Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Friday 19 May 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. The quadratic equation

$$3x^2 - 5x + 1 = 0$$

has roots α and β.

Without solving the quadratic equation, find the exact value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

(4)

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May 2017
WFM01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$3x^2 - 5x + 1 = 0$ has roots α, β		
	$\alpha + \beta = \frac{5}{3},\ \alpha\beta = \frac{1}{3}$	**Both** $\alpha + \beta = \frac{5}{3}$ **and** $\alpha\beta = \frac{1}{3}$, seen or implied	B1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \dots$	Attempts to substitute at least one of their $(\alpha^2 + \beta^2)$ or their $\alpha\beta$ into $\frac{\alpha^2 + \beta^2}{\alpha\beta}$	M1
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dots\dots$	**Use** of a **correct** identity for $\alpha^2 + \beta^2$ (May be implied by their work)	M1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\left(\frac{5}{3}\right)^2 - 2\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)} = \frac{\frac{19}{9}}{\frac{1}{3}} = \frac{19}{3}$	**dependent on ALL previous marks being awarded** $\frac{19}{3}$ or $\frac{57}{9}$ or $6\frac{1}{3}$ or $6.\dot{3}$ o.e. **from correct working**	A1 cso
			(4)
			4

Question 1 Notes

1.	**Note**	Finding $\alpha + \beta = \frac{5}{3}, \alpha\beta = \frac{1}{3}$ by writing down $\alpha, \beta = \frac{5+\sqrt{13}}{6}, \frac{5-\sqrt{13}}{6}$ or by applying $\alpha + \beta = \left(\frac{5+\sqrt{13}}{6}\right) + \left(\frac{5-\sqrt{13}}{6}\right) = \frac{5}{3}$ and $\alpha\beta = \left(\frac{5+\sqrt{13}}{6}\right)\left(\frac{5-\sqrt{13}}{6}\right) = \frac{1}{3}$ scores B0.
	Note	Those candidates who then apply $\alpha + \beta = \frac{5}{3}, \alpha\beta = \frac{1}{3}$ having written down/applied $\alpha, \beta = \frac{5+\sqrt{13}}{6}, \frac{5-\sqrt{13}}{6}$ in part (a) can only score the M marks.
	Note	Give M0M0A0 for $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\left(\frac{5+\sqrt{13}}{6}\right)}{\left(\frac{5-\sqrt{13}}{6}\right)} + \frac{\left(\frac{5-\sqrt{13}}{6}\right)}{\left(\frac{5+\sqrt{13}}{6}\right)} = \frac{19}{3}$
	Note	Give M0M0A0 for $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\left(\frac{5+\sqrt{13}}{6}\right)^2 + \left(\frac{5-\sqrt{13}}{6}\right)^2}{\left(\frac{5+\sqrt{13}}{6}\right)\left(\frac{5-\sqrt{13}}{6}\right)} = \frac{19}{3}$
	Note	Give M0M0A0 for $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\left(\frac{5+\sqrt{13}}{6}\right) + \left(\frac{5-\sqrt{13}}{6}\right)\right)^2 - 2\left(\frac{5+\sqrt{13}}{6}\right)\left(\frac{5-\sqrt{13}}{6}\right)}{\left(\frac{5+\sqrt{13}}{6}\right)\left(\frac{5-\sqrt{13}}{6}\right)} = \frac{19}{3}$
	Note	Allow B1 for **both** $S = \frac{5}{3}$ **and** $P = \frac{1}{3}$ or for $\sum\alpha = \frac{5}{3}$ **and** $\prod\alpha = \frac{1}{3}$
	Note	Give final A0 for 6.3 or 6.33 **without reference** to $\frac{19}{3}$ or $\frac{57}{9}$ or $6\frac{1}{3}$

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2. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 4 \\ -k & 2k \\ 3 & 0 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) find the matrix **AB**,

(2)

(b) find the exact value of k for which det(**AB**) = 0

(2)

Question Number	Scheme	Notes	Marks
2. (a)	$\mathbf{AB} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -k & 2k \\ 3 & 0 \end{pmatrix}$		
	$= \begin{pmatrix} 6-k-6 & 12+2k-0 \\ -2+0+15 & -4+0+0 \end{pmatrix}$	Obtains a 2×2 matrix consisting of 4 elements with at least two correct elements which can be simplified or un-simplified	M1
		Correct ***un-simplified*** matrix for **AB**	A1
	$= \begin{pmatrix} -k & 12+2k \\ 13 & -4 \end{pmatrix}$		**(2)**
(b)	$\{\det(\mathbf{AB}) = 0 \Rightarrow\}$		
	$(-k)(-4) - 13(12+2k) = 0$ $\Rightarrow 4k - 156 - 26k = 0$ $\Rightarrow -22k = 156$ $\Rightarrow k = -\frac{156}{22}$ or $-\frac{78}{11}$ or $-7\frac{1}{11}$	Applies $"ad - bc" = 0$ on their 2×2 matrix for **AB** and solves the resulting equation to give $k = \ldots$	M1
		$k = -\frac{156}{22}$ or $-\frac{78}{11}$ or $-7\frac{1}{11}$ Accept any exact equivalent form for k Condone $-7.\dot{0}\dot{9}$	A1
			(2)
			4
	Question 2 Notes		
2. (a)	**Note**	Give A1 (ignore subsequent working) for a correct un-simplified answer which is later followed by an incorrect simplified answer.	
(b)	**Note**	Give M1A1 for sight of the correct answer in part (b).	
	Note	Condone the sign error in applying $\ldots - 13(12+2k) = 0$ to give $\ldots - 156 + 26k = 0$ (o.e.) E.g. Allow M1 for $\begin{vmatrix} -k & 12+2k \\ 13 & -4 \end{vmatrix} = 0 \Rightarrow 4k - 156 + 26k = 0 \Rightarrow k = \ldots$	
	Note	Give final A0 for -7.0 or -7.1 or -7.09 **without reference** to $-\frac{156}{22}$ or $-\frac{78}{11}$ or $-7\frac{1}{11}$	

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3. Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

(5)

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Question Number	Scheme	Notes	Marks
3.	Required to prove by induction the result $\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}, \; n \in \mathbb{Z}^+$		
Way 1	$n=1$: LHS $= \frac{1}{3}$, RHS $= \frac{1}{2} - \frac{1}{(2)(3)} = \frac{1}{3}$	Shows or states LHS $= \frac{1}{3}$ **and shows either** RHS $= \frac{1}{2} - \frac{1}{(1+1)(2+1)} = \frac{1}{3}$ **or** RHS $= \frac{1}{2} - \frac{1}{(2)(3)} = \frac{1}{3}$ **or** RHS $= \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$	B1
	(Assume the result is true for $n=k$)		
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= \frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+2)(k+3)}$		
	$= \frac{1}{2} - \frac{(k+3)}{(k+1)(k+2)(k+3)} + \frac{2}{(k+1)(k+2)(k+3)}$ **or** $= \frac{1}{2} - \left(\frac{(k+3)-2}{(k+1)(k+2)(k+3)}\right)$	**dependent on the previous M mark** Makes $(k+1)(k+2)(k+3)$ a common denominator for their second and third fractions	dM1
	$= \frac{1}{2} - \frac{1}{(k+2)(k+3)}$	Obtains $\frac{1}{2} - \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} - \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 **cso**
	Final A1 is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.		**(5)**
			5
Way 2	**The M1dM1A1 marks for Alternative Way 2**		
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= \frac{(k+1)(k+2)(k+3) - 2(k+3) + 2(2)}{2(k+1)(k+2)(k+3)}$	**dependent on the previous M mark** Makes $2(k+1)(k+2)(k+3)$ a common denominator for their three fractions	dM1
	$= \frac{k^3+6k^2+9k+4}{2(k+1)(k+2)(k+3)} = \frac{(k+1)(k^2+5k+4)}{2(k+1)(k+2)(k+3)} = \frac{k^2+5k+4}{2(k+2)(k+3)} = \frac{(k+2)(k+3)-2}{2(k+2)(k+3)}$		
	$= \frac{1}{2} - \frac{1}{(k+2)(k+3)}$	Obtains $\frac{1}{2} - \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} - \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1

		Question 3 Notes
3.	**Note**	LHS = RHS by itself or LHS = RHS = $\frac{1}{3}$ is not sufficient for the 1st B1 mark.
	Note Way 2	The 1st A1 can be obtained by e.g. using algebra to show that $\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)}$ gives $\frac{(k^2+5k+4)}{2(k+2)(k+3)}$ and by using algebra to show that $\frac{1}{2} - \frac{1}{(k+2)(k+3)}$ also gives $\frac{(k^2+5k+4)}{2(k+2)(k+3)}$
	Note	Moving from $\frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+2)(k+3)}$ to $\frac{1}{2} - \frac{1}{(k+2)(k+3)}$ ***with no intermediate working*** is 2nd M0 1st A0 2nd A0.

Way 3	**The M1dM1A1 marks for Alternative Way 3**		
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$	Adds the $(k+1)^{th}$ term to the sum of k terms	M1
	$= \frac{1}{2} - \frac{1}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} - \frac{1}{(k+2)(k+3)}$	**dependent on the previous M mark** This step must be seen in Way 3	dM1
	$= \frac{1}{2} - \frac{1}{(k+2)(k+3)}$	Obtains $\frac{1}{2} - \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} - \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1

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4. The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t}$$

The straight line with equation $3y - 2x = 10$ intersects H at the points A and B.

Given that the point A is above the x-axis,

(a) find the coordinates of the point A and the coordinates of the point B.

(5)

(b) Find the coordinates of the midpoint of AB.

(2)

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Question Number	Scheme	Notes	Marks
4. (a) **Way 1**	$\left\{x = 4t,\ y = \frac{4}{t} \Rightarrow\right\}\ 3\left(\frac{4}{t}\right) - 2(4t) = 10$	Substitutes $x = 4t$ **and** $y = \frac{4}{t}$ into the printed equation to obtain an equation in t only	M1
	$8t^2 + 10t - 12 = 0$ or $4t^2 + 5t - 6 = 0$ **(*can be implied*)**	A correct 3 term quadratic **Note:** E.g. $12 - 8t^2 = 10t$, $8t^2 + 10t - 12\{=0\}$ or $8t^2 + 10t = 12$ are acceptable for this mark	A1
	$(8t - 6)(t + 2) = 0 \Rightarrow t = ...$ or $(4t - 3)(2t + 4) = 0 \Rightarrow t = ...$ or $(4t - 3)(t + 2) = 0 \Rightarrow t = ...$	**dependent on the previous M mark** Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find $t = ...$	dM1
	• $x = 4\left(\frac{3}{4}\right) = 3$ **and** $y = \frac{4}{\left(\frac{3}{4}\right)} = \frac{16}{3}$ • $x = 4(-2) = -8$ **and** $y = \frac{4}{(-2)} = -2$	**dependent on both the previous M marks** Correct substitution at least one of their values for t into the given parametric equations **and** obtains ***two sets*** of corresponding values for $x = ...$ **and** $y = ...$	ddM1
	$A\left(3, \frac{16}{3}\right), B(-8, -2)$ **or** $A{:}\ x = 3,\ y = \frac{16}{3}$ **and** $B{:}\ x = -8,\ y = -2$	Identifies the correct coordinates for A and B	A1 **cao**
			(5)
(a) **Way 2**	$x\left(\frac{10 + 2x}{3}\right) = 16$ $\left(\frac{3y - 10}{2}\right)y = 16$ $3\left(\frac{16}{x}\right) - 2x = 10$ $3y - 2\left(\frac{16}{y}\right) = 10$	**Either** substitutes their rearranged $3y - 2x = 10$ into $xy = k$ **or** substitutes either $y = \frac{k}{x}$ or $x = \frac{k}{y}$, $k \neq 0$, into $3y - 2x = 10$ to form an equation in either x only or y only	M1
	$2x^2 + 10x - 48 = 0$ or $x^2 + 5x - 24 = 0$ or $\frac{2}{3}x^2 + \frac{10}{3}x - 16 = 0$ or $\frac{3}{2}y^2 - 5y - 16 = 0$ or $3y^2 - 10y - 32 = 0$ **(*can be implied*)**	A correct 3 term quadratic **Note:** $10x + 2x^2 = 48$, $3y^2 - 10y = 32$ or $x^2 + 5x - 24\{=0\}$ are acceptable for this mark	A1
	e.g. $(2x + 16)(x - 3) = 0 \Rightarrow x = ...$ or $(x + 8)(x - 3) = 0 \Rightarrow x = ...$ or $(3y - 16)(y + 2) = 0 \Rightarrow y = ...$	**dependent on the previous M mark** Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find either $x = ...$ or $y = ...$	dM1
	E.g. $x = 3 \Rightarrow y = \frac{16}{3}$ $x = -8 \Rightarrow y = \frac{16}{-8} = -2$	**dependent on both the previous M marks.** Correct substitution of at least one of their values for x or y into either $3y - 2x = 10$ **or** their rearranged $3y - 2x = 10$ or $y = \frac{k}{x}$ or $x = \frac{k}{y}$, $k \neq 0$, **and** obtains ***two sets*** of corresponding values for $x = ...$ **and** $y = ...$	ddM1
	$A\left(3, \frac{16}{3}\right), B(-8, -2)$ **or** $A{:}\ x = 3,\ y = \frac{16}{3}$ **and** $B{:}\ x = -8,\ y = -2$	Identifies the correct coordinates for A and B	A1 **cao**
			(5)
(b)	$\left(\frac{3 + (-8)}{2}, \frac{\frac{16}{3} + (-2)}{2}\right); = \left(-\frac{5}{2}, \frac{5}{3}\right)$	Uses their (x_1, y_1) and (x_2, y_2) from part (a) to apply $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ o.e.	M1;
		Correct answer	A1
			(2)
			7

		Question 4 Notes
4. (a)	**SC**	If the two previous M marks have been gained then award Special Case ddM1 for finding their correct points by writing either $x=3,\ y=\frac{16}{3}$ **or** $x=-8,\ y=-2$ **or** $\left(3, \frac{16}{3}\right)$ **or** $\left(-8, -2\right)$
	Note	A decimal answer of e.g. $A\left(3, 5.33\right),\ B\left(-8, -2\right)$ (without a correct exact answer) is 2nd A0
	Note	**<u>Writing coordinates the wrong way round</u>** E.g. writing $x=3,\ y=\frac{16}{3}$ **and** $x=-8,\ y=-2$ followed by $A\left(\frac{16}{3}, 3\right),\ B\left(-8, -2\right)$ is 2nd A0
	Note	Imply the dM1 mark for ***writing down*** the ***correct*** roots for ***their*** quadratic equation. E.g. • $2x^2+10x-48=0$ or $x^2+5x-24=0$ or $\frac{2}{3}x^2+\frac{10}{3}x=16 \rightarrow x=3, -8$ • $\frac{3}{2}y^2-5y-16=0$ or $3y^2-10y-32=0 \rightarrow y=\frac{16}{3}, -2$ • $8t^2+10t=12$ or $4t^2+5t-6=0 \rightarrow t=\frac{3}{4}, -2$
	Note	For example, give dM0 for • $8t^2+10t=12$ or $4t^2+5t-6=0 \rightarrow t=\frac{1}{4}, -2$ [incorrect solution] with no intermediate working.
	Note	You can also imply the 1st A1 dM1 marks for either • $x\left(\frac{10+2x}{3}\right)=16$ or $3\left(\frac{16}{x}\right)-2x=10 \rightarrow x=3, -8$ • $\left(\frac{3y-10}{2}\right)y=16$ or $3y-2\left(\frac{16}{y}\right)=10 \rightarrow y=\frac{16}{3}, -2$ • $3\left(\frac{4}{t}\right)-2(4t)=10 \rightarrow x=3, -8$ • $3\left(\frac{4}{t}\right)-2(4t)=10 \rightarrow y=\frac{16}{3}, -2$ with no intermediate working.
	Note	You can imply the 1st A1 dM1 ddM1 marks for either • $x\left(\frac{10+2x}{3}\right)=16$ or $3\left(\frac{16}{x}\right)-2x=10 \rightarrow x=3, -8$ **and** $y=\frac{16}{3}, -2$ • $3\left(\frac{4}{t}\right)-2(4t)=10 \rightarrow x=3, -8$ **and** $y=\frac{16}{3}, -2$ with no intermediate working. You can then imply the final A1 mark if they correctly identify the correct pairs of values or coordinates which relate to the point *A* and the point *B*.
	Note	Give 2nd A0 for a final answer of **both** $A\left(3, \frac{16}{3}\right),\ B\left(-8, -2\right)$ **and** $A\left(-8, -2\right),\ B\left(3, \frac{16}{3}\right)$,
(b)	**Note**	A decimal answer of e.g. $\left(-2.5, 1.67\right)$ (without a correct exact answer) is A0
	Note	Allow A1 for $\left(-\frac{5}{2}, \frac{10}{6}\right)$ or $\left(-2\frac{1}{2}, -1\frac{2}{3}\right)$ or exact equivalent.

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5. $$f(x) = 30 + \frac{7}{\sqrt{x}} - x^5, \qquad x > 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval [2, 2.1].

(a) Starting with the interval [2, 2.1], use interval bisection twice to find an interval of width 0.025 that contains α.

(4)

(b) Taking 2 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(5)

Question Number	Scheme	Notes	Marks
5.	Given $f(x) = 30 - \frac{7}{\sqrt{x}} - x^5$, $x > 0$ and root of $f(x) = 0$ lies in the interval $[2, 2.1]$		
(a) **Way 1**	$f(2) = 2.9497...$ **or** $f(2.1) = -6.0105...$	Attempts to evaluate ***at least one*** of $f(2)$ ***or*** $f(2.1)$ **and** evaluates $f(2.05)$	M1
	$f(2.05) = -1.3160...$	$f(2)$ ***or*** $f(2.1)$ correct awrt (or truncated) to 1 sf **and** $f(2.05)$ correct awrt (or truncated) to 1 sf	A1
	$f(2.025) = ...$	**dependent on the previous M mark** Evaluates $f(2.025)$ (and not $f(2.075)$)	dM1
	$f(2.025) = 0.86846...$ so interval is $(2.025, 2.05)$ or $(2.025, 2.050)$	$f(2.025)$ correct awrt (or truncated) to 1 sf **and** correct interval. Allow $2.025 \leqslant x \leqslant 2.05$ or $2.025 < x < 2.05$ or $2.025 \leqslant \alpha \leqslant 2.05$ or $2.025 < \alpha < 2.05$ or $[2.025, 2.05]$ or $(2.025, 2.05)$ equivalent in words. Condone $2.025 - 2.05$. Allow a mixture of "ends". Do not allow incorrect statements such as $2.05 < \alpha < 2.025$ or $(2.05, 2.025)$ or $2.05 - 2.025$ unless they are recovered. Ignore the subsequent iteration of $f(2.0375)$	A1
	Note that some candidates only indicate the sign of f and not its value. In this case the M marks can still score as defined but not the A marks.		**(4)**
(a) **Way 2**	**Common approach in the form of a table (use the mark scheme above)**		

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	2.9497...	2.1	$-6.0105...$	2.05	$-1.3160...$
2	2.9497...	2.05	$-1.3160...$	2.025	0.86846...

so interval is $2.025 < \alpha < 2.05$ **would score full marks in part (a)**

Question Number	Scheme	Notes	Marks
(b)	$f'(x) = -\frac{7}{2}x^{-\frac{3}{2}} - 5x^4$	At least one of either $-\frac{7}{\sqrt{x}} \to \pm Ax^{-\frac{3}{2}}$ or $-x^5 \to \pm Bx^4$ where A and B are non-zero constants.	M1
		At least one of either $-\frac{7}{2}x^{-\frac{3}{2}}$ or $-5x^4$ simplified or un-simplified	A1
		Correct differentiation simplified or un-simplified	A1
	$\left\{\alpha \simeq 2 - \frac{f(2)}{f'(2)}\right\} \Rightarrow \alpha \simeq 2 - \frac{2.949747468...}{-81.23743687...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $f(2)$ and $f'(2)$	dM1
	$\{\alpha = 2.036310199...\} \Rightarrow \alpha = 2.04$ (2 dp)	**dependent on all 4 previous marks** 2.04 on their first iteration (Ignore any subsequent iterations)	A1 **cso cao**
	Correct differentiation followed by a correct answer of 2.04 scores full marks in part (b) **Correct answer with <u>no</u> working scores no marks in part (b)**		**(5)**
			9
	Question 5 Notes		

5. (a)	**Note**	Give 2nd M0 for evaluating both $f(2.025)$ and $f(2.075)$
	Note	Do not allow "interval = $f(2.025)$ to $f(2.05)$" unless recovered.
	Note	A method of evaluating $f(2.05)$ followed by $f(2.025)$ with ***no evidence*** of evaluating ***at least one of either*** $f(2)$ or $f(2.1)$ is M0A0M0A0

		Question 5 Notes Continued
5. (b)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	Final dM1	This mark can be implied by applying at least one correct ***value*** of either $\text{f}(2)$ or $\text{f}'(2)$ in $2 - \frac{\text{f}(2)}{\text{f}'(2)}$. So just $2 - \frac{\text{f}(2)}{\text{f}'(2)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	You can imply the M1A1A1 marks for algebraic differentiation for either • $\text{f}'(2) = -\frac{7}{2}(2)^{-\frac{3}{2}} - 5(2)^4$ • $\text{f}'(2)$ applied correctly in $\alpha \simeq 2 - \dfrac{30 - 7(2)^{-\frac{1}{2}} - (2)^5}{-\frac{7}{2}(2)^{-\frac{3}{2}} - 5(2)^4}$
	Note	**Differentiating INCORRECTLY to give** $\text{f}'(x) = -\frac{7}{2}x^{-2} - 5x^4$ leads to $\alpha \simeq 2 - \frac{2.949747468...}{-81.75} = 2.036082538... = 2.04$ (2 dp) **This response should be awarded M1A1A0M1A0**

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6. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{n}{a}(n+1)(n+2)(3n+b)$$

where a and b are integers to be found.

(4)

(b) Hence find the value of

$$\sum_{r=25}^{49} (r^2(r+1)+2)$$

(4)

Question Number	Scheme	Notes	Marks
6. (a)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	{**Note:** Let $f(n) = \frac{1}{12}n(n+1)(n+2)(3n+1)$ ***or their*** answer to part (a).}	
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	Attempts to expand $r^2(r+1)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{12}n(n+1)\left[3n(n+1) + 2(2n+1)\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both standard formulae.	dM1
	$= \frac{1}{12}n(n+1)\left[3n^2 + 7n + 2\right]$	{this step does not have to be written}	
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	Correct completion with no errors. **Note:** $a = 12, b = 1$	A1 **cso**
			(4)
(b) **Way 1**	$\left\{\sum_{r=25}^{49} r^2(r+1)\right\}$	Attempts to find either $f(49) - f(24)$ or $f(49) - f(25)$. This mark can be implied.	M1
	$= \left(\frac{1}{12}(49)(50)(51)(148)\right) - \left(\frac{1}{12}(24)(25)(26)(73)\right)$ $\{= 1541050 - 94900 = 1446150\}$	Correct numerical expression for $f(49) - f(24)$ which can be simplified or un-simplified. **Note:** This mark can be implied by seeing 1446150	A1
	$\left\{\sum_{r=25}^{49}\left(r^2(r+1) + 2\right)\right\}$ $= "1446150" + 25(2); = 1446200$	Adds $25(2)$ or equivalent to their $\sum_{r=25}^{49} r^2(r+1)$ or clear evidence that $\sum_{r=25}^{49} 2 = 2(49) - 2(24)$ or 50	M1
		1446200	A1 **cao**
			(4)
(b) **Way 2**	$\left\{\sum_{r=25}^{49}\left(r^2(r+1) + 2\right)\right\} = \left(\underline{\underline{\frac{1}{12}(49)(50)(51)(148)}} + \underline{2(49)}\right) - \left(\underline{\underline{\frac{1}{12}(24)(25)(26)(73)}} + \underline{2(24)}\right)$ $= (\underline{\underline{1541050}} + \underline{98}) - (\underline{\underline{94900}} + \underline{48}) = 1541148 - 94948 = 1446200$		
	Attempts to find either $\underline{\underline{f(49) - f(24)}}$ or $\underline{\underline{f(49) - f(25)}}$		M1
	Correct numerical expression for $f(49) - f(24)$ which can be simplified or un-simplified. **Note:** This mark can be implied by $(\underline{\underline{1541050}} + ...) - (\underline{\underline{94900}} + ...)$ or $1541148 - 94948$		A1
	Adds 50 or equivalent to their $\sum_{r=25}^{49} r^2(r+1)$ or clear evidence that $\sum_{r=25}^{49} 2 = 2(49) - 2(24)$ or 50. **Note:** This mark can be implied by $(\underline{...} + \underline{2(49)}) - (\underline{...} + \underline{2(24)})$ or $1541148 - 94948$		M1
	1446200		A1 **cao**
			(4)
			8

Question Number	Scheme	Notes	Marks
6. (b) **Way 3**	$\left\{\sum_{r=25}^{49}\left(r^2(r+1)+2\right)\right\} = \sum_{r=25}^{49} r^3 + \sum_{r=25}^{49} r^2 + \sum_{r=25}^{49} 2$ $= \left(\frac{1}{4}(49)^2(50)^2 - \frac{1}{4}(24)^2(25)^2\right) + \left(\frac{1}{6}(49)(50)(99) - \frac{1}{6}(24)(25)(49)\right) + (98 - 48)$ $= (1500625 - 90000) + (40425 - 4900) + 50 = 1410625 + 35525 + 50 = 1446200$ **or** $= \sum_{r=25}^{49}\left(r^3 + r^2 + 2\right)$ $= \left(\frac{1}{4}(49)^2(50)^2 + \frac{1}{6}(49)(50)(99) + 2(49)\right) - \left(\frac{1}{4}(24)^2(25)^2 + \frac{1}{6}(24)(25)(49) + 2(24)\right)$ $= (1500625 + 40425 + 98) - (90000 + 4900 + 48) = 1541148 - 94948 = 1446200$		
	Attempts to find either $f(49) - f(24)$ or $f(49) - f(25)$		M1
	Correct numerical expression for $f(49) - f(24)$ which can be simplified or un-simplified.		A1
	Adds 50 or equivalent to their $\sum_{r=25}^{49} r^2(r+1)$ or clear evidence that $\sum_{r=25}^{49} 2 = 2(49) - 2(24)$ or 50		M1
	1446200		A1 **cao**
			(4)

	Question 6 Notes	
6. (a)	**Note**	Applying e.g. $n=1,\ n=2$ to the printed equation without applying the standard formulae to give $a=12,\ b=1$ is M0A0M0A0
	Alt 1	**Alt Method 1:** Using $\frac{1}{4}n^4 + \frac{5}{6}n^3 + \frac{3}{4}n^2 + \frac{1}{6}n \equiv \frac{3}{a}n^4 + \frac{(9+b)}{a}n^3 + \frac{(6+3b)}{a}n^2 + \frac{2b}{a}n$ o.e.
	dM1	Equating coefficients to find both $a = ...$ and $b = ...$ **and** at least one of $a=12,\ b=1$
	A1 cso	Finds $a=12,\ b=1$ and demonstrates the identity works for all of its terms.
	Alt 2	**Alt Method 2:** $\frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1) \equiv \frac{1}{a}n(n+1)(n+2)(3n+b)$
	dM1	Substitutes $n=1,\ n=2,$ into this identity o.e. to find both $a = ...$ and $b = ...$ **and** at least one of $a=12,\ b=1$
	A1	Finds $a=12,\ b=1$
	Note	Allow final dM1A1 for $\frac{1}{4}n^4 + \frac{5}{6}n^3 + \frac{3}{4}n^2 + \frac{1}{6}n$ or $\frac{1}{12}n(3n^3 + 10n^2 + 9n + 2)$ or $\frac{1}{12}(3n^4 + 10n^3 + 9n^2 + 2n) \rightarrow \frac{1}{12}n(n+1)(n+2)(3n+1)$ from no incorrect working.

	Question 6 Notes Continued	
6. (b)	**Note**	Give 1st M1 1st A0 for applying $\mathrm{f}(49) - \mathrm{f}(25)$. i.e. $1541050 - 111150\{= 1429900\}$
	Note	You cannot follow through their incorrect answer from part (a) for the 1st A1 mark.
	Note	Give M1A0M1A0 for applying $\left[\mathrm{f}(49) + 2(49)\right] - \left[\mathrm{f}(25) + 2(24)\right]$ i.e. $1541148 - 111198\{= 1429950\}$
	Note	Give M1A0M0A0 for applying $\left[\mathrm{f}(49) + 2(49)\right] - \left[\mathrm{f}(25) + 2(25)\right]$ i.e. $1541148 - 111200\{= 1429948\}$
	Note	Give 1st M0 1st A0 for applying $(49)^2(50) - (24)^2(25) = 120050 - 14400 = 105650$
	Note	Give 1st M0 1st A0 for applying $(49)^2(50) - (25)^2(26) = 120050 - 16250 = 103800$
	Note	Give M0A0M0A0 for listing individual terms. e.g. $16250 + 18252 + \dots + 112896 + 120050 = 1446200$
	Note	Give 2nd M0 for lack of bracketing in $\frac{1}{12}(49)(50)(51)(148) + 2(49) - \frac{1}{12}(24)(25)(26)(73) + 2(24)$ unless recovered
	Note	Give M0A0M0A0 for writing down 1446200 without any working.
	Note	Applying $\mathrm{f}(49) - \mathrm{f}(24)$ for $\frac{1}{4}n(n+1)(n+2)(3n+1)$ is $4623150 - 284700 = 4338450$ is 1st M1 1st A0

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7.

$$f(z) = z^4 + 4z^3 + 6z^2 + 4z + a$$

where a is a real constant.

Given that $1 + 2i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation.

(1)

(b) (i) Hence, find the other roots of the equation $f(z) = 0$

(ii) State the value of a.

(7)

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Question Number	Scheme	Notes	Marks
7.	$\mathrm{f}(z) = z^4 + 4z^3 + 6z^2 + 4z + a$, a is a real constant. $z_1 = 1 + 2\mathrm{i}$ satisfies $\mathrm{f}(z) = 0$		
(a)	$\{z_2 =\}\ 1 - 2\mathrm{i}$	$1 - 2\mathrm{i}$	B1
			(1)
(b)(i)	$z^2 - 2z + 5$	Attempt to expand $(z - (1+2\mathrm{i}))(z - (1-2\mathrm{i}))$ or $(z - (1+2\mathrm{i}))(z - (\text{their complex } z_2))$ or any valid method ***to establish a quadratic factor*** e.g. $z = 1 \pm 2\mathrm{i} \Rightarrow z - 1 = \pm 2\mathrm{i} \Rightarrow z^2 - 2z + 1 = -4$ or sum of roots 2, product of roots 5 to give $z^2 \pm (\text{their sum})z + (\text{their product})$	M1
		$z^2 - 2z + 5$	A1
	$\mathrm{f}(x) = (z^2 - 2z + 5)(z^2 + 6z + 13)$	Attempts to find the other quadratic factor. e.g. using long division to obtain either $z^2 \pm kz + ..., k \neq 0$ or $z^2 \pm \alpha z + \beta, \beta \neq 0, \alpha$ can be 0 or factorising e.g. $\mathrm{f}(z) = (z^2 - 2z + 5)(z^2 \pm kz \pm c), k \neq 0$ or $\mathrm{f}(z) = (z^2 - 2z + 5)(z^2 \pm \alpha z \pm \beta), \beta \neq 0, \alpha$ can be 0	M1
		$z^2 + 6z + 13$	A1
	$\{z^2 + 6z + 13 = 0 \Rightarrow\}$		
	Either • $z = \dfrac{-6 \pm \sqrt{36 - 4(1)(13)}}{2(1)}$ • $(z+3)^2 - 9 + 13 = 0 \Rightarrow z = ...$	**dependent on only the previous M mark** Correct method of applying the quadratic formula or completing the square for solving a 3TQ on their 2nd quadratic factor	dM1
	$\{z =\}\ -3 + 2\mathrm{i}, -3 - 2\mathrm{i}$	$-3 + 2\mathrm{i}$ **and** $-3 - 2\mathrm{i}$	A1
			(6)
(ii)	$\{a =\}\ 65$	65 or $a = 65$ stated anywhere in (b)	B1
			(1)
			8

Question 7 Notes

7. (b)(i)	**Note**	No working leading to $x = -3 + 2\mathrm{i}, -3 - 2\mathrm{i}$ is M0A0M0A0M0A0.
	Note	You can assume $x \equiv z$ for solutions in this question.
	Note	Give dM1A1 for $z^2 + 6z + 13 = 0 \Rightarrow z = -3 + 2\mathrm{i}, -3 - 2\mathrm{i}$ with no intermediate working.
	Note	<u>**Special Case:**</u> If their second ***3 term quadratic*** factor **can** be factorised then give Special Case dM1 for correct factorisation leading to $z = ...$
	Note	Otherwise, give 3rd dM0 for applying a method of factorising to solve their 3TQ.
	Note	**Reminder:** Method Mark for solving a 3TQ, "$az^2 + bz + c = 0$" **Formula:** Attempt to use the correct formula (with values for *a*, *b* and *c*) **Completing the square** $\left(z \pm \dfrac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $z = ...$

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8. The parabola C has cartesian equation $y^2 = 36x$. The point $P(9p^2, 18p)$, where p is a positive constant, lies on C.

(a) Using calculus, show that an equation of the tangent to C at P is

$$py - x = 9p^2$$

(4)

This tangent cuts the directrix of C at the point $A(-a, 6)$, where a is a constant.

(b) Write down the value of a.

(1)

(c) Find the exact value of p.

(3)

(d) Hence find the exact coordinates of the point P, giving each coordinate as a simplified surd.

(3)

Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 36x,\ P(9p^2, 18p)$ lies on C, where p is a constant.		
(a)	$y = 6x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(6)x^{-\frac{1}{2}} = \frac{3}{\sqrt{x}}$	$\frac{dy}{dx} = \pm kx^{-\frac{1}{2}}$	M1
	$y^2 = 36x \Rightarrow 2y\frac{dy}{dx} = 36$	$py\frac{dy}{dx} = q$	
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 18\left(\frac{1}{18p}\right)$	their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dx}{dt}}$	
	So at P, $m_T = \frac{1}{p}$	Correct calculus work leading to $m_T = \frac{1}{p}$	A1
	$y - 18p = \frac{1}{p}(x - 9p^2)$ **or** $y = \frac{1}{p}x + 9p$	Correct straight line method for an equation of a **tangent** where $m_T \left(\neq m_N\right)$ is found by using calculus. **Note:** m_T must be a function of p	M1
	leading to $py - x = 9p^2$ (*)	Correct solution only	A1 *
			(4)
(b)	(Directrix: $x = -9 \Rightarrow$) $a = 9$	$a = 9$ **or** $a = 9$ stated anywhere in this question	B1
			(1)
(c)	Tangent goes through $(-a, 6) \Rightarrow$		
	$6p + 9 = 9p^2$	Substitutes their value $x = -$"a" or their value $x =$ "a" and $y = 6$ into either $py - x = 9p^2$ **or** $py - x = -9p^2$	M1
	$9p^2 - 6p - 9 = 0$ or $3p^2 - 2p - 3 = 0$		
	E.g. $p = \frac{6 \pm \sqrt{36 - 4(9)(-9)}}{2(9)}$	**dependent on the previous M mark** Correct method of solving their 3TQ	dM1
	{as $p > 0$} $p = \frac{1+\sqrt{10}}{3}$	$p = \frac{1+\sqrt{10}}{3}$ or $\frac{6+\sqrt{360}}{18}$ or $\frac{6+6\sqrt{10}}{18}$ etc.	A1
	Note: Give A0 for giving two values for p as their answer to part (c)		**(3)**
(d)	$x = 9\left(\frac{1+\sqrt{10}}{3}\right)^2,\ y = 18\left(\frac{1+\sqrt{10}}{3}\right)$	Uses a **real** value of p, which is the result of substituting $(\pm a, 6)$ into $py - x = \pm 9p^2$, and substitutes p into at least one of either $x = 9p^2$ or $y = 18p$	M1
	$\left(11 + 2\sqrt{10},\ 6 + 6\sqrt{10}\right)$ **or** $\left(11 + 2\sqrt{10},\ 6\left(1+\sqrt{10}\right)\right)$	Either $x = 11 + 2\sqrt{10}$ or $y = 6 + 6\sqrt{10}$ or $y = 6\left(1+\sqrt{10}\right)$	A1
		Correct coordinates of P. Condone $x = ...,\ y = ...$	A1
	Note: Give 2nd A0 for two sets of coordinates for P		**(3)**
			11

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9. $$z = \frac{1}{5} - \frac{2}{5}\text{i}$$

(a) Find the modulus and the argument of z, giving the modulus as an exact answer and giving the argument in radians to 2 decimal places.

(3)

Given that

$$zw = \lambda\text{i}$$

where λ is a real constant,

(b) find w in the form $a + \text{i}b$, where a and b are real. Give your answer in terms of λ.

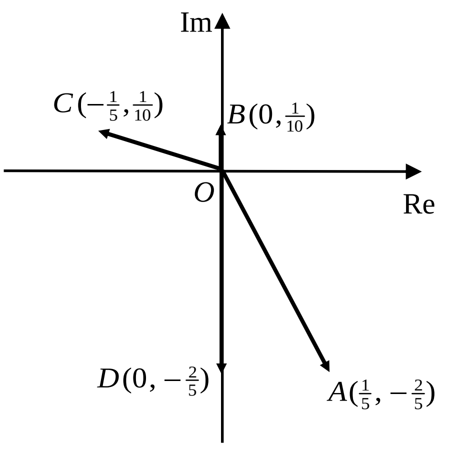
(3)

(c) Given that $\lambda = \frac{1}{10}$

(i) find $\frac{4}{3}(z + w)$,

(ii) plot the points A, B, C and D, representing z, zw, w and $\frac{4}{3}(z + w)$ respectively, on a single Argand diagram.

(4)

Question Number	Scheme	Notes	Marks
9. (a)	$\left\{\lvert z\rvert =\right\}\ \sqrt{\left(\frac{1}{5}\right)^2+\left(-\frac{2}{5}\right)^2}$; $=\frac{\sqrt{5}}{5}$ or $\frac{1}{\sqrt{5}}$ or $\sqrt{\frac{1}{5}}$	$\sqrt{\left(\frac{1}{5}\right)^2+\left(-\frac{2}{5}\right)^2}$ or $\sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{2}{5}\right)^2}$ which can be implied.	M1
		Correct **exact** answer	A1
	$\left\{\arg z = \arctan(-2) = -1.107148718...\right\} = -1.11$ (2 dp)	-1.11 **cao** or 5.18 **cao**	B1
			(3)
(b) **Way 1**	$w=\frac{/\,\mathrm{i}}{z}=\frac{/\,\mathrm{i}}{(\frac{1}{5}-\frac{2}{5}\mathrm{i})}$ **or** $w=\frac{5/\,\mathrm{i}}{5z}=\frac{5/\,\mathrm{i}}{(1-2\mathrm{i})}$	Correct method of making *w* the subject and substituting for *z*	M1
	$=\frac{/\,\mathrm{i}(\frac{1}{5}+\frac{2}{5}\mathrm{i})}{(\frac{1}{5}-\frac{2}{5}\mathrm{i})(\frac{1}{5}+\frac{2}{5}\mathrm{i})}$ $=\frac{-\frac{2}{5}+\frac{1}{5}/\,\mathrm{i}}{\frac{1}{25}+\frac{4}{25}}$ **or** $=\frac{5/\,\mathrm{i}(1+2\mathrm{i})}{(1-2\mathrm{i})(1+2\mathrm{i})}$ $=\frac{-10/+5/\,\mathrm{i}}{1+4}$	**dependent on the previous M mark** Multiplies numerator and denominator of right hand side by $(\frac{1}{5}+\frac{2}{5}\mathrm{i})$ or $(1+2\mathrm{i})$ to give an expression in terms of / which contains a real denominator	dM1
	$=-2/+/\,\mathrm{i}$ **or** $=-2/+/\,\mathrm{i}$	$-2/+/\,\mathrm{i}$ or $/\,\mathrm{i}-2/$	A1
			(3)
(b) **Way 2**	$(\frac{1}{5}-\frac{2}{5}\mathrm{i})(a+b\mathrm{i})=/\,\mathrm{i}$ Þ $\frac{1}{5}a+\frac{1}{5}b\mathrm{i}-\frac{2}{5}a\mathrm{i}+\frac{2}{5}b=/\,\mathrm{i}$ $\frac{1}{5}a+\frac{2}{5}b=0$ or $-\frac{2}{5}a+\frac{1}{5}b=/$	Substitutes *z* and *w* into $zw=/\,\mathrm{i}$, expands *zw* and attempts to equate either the real part of the imaginary part of the resulting equation.	M1
	$\frac{1}{5}a+\frac{2}{5}b=0$, $-\frac{2}{5}a+\frac{1}{5}b=/$ Þ $a=...$ or $b=...$	**dependent on the previous M mark** Obtains an equation in terms of *a* and *b* and obtains a second equation in terms of *a*, *b* and / and solves them simultaneously to give at least one of $a=...$ or $b=...$	dM1
	$\left\{a=-2/,\ b=/\ \text{Þ}\right\}\ w=-2/+/\,\mathrm{i}$	$-2/+/\,\mathrm{i}$ or $/\,\mathrm{i}-2/$	A1
			(3)
(c)	$\left\{\frac{4}{3}(z+w)=\right\}\ \frac{4}{3}\left(\left(\frac{1}{5}-\frac{2}{5}\mathrm{i}\right)+\left(-\frac{2}{10}+\frac{1}{10}\mathrm{i}\right)\right)$; $=-\frac{2}{5}\mathrm{i}$	Substitutes *z*, / and their *w* into $\frac{4}{3}(z+w)$	M1
		$-\frac{2}{5}\mathrm{i}$ or $-\frac{6}{15}\mathrm{i}$ or $-0.4\mathrm{i}$ o.e.	A1
			(2)
(d)		**Criteria** • plots $(\frac{1}{5},-\frac{2}{5})$ in quadrant 4 • plots $(0,\frac{1}{10})$ on the positive imaginary axis • plots $(-\frac{1}{5},\frac{1}{10})$ in quadrant 2 • plots $(0,-\frac{2}{5})$ on the negative imaginary axis	
		Satisfies at least two of the four criteria	B1
		Satisfies all four criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1
			(2)
			10

	Question 9 Notes	
9. (a)	**Note**	M1 can be implied by awrt 0.45 or a truncated 0.44
	Note	Give A0 for 0.4472… **without reference** to $\frac{\sqrt{5}}{5}$ or $\frac{1}{\sqrt{5}}$ or $\sqrt{\frac{1}{5}}$
	Note	Give B0 for -1.11 **followed by a final answer of** 1.11
(b)	**Note**	**Be aware** that $\frac{1}{(\frac{1}{5}-\frac{2}{5}\mathrm{i})} = 1 + 2\mathrm{i}$

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10. *In your answers to this question, the elements of each matrix should be expressed in exact form in surds where necessary.*

The transformation U, represented by the 2×2 matrix **P**, is a rotation through 45° anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through 60° anticlockwise about the origin.

(b) Write down the matrix **Q**.

(1)

The transformation U followed by the transformation V is the transformation T.
The transformation T is represented by the matrix **R**.

(c) Use your matrices from parts (a) and (b) to find the matrix **R**.

(3)

(d) Give a full geometric description of T as a single transformation.

(2)

(e) Deduce from your answers to parts (c) and (d) that $\sin 75° = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$ and find the exact value of cos 75°, explaining your answers fully.

(2)

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Question Number	Scheme	Notes	Marks
10. (a)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	Correct matrix which is expressed in exact surds	B1
			(1)
(b)	$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct matrix which is expressed in exact surds	B1
			(1)
(c)	$\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \right\} \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} = \ldots$	Multiplies their matrix from part (a) by their matrix from part (b) **[either way round]** and finds at least one element in the resulting matrix	M1
	$= \begin{pmatrix} \frac{\sqrt{2}-\sqrt{6}}{4} & \frac{-\sqrt{2}-\sqrt{6}}{4} \\ \frac{\sqrt{2}+\sqrt{6}}{4} & \frac{\sqrt{2}-\sqrt{6}}{4} \end{pmatrix}$ or $\begin{pmatrix} \frac{1-\sqrt{3}}{2\sqrt{2}} & \frac{-1-\sqrt{3}}{2\sqrt{2}} \\ \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{1-\sqrt{3}}{2\sqrt{2}} \end{pmatrix}$	At least 3 correct exact elements	A1
		Correct exact matrix **Note:** Allow multiplication either way round	A1
			(3)
(d)	Rotation about (0, 0)	Rotation (condone turn) **and** about (0, 0) or about *O* or about the origin	B1
	105 degrees (anticlockwise)	105 degrees or $\frac{7\pi}{12}$ (anticlockwise) or 255 degrees clockwise or $\frac{17\pi}{12}$ clockwise	B1 o.e.
	Note: Give 2nd B0 for 105 degrees clockwise **Note:** Give B0B0 for combinations of transformations		**(2)**
(e)	Either • $\sin 75° = \sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ • $\sin 75° = \sin 105° = \frac{\sqrt{2}+\sqrt{6}}{4} = \frac{\sqrt{3}+1}{2\sqrt{2}}$	**dependent on the 1st A mark in part (c)** and states $\sin 75° = \sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}}$	dB1
	$\cos 75° = -\cos 105° = -\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)$ or $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$	States $\cos 75° = -\cos 105°$ and deduces a correct exact value for $\cos 75°$	B1
			(2)
			9

Question 10 Notes

10. (e)	**ALT 1**	Comparing their matrix found in part (c) with a correct $\begin{pmatrix} -\cos 75 & -\sin 75 \\ \sin 75 & -\cos 75 \end{pmatrix}$ (representing a rotation $105°$ anti-clockwise about *O*) gives	
		$\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ (with the 1st A mark scored in part (c))	B1
		$\cos 75° = -\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)$ or $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$	B1
			(2)