Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 15 January 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1.

$$f(x) = 3x^2 - \frac{5}{3\sqrt{x}} - 6, \quad x > 0$$

The single root α of the equation $f(x) = 0$ lies in the interval [1.5, 1.6].

(a) Taking 1.5 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

(b) Use linear interpolation once on the interval [1.5, 1.6] to find another approximation to α. Give your answer to 3 decimal places.

(3)

January 2018 WFM01 Further Pure Mathematics F1 Mark Scheme

Question Number	Scheme	Notes	Marks
1.	Given $\mathrm{f}(x) = 3x^2 - \dfrac{5}{3\sqrt{x}} - 6,\ x > 0$ and root, α, of $\mathrm{f}(x) = 0$ lies in the interval $[1.5, 1.6]$		
(a)	$\mathrm{f}'(x) = 6x + \dfrac{5}{6}x^{-\frac{3}{2}}$	At least one of either $3x^2 \to \pm Ax$ or $-\dfrac{5}{3\sqrt{x}} \to \pm Bx^{-\frac{3}{2}}$ where A and B are non-zero constants.	M1
		Correct differentiation which can be simplified or un-simplified	A1
	$\left\{\alpha \simeq 1.5 - \dfrac{\mathrm{f}(1.5)}{\mathrm{f}'(1.5)}\right\} \Rightarrow \alpha \simeq 1.5 - \dfrac{-0.6108276349...}{9.453609212...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $\mathrm{f}(1.5)$ and $\mathrm{f}'(1.5)$	dM1
	$\{\alpha = 1.564613167...\} \Rightarrow \alpha = 1.565$ (3 dp)	**dependent on all 3 previous marks** 1.565 on their first iteration (Ignore any subsequent iterations)	A1 **cso**
	Correct differentiation followed by a correct answer of 1.565 scores full marks in part (a) **Correct answer with <u>no</u> working scores no marks in part (a)**		**(4)**
(b)	**Either** • $\dfrac{\alpha - 1.5}{"0.6108276349..."} = \dfrac{1.6 - \alpha}{"0.3623843083..."}$ • $\dfrac{\alpha - 1.5}{1.6 - \alpha} = \dfrac{"0.6108276349..."}{"0.3623843083..."}$ • $\dfrac{\alpha - 1.5}{"0.6108276349..."} = \dfrac{1.6 - 1.5}{"0.3623843083..." + "0.6108276349..."}$	A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.	M1
	Either • $\alpha = \left(\dfrac{(1.6)("0.6108276349...") + (1.5)("0.3623843083...")}{"0.3623843083..." + "0.6108276349..."}\right)$ • $\alpha = 1.5 + \left(\dfrac{"0.6108276349..."}{"0.3623843083..." + "0.6108276349..."}\right)(0.1)$ • $\alpha = 1.5 + \left(\dfrac{"-0.6108276349..."}{"-0.3623843083..." + "-0.6108276349..."}\right)(0.1)$	**dependent on the previous M mark** Rearranges to make $\alpha = ...$	dM1
	$\{\alpha = 1.562764092...\} \Rightarrow \alpha = 1.563$ (3 dp)	1.563 (Ignore any subsequent iterations)	A1 **cao**
			(3)
(b) **Way 2**	$\dfrac{x}{"0.6108276349..."} = \dfrac{0.1 - x}{"0.3623843083..."} \Rightarrow x = \dfrac{(0.1)("0.6108276349...")}{0.9732119432...} = 0.062764092...$		
	$\alpha = 1.5 + 0.062764092...$	Finds x using a correct method of similar triangles and applies "1.5 + their x"	M1 dM1
	$\{\alpha = 1.562764092...\} \Rightarrow \alpha = 1.563$ (3 dp)	1.563	A1 **cao**
(b) **Way 3**	$\dfrac{0.1 - x}{"0.6108276349..."} = \dfrac{x}{"0.3623843083..."} \Rightarrow x = \dfrac{(0.1)("0.3623843083...")}{0.9732119432...} = 0.037235908...$		
	$\alpha = 1.6 - 0.037235908...$	Finds x using a correct method of similar triangles and applies "1.6 − their x"	M1 dM1
	$\{\alpha = 1.562764092...\} \Rightarrow \alpha = 1.563$ (3 dp)	1.563	A1 **cao**
			7

		Question 1 Notes
1. (a)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	dM1	This mark can be implied by applying at least one correct ***value*** of either $\text{f}(1.5)$ or $\text{f}'(1.5)$ to 1 significant figure in $1.5 - \frac{\text{f}(1.5)}{\text{f}'(1.5)}$. So just $1.5 - \frac{\text{f}(1.5)}{\text{f}'(1.5)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	You can imply the M1A1 marks for algebraic differentiation for either • $\text{f}'(1.5) = 6(1.5) + \frac{5}{6}(1.5)^{-\frac{3}{2}}$ • $\text{f}'(1.5)$ applied correctly in $\alpha \simeq 1.5 - \dfrac{3(1.5)^2 - \frac{5}{3}(1.5)^{-\frac{1}{2}} - 6}{6(1.5) + \frac{5}{6}(1.5)^{-\frac{3}{2}}}$
	Note	**Differentiating INCORRECTLY to give** $\text{f}'(x) = 6x - \frac{5}{6}x^{-2}$ leads to $\alpha \simeq 1.5 - \frac{-0.6108276349...}{9.3703703704...} = 1.565187139... = 1.565$ (3 dp) **This response should be awarded M1 A0 dM1 A0**
	Note	**Differentiating INCORRECTLY to give** $\text{f}'(x) = 6x - \frac{5}{6}x^{-\frac{3}{2}}$ leads to $\alpha \simeq 1.5 - \frac{-0.6108276349...}{8.546390788...} = 1.571471999... = 1.571$ (3 dp) **This response should be awarded M1 A0 dM1 A0**
	S.C.	**Special Case: Differentiating INCORRECTLY to give** $\text{f}'(x) = 6x - \frac{5}{6}x^{-\frac{3}{2}}$ and $\alpha \simeq 1.5 - \frac{\text{f}(1.5)}{\text{f}'(1.5)} = 1.571$ is **M1 A0 dM1 A0**
1. (b)	**Note**	$\frac{\alpha - 1.5}{1.6 - \alpha} = \left\lvert\frac{"-0.6108276349..."}{"0.3623843083..."}\right\rvert$ is a valid method for the first M mark
	Note	$\frac{\alpha - 1.5}{1.6 - \alpha} = \frac{"0.6108276349..."}{"0.3623843083..."} \Rightarrow \alpha = 1.563$ with no intermediate working is M1 dM1 A1
	Note	$\frac{\alpha - 1.5}{-0.6108276349...} = \frac{1.6 - \alpha}{0.3623843083...} \Rightarrow \alpha = 1.745861961... = 1.745$ (3 dp) is M0 dM0 A0
	Note	$\frac{\alpha - 1.5}{-0.6108276349...} = \frac{1.6 - \alpha}{-0.3623843083...} \Rightarrow \alpha = 1.562764092... = 1.563$ (3 dp) is M1 dM1 A1

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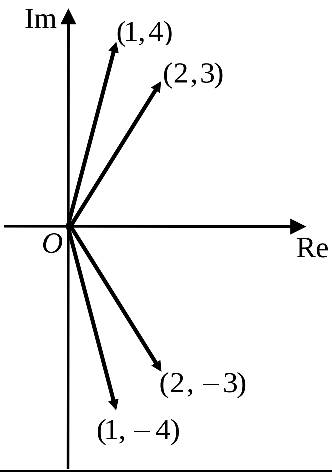
2. $$f(z) = z^4 - 6z^3 + 38z^2 - 94z + 221$$

(a) Given that $z = 2 + 3i$ is a root of the equation $f(z) = 0$, use algebra to find the three other roots of $f(z) = 0$

(7)

(b) Show the four roots of $f(z) = 0$ on a single Argand diagram.

(2)

Question Number	Scheme	Notes	Marks
2.	$\mathrm{f}(z) = z^4 - 6z^3 + 38z^2 - 94z + 221,\ z_1 = 2 + 3\mathrm{i}$ satisfies $\mathrm{f}(z) = 0$		
(a)	$\{z_2 =\}\ 2 - 3\mathrm{i}$	$2 - 3\mathrm{i}$ seen or used in part (a)	B1
	$z^2 - 4z + 13$	Attempt to expand $(z-(2+3\mathrm{i}))(z-(2-3\mathrm{i}))$ or $(z-(2+3\mathrm{i}))(z-(\text{their complex } z_2))$ or any valid method ***to establish a quadratic factor*** e.g. $z = 2 \pm 3\mathrm{i} \Rightarrow z - 2 = \pm 3\mathrm{i} \Rightarrow z^2 - 4z + 4 = -9$ or sum of roots $= 4$, product of roots 13 to give $z^2 \pm (\text{their sum})z + (\text{their product})$	M1
		$z^2 - 4z + 13$	A1
	$(z^2 - 4z + 13)(z^2 - 2z + 17)$	Attempts to find the other quadratic factor. e.g. using long division to obtain either $z^2 \pm kz + ...,\ k = \text{value} \neq 0$ or $z^2 \pm \alpha z + \beta,\ \beta = \text{value} \neq 0,\ \alpha$ can be 0 or e.g. factorising to obtain either $\mathrm{f}(z) = (z^2 - 2z + 5)(z^2 \pm kz \pm c),\ k = \text{value} \neq 0$ or $\mathrm{f}(z) = (z^2 - 2z + 5)(z^2 \pm \alpha z \pm \beta),\ \beta = \text{value} \neq 0,\ \alpha$ can be 0	M1
		$z^2 - 2z + 17$	A1
	$\{z^2 - 2z + 17 = 0 \Rightarrow\}$		
	Either • $z = \dfrac{--2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$ • $(z-1)^2 - 1 + 17 = 0 \Rightarrow z = ...$	**dependent on only the previous M mark** Correct method of applying the quadratic formula or completing the square for solving a 3TQ on their 2nd quadratic factor	dM1
	$\{z =\}\ 1 + 4\mathrm{i},\ 1 - 4\mathrm{i}$	$1 + 4\mathrm{i}$ **and** $1 - 4\mathrm{i}$	A1
			(7)
(b)		<u>**Criteria**</u> • $2 \pm 3\mathrm{i}$ plotted correctly in quadrants 1 and 4 • **Dependent on the final M mark being awarded in part (a).** Their final two roots are plotted correctly	
		Satisfies at least one of the criteria	B1ft
		Satisfies both criteria with some indication of scale or coordinates stated with at least one pair of roots symmetrical about the real axis	B1ft
			(2)
			9

		Question 2 Notes
2. (a)	**Note**	No working leading to $x = 1+4\text{i},\ 1-4\text{i}$ is M0A0M0A0M0A0.
	Note	You can assume $x \equiv z$ for solutions in this question.
	Note	Give dM1A1 for $z^2 - 2z + 17 = 0 \Rightarrow z = 1 + 4\text{i}, 1 - 4\text{i}$ with no intermediate working.
	Note **Note**	**<u>Special Case:</u>** If their second ***3 term quadratic*** factor **can** be factorised then give Special Case dM1 for correct factorisation leading to $z = ...$ Otherwise, give 3rd dM0 for applying a method of factorising to solve their 3TQ.
	Note	**Reminder:** Method Mark for solving a 3TQ, "$az^2 + bz + c = 0$" **Formula:** Attempt to use the correct formula (with values for *a*, *b* and *c*) **Completing the square:** $\left(z \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ q \neq 0,$ leading to $z = ...$

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3. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where a and b are integers to be determined.

(4)

(b) Given that

$$\sum_{r=5}^{25} r^2(r+1) + \sum_{r=1}^{k} 3^r = 140543$$

find the value of the integer k.

(4)

Question Number	Scheme	Notes	Marks
3. (a)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$		
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	Attempts to expand $r^2(r+1)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{12}n(n+1)\left[3n(n+1) + 2(2n+1)\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both standard formulae.	dM1
	$= \frac{1}{12}n(n+1)\left[3n^2 + 7n + 2\right]$	{this step does not have to be written}	
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	Correct completion with no errors. **Note:** $a=3, b=1$	A1
			(4)
(b)	$\sum_{r=5}^{25} r^2(r+1) + \sum_{r=1}^{k} 3^r = 140543$	{**Note:** Let $f(n) = \frac{1}{12}n(n+1)(n+2)(3n+1)$ ***or their*** answer to part (a).}	
	$\left\{\sum_{r=5}^{25} r^2(r+1)\right\} = \left(\frac{1}{12}(25)(26)(27)(76)\right) - \left(\frac{1}{12}(4)(5)(6)(13)\right)$ $\{= 111150 - 130 = 111020\}$	Attempts to find either $f(25) - f(4)$ or $f(25) - f(5)$ This mark can be implied	M1
	$\sum_{r=1}^{k} 3^r = 140543 - \text{"111020"}$ $\{= 29523\}$	**dependent on the previous M mark** their $\sum_{r=1}^{k} 3^r = 140543 - \text{"111020"}$ This mark can be implied	dM1
	$\frac{3(1-3^k)}{1-3}$ or $\frac{3(3^k-1)}{3-1}$	Correct GP sum formula with $a=3, r=3, n=k$	M1
	$\left\{\frac{3(1-3^k)}{1-3} = 29523 \Rightarrow 3^k = 19683 \Rightarrow\right\}$ $k=9$	$k=9$ from a correct solution	A1 **cso**
			(4)
(b) **Alt 1**	**<u>Alt 1 Method for the final 2 marks</u>**		
	$\sum_{r=1}^{k} 3^r = 29523$ $\Rightarrow 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7 + 3^8 + 3^9$ or $3+9+27+81+243+729+2187+6561+19683$	Attempts to solve $\sum_{r=1}^{k} 3^r =$ value by evaluating 3^r from $r=1$ to at least as far as $r=9$	M1
	$= 29523$, so $k=9$	$k=9$ from a correct solution	A1 **cso**
(b) **Alt 2**	**<u>Alt 2 Method for the final 2 marks</u>**		
	$\sum_{r=1}^{k} 3^r = 29523 \Rightarrow 3(1 + 3 + 3^2 + 3^3 + ... + 3^{k-1}) = 29523$		
	$\left\{\sum_{r=1}^{k} 3^r = \sum_{r=1}^{k-1} 3^r + 3^k =\right\}$ $\frac{\text{"29523"}}{3} - 1 + 3^k = \text{"29523"}$	$\frac{\text{"29523"}}{3} - 1 + 3^k = \text{"29523"}$	M1
	$\{3^k = 19683 \Rightarrow\}$ $k=9$	$k=9$ from a correct solution	A1 **cso**
			8

		Question 3 Notes
3. (a)	**Note**	Applying e.g. $n=1,\ n=2$ to the printed equation without applying the standard formulae to give $a=3,\ b=1$ is M0A0M0A0
	Alt 1	**Alt Method 1 (Award the first two marks using the main scheme)** Using $\frac{1}{12}\left(3n^4+10n^3+9n^2+2n\right)\equiv\frac{1}{12}\left(an^4+(3a+b)n^3+(2a+3b)n^2+2bn\right)$ o.e.
	dM1	Equating coefficients to find both $a=...$ and $b=...$ **and** at least one of $a=3,\ b=1$
	A1 cso	Finds $a=3,\ b=1$ and demonstrates the identity works for all of its terms.
	Alt 2	**Alt Method 2: (Award the first two marks using the main scheme)** $\frac{1}{4}n^2(n+1)^2+\frac{1}{6}n(n+1)(2n+1)\equiv\frac{1}{12}n(n+1)(n+2)(an+b)$
	dM1	Substitutes $n=1,\ n=2,$ into this identity o.e. and solves to find both $a=...$ and $b=...$ **and** at least one of $a=3,\ b=1$. **Note:** $n=1$ gives $4=a+b$ and $n=2$ gives $7=2a+b$
	A1	Finds $a=3,\ b=1$
	Note	Allow final dM1A1 for $\frac{1}{4}n^4+\frac{5}{6}n^3+\frac{3}{4}n^2+\frac{1}{6}n$ or $\frac{1}{12}n(3n^3+10n^2+9n+2)$ or $\frac{1}{12}(3n^4+10n^3+9n^2+2n)\to\frac{1}{12}n(n+1)(n+2)(3n+1)$ with no incorrect working.
	Note	A correct proof $\sum_{r=1}^{n}r^2(r+1)=\frac{1}{12}n(n+1)(n+2)(3n+1)$ followed by stating an incorrect e.g. $a=1,\ b=3$ is M1A1dM1A1 (ignore subsequent working)
(b)	**Note**	Using $\mathrm{f}(25)-\mathrm{f}(5)$ gives • $\mathrm{f}(25)-\mathrm{f}(5)=111150-280=110870$ • $\sum_{r=1}^{k}3^r=140543-\text{"110870"}=29673$
	Note	Allow 1st M1 for either • $\left\{\sum_{r=5}^{25}r^2(r+1)\right\}=\left(\frac{1}{4}(25)^2(26)^2+\frac{1}{6}(25)(26)(51)\right)-\left(\frac{1}{4}(4)^2(5)^2+\frac{1}{6}(4)(5)(9)\right)$ $\{=(105625+5525)-(100+30)=111150-130=111020\}$ • $\left\{\sum_{r=5}^{25}r^2(r+1)\right\}=\left(\frac{1}{4}(25)^2(26)^2+\frac{1}{6}(25)(26)(51)\right)-\left(\frac{1}{4}(5)^2(6)^2+\frac{1}{6}(5)(6)(11)\right)$ $\{=(105625+5525)-(225+55)=111150-280=110870\}$
	Note	$\frac{3\left(1-3^k\right)}{1-3}$ or $\frac{3\left(3^k-1\right)}{3-1}=29523\Rightarrow k=9$ with no intermediate working is 2nd M1 2nd A1
	Note	$\sum_{r=1}^{k}3^r=29523\Rightarrow k=9$ with no intermediate working is 2nd M1 2nd A1

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4. The quadratic equation

$$3x^2 + 2x + 5 = 0$$

has roots α and β.

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$

(2)

(b) show that $\alpha^3 + \beta^3 = \frac{82}{27}$

(2)

(c) find a quadratic equation which has roots

$$\left(\alpha + \frac{\alpha}{\beta^2}\right) \text{ and } \left(\beta + \frac{\beta}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers.

(4)

Question Number	Scheme	Notes	Marks
4.	$3x^2+2x+5=0$ has roots α, β		
(a)	$\alpha+\beta=-\frac{2}{3},\ \alpha\beta=\frac{5}{3}$		
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=......$	**Use** of the **correct** identity for $\alpha^2+\beta^2$ (May be implied by their work)	M1
	$=\left(-\frac{2}{3}\right)^2-2\left(\frac{5}{3}\right)=-\frac{26}{9}$	$-\frac{26}{9}$ or $-2\frac{8}{9}$ from correct working	A1 **cso**
			(2)
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$ or $=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=......$	**Use** of an **appropriate** and **correct** identity for $\alpha^3+\beta^3$ (May be implied by their work)	M1
	$=\left(-\frac{2}{3}\right)^3-3\left(\frac{5}{3}\right)\left(-\frac{2}{3}\right)=\frac{82}{27}$ * or $=\left(-\frac{2}{3}\right)\left(-\frac{26}{9}-\frac{5}{3}\right)=\frac{82}{27}$ *	$\frac{82}{27}$ **from correct working**	A1 * **cso**
			(2)
(c)	Sum $=\alpha+\frac{\alpha}{\beta^2}+\beta+\frac{\beta}{\alpha^2}$ $=\alpha+\beta+\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$ or $=\frac{\alpha\beta^2+\alpha}{\beta^2}+\frac{\alpha^2\beta+\beta}{\alpha^2}$ $=\frac{\alpha^3+\beta^3+\alpha^2\beta^2(\alpha+\beta)}{\alpha^2\beta^2}$ $=\left(-\frac{2}{3}\right)+\frac{\left(\frac{82}{27}\right)}{\left(\frac{5}{3}\right)^2}$ $\left\{=-\frac{2}{3}+\frac{82}{75}=\frac{32}{75}\right\}$	Simplifies $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$ to give either $\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$ or $\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ **and** substitutes at least one of their $\alpha+\beta,\ \alpha^3+\beta^3$ or $\alpha\beta$ into an expression for the sum of $\left(\alpha+\frac{\alpha}{\beta^2}\right)$ and $\left(\beta+\frac{\beta}{\alpha^2}\right)$	M1
	Product $=\left(\alpha+\frac{\alpha}{\beta^2}\right)\left(\beta+\frac{\beta}{\alpha^2}\right)$ $=\alpha\beta+\frac{\alpha\beta}{\alpha^2}+\frac{\alpha\beta}{\beta^2}+\frac{\alpha\beta}{\alpha^2\beta^2}$ $=\alpha\beta+\frac{\beta}{\alpha}+\frac{\alpha}{\beta}+\frac{1}{\alpha\beta}$ $=\alpha\beta+\frac{\beta^2+\alpha^2}{\alpha\beta}+\frac{1}{\alpha\beta}$ or $=\left(\frac{\alpha\beta^2+\alpha}{\beta^2}\right)\left(\frac{\alpha^2\beta+\beta}{\alpha^2}\right)$ $=\frac{\alpha^3\beta^3+\alpha\beta^3+\alpha^3\beta+\alpha\beta}{\alpha^2\beta^2}$ $=\frac{\alpha^3\beta^3+\alpha\beta(\beta^2+\alpha^2)+\alpha\beta}{\alpha^2\beta^2}$ $=\left(\frac{5}{3}\right)+\frac{\left(-\frac{26}{9}\right)}{\left(\frac{5}{3}\right)}+\frac{1}{\left(\frac{5}{3}\right)}$ $\left\{=\frac{5}{3}-\frac{26}{15}+\frac{3}{5}=\frac{8}{15}\right\}$	Expands $\left(\alpha+\frac{\alpha}{\beta^2}\right)\left(\beta+\frac{\beta}{\alpha^2}\right)$ to give 4 terms and substitutes either their $\alpha\beta$ at least once or their $\alpha^2+\beta^2$ into their resulting expression	M1
	$x^2-\frac{32}{75}x+\frac{8}{15}=0$	Applies $x^2-(\text{sum})x+\text{product}$ (can be implied), where sum and product are numerical values. **Note:** "=0" not required for this mark	M1
	$75x^2-32x+40=0$	Any integer multiple of $75x^2-32x+40=0$, including the "=0"	A1
			(4)
			8

		Question 4 Notes
4. (a)	**Note**	Writing a correct $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ without attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^2 - 2\alpha\beta$ is M0
	Note	Give M1A0 for $\alpha+\beta = \frac{2}{3},\ \alpha\beta = \frac{5}{3}$ leading to $\alpha^2+\beta^2 = \left(\frac{2}{3}\right)^2 - 2\left(\frac{5}{3}\right) = -\frac{26}{9}$
	Note	Give M1A1 for writing $\alpha^2+\beta^2 = -\frac{26}{9}$ with no evidence of applying $\alpha+\beta = -\frac{2}{3},\ \alpha\beta = \frac{5}{3}$
(b)	**Note**	Allow M1 A1 for $\alpha^3+\beta^3 = (\alpha^2+\beta^2)(\alpha+\beta) - \alpha\beta(\alpha+\beta)$ $= \left(-\frac{26}{9}\right)\left(-\frac{2}{3}\right) - \left(-\frac{2}{3}\right)\left(\frac{5}{3}\right) \left\{= \frac{52}{27} + \frac{10}{9}\right\} = \frac{82}{27}$ *
	Note	Writing a correct $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ without attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ is M0
	Note	Writing a correct $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ without attempting to substitute at least one of either their $\alpha+\beta$, their $\alpha^2+\beta^2$ or their $\alpha\beta$ into $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ is M0
(a), (b)	**Note**	Applying $\frac{-1+\sqrt{14}\,i}{3}, \frac{-1-\sqrt{14}\,i}{3}$ explicitly will score (a) M0A0, (b) M0A0 • E.g. In part (a), give no credit for $\left(\frac{-1+\sqrt{14}\,i}{3}\right)^2 + \left(\frac{-1-\sqrt{14}\,i}{3}\right)^2 = -\frac{26}{9}$ • E.g. In part (b), give no credit for $\left(\frac{-1+\sqrt{14}\,i}{3}\right)^3 + \left(\frac{-1-\sqrt{14}\,i}{3}\right)^3 = \frac{82}{17}$
	Note	Using $\frac{-1+\sqrt{14}\,i}{3}, \frac{-1-\sqrt{14}\,i}{3}$ to find $\alpha+\beta = -\frac{2}{3},\ \alpha\beta = \frac{5}{3}$ followed by • $\alpha^2+\beta^2 = \left(\frac{2}{3}\right)^2 - 2\left(\frac{5}{3}\right) = -\frac{26}{9}$, scores M1A0 in part (a) • $\alpha^3+\beta^3 = \left(-\frac{2}{3}\right)^3 - 3\left(\frac{5}{3}\right)\left(-\frac{2}{3}\right) = \frac{82}{27}$, scores M1A0 in part (b)
(c)	**Note**	A correct method leading to $a = 75, b = -32, c = 40$ without writing a final answer of $75x^2 - 32x + 40 = 0$ is final M1A0.
	Note	Using $\frac{-1+\sqrt{14}\,i}{3}, \frac{-1-\sqrt{14}\,i}{3}$ ***explicitly*** to find the sum and product of $\left(\alpha + \frac{\alpha}{\beta^2}\right)$ and $\left(\beta + \frac{\beta}{\alpha^2}\right)$ scores M0M0M0A0 in part (c).
	Note	Using $\frac{-1+\sqrt{14}\,i}{3}, \frac{-1-\sqrt{14}\,i}{3}$ to find $\alpha+\beta = -\frac{2}{3},\ \alpha\beta = \frac{5}{3}$ **and applying** $\alpha+\beta = -\frac{2}{3},\ \alpha\beta = \frac{5}{3}$ can potentially score full marks in part (c). E.g. • Sum $= \alpha+\beta+\frac{\alpha^3+\beta^3}{(\alpha\beta)^2} = \left(-\frac{2}{3}\right) + \frac{\left(\frac{82}{27}\right)}{\left(\frac{5}{3}\right)^2} = \frac{32}{75}$ • Product $= \alpha\beta + \frac{\beta^2+\alpha^2}{\alpha\beta} + \frac{1}{\alpha\beta} = \left(\frac{5}{3}\right) + \frac{\left(-\frac{26}{9}\right)}{\left(\frac{5}{3}\right)} + \frac{1}{\left(\frac{5}{3}\right)} = \frac{8}{15}$ • $x^2 - \frac{32}{75}x + \frac{8}{15} = 0 \Rightarrow 75x^2 - 32x + 40 = 0$

Leave blank

5. (i) Given that

$$\frac{2z+3}{z+5-2\text{i}} = 1+\text{i}$$

find z, giving your answer in the form $a + b\text{i}$, where a and b are real constants.

(5)

(ii) Given that

$$w = (3 + \lambda\text{i})(2 + \text{i})$$

where λ is a real constant, and that

$$|w| = 15$$

find the possible values of λ.

(4)

Question Number	Scheme	Notes	Marks
5.	(i) $\dfrac{2z+3}{z+5-2\text{i}} = 1+\text{i}$ (ii) $w = (3+\lambda\text{i})(2+\text{i})$ and $\lvert w\rvert = 15$		
(i)	$2z+3 = (1+\text{i})(z+5-2\text{i})$	Multiplies both sides by $(z+5-2\text{i})$	M1
	$2z+3 = z+5-2\text{i}+\text{i}z+5\text{i}+2 = z+\text{i}z+7+3\text{i}$		
	E.g. • $2z - z(1+\text{i}) = (1+\text{i})(5-2\text{i}) - 3$ • $z - \text{i}z = 4+3\text{i}$	**dependent on the previous M mark** Collects terms in z to one side	dM1
	$z = \dfrac{4+3\text{i}}{1-\text{i}}$	Correct expression for $z = ...$	A1
	$z = \dfrac{(4+3\text{i})}{(1-\text{i})}\dfrac{(1+\text{i})}{(1+\text{i})} = \dfrac{1}{2}+\dfrac{7}{2}\text{i}$	**dependent on both previous M marks** Multiplies numerator and denominator by the conjugate of the denominator and attempts to find $z = ...$	ddM1
		e.g. $\dfrac{1}{2}+\dfrac{7}{2}\text{i}$ **or** $\dfrac{7}{2}\text{i}+\dfrac{1}{2}$ **or** $0.5+3.5\text{i}$ **or** $a=\dfrac{1}{2}, b=\dfrac{7}{2}$	A1 **cao**
			(5)
(i) **Way 2**	$2z+3 = (1+\text{i})(z+5-2\text{i})$	Multiplies both sides by $(z+5-2\text{i})$	M1
	$2(a+b\text{i})+3 = (1+\text{i})(a+b\text{i}+5-2\text{i})$ $(2a+3)+2b\text{i} = a+b\text{i}+5-2\text{i}+a\text{i}-b+5\text{i}+2$ $(2a+3)+2b\text{i} = (a-b+7)+(b+a+3)\text{i}$	**dependent on the previous M mark** Applies $z = a+b\text{i}$, multiplies out and attempts to equate **either** the real part **or** the imaginary part of the resulting equation	dM1
	$\{\text{Real} \Rightarrow\}$ $2a+3 = a-b+7$ $\{\text{Imaginary} \Rightarrow\}$ $2b = b+a+3$	Both correct equations which can be simplified or un-simplified	A1
	$\left\{\begin{matrix} a+b=4 \\ -a+b=3\end{matrix}\right\} \Rightarrow b=\dfrac{7}{2}, a=\dfrac{1}{2}$	**dependent on both previous M marks.** Obtains two equations both in terms of a and b and solves them simultaneously to give at least one of $a = ...$ or $b = ...$	ddM1
		e.g. $a=\dfrac{1}{2}, b=\dfrac{7}{2}$ **or** $\dfrac{1}{2}+\dfrac{7}{2}\text{i}$ **or** $\dfrac{7}{2}\text{i}+\dfrac{1}{2}$ **or** $0.5+3.5\text{i}$	A1 **cao**
			(5)
(ii)	$w = 6+3\text{i}+2\text{i}\lambda-\lambda$ $w = (6-\lambda)+(3+2\lambda)\text{i}$ $(15)^2 = (6-\lambda)^2+(3+2\lambda)^2$	Squares and adds the real and imaginary parts of w and sets equal to either 15^2 or 15	M1
		Correct equation which can be simplified or un-simplified	A1
	$\{225 = 36-12\lambda+\lambda^2+9+12\lambda+4\lambda^2\}$ $225 = 45+5\lambda^2 \Rightarrow \lambda^2 = 36$	**dependent on the previous M mark** Solves their quadratic in λ to give $\lambda^2 = ...$ or $\lambda = ...$	dM1
	$\lambda = 6, -6$	$\lambda = 6, -6$	A1
			(4)
(ii) **Way 2**	$\{\lvert(3+\lambda\text{i})(2+\text{i})\rvert = 15 \Rightarrow\}$ $\sqrt{(3^2+\lambda^2)}\sqrt{(2^2+1^2)} = 15$ or $(3^2+\lambda^2)(5) = (15)^2$	$\sqrt{(3^2+\lambda^2)}\sqrt{(2^2+1^2)} = 15$ or $(3^2+\lambda^2)(2^2+1^2) = 15$	M1
		Correct equation which can be simplified or un-simplified	A1
	$45 = 9+\lambda^2 \Rightarrow \lambda^2 = 36$	**dependent on the previous M mark** Solves their quadratic in λ to give $\lambda^2 = ...$ or $\lambda = ...$	dM1
	$\lambda = 6, -6$	$\lambda = 6, -6$	A1
			(4)
			9

Question Number	Scheme	Notes	Marks
5.	$\frac{2z+3}{z+5-2\mathrm{i}} = 1+\mathrm{i}$		
(i) **Way 3**	$\frac{2z+10-4\mathrm{i}-7+4\mathrm{i}}{z+5-2\mathrm{i}} = 1+\mathrm{i}$		
	$2 + \frac{-7+4\mathrm{i}}{z+5-2\mathrm{i}} = 1+\mathrm{i}$	$\frac{2z+3}{z+5-2\mathrm{i}} \to 2 \pm \frac{k}{z+5-2\mathrm{i}},\ k \in \mathbb{C}$	M1
	$1-\mathrm{i} = \frac{7-4\mathrm{i}}{z+5-2\mathrm{i}}$		
	$z+5-2\mathrm{i} = \frac{7-4\mathrm{i}}{1-\mathrm{i}}$	**dependent on the previous M mark** Rearranges to give $z+5-2\mathrm{i} = ...$	dM1
		Correct expression for $z+5-2\mathrm{i} = ...$	A1
	$z+5-2\mathrm{i} = \frac{(7-4\mathrm{i})}{(1-\mathrm{i})}\frac{(1+\mathrm{i})}{(1+\mathrm{i})} \Rightarrow z = ...$	**dependent on both previous M marks** Multiplies numerator and denominator by the conjugate of the denominator and attempts to find $z = ...$	ddM1
	$\left\{z+5-2\mathrm{i} = \frac{11}{2}+\frac{3}{2}\mathrm{i} \Rightarrow\right\}\ z = \frac{1}{2}+\frac{7}{2}\mathrm{i}$	e.g. $\frac{1}{2}+\frac{7}{2}\mathrm{i}$ **or** $\frac{7}{2}\mathrm{i}+\frac{1}{2}$ **or** $0.5+3.5\mathrm{i}$	A1
			(5)
(i) **Way 4**	$\frac{2(a+b\mathrm{i})+3}{a+b\mathrm{i}+5-2\mathrm{i}} = 1+\mathrm{i} \Rightarrow \frac{(2a+3)+2b\mathrm{i}}{(a+5)+(b-2)\mathrm{i}} = 1+\mathrm{i}$		
	$\left(\frac{(2a+3)+2b\mathrm{i}}{(a+5)+(b-2)\mathrm{i}}\right)\left(\frac{(a+5)-(b-2)\mathrm{i}}{(a+5)-(b-2)\mathrm{i}}\right) = 1+\mathrm{i}$		
	$\frac{[(2a+3)(a+5)+2b(b-2)]+\mathrm{i}[2b(a+5)-(2a+3)(b-2)]}{(a+5)^2+(b-2)^2} = 1+\mathrm{i}$		
	$\{\text{Real} \Rightarrow\}\ \frac{(2a+3)(a+5)+2b(b-2)}{(a+5)^2+(b-2)^2} = 1$ $\{\text{Imaginary} \Rightarrow\}\ \frac{2b(a+5)-(2a+3)(b-2)}{(a+5)^2+(b-2)^2} = 1$	Applies $z = a+b\mathrm{i}$ and a full method leading to equating both the real part and the imaginary part	M1
	$\{\text{Real} \Rightarrow\}\ a^2+b^2+3a-14=0$ $\{\text{Imaginary} \Rightarrow\}\ a^2+b^2+6a-11b+23=0$	**dependent on the previous M mark** Manipulates both their real part and their imaginary part into their simplest forms	dM1
		Both correct simplified equations	A1
	"Real - Imaginary" gives $-3a+11b-37=0$ **and** e.g. • $a = \frac{11b-37}{3} \Rightarrow \left(\frac{11b-37}{3}\right)^2 + b^2 + 3\left(\frac{11b-37}{3}\right) - 14 = 0$ $\Rightarrow 2b^2 - 11b + 14 = 0 \Rightarrow (b-2)(2b-7) = 0 \Rightarrow b = ...$ • $b = \frac{3a+37}{11} \Rightarrow a^2 + \left(\frac{3a+37}{11}\right)^2 + 3a - 14 = 0$ $\Rightarrow 2a^2 + 9a - 5 = 0 \Rightarrow (a+5)(2a-1) = 0 \Rightarrow a = ...$	**dependent on both previous M marks.** Solves their equations simultaneously to obtain at least one value of $b = ...$ or $a = ...$	ddM1
	$z = \frac{1}{2}+\frac{7}{2}\mathrm{i}$ **only**	e.g. $\frac{1}{2}+\frac{7}{2}\mathrm{i}$ **or** $\frac{7}{2}\mathrm{i}+\frac{1}{2}$ **or** $0.5+3.5\mathrm{i}$	A1
			(5)

Question Number	Scheme	Notes	Marks
5.	$\dfrac{2z+3}{z+5-2\text{i}} = 1+\text{i}$		
(i) **Way 5**	$\dfrac{2z+3}{1+\text{i}} = z+5-2\text{i}$		
	$\dfrac{(2z+3)}{(1+\text{i})}\dfrac{(1-\text{i})}{(1-\text{i})} = z+5-2\text{i}$	Multiplies $\dfrac{(2z+3)}{(1+\text{i})}$ by $\dfrac{(1-\text{i})}{(1-\text{i})}$ and sets equal to $z+5-2\text{i}$	M1
	$\dfrac{(2z+3)(1-\text{i})}{2} = z+5-2\text{i}$		
	$2z+3-2\text{i}z-3\text{i} = 2z+10-4\text{i}$		
	$2\text{i}z = -7+\text{i}$	**dependent on the previous M mark** Rearranges to make $2\text{i}z = \dots$	dM1
		Correct expression for $2\text{i}z = \dots$	A1
	$-2z = -7\text{i}-1 \Rightarrow z = \dots$	**dependent on both previous M marks** Multiplies both sides by i and attempts to find $z = \dots$	ddM1
	$z = \dfrac{1}{2}+\dfrac{7}{2}\text{i}$	e.g. $\dfrac{1}{2}+\dfrac{7}{2}\text{i}$ **or** $\dfrac{7}{2}\text{i}+\dfrac{1}{2}$ **or** $0.5+3.5\text{i}$	A1
			(5)

	Question 5 Notes		
5. (i)	**Note**	Way 4 method generates $z=\dfrac{1}{2}+\dfrac{7}{2}\text{i}$ and $z=-5+2\text{i}$ but $z=\dfrac{1}{2}+\dfrac{7}{2}\text{i}$ must be stated as the only answer for the final A mark	
	Note	Give final A0 for a correct $a=\dfrac{1}{2}, b=\dfrac{7}{2}$ followed by an incorrect $\{z=\}\ \dfrac{7}{2}+\dfrac{1}{2}\text{i}$	
	Note	$\{z=\}\ \dfrac{1}{2}+\text{i}\dfrac{7}{2}$ is fine for the final A mark	
	Note	Give final A0 for $\{z=\}\ \dfrac{1+7\text{i}}{2}$ without reference to e.g. $a=\dfrac{1}{2}, b=\dfrac{7}{2}$ **or** $\dfrac{1}{2}+\dfrac{7}{2}\text{i}$, etc.	
(ii)	**Note**	$w=(6-\lambda)+(3+2\lambda)\text{i} \Rightarrow (15)^2=(6-\lambda)^2-(3+2\lambda)^2$ is 1st M0	
	Note	$\lvert(3+\lambda\text{i})(2+\text{i})\rvert=15 \Rightarrow \sqrt{(3^2-\lambda^2)}\sqrt{(2^2-1^2)}=15$ is 1st M0	
	Note	Give final A0 for either • $\lambda=6,-6 \Rightarrow \lambda=6$ • $\lambda=6,-6 \Rightarrow \lambda=-6$	

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6. The parabola C has equation $y^2 = 32x$ and the point S is the focus of this parabola. The point $P(2, 8)$ lies on C and the point T lies on the directrix of C. The line segment PT is parallel to the x-axis.

(a) Write down the coordinates of S.

(1)

(b) Find the length of PT.

(1)

(c) Using calculus, show that the tangent to C at the point P has equation

$$y = 2x + 4$$

(4)

The hyperbola H has equation $xy = 4$. The tangent to C at P meets H at the points L and M.

(d) Find the exact coordinates of the points L and M, giving your answers in their simplest form.

(6)

Question Number	Scheme	Notes	Marks
6.	$C: y^2 = 32x$; S is the focus of C; $P(2, 8)$ lies on C; T lies on the direcrix of C. $H: xy = 4$		
(a)	S has coordinates $(8, 0)$	$(8, 0)$	B1 **cao**
			(1)
(b)	$\{PT$ is parallel to the x-axis $\Rightarrow\}$ $T(-8, 8) \Rightarrow PT = 2 - -8 = 10$ Focus-directrix Property $\Rightarrow PT = \sqrt{8^2 + (8-2)^2} = 10$	$PT = 10$	B1 **cao**
			(1)
(c)	$y = \sqrt{32}\, x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{32}\, x^{-\frac{1}{2}}$ or $2\sqrt{2}\, x^{-\frac{1}{2}}$	$\frac{dy}{dx} = \pm k x^{-\frac{1}{2}}; k \neq 0$	M1
	$y^2 = 32x \Rightarrow 2y\frac{dy}{dx} = 32$	$\lambda y \frac{dy}{dx} = \mu;\ \lambda, \mu \neq 0$	
	$x = 8t^2,\ y = 16t \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 16\left(\frac{1}{16t}\right)$	$x = at^2,\ y = 2at \Rightarrow$ their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dy}{dt}}$; $a \neq 0$	
	So at P, $m_T = 2$	Correct calculus work leading to $m_T = 2$	A1
	Either • $y - 8 = "2"(x-2)$ • $8 = "2"(2) + c \Rightarrow y = "2"x +$ their c	Correct straight line method using their gradient $m_T\ (\neq m_N)$ which is found by using calculus. **Note:** m_T must be a value	M1
	Correct algebra leading to $y = 2x + 4$ *	Correct solution only	A1 *
			(4)
(d)	$x(2x+4) = 4$ or $\left(\frac{y-4}{2}\right)y = 4$ or $\frac{4}{x} = 2x + 4$ or $y = 2\left(\frac{4}{y}\right) + 4$ or $\frac{2}{t} = 2(2t) + 4$	Subsitutes either • $y = 2x + 4$ into $xy = 4$ • $y = \frac{4}{x}$ or $x = \frac{4}{y}$ into $y = 2x + 4$ • $x = 2t$ and $y = \frac{2}{t}$ into $y = 2x + 4$ to form an equation in either x only, y only or t only	M1
	$2x^2 + 4x - 4 = 0$ or $x^2 + 2x - 2 = 0$ or $\frac{1}{2}y^2 - 2y - 4 = 0$ or $y^2 - 4y - 8 = 0$ or $4t^2 + 4t - 2 = 0$ or $2t^2 + 2t - 1 = 0$	A correct 3 term quadratic **Note:** $2x^2 + 4x = 4$, $\frac{1}{2}y^2 - 2y - 4 = 0$, $2 = 4t^2 + 4t$ or $2x^2 + 4x - 4\ \{= 0\}$ are acceptable for this mark	A1
	• $\{x^2 + 2x - 2 = 0 \Rightarrow\}\ (x+1)^2 - 1 - 2 = 0 \Rightarrow x = ...$ • $\{2t^2 + 2t - 1 = 0 \Rightarrow\}\ t = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)}$ **and** either $x = 2\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{3}\right)$ or $y = \frac{2}{\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{3}\right)}$ • $\{y^2 - 4y - 8 = 0 \Rightarrow\}\ y = \frac{--4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$	**dependent on the previous M mark** Correct method (e.g. completing the square, applying the quadratic formula or factorising) of solving a 3TQ to find either $x = ...$ or $y = ...$	dM1
	Either $x = -1 \pm \sqrt{3}$ or $y = 2 \pm 2\sqrt{3}$	Both correct x coordinates or both correct y coordinates. **(See note)**	A1
	E.g. $y = 2(-1+\sqrt{3}) + 4$ or $y = \frac{4}{(-1+\sqrt{3})}$, etc.	**dependent on the first M mark** At least one attempt to find the other coordinate	dM1
	Either $(-1+\sqrt{3},\ 2+2\sqrt{3}), (-1-\sqrt{3},\ 2-2\sqrt{3})$ **or** $x = -1+\sqrt{3},\ y = 2+2\sqrt{3}$ and $x = -1-\sqrt{3},\ y = 2-2\sqrt{3}$	All correct and paired	A1
			(6)
			12

	Question 6 Notes	
6. (d)	**Note**	Condone $y = 2 \pm \sqrt{12}$ for the 2nd A1 mark.
	Note	Do not allow $(-1+\sqrt{3},\ 2+\sqrt{12}),\ (-1-\sqrt{3},\ 2-\sqrt{12})$ for the final A mark.
	Note	Writing $x = -1 \pm \sqrt{3},\ y = 2 \pm 2\sqrt{3}$ without any evidence of the correct coordinate pairings is final A0
	Note	**<u>Writing coordinates the wrong way round</u>** E.g. writing $x = -1+\sqrt{3},\ y = 2+2\sqrt{3}$ and $x = -1-\sqrt{3},\ y = 2-2\sqrt{3}$ followed by $(-1+\sqrt{3},\ 2-2\sqrt{3}),\ (-1-\sqrt{3},\ 2+2\sqrt{3})$ is final A0
	Note	Imply the 1st dM1 mark for ***writing down*** the ***correct*** roots for ***their*** quadratic equation. E.g. • $2x^2 + 4x - 4 = 0$ or $x^2 + 2x - 2 = 0$ or $2x^2 + 4x = 4 \rightarrow x = -1 \pm \sqrt{3}$ • $\frac{1}{2}y^2 - 2y - 4 = 0$ or $y^2 - 4y - 8 = 0 \rightarrow y = 2 \pm 2\sqrt{3}$
	Note	You can imply the 1st A1, 1st dM1, 2nd A1 marks for either • $x(2x+4) = 4$ or $\frac{4}{x} = 2x + 4 \rightarrow x = -1 \pm \sqrt{3}$ • $\left(\frac{y-4}{2}\right)y = 4$ or $y = 2\left(\frac{4}{y}\right) + 4 \rightarrow y = 2 \pm 2\sqrt{3}$ with no intermediate working.
	Note	You can imply the 1st A1, 1st dM1, 2nd A1, 2nd dM1 marks for either • $x(2x+4) = 4$ or $\frac{4}{x} = 2x + 4 \rightarrow x = -1 \pm \sqrt{3}$ **and** $y = 2 \pm 2\sqrt{3}$ • $\left(\frac{y-4}{2}\right)y = 4$ or $y = 2\left(\frac{4}{y}\right) + 4 \rightarrow y = 2 \pm 2\sqrt{3}$ **and** $x = -1 \pm \sqrt{3}$ with no intermediate working. You can then imply the final A1 mark if they correctly state the correct coordinate pairings.
	Note	**2nd A1:** Allow this mark for both correct *x* coordinates or both correct *y* coordinates which are in the form $\frac{a \pm b\sqrt{c}}{d}$, where *a*, *b*, *c* and *d* are simplified integers

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7. (i)

$$\mathbf{A} = \begin{pmatrix} 6 & k \\ -3 & -4 \end{pmatrix}, \text{ where } k \text{ is a real constant, } k \neq 8$$

Find, in terms of k,

(a) $\mathbf{A}^{-1}$ **(3)**

(b) $\mathbf{A}^2$ **(1)**

Given that $\mathbf{A}^2 + 3\mathbf{A}^{-1} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$

(c) find the value of k. **(3)**

(ii)

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$$

The matrix **M** represents a one way stretch, parallel to the y-axis, scale factor p, where $p > 0$, followed by a rotation anticlockwise through an angle θ about (0, 0).

(a) Find the value of p. **(2)**

(b) Find the value of θ. **(2)**

Question Number	Scheme	Notes	Marks
7.	$\mathbf{A}=\begin{pmatrix}6 & k\\ -3 & -4\end{pmatrix}, k\neq 8;\ \mathbf{A}^2+3\mathbf{A}^{-1}=\begin{pmatrix}5 & 9\\ -3 & -5\end{pmatrix};\ \mathbf{M}=\begin{pmatrix}-\frac{1}{2} & -\sqrt{3}\\ \frac{\sqrt{3}}{2} & -1\end{pmatrix}$		
(i)(a)	$\det(\mathbf{A})=6(-4)-(k)(-3)\ \{=-24+3k\}$	Correct $\det(\mathbf{A})$ which can be un-simplifed or simplified	B1
	$\{\mathbf{A}^{-1}=\}\ \dfrac{1}{3k-24}\begin{pmatrix}-4 & -k\\ 3 & 6\end{pmatrix}$	$\begin{pmatrix}-4 & -k\\ 3 & 6\end{pmatrix}$	M1
		Correct $\mathbf{A}^{-1}$	A1
			(3)
(b)	$\{\mathbf{A}^2=\}\ \begin{pmatrix}36-3k & 6k-4k\\ -18+12 & -3k+16\end{pmatrix}\ \left\{=\begin{pmatrix}36-3k & 2k\\ -6 & -3k+16\end{pmatrix}\right\}$	Correct $\mathbf{A}^2$ which can be un-simplifed or simplified	B1
			(1)
(c)	• $\begin{pmatrix}36-3k & 2k\\ -6 & -3k+16\end{pmatrix}+\dfrac{3}{3k-24}\begin{pmatrix}-4 & -k\\ 3 & 6\end{pmatrix}=\begin{pmatrix}5 & 9\\ -3 & -5\end{pmatrix}$ • $36-3k-\dfrac{12}{3k-24}=5$ • $2k-\dfrac{3k}{3k-24}=9$ • $-6+\dfrac{9}{3k-24}=-3$ • $-3k+16+\dfrac{18}{3k-24}=-5$ **Either** • attempts to form an equation for (their $\mathbf{A}^2$) + 3(their $\mathbf{A}^{-1}$) $=\begin{pmatrix}5 & 9\\ -3 & -5\end{pmatrix}$ in k • **or** attempts to add an element of (their $\mathbf{A}^2$) to the corresponding element of 3(their $\mathbf{A}^{-1}$) and equates to the corresponding element of the given matrix to form an equation in k		M1
	$\left\{\text{e.g. } -6+\dfrac{9}{3k-24}=-3\right\}\Rightarrow\ k=9$	**dependent on the previous M mark** Solves their equation to give $k=\ldots$	dM1
		Final answer of $k=9$ ***only***	A1
			(3)
	Note: Parts (ii)(a) and (ii)(b) can be marked together **Please refer to the notes on the next page when marking (ii)(a) and (ii)(b)**		
(ii)(a)	• $p=\left(-\dfrac{1}{2}\right)(-1)-\left(-\sqrt{3}\right)\left(\dfrac{\sqrt{3}}{2}\right)=2$ • $-p\sin\theta=-\sqrt{3}$, $p\cos\theta=-1$ ○ $p=\sqrt{(\pm\sqrt{3})^2+(-1)^2}=2$ ○ $p=\dfrac{-\sqrt{3}}{-\sin\text{"}120°\text{"}}=2$ or $p=\dfrac{-1}{\cos\text{"}120°\text{"}}=2$	Attempts $p=\pm\dfrac{1}{2}\pm\left(\sqrt{3}\right)\left(\dfrac{\sqrt{3}}{2}\right)$ **or** uses a full method of trigonometry to find $p=\ldots$	M1
		$p=2$ only	A1
			(2)
(b)	$\cos\theta=-\dfrac{1}{2}, \sin\theta=\dfrac{\sqrt{3}}{2}, \tan\theta=-\sqrt{3}$ E.g. • $\Rightarrow\ \theta=120°$ • $\Rightarrow\ \theta=180-\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)=120°$ • $\Rightarrow\ \theta=180-\tan^{-1}\left(\sqrt{3}\right)=120°$	Uses trigonometry to find an expression or value for θ which is in the range $(1.57..., 3.14...)$ or $(90°, 180°)$ $(-3.14..., -4.71...)$ or $(-180°, -270°)$	M1
		$120°$ or $-240°$ or $\dfrac{2\pi}{3}$ or $-\dfrac{4\pi}{3}$ or awrt 2.09 or awrt -4.19	A1
			(2)
			11

		Question 7 Notes
7. (i)(c)	**Note**	Give 1st M1 for $\begin{pmatrix} 36-3k-\frac{12}{3k-24} & 2k-\frac{3k}{3k-24} \\ -6+\frac{9}{3k-24} & -3k+16-\frac{18}{3k-24} \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$
	Note	• $36-3k-\frac{12}{3k-24}=5 \to 3k^2-55k+252=0 \to (k-9)(3k-28)=0 \to k=9, \frac{28}{3}$ • $2k-\frac{3k}{3k-24}=9 \to k^2-13k+36=0 \to (k-9)(k-4)=0 \to k=9, 4$ • $-6+\frac{9}{3k-24}=-3 \to k=9$ • $-3k+16-\frac{18}{3k-24}=-5 \to k^2-15k+54=0 \to (k-9)(k-6)=0 \to k=9, 6$
	Note	Uses a correct element equation in part (c) leading to $k=9$ is M1 dM1 A1 even if they have followed through an incorrect $\mathbf{A}^{-1}$ in (i)(a) or an incorrect $\mathbf{A}^2$ in (ii)(b).
	Note	Give M0 dM0 A0 for an incorrect method of $36-3k-4=5 \Rightarrow k=9$
(ii)	**Note**	$\mathbf{M}=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\theta & -p\sin\theta \\ \sin\theta & p\cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$
	Note	**<u>IMPORTANT NOTE</u>** Give (ii)(a) M0A0 (b) M0A0 for a method of $\mathbf{M}=\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ p\sin\theta & p\cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$ leading to (ii)(a) $p=...$, (ii)(b) $\theta=...$
(ii)(a)	**Note**	$\det(\mathbf{M})=\left(-\frac{1}{2}\right)(-1)-\left(-\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)=2$ followed by $p=\sqrt{2}$ is M0 A0
	Note	$p=\det(\mathbf{M})=\left(-\frac{1}{2}\right)(-1)-\left(-\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)=2$ is M1 A1
	Note	$p=\dfrac{\sqrt{(\pm\sqrt{3})^2+(-1)^2}}{\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}=2$ is M1 A1

Leave blank

8. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = u_n + 3n - 2 \qquad n \geqslant 1$$

Prove by induction that, for all positive integers n,

$$u_n = \frac{3}{2}n^2 - \frac{7}{2}n + 5$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$\mathrm{f}(n) = 3^{2n+3} + 40n - 27 \text{ is divisible by } 64$$

(6)

Question Number	Scheme	Notes	Marks
8.	(i) $u_1 = 3,\ u_{n+1} = u_n + 3n - 2\ ,\ u_n = \frac{3}{2}n^2 - \frac{7}{2}n + 5$	(ii) $f(n) = 3^{2n+3} + 40n - 27$ is divisible by 64	
(i)	$n = 1,\ u_1 = \frac{3}{2} - \frac{7}{2} + 5 = 3$	Uses $u_n = \frac{3}{2}n^2 - \frac{7}{2}n + 5$ to show that $u_1 = 3$	B1
	(Assume the result is true for $n = k$)		
	$\{u_{k+1} = u_k + 3k - 2 \Rightarrow\}$ $u_{k+1} = \frac{3}{2}k^2 - \frac{7}{2}k + 5 + 3k - 2\ \left\{= \frac{3}{2}k^2 - \frac{1}{2}k + 3\right\}$	Finds u_{k+1} by attempting to substitute $u_k = \frac{3}{2}k^2 - \frac{7}{2}k + 5$ into $u_{k+1} = u_k + 3k - 2.$ Condone one slip.	M1
	$= \frac{3}{2}(k+1)^2 - 3k - \frac{3}{2} - \frac{1}{2}k + 3$	**dependent on the previous M mark**. Attempts to write u_{k+1} in terms of $(k+1)$	dM1
	$= \frac{3}{2}(k+1)^2 - \frac{7}{2}k + \frac{3}{2}$		
	$= \frac{3}{2}(k+1)^2 - \frac{7}{2}(k+1) + 5$	***Uses algebra*** to achieve this result with no errors	A1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**
			(5)
(ii) **Way 1**	$f(1) = 3^5 + 40 - 27 = 256$	$f(1) = 256$ is the minimum	B1
	$f(k+1) - f(k) = (3^{2(k+1)+3} + 40(k+1) - 27) - (3^{2k+3} + 40k - 27)$	Attempts $f(k+1)$ - $f(k)$	M1
	$f(k+1) - f(k) = 8(3^{2k+3}) + 40$		
	$= 8(3^{2k+3} + 40k - 27) - 64(5k - 4)$	$8(3^{2k+3} + 40k - 27)$ or $8f(k)$	A1
	or $= 8(3^{2k+3} + 40k - 27) - 320k + 256$	$-64(5k-4)$ or $-320k + 256$	A1
	$f(k+1) = 8f(k) - 64(5k-4) + f(k)$ or $f(k+1) = 8f(k) - 320k + 256 + f(k)$ or $f(k+1) = 9(3^{2k+3} + 40k - 27) - 320k + 256$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(3^{2k+3} + 40k - 27)$	dM1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$,</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**
			(6)
(ii) **Way 2**	$f(1) = 3^5 + 40 - 27 = 256$	$f(1) = 256$ is the minimum	B1
	$f(k+1) = 3^{2(k+1)+3} + 40(k+1) - 27$	Attempts $f(k+1)$	M1
	$f(k+1) = 9(3^{2k+3}) + 40k + 13$		
	$= 9(3^{2k+3} + 40k - 27) - 64(5k - 4)$	$9(3^{2k+3} + 40k - 27)$ or $9f(k)$	A1
	or $= 9(3^{2k+3} + 40k - 27) - 320k + 256$	$-64(5k-4)$ or $-320k + 256$	A1
	$f(k+1) = 9f(k) - 64(5k-4)$ or $f(k+1) = 9f(k) - 320k + 256$ or $f(k+1) = 9(3^{2k+3} + 40k - 27) - 320k + 256$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(3^{2k+3} + 40k - 27)$	dM1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$,</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**
			11

Question Number	Scheme	Notes	Marks
8.	(ii) $f(n) = 3^{2n+3} + 40n - 27$ is divisible by 64		
(ii) **Way 3**	**General Method:** Using $f(k+1) - mf(k)$; where m is an integer		
	$f(1) = 3^5 + 40 - 27 = 256$	$f(1) = 256$ is the minimum	B1
	$f(k+1) - mf(k) = (3^{2(k+1)+3} + 40(k+1) - 27) - m(3^{2k+3} + 40k - 27)$	Attempts $f(k+1) - mf(k)$	M1
	$f(k+1) - mf(k) = (9-m)(3^{2k+3}) + 40k(1-m) + (13+27m)$		
	$= (9-m)(3^{2k+3} + 40k - 27) - 64(5k-4)$	$(9-m)(3^{2k+3} + 40k - 27)$ or $(9-m)f(k)$	A1
	or $= (9-m)(3^{2k+3} + 40k - 27) - 320k + 256$	$-64(5k-4)$ or $-320k + 256$	A1
	$f(k+1) = (9-m)f(k) - 64(5k-4) + mf(k)$ or $f(k+1) = (9-m)f(k) - 320k + 256 + mf(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(3^{2k+3} + 40k - 27)$	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>, As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**
(ii) **Way 4**	**General Method:** Using $f(k+1) - mf(k)$		
	$f(1) = 3^5 + 40 - 27 = 256$	$f(1) = 256$ is the minimum	B1
	$f(k+1) - mf(k) = (3^{2(k+1)+3} + 40(k+1) - 27) - m(3^{2k+3} + 40k - 27)$	Attempts $f(k+1) - mf(k)$	M1
	$f(k+1) - mf(k) = (9-m)(3^{2k+3}) + 40k(1-m) + (13+27m)$		
	$m = -55 \Rightarrow f(k+1) + 55f(k) = 64(3^{2k+3}) - 2240k + 1472$	$m = -55$ and $64(3^{2k+3})$	A1
		$m = -55$ and $-2240k + 1472$	A1
	$f(k+1) = 64(3^{2k+3}) - 2240k + 1472 - 55f(k)$ or $f(k+1) = 64(3^{2k+3}) - 64(35k - 23) - 55f(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>, As the result has been shown to be <u>true for $n = 1$</u>, then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**

Question 8 Notes

(i) & (ii)	**Note**	**Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
(i)	**Note**	Moving from either $u_{k+1} = \frac{3}{2}k^2 - \frac{7}{2}k + 5 + 3k - 2$ or $u_{k+1} = \frac{3}{2}k^2 - \frac{1}{2}k + 3$ to $u_{k+1} = \frac{3}{2}(k+1)^2 - \frac{7}{2}(k+1) + 5$ ***with no intermediate stage involving either*** • writing u_{k+1} as a function of $(k+1)$ • **or** writing u_{k+1} as $u_{k+1} = \frac{3}{2}k^2 + 3k + \frac{3}{2} - \frac{7}{2}k - \frac{7}{2} + 5$ is dM1A0A0
	Note	Some candidates will write down $u_{k+1} = \frac{3}{2}k^2 - \frac{7}{2}k + 5 + 3k - 2$ **(give 1st M1)** and simplify this to $u_{k+1} = \frac{3}{2}k^2 - \frac{1}{2}k + 3$ They will then write $u_{k+1} = \frac{3}{2}(k+1)^2 - \frac{7}{2}(k+1) + 5$ **(give 2nd M1)** and use algebra to show that $u_{k+1} = \frac{3}{2}(k+1)^2 - \frac{7}{2}(k+1) + 5 = \frac{3}{2}(k^2 + 2k + 1) - \frac{7}{2}k - \frac{7}{2} + 5 = \frac{3}{2}k^2 - \frac{1}{2}k + 3$ **(give 1st A1)**

	Question 8 Notes Continued	
8. (ii)	**Note**	Some candidates may set $\mathrm{f}(k) = 64M$ and so may prove the following general result • $\{\mathrm{f}(k+1) = 9\mathrm{f}(k) - 64(5k-4)\} \Rightarrow \mathrm{f}(k+1) = 576M - 64(5k-4)$ • $\{\mathrm{f}(k+1) = 9\mathrm{f}(k) - 320k + 256\} \Rightarrow \mathrm{f}(k+1) = 576M - 320k + 256$
	Note	$\mathrm{f}(n) = 3^{2n+3} + 40n - 27$ can be rewritten as either $\mathrm{f}(n) = 27(3^{2n}) + 40n - 27$ or $\mathrm{f}(n) = 27(9^n) + 40n - 27$
	Note	In part (ii), Way 4 there are many alternatives where candidates focus on isolating $\beta(3^{2k+3})$ where β is a multiple of 64. Listed below are some alternative results: • $\mathrm{f}(k+1) = 128(3^{2k+3}) - 119\mathrm{f}(k) + 4800k - 3200$ • $\mathrm{f}(k+1) = -64(3^{2k+3}) + 73\mathrm{f}(k) - 2880k + 1984$ See below for how these are derived.

8. (ii)	(ii) $\mathrm{f}(n) = 3^{2n+3} + 40n - 27$ is divisible by 64		
	The A1A1dM1 marks for Alternatives using $\mathrm{f}(k+1) - m\mathrm{f}(k)$		
Way 4.1	$\mathrm{f}(k+1) = 9(3^{2k+3}) + 40k + 13$		
	$= 128(3^{2k+3}) - 119(3^{2k+3}) + 40k + 13$		
	$= 128(3^{2k+3}) - 119[3^{2k+3} + 40k - 27] + 4800k - 3200$	$m = -119$ and $128(3^{2k+3})$	A1
		$m = -119$ and $4800k - 3200$	A1
	$\mathrm{f}(k+1) = 128(3^{2k+3}) - 119\mathrm{f}(k) + 4800k - 3200$ or $\mathrm{f}(k+1) = 128(3^{2k+3}) - 119[3^{2k+3} + 40k - 27] + 4800k - 3200$	as before	dM1
Way 4.2	$\mathrm{f}(k+1) = 9(3^{2k+3}) + 40k + 13$		
	$= -64(3^{2k+3}) + 73(3^{2k+3}) + 40k + 13$		
	$= -64(3^{2k+3}) + 73[3^{2k+3} + 40k - 27] - 2880k + 1984$	$m = 73$ and $-64(3^{2k+3})$	A1
		$m = 73$ and $-2880k + 1984$	A1
	$\mathrm{f}(k+1) = -64(3^{2k+3}) + 73\mathrm{f}(k) - 2880k + 1984$ or $\mathrm{f}(k+1) = -64(3^{2k+3}) + 73[3^{2k+3} + 40k - 27] - 2880k + 1984$	as before	dM1