in \mathbb{Z}^+)$</u>		A1 **cso**

	Note	Some candidates may set $\mathrm{f}(k) = 9M$ and so may prove the following general result • $\{\mathrm{f}(k+1) = 25\mathrm{f}(k) - 9(8k - 3)\} \Rightarrow \mathrm{f}(k+1) = 225M - 9(8k - 3)$ • $\{\mathrm{f}(k+1) = 25\mathrm{f}(k) - 72k + 27\} \Rightarrow \mathrm{f}(k+1) = 225M - 72k + 27$
	Question 9 Notes	
(i)	**Note**	LHS = RHS by itself is not sufficient for the 1st B1 mark in part (i).
(i) & (ii)	**Note**	**Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
(ii)	**Note**	In part (ii), Way 4 there are many alternatives where candidates focus on isolating $b(5^{2k})$, where b is a multiple of 9. Listed below are some alternative results: $\mathrm{f}(k+1) = 36(5^{2k}) - 11\mathrm{f}(k) + 36k - 9$ $\mathrm{f}(k+1) = 18(5^{2k}) + 7\mathrm{f}(k) - 18k + 9$ $\mathrm{f}(k+1) = 27(5^{2k}) - 2\mathrm{f}(k) + 9k$ $\mathrm{f}(k+1) = 9(5^{2k}) + 16\mathrm{f}(k) - 45k + 18$ See the next page for how these are derived.

	Question 9 Notes Continued			
	(ii) $\mathrm{f}(n) = 5^{2n} + 3n - 1$ is divisible by 9			
9. (ii)	**The A1A1dM1 marks for Alternatives using** $\mathrm{f}(k+1) - m\mathrm{f}(k)$			
	Way 4.1	$\mathrm{f}(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 36(5^{2k}) - 11(5^{2k}) + 3k + 2$		
		$= 36(5^{2k}) - 11[(5^{2k}) + 3k - 1] + 36k - 9$	$m = -11$ and $36(5^{2k})$	A1
			$m = -11$ and $36k - 9$	A1
		$\mathrm{f}(k+1) = 36(5^{2k}) - 11\mathrm{f}(k) + 36k - 9$ or $\mathrm{f}(k+1) = 36(5^{2k}) - 11[(5^{2k}) + 3k - 1] + 36k - 9$	as before	dM1
	Way 4.2	$\mathrm{f}(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 27(5^{2k}) - 2(5^{2k}) + 3k + 2$		
		$= 27(5^{2k}) - 2[(5^{2k}) + 3k - 1] + 9k$	$m = -2$ and $27(5^{2k})$	A1
			$m = -2$ and $9k$	A1
		$\mathrm{f}(k+1) = 27(5^{2k}) - 2\mathrm{f}(k) + 9k$ or $\mathrm{f}(k+1) = 27(5^{2k}) - 2[(5^{2k}) + 3k - 1] + 9k$	as before	dM1
	Way 4.3	$\mathrm{f}(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 18(5^{2k}) + 7(5^{2k}) + 3k + 2$		
		$= 18(5^{2k}) + 7[(5^{2k}) + 3k - 1] - 18k + 9$	$m = 7$ and $18(5^{2k})$	A1
			$m = 7$ and $-18k + 9$	A1
		$\mathrm{f}(k+1) = 18(5^{2k}) + 7\mathrm{f}(k) - 18k + 9$ or $\mathrm{f}(k+1) = 18(5^{2k}) + 7[(5^{2k}) + 3k - 1] - 18k + 9$	as before	dM1
	Way 4.4	$\mathrm{f}(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 9(5^{2k}) + 16(5^{2k}) + 3k + 2$		
		$= 9(5^{2k}) + 16[(5^{2k}) + 3k - 1] - 45k + 18$	$m = 16$ and $9(5^{2k})$	A1
			$m = 16$ and $-45k + 18$	A1
		$\mathrm{f}(k+1) = 9(5^{2k}) + 16\mathrm{f}(k) - 45k + 18$ or $\mathrm{f}(k+1) = 9(5^{2k}) + 16[(5^{2k}) + 3k - 1] - 45k + 18$	as before	dM1