Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Monday 14 May 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+3) = \frac{n}{a}(n+1)(n+b)$$

where a and b are integers to be found.

(4)

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June 2018
WFM01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\sum_{r=1}^{n} r(r+3) = \sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r$		
	$= \frac{1}{6}n(n+1)(2n+1) + 3\left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(r+3)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{6}n(n+1)\left[(2n+1)+9\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both correct standard formulae.	dM1
	$= \frac{1}{6}n(n+1)(2n+10)$	{this step does not have to be written}	
	$= \frac{n}{3}(n+1)(n+5)$ or $\frac{1}{3}n(n+1)(n+5)$	Correct completion with no errors. **Note:** $a=3, b=5$	A1
			(4)
			4

Question 1 Notes

1.	**Note**	Applying e.g. $n=1, n=2$ to the printed equation without applying the standard formulae to give $a=3, b=5$ is M0A0M0A0
	Alt 1	**Alt Method 1 (Award the first two marks using the main scheme)** Using $\frac{1}{3}n^3 + 2n^2 + \frac{5}{3}n \equiv \frac{1}{a}n^3 + \left(\frac{b+1}{a}\right)n^2 + \frac{b}{a}n$ o.e.
	dM1	Equating coefficients to find both $a = \ldots$ and $b = \ldots$ **and** at least one correct of $a=3$ or $b=5$
	A1	Finds $a=3$ and $b=5$
	Alt 2	**Alt Method 2: (Award the first two marks using the main scheme)** $\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) \equiv \frac{n}{a}(n+1)(n+b)$
	dM1	Substitutes $n=1, n=2$, into this identity o.e. and solves to find both $a = \ldots$ and $b = \ldots$ **and** at least one correct of $a=3, b=5$. **Note:** $n=1$ gives $4 = \frac{2(1+b)}{a}$ or $2a-b=1$ and $n=2$ gives $14 = \frac{6(2+b)}{a}$ or $7a-3b=6$
	A1	Finds $a=3$ and $b=5$
	Note	Allow final dM1A1 for $\frac{1}{3}n^3 + 2n^2 + \frac{5}{3}n$ or $\frac{1}{3}(n^3 + 6n^2 + 5n) \rightarrow \frac{n}{3}(n+1)(n+5)$ with no incorrect working.
	Note	A correct proof $\sum_{r=1}^{n} r(r+3) = \frac{n}{3}(n+1)(n+5)$ followed by stating an incorrect e.g. $a=5, b=3$ is M1A1dM1A1 (ignore subsequent working)
	Note	Give A0 for $\frac{2}{6}n(n+1)(n+5)$ without reference to $a=3$ or $\frac{n}{3}(n+1)(n+5)$ or $\frac{1}{3}n(n+1)(n+5)$

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2. The transformation represented by the 2×2 matrix **P** is an anticlockwise rotation about the origin through 45 degrees.

(a) Write down the matrix **P**, giving the exact numerical value of each element.

(1)

$$\mathbf{Q} = \begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix}, \text{ where } k \text{ is a constant and } k > 0$$

(b) Describe fully the single geometrical transformation represented by the matrix **Q**.

(2)

The combined transformation represented by the matrix **PQ** transforms the rhombus R_1 onto the rhombus R_2.

The area of the rhombus R_1 is 6 and the area of the rhombus R_2 is 147

(c) Find the value of the constant k.

(4)

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Question Number	Scheme	Notes	Marks
2.	**P** represents an anti-clockwise rotation about the origin through 45 degrees		
(a)	$\{\mathbf{P} = \}\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ or e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$	Correct matrix which is expressed in exact surds	B1
			(1)
(b)	Enlargement	Enlargement or enlarge	M1
	Centre $(0, 0)$ with scale factor $k\sqrt{2}$	About $(0, 0)$ or about O or about the origin **and** scale or factor or times **and** $k\sqrt{2}$ **Note:** Allow $\sqrt{2k^2}$ in place of $k\sqrt{2}$	A1
	Note: Give M0A0 for combinations of transformations		**(2)**
(c) **Way 1**	$\{\mathbf{PQ} =\}\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix} = \begin{pmatrix} k & -k \\ k & k \end{pmatrix}$	Multiplies their matrix from part (a) by **Q [either way round]** and applies "$ad - bc$" to the resulting matrix	M1
	$\{\det \mathbf{PQ} =\}\ (k)(k) - (-k)(k) = 2k^2$	to give $2k^2$ or states $\lvert \text{their } \det \mathbf{PQ} \rvert = 2k^2$ Condone $\{\det \mathbf{PQ} =\}\ k^2 + k^2$	A1
	$6(2k^2) = 147$ or $2k^2 = \frac{147}{6}$	$6(\text{their determinant}) = 147$ **or** puts their determinant **equal to** $\frac{147}{6}$	M1
	$\left\{\Rightarrow k^2 = \frac{49}{4} \Rightarrow\right\}\ k = \frac{7}{2}$	Obtains $k = 3.5$, o.e.	A1
			(4)
(c) **Way 2**	$\det \mathbf{Q} = (k\sqrt{2})(k\sqrt{2}) - (0)(0)$ or $\det \mathbf{Q} = (k\sqrt{2})(k\sqrt{2})$	applies "$ad - bc$" to **Q** or applies $(k\sqrt{2})^2$	M1
	$\{\det \mathbf{P} = 1 \Rightarrow\}\ \det \mathbf{PQ} = (1)(2k^2) = 2k^2$ or $\det \mathbf{Q} = 2k^2$	and deduces that $\det \mathbf{PQ} = 2k^2$ or states $\lvert \text{their } \det \mathbf{PQ} \rvert = 2k^2$ or $\det \mathbf{Q} = 2k^2$	A1
	$6(2k^2) = 147$ or $2k^2 = \frac{147}{6}$	$6(\text{their } \det(\mathbf{PQ})) = 147$ **or** $(\text{their } \det(\mathbf{PQ})) = \frac{147}{6}$ **or** $6(\text{their } \det(\mathbf{Q})) = 147$ **or** $(\text{their } \det(\mathbf{PQ})) = \frac{147}{6}$	M1
	$\left\{\Rightarrow k^2 = \frac{49}{4} \Rightarrow\right\}\ k = \frac{7}{2}$	Obtains $k = 3.5$, o.e.	A1
			(4)
			7

		Question 2 Notes
2. (b)	**Note**	"original point" is not acceptable in place of the word "origin".
	Note	"expand" is not acceptable for M1
	Note	"enlarge x by $k\sqrt{2}$ and no change in y" is M0A0
(c)	**Note**	Obtaining $k = \pm 3.5$ with no evidence of $k = 3.5$ {only} is A0
	Way 2 Note 1	Give M1A1M0A0 for writing down $147(2k^2) = 6$ **or** $\frac{1}{2k^2} = \frac{147}{6}$ **or** $6\left(\frac{1}{2k^2}\right) = 147$, o.e. with no other supporting working.
	Way 2 Note 2	Give M0A0M1A0 for writing $\det \mathbf{Q} = \frac{1}{k^2 - (-k^2)}$ or $\frac{1}{2k^2}$, followed by $6\left(\frac{1}{2k^2}\right) = 147$
	Note	Allow M1A1 for an incorrect rotation matrix **P**, leading to $\det \mathbf{PQ} = 2k^2$
	Note	Allow M1A1M1A1 for an incorrect rotation matrix **P**, leading to $\det \mathbf{PQ} = 2k^2$ and $k = 3.5$, o.e.
	Note	Using the scale factor of enlargement to write down $k\sqrt{2} = \sqrt{\frac{147}{6}} \Rightarrow k = 3.5$ is M1A1dM1A1
	Note	Using the scale factor of enlargement to write down $k\sqrt{2} = \sqrt{\frac{6}{147}}$ is M1A1dM0

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3.

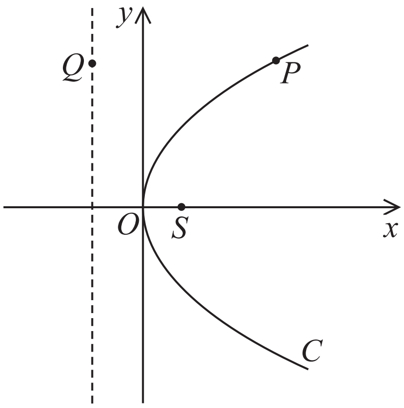


Figure 1

Figure 1 shows the parabola C which has cartesian equation $y^2 = 6x$. The point S is the focus of C.

(a) Find the coordinates of the point S.

(1)

The point P lies on the parabola C, and the point Q lies on the directrix of C. PQ is parallel to the x-axis with distance $PQ = 14$

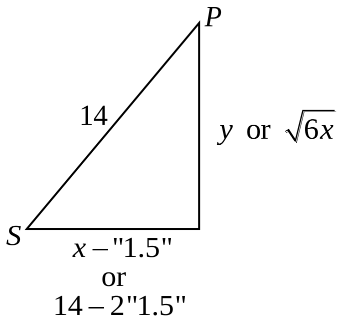
(b) State the distance SP.

(1)

Given that the point P is above the x-axis,

(c) find the exact coordinates of P.

(3)

Question Number	Scheme	Notes	Marks
3.	$C: y^2 = 6x$; S is the focus of C; $y^2 = 4ax$; $P(at^2, 2at)$; Q lies on the directrix of C. $PQ = 14$		
(a)	$\{a = 1.5 \Rightarrow\}$ S has coordinates $(1.5, 0)$	$(1.5, 0)$ or $\left(\frac{3}{2}, 0\right)$ or $\left(\frac{6}{4}, 0\right)$	B1 **cao**
	Note: You can recover this mark for $S(1.5, 0)$ stated either parts (b) or part (c)		**(1)**
(b)	$\{PQ$ is parallel to the x-axis $\Rightarrow\}$ Focus-directrix Property $\Rightarrow SP\ \{= PQ\} = 14$	$SP = 14$ **or** 14 stated by itself in (b)	B1 **cao**
	Note: $PQ = 14$ stated by itself without reference to $SP = 14$ is B0		**(1)**
(c) **Way 1**	$\left\{\text{directrix } x = -\frac{3}{2}\ \&\ PQ = 14 \Rightarrow\right\}$ $x_P = 14 - \frac{3}{2}$ $\{= 12.5\}$	$x = 14 -$ their "a"	M1
	$y_P^{\ 2} = 6(12.5) \Rightarrow y_P = ...$	**dependent on the previous M mark** Substitutes their x into $y^2 = 6x$ and finds $y = ...$	dM1
	Either $x = 12.5,\ y = 5\sqrt{3}$ **or** $\left(12.5, 5\sqrt{3}\right)$	Correct and paired. Accept $\left(12.5, \sqrt{75}\right)$	A1
			(3)
			
(c) **Way 2**	$(x - 1.5)^2 + (6x) = 14^2$ $\Rightarrow x^2 + 3x - 193.75 = 0 \Rightarrow x = ...$	Applies Pythagoras to $x -$ "a", $\sqrt{6x}$ and 14, then forms and solves quadratic equation in x to give $x = ...$	M1
		As in Way 1	dM1 A1
			(3)
(c) **Way 3**	$11^2 + y^2 = 14^2 \Rightarrow y = ...$	Applies Pythagoras to $14 -$ "$2a$", y and 14, and solves to give $y = ...$	M1
	$\left(\sqrt{75}\right)^2 = 6x \Rightarrow x = ...$	**dependent on the previous M mark** Substitutes their y into $y^2 = 6x$ and finds $x = ...$	dM1
	Either $x = 12.5,\ y = 5\sqrt{3}$ **or** $\left(12.5, 5\sqrt{3}\right)$	Correct and paired. Accept $\left(12.5, \sqrt{75}\right)$	A1
			(3)
(c) **Way 4**	$(1.5t^2 - 1.5)^2 + (3t)^2 = 14^2$ $\Rightarrow 2.25t^4 + 4.5t^2 - 193.75 = 0$ $\{\text{or } 9t^4 + 18t^2 - 775 = 0\}$ $\Rightarrow t^2 = \frac{25}{3} \Rightarrow t = \frac{5\sqrt{3}}{3}$	Applies Pythagoras to "1.5"t^2 − "1.5", 2("1.5")t and 14, forms and solves a quadratic equation in t^2 to give $t^2 = ...$ or $t = ...$, and finds at leasts one of $x = ...$ or $y = ...$ by using $x =$ "1.5"t^2 or $y = 2$("1.5")t	M1
	$\Rightarrow x = 1.5\left(\frac{5\sqrt{3}}{3}\right)^2,\ y = 3\left(\frac{5\sqrt{3}}{3}\right)$	**dependent on the previous M mark** Finds both $x = ...$ and $y = ...$ by using $x =$ "1.5"t^2 and $y = 2$("1.5")t	dM1
	Either $x = 12.5,\ y = 5\sqrt{3}$ **or** $\left(12.5, 5\sqrt{3}\right)$	Correct and paired. Accept $\left(12.5, \sqrt{75}\right)$	A1
			(3)
			5

Question Number	Scheme	Notes	Marks
3.	$C: y^2 = 6x$; S is the focus of C; $y^2 = 4ax$; $P(at^2, 2at)$; Q lies on the directrix of C. $PQ = 14$		
(c) **Way 5**	$\left\{x_P = \frac{3}{2}t^2,\ x_Q = -\frac{3}{2},\ PQ = 14 \Rightarrow\right\}$ $(1.5t^2 - -1.5) = 14 \Rightarrow 1.5t^2 = 12.5$ $\Rightarrow t^2 = \frac{25}{3} \Rightarrow t = \frac{5\sqrt{3}}{3}$ $\Rightarrow x = 1.5\left(\frac{5\sqrt{3}}{3}\right)^2,\ y = 3\left(\frac{5\sqrt{3}}{3}\right)$	Uses horizontal distance $PQ = 14$ to form and solve the equation $"1.5"t^2 - "-1.5" = 14$ to give $t^2 = ...$ or $t = ...$, and finds at leasts one of $x = ...$ or $y = ...$ by using $x = "1.5"t^2$ or $y = 2("1.5")t$	M1
		dependent on the previous M mark Finds both $x = ...$ and $y = ...$ by using $x = "1.5"t^2$ and $y = 2("1.5")t$	dM1
	Either $x = 12.5,\ y = 5\sqrt{3}$ **or** $(12.5,\ 5\sqrt{3})$	Correct and paired. Accept $(12.5,\ \sqrt{75})$	A1
			(3)
(c) **Way 6**	$\left\{S(1.5, 0),\ P\left(\frac{y^2}{6}, y\right),\ SP = 14 \Rightarrow\right\}$ $\left(\frac{1}{6}y^2 - \frac{3}{2}\right)^2 + y^2 = 14^2 \Rightarrow y = ...$ $\{y^4 + 18y^2 - 6975 = 0\}$	Applies Pythagoras to $\frac{y^2}{6} - "1.5"$, y and 14, and solves to give $y = ...$	M1
	$\left(\sqrt{75}\right)^2 = 6x \Rightarrow x = ...$	**dependent on the previous M mark** Substitutes their y into $y^2 = 6x$ and finds $x = ...$	dM1
	Either $x = 12.5,\ y = 5\sqrt{3}$ **or** $(12.5,\ 5\sqrt{3})$	Correct and paired. Accept $(12.5,\ \sqrt{75})$	A1
			(3)

	Question 3 Notes	
3. (c)	**Note**	**<u>Writing coordinates the wrong way round</u>** E.g. writing $x = 12.5,\ y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 12.5)$ is final A0
	Note	Obtaining both $(12.5,\ 5\sqrt{3})$ and $(12.5,\ -5\sqrt{3})$ with no evidence of only $(12.5,\ 5\sqrt{3})$ is A0
	Note	Give final A1 for $(12.5,$ awrt $8.66)$, with either $y = \sqrt{75}$ or $y = 5\sqrt{3}$ seen in their working
	Note	You can mark part (b) and part (c) together

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4.

$$\mathbf{A} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}$$

where p and q are non-zero real constants.

(a) Find $\mathbf{A}^{-1}$ in terms of p and q.

(3)

Given $\mathbf{XA} = \mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$$

(b) find the matrix $\mathbf{X}$, giving your answer in its simplest form.

(4)

Question Number	Scheme	Notes	Marks
4.	$\mathbf{A}=\begin{pmatrix}2p & 3q\\ 3p & 5q\end{pmatrix}$; $\mathbf{XA}=\mathbf{B}$; $\mathbf{B}=\begin{pmatrix}p & q\\ 6p & 11q\\ 5p & 8q\end{pmatrix}$		
(a)	$\{\det(\mathbf{A})=\}\ 2p(5q)-(3p)(3q)\ \{=pq\}$	$2p(5q)-(3p)(3q)$ which can be un-simplifed or simplifed	B1
	$\{\mathbf{A}^{-1}=\}\ \frac{1}{pq}\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}$ or $\begin{pmatrix}\frac{5}{p} & -\frac{3}{p}\\ -\frac{3}{q} & \frac{2}{q}\end{pmatrix}$	$\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}$	M1
		Correct $\mathbf{A}^{-1}$	A1
			(3)
(b) **Way 1**	$\{\mathbf{X}=\mathbf{BA}^{-1}=\}$ $\begin{pmatrix}p & q\\ 6p & 11q\\ 5p & 8q\end{pmatrix}\frac{1}{pq}\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}=\ldots$	Attempts $\mathbf{BA}^{-1}$ and finds at least one element (or at least one element calculation) of their matrix **X** **Note:** Allow one slip in copying down **B** **Note:** Allow one slip in copying down $\mathbf{A}^{-1}$	M1
	$=\frac{1}{pq}\begin{pmatrix}2pq & -pq\\ -3pq & 4pq\\ pq & pq\end{pmatrix}$	At least 4 correct elements (need not be in a matrix)	A1
		dependent on the first M mark Finds a 3×2 matrix of 6 elements	dM1
	$=\begin{pmatrix}2 & -1\\ -3 & 4\\ 1 & 1\end{pmatrix}$	Correct simplified matrix for **X**	A1
			(4)
(b) **Way 2**	$\{\mathbf{XA}=\mathbf{B}\Rightarrow\}\begin{pmatrix}a & b\\ c & d\\ e & f\end{pmatrix}\begin{pmatrix}2p & 3q\\ 3p & 5q\end{pmatrix}=\begin{pmatrix}p & q\\ 6p & 11q\\ 5p & 8q\end{pmatrix}$ $2pa+3pb=p,\quad 3qa+5qb=q$ **or** $2pc+3pd=6p,\quad 3qc+5qd=11q$ **or** $2pe+3pf=5p,\quad 3qe+5qf=8q$ **and** finds at least one of *a*, *b*, *c*, *d*, *e* or *f*	Applies $\mathbf{XA}=\mathbf{B}$ for a 3×2 matrix **X** and attempts simultaneous equations in *a* and *b* **or** *c* and *d* **or** *e* and *f* to find at least one of *a*, *b*, *c*, *d*, *e* or *f* **Note:** Allow one slip in copying down **A** **Note:** Allow one slip in copying down **B**	M1
	$\begin{Bmatrix}2a+3b=1, & 3a+5b=1\\ 2c+3d=6, & 3c+5d=11\\ 2e+3f=5, & 3e+5f=8\end{Bmatrix}\Rightarrow\begin{matrix}a=2,\ b=-1\\ c=-3,\ d=4\\ e=1,\ f=1\end{matrix}$	At least 4 correct elements	A1
		dependent on the first M mark Finds all 6 elements for the 3×2 matrix **X**	dM1
	$\Rightarrow\mathbf{X}=\begin{pmatrix}2 & -1\\ -3 & 4\\ 1 & 1\end{pmatrix}$	Correct simplified matrix for **X**	A1
			(4)
			7

	Question 4 Notes	
4. (a)	**Note**	Condone $\frac{1}{10pq-9pq}\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}$ or $\frac{1}{2p(5q)-(3p)(3q)}\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}$ for A1
	Note	Condone $\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}\frac{1}{pq}$ or $\begin{pmatrix}5q & -3q\\ -3p & 2p\end{pmatrix}\frac{1}{2p(5q)-(3p)(3q)}$ for A1
	Note	Condone $\begin{pmatrix}\frac{5q}{pq} & -\frac{3q}{pq}\\ -\frac{3p}{pq} & \frac{2p}{pq}\end{pmatrix}$ for A1
(b)	**Note**	**Way 1:** Allow SC 1st A1 for at least 4 correct elements in $\begin{pmatrix}\frac{2pq}{\text{their det }\mathbf{A}} & \frac{-pq}{\text{their det }\mathbf{A}}\\ \frac{-3pq}{\text{their det }\mathbf{A}} & \frac{4pq}{\text{their det }\mathbf{A}}\\ \frac{pq}{\text{their det }\mathbf{A}} & \frac{pq}{\text{their det }\mathbf{A}}\end{pmatrix}$ or for at least 4 of these elements seen in their calculations

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5. Given that

$$z^4 - 6z^3 + 34z^2 - 54z + 225 \equiv (z^2 + 9)(z^2 + az + b)$$

where a and b are real numbers,

(a) find the value of a and the value of b.

(2)

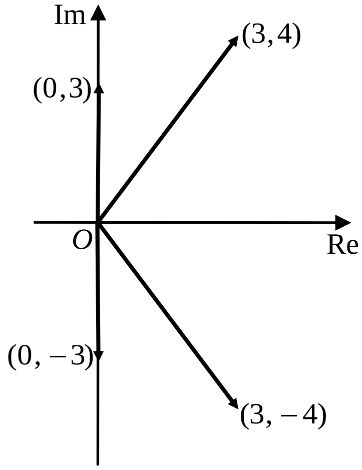
(b) Hence find the exact roots of the equation

$$z^4 - 6z^3 + 34z^2 - 54z + 225 = 0$$

(4)

(c) Show your roots on a single Argand diagram.

(2)

Question Number	Scheme	Notes	Marks
5.	$z^4 - 6z^3 + 34z^2 - 54z + 225 \equiv (z^2+9)(z^2+az+b)$; a, b are real numbers		
(a)	$a = -6,\ b = 25$	At least one of $a = -6$ or $b = 25$	B1
		Both $a = -6$ and $b = 25$	B1
			(2)
(b)	$\{z^2 + 9 = 0 \Rightarrow\}\ z = 3\text{i}, -3\text{i}$	At least one of $3\text{i}, -3\text{i}, \sqrt{9}\,\text{i}$ or $-\sqrt{9}\,\text{i}$	M1
		Both 3i **and** -3i	A1
	$\{z^2 - 6z + 25 = 0 \Rightarrow\}$ • $z = \dfrac{--6 \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$ **or** • $(z-3)^2 - 9 + 25 = 0 \Rightarrow z = ...$	Correct method of applying the quadratic formula or completing the square for solving their $z^2 + az + b = 0;\ a, b \neq 0$	M1
	$\{z =\}\ 3 + 4\text{i}, 3 - 4\text{i}$	$3 + 4\text{i}$ **and** $3 - 4\text{i}$	A1
			(4)
(c)		**Criteria** • $\pm 3\text{i}$ or $\pm$(their k)i plotted correctly on the imaginary axis, where $k \in \mathbb{R}, k > 0$ • **dependent on the final M mark being awarded in part (b)** Their final two roots of the form $\lambda \pm \mu\text{i},\ \lambda, \mu \neq 0$, are plotted correctly	
		Satisfies at least one of the criteria	B1ft
		Satisfies both criteria with some indication of scale or coordinates stated with at least one pair of roots symmetrical about the real axis	B1ft
			(2)
			8

Question 5 Notes

5. (a)	**Note**	Give B1B0 for writing down a correct $(z^2 - 6z + 25)$, followed by $a = 25, b = -6$
	Note	If the values of *a* and *b* **are not stated**, then • give B1B1 for writing down a correct $(z^2 - 6z + 25)$, • give B1B0 for writing down $(z^2 + \text{their "}a\text{"}z + \text{their "}b\text{"})$, with exactly one of their *a* or their *b* correct
(b)	**Note**	No working leading to $z = 3\text{i}, -3\text{i}$ is 1st M1 1st A1
	Note	$z = \pm\sqrt{9\text{i}}$ unless recovered is 1st M0 1st A0
	Note	You can assume $x \equiv z$ for solutions in this question
	Note	• Give 2nd M1 2nd A1 for $z^2 - 6z + 25 = 0 \Rightarrow z = 3 + 4\text{i}, 3 - 4\text{i}$ with no intermediate working. • Give 2nd M1 2nd A1 for $z = 3 + 4\text{i}, 3 - 4\text{i}$ with no intermediate working having stated $a = -6, b = 25$ in part (a) or part (b). • Otherwise, give 2nd M0 2nd A0 for $z = 3 + 4\text{i}, 3 - 4\text{i}$ with no intermediate working.

	Question 5 Notes Continued	
5. (b)	**Note**	**<u>Special Case:</u>** If their ***3-term quadratic*** factor $z^2 + "a"z + "b"$ **can** be factorised then give Special Case 2nd M1 for correct factorisation leading to $z = ...$
	Note	Otherwise, give 2nd M0 for applying a method of factorisation to solve their 3TQ.
	Note	**<u>Reminder:</u>** Method Mark for solving a 3TQ, "$az^2 + bz + c = 0$" **Formula:** Attempt to use the correct formula (with values for *a*, *b* and *c*) **Completing the square:** $\left(z \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ q \neq 0,$ leading to $z = ...$
5. (b)(c)	**Note**	You can mark part (b) and part (c) together

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6.

$$f(x) = \frac{2(x^3 + 3)}{\sqrt{x}} - 9, \quad x > 0$$

The equation $f(x) = 0$ has two real roots α and β, where $0.4 < \alpha < 0.5$ and $1.2 < \beta < 1.3$

(a) Taking 0.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.
(5)

(b) Use linear interpolation once on the interval [1.2, 1.3] to find an approximation to β, giving your answer to 3 decimal places.
(4)

Question Number	Scheme	Notes	Marks
6.	Given $f(x)=\dfrac{2(x^3+3)}{\sqrt{x}}-9,\ x>0$; Roots α, β: $0.4<\alpha<0.5$ and $1.2<\beta<1.3$		
(a)	$\left\{f(x)=2x^{\frac{5}{2}}+6x^{-\frac{1}{2}}-9 \Rightarrow\right\}$	Some evidence of $\pm\lambda x^n \to \pm\mu x^{n-1}$; $\lambda, \mu \neq 0$	M1
		Differentiates to give $\pm Ax^{\frac{3}{2}} \pm Bx^{-\frac{3}{2}}$; $A, B \neq 0$	M1
	$f'(x)=5x^{\frac{3}{2}}-3x^{-\frac{3}{2}}$	Correct differentiation which can be simplified or un-simplified	A1
	$\left\{\alpha \simeq 0.45-\dfrac{f(0.45)}{f'(0.45)}\right\} \Rightarrow \alpha \simeq 0.45-\dfrac{0.2159541693...}{-8.428734015...}$	Valid attempt at Newton-Raphson using their values of $f(0.45)$ and $f'(0.45)$	M1
	$\{\alpha=0.4756211869...\} \Rightarrow \alpha=0.476$ (3 dp)	**dependent on all 4 previous marks** 0.476 on their first iteration (Ignore any subsequent iterations)	A1 **cso**
	Correct differentiation followed by a correct answer of 0.476 scores full marks in part (a) **Correct answer with <u>no</u> working scores no marks in part (a)**		**(5)**
(a) **Alt 1**	**<u>Alternative method 1 for the first 3 marks</u>**		
	$\left\{\begin{matrix} u=2x^3+6 & v=\sqrt{x} \\ u'=6x^2 & v'=\frac{1}{2}x^{-\frac{1}{2}}\end{matrix}\right\} \Rightarrow$	Some evidence of $\pm\lambda x^n \to \pm\mu x^{n-1}$; $\lambda, \mu \neq 0$	M1
		Differentiates to give $\dfrac{\pm Ax^2(\sqrt{x}) \pm Bx^{-\frac{1}{2}}(2x^3+6)}{x}$; $A, B \neq 0$	M1
	$f'(x)=\dfrac{6x^2(\sqrt{x})-\frac{1}{2}x^{-\frac{1}{2}}(2x^3+6)}{x}$	Correct differentiation which can be simplified or un-simplified	A1
(b)	**Either** • $\dfrac{\beta-1.2}{\text{"0.3678924937..."}}=\dfrac{1.3-\beta}{\text{"0.1161410527..."}}$	At least one of either $\pm$(awrt 0.37, trunc. 0.36, awrt 0.12, or trunc. 0.11) This mark may be implied.	B1
	• $\dfrac{\beta-1.2}{1.3-\beta}=\dfrac{\text{"0.3678924937..."}}{\text{"0.1161410527..."}}$ • $\dfrac{\beta-1.2}{\text{"0.3678924937..."}}=\dfrac{1.3-1.2}{\text{"0.1161410527..."}+\text{"0.3678924937..."}}$	A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.	M1
	• $\beta=\left(\dfrac{(1.3)(\text{"0.3678924937..."})+(1.2)(\text{"0.1161410527..."})}{\text{"0.1161410527..."}+\text{"0.3678924937..."}}\right)$ $=\left(\dfrac{0.4782602418...+0.1393692632...}{0.4840335464...}\right)=\left(\dfrac{0.617629505...}{0.484033546...}\right)$ • $\beta=1.2+\left(\dfrac{\text{"0.3678924937..."}}{\text{"0.1161410527..."}+\text{"0.3678924937..."}}\right)(0.1)$ • $\beta=1.2+\left(\dfrac{\text{"}-0.3678924937...\text{"}}{\text{"}-0.1161410527...\text{"}+\text{"}-0.3678924937...\text{"}}\right)(0.1)$	**dependent on the previous M mark** Rearranges to give $\beta=...$	dM1
	$\{\beta=1.276005578...\} \Rightarrow \beta=1.276$ (3 dp)	1.276 (Ignore any subsequent iterations)	A1 **cao**
			(4)
			9

Question Number	Scheme	Notes	Marks
6. (b) **Way 2**	$\dfrac{x}{\text{"0.3678924937..."}} = \dfrac{0.1 - x}{\text{"0.1161410527..."}}$	At least one of either $\pm$(awrt 0.37, trunc. 0.36, awrt 0.12, or trunc. 0.11) This mark may be implied.	B1
	$x = \dfrac{(0.1)(\text{"0.3678924937..."})}{0.4840335464...} = 0.0760055778...$ $\Rightarrow \beta = 1.2 + 0.0760055778...$	Finds x using a correct method of similar triangles and applies "1.5 + their x"	M1 dM1
	$\{\beta = 1.276005578...\} \Rightarrow \beta = 1.276$ (3 dp)	1.276	A1 **cao**
			(4)
(b) **Way 3**	$\dfrac{0.1 - x}{\text{"0.3678924937..."}} = \dfrac{x}{\text{"0.1161410527..."}}$	At least one of either $\pm$(awrt 0.37, trunc. 0.36, awrt 0.12, or trunc. 0.11) This mark may be implied.	B1
	$x = \dfrac{(0.1)(\text{"0.1161410527..."})}{0.4840335464...} = 0.0239944222...$ $\Rightarrow \beta = 1.3 - 0.0239944222...$	Finds x using a correct method of similar triangles and applies "1.6 − their x"	M1 dM1
	$\{\beta = 1.276005578...\} \Rightarrow \beta = 1.276$ (3 dp)	1.276	A1 **cao**
			(4)

	Question 6 Notes	
6. (a)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	M1	This mark can be implied by applying at least one correct ***value*** of either $\mathrm{f}(0.45)$ or $\mathrm{f}'(0.45)$ to 1 significant figure in $0.45 - \dfrac{\mathrm{f}(0.45)}{\mathrm{f}'(0.45)}$. So just $0.45 - \dfrac{\mathrm{f}(0.45)}{\mathrm{f}'(0.45)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	You can imply the M1A1A1 marks for algebraic differentiation for either • $\mathrm{f}'(0.45) = 5(0.45)^{\frac{3}{2}} - 3(0.45)^{-\frac{3}{2}}$ • $\mathrm{f}'(1.5)$ applied correctly in $\alpha \simeq 0.45 - \dfrac{\dfrac{2((0.45)^3+3)}{\sqrt{0.45}} - 9}{5(0.45)^{\frac{3}{2}} - 3(0.45)^{-\frac{3}{2}}}$

(a) **Alt 2**	<u>**Alternative method 2 for the first 3 marks**</u>		
	$\left\{\begin{array}{ll} u = 2x^3+6 & v = x^{-\frac{1}{2}} \\ u' = 6x^2 & v' = -\frac{1}{2}x^{-\frac{3}{2}} \end{array}\right\} \Rightarrow$	Some evidence of $\pm\lambda x^n \to \pm\mu x^{n-1}$; $\lambda, \mu \neq 0$ **Note:** Allow M1 for either $\pm Ax^2(x^{-\frac{1}{2}})$ or $\pm Bx^{-\frac{3}{2}}(2x^3+6)$ or $\pm Bx^{-\frac{3}{2}}(x^3+3)$; $A, B \neq 0$	M1
		Differentiates to give $\pm Ax^2(x^{-\frac{1}{2}}) \pm Bx^{-\frac{3}{2}}(2x^3+6)$; $A, B \neq 0$	M1
	$\mathrm{f}'(x) = 6x^2(x^{-\frac{1}{2}}) - \dfrac{1}{2}x^{-\frac{3}{2}}(2x^3+6)$	Correct differentiation which can be simplified or un-simplified	A1

		Question 6 Notes Continued
6. (b)	**Note**	Condone writing the symbol α in place of β in part (b)
	Note	$\frac{\beta - 1.2}{1.3 - \beta} = \left\lvert\frac{\text{"}-0.3678924937...\text{"}}{\text{"}0.1161410527...\text{"}}\right\rvert$ is a valid method for the first M mark
	Note	Give 1st M1 for either $\frac{-\text{f}(1.2)}{\text{f}(1.3)} = \frac{\beta - 1.2}{1.3 - \beta}$ or $\frac{\lvert\text{f}(1.2)\rvert}{\text{f}(1.3)} = \frac{\beta - 1.2}{1.3 - \beta}$ or $\frac{\lvert\text{f}(1.2)\rvert}{\lvert\text{f}(1.3)\rvert} = \frac{\beta - 1.2}{1.3 - \beta}$
	Note	Give M1M1 for the correct statement $\frac{1.3\lvert\text{f}(1.2)\rvert + 1.2\text{f}(1.3)}{\text{f}(1.3) + \lvert\text{f}(1.2)\rvert}$
	Note	Give M1M1 for the correct statement $\beta = \frac{1.3 + 1.2k}{k+1}$, where $k = \frac{\text{f}(1.3)}{\lvert\text{f}(1.2)\rvert} = \frac{0.116141...}{0.367892...} = 0.31569...$
	Note	$\frac{\beta - 1.2}{1.3 - \beta} = \frac{\text{"}0.3678924937...\text{"}}{\text{"}0.1161410527...\text{"}} \Rightarrow \beta = 1.276$ with no intermediate working is B1 M1 dM1 A1
	Note	$\frac{\beta - 1.2}{-0.3678924937...} = \frac{1.3 - \beta}{0.1161410527...} \Rightarrow \beta = 1.34613... = 1.346$ (3 dp) is B1 M0 dM0 A0
	Note	$\frac{\beta - 1.2}{-0.3678924937...} = \frac{1.3 - \beta}{-0.1161410527...} \Rightarrow \beta = 1.276$ (3 dp) is B1 M1 dM1 A1

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7. It is given that α and β are roots of the equation $5x^2 - 4x + 3 = 0$

Without solving the quadratic equation,

(a) find the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(5)

(b) find a quadratic equation which has roots $\frac{3}{\alpha^2}$ and $\frac{3}{\beta^2}$

giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers to be found.

(4)

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Question Number	Scheme	Notes	Marks
7.	$5x^2 - 4x + 3 = 0$ has roots α, β		
(a)	$\alpha + \beta = \frac{4}{5},\ \alpha\beta = \frac{3}{5}$	**Both** $\alpha + \beta = \frac{4}{5}$ **and** $\alpha\beta = \frac{3}{5}$, seen or implied	B1
	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$	**States or uses** $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$	M1
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = ...$	**Use** of the **correct** identity for $\alpha^2 + \beta^2$ (May be implied by their work)	M1
	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{\left(\frac{4}{5}\right)^2 - 2\left(\frac{3}{5}\right)}{\left(\frac{3}{5}\right)^2}$	Applies $\alpha^2\beta^2 = (\alpha\beta)^2$ correctly in the denominator of $\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$ using their value of $\alpha\beta$	M1
	$= \frac{-\left(\frac{14}{25}\right)}{\left(\frac{9}{25}\right)} = -\frac{14}{9}$	**dependent on ALL previous marks being awarded** $-\frac{14}{9}$ or $-1\frac{5}{9}$ or $-1.\dot{5}$ **from correct working**	A1 **cso**
			(5)
(b) **Way 1**	$\{\text{Sum} =\}\ \frac{3}{\alpha^2} + \frac{3}{\beta^2} = 3\left(-\frac{14}{9}\right)\ \left\{= -\frac{14}{3} \text{ or } -\frac{42}{9}\right\}$	Simplifies $\frac{3}{\alpha^2} + \frac{3}{\beta^2}$ to give 3(their answer to (a))	M1
	$\{\text{Product} =\}\ \left(\frac{3}{\alpha^2}\right)\left(\frac{3}{\beta^2}\right) = \frac{9}{\left(\frac{3}{5}\right)^2}\ \{= 25\}$	Applies $\frac{9}{(\text{their } \alpha\beta)^2}$ using their value of $\alpha\beta$	M1
	$x^2 + \frac{14}{3}x + 25 = 0$	Applies $x^2 - (\text{sum})x + \text{product}$ (can be implied), where sum and product are numerical values. **Note:** "$=0$" is not required for this mark	M1
	$3x^2 + 14x + 75 = 0$	***Any integer multiple*** of $3x^2 + 14x + 75 = 0$, including the "$=0$"	A1
			(4)
			9

Question 7 Notes

7. (a)	**Note**	Writing a correct $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ without attempting to substitute at least one of either their $\alpha + \beta$ or their $\alpha\beta$ into $(\alpha + \beta)^2 - 2\alpha\beta$ is 2nd M0
	Note	Give B0M1M1M1A0 for $\alpha + \beta = -\frac{4}{5},\ \alpha\beta = \frac{3}{5}$ leading to $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\left(-\frac{4}{5}\right)^2 - 2\left(\frac{3}{5}\right)}{\left(\frac{3}{5}\right)^2} = -\frac{14}{9}$
	Note	Writing down $\alpha, \beta = \frac{2 + \sqrt{11}\text{i}}{5}, \frac{2 - \sqrt{11}\text{i}}{5}$ and then stating $\alpha + \beta = \frac{4}{5},\ \alpha\beta = \frac{3}{5}$ **or** applying $\alpha + \beta = \frac{2 + \sqrt{11}\text{i}}{5} + \frac{2 - \sqrt{11}\text{i}}{5} = \frac{4}{5}$ and $\alpha\beta = \left(\frac{2 + \sqrt{11}\text{i}}{5}\right)\left(\frac{2 - \sqrt{11}\text{i}}{5}\right) = \frac{3}{5}$ scores B0
	Note	Those candidates who then apply $\alpha + \beta = \frac{4}{5},\ \alpha\beta = \frac{3}{5}$, having written down/applied $\alpha, \beta = \frac{2 + \sqrt{11}\text{i}}{5}, \frac{2 - \sqrt{11}\text{i}}{5}$, can only score the M marks in part (a)
	Note	Give B0M0M0M0A0 for $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{\left(\frac{2+\sqrt{11}\text{i}}{5}\right)^2} + \frac{1}{\left(\frac{2-\sqrt{11}\text{i}}{5}\right)^2} = -\frac{14}{9}$

		Question 7 Notes Continued
7. (a)	**Note**	Give B0M1M0M0A0 for $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{\left(\frac{2+\sqrt{11}\mathrm{i}}{5}\right)^2+\left(\frac{2-\sqrt{11}\mathrm{i}}{5}\right)^2}{\left(\frac{2+\sqrt{11}\mathrm{i}}{5}\right)^2\left(\frac{2-\sqrt{11}\mathrm{i}}{5}\right)^2} = -\frac{14}{9}$
	Note	Give B0M1M0M0A0 for $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2} = \frac{\left(\frac{2+\sqrt{11}\mathrm{i}}{5}+\frac{2-\sqrt{11}\mathrm{i}}{5}\right)^2-2\left(\frac{2+\sqrt{11}\mathrm{i}}{5}\right)\left(\frac{2-\sqrt{11}\mathrm{i}}{5}\right)}{\left(\frac{2+\sqrt{11}\mathrm{i}}{5}\right)^2\left(\frac{2-\sqrt{11}\mathrm{i}}{5}\right)^2} = -\frac{14}{9}$
	Note	Allow B1 for both $\mathrm{S}=\frac{4}{5}$ **and** $\mathrm{P}=\frac{3}{5}$ or for $\sum=\frac{4}{5}$ **and** $\prod=\frac{3}{5}$
	Note	Give final A0 for e.g. -1.55 or -1.5556 without reference to $-\frac{14}{9}$ or $-1\frac{5}{9}$ or $-1.\dot{5}$
	Note	Give 2nd M1 for applying their $\alpha+\beta=\frac{4}{5}$ on $5\alpha^2-4\alpha+3=0,\ 5\beta^2-4\beta+3=0 \Rightarrow 5(\alpha^2+\beta^2)-4(\alpha+\beta)+6=0$ to give $5(\alpha^2+\beta^2)-4\left(\frac{4}{5}\right)+6=0$ $\left\{\Rightarrow \alpha^2+\beta^2=\frac{-6+\frac{16}{5}}{5} = -\frac{14}{25}\right\}$
(b)	**Note**	A correct method leading to $a=3, b=14, c=75$ without writing a final answer of $3x^2+14x+75=0$ is final M1A0
	Note	Using $\frac{2+\sqrt{11}\mathrm{i}}{5}, \frac{2-\sqrt{11}\mathrm{i}}{5}$ ***explicitly,*** to find the sum and product of $\frac{3}{\alpha^2}$ and $\frac{3}{\beta^2}$ to give $x^2+\frac{14}{3}x+25=0 \Rightarrow 3x^2+14x+75=0$ scores M0M0M1A0 in part (b)
	Note	Using $\frac{2+\sqrt{11}\mathrm{i}}{5}, \frac{2-\sqrt{11}\mathrm{i}}{5}$ to find $\alpha+\beta=\frac{4}{5},\ \alpha\beta=\frac{3}{5},\ \frac{1}{\alpha^2}+\frac{1}{\beta^2}=-\frac{14}{9}$ **and applying** $\left\{\alpha+\beta=\frac{4}{5},\right\}\ \alpha\beta=\frac{3}{5},\ \frac{1}{\alpha^2}+\frac{1}{\beta^2}=-\frac{14}{9}$ can potentially score full marks in part (b). E.g. • Sum $=\frac{3}{\alpha^2}+\frac{3}{\beta^2}=3\left(-\frac{14}{9}\right) = -\frac{14}{3}$ • Product $=\left(\frac{3}{\alpha^2}\right)\left(\frac{3}{\beta^2}\right)=\frac{9}{\left(\frac{3}{5}\right)^2} = 25$ • $x^2+\frac{14}{3}x+25=0 \Rightarrow 3x^2+14x+75=0$
	Note	Finding $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=-\frac{14}{9}$ and correctly writing $x^2-3\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x+\frac{9}{(\alpha\beta)^2}=0$ followed by $x^2-\frac{14}{3}x+25=0 \Rightarrow 3x^2-14x+75=0$ (incorrect substitution of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=-\frac{14}{9}$) is M0M1M1A0

Question Number	Scheme	Notes	Marks
7.	$5x^2 - 4x + 3 = 0$ has roots α, β		
(b) **Way 2**	$y = \frac{3}{x^2} \Rightarrow x = \frac{3}{y^2} \Rightarrow 5\left(\frac{3}{y}\right) - 4\sqrt{\frac{3}{y}} + 3 = 0$	Substitutes $x^2 = \frac{3}{y}$ into $5x^2 - 4x + 3 = 0$	M1
	$\frac{15}{y} + 3 = 4\sqrt{\frac{3}{y}} \Rightarrow \left(\frac{15}{y} + 3\right)^2 = \left(4\sqrt{\frac{3}{y}}\right)^2$	**dependent on the previous M mark** Correct method for squaring both sides of their equation	dM1
	$\frac{225}{y^2} + \frac{45}{y} + \frac{45}{y} + 9 = 16\left(\frac{3}{y}\right)$		
	$\frac{225}{y^2} + \frac{42}{y} + 9 = 0$		
	$9y^2 + 42y + 225 = 0$	**dependent on the previous M mark** Obtains an expression of the form $ay^2 + by + c,\ a, b, c \neq 0$ **Note:** $"=0"$ not required for this mark	dM1
		Any integer multiple of $3y^2 + 14y + 75 = 0$, or $3x^2 + 14x + 75 = 0$, including the $"=0"$	A1
			(4)

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8. Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ \dfrac{a^n - b^n}{a - b} & b^n \end{pmatrix}$$

where a and b are constants and $a \neq b$.

(5)

Question Number	Scheme	Notes	Marks
8.	$\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ \dfrac{a^n - b^n}{a-b} & b^n \end{pmatrix};\ n \in \mathbb{Z}^+;\ a \neq b$		
	$n=1$, LHS $= \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$, RHS $= \begin{pmatrix} a & 0 \\ \dfrac{a-b}{a-b} & b \end{pmatrix} = \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$	Shows or states that **either** LHS = RHS $= \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$ **or** LHS $= \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$ or $\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^1$, RHS $= \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$	B1
	(Assume the result is true for $n = k$)		
	$\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^{k+1} = \begin{pmatrix} a^k & 0 \\ \dfrac{a^k - b^k}{a-b} & b^k \end{pmatrix}\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$ **or** $\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}\begin{pmatrix} a^k & 0 \\ \dfrac{a^k - b^k}{a-b} & b^k \end{pmatrix}$	$\begin{pmatrix} a^k & 0 \\ \dfrac{a^k - b^k}{a-b} & b^k \end{pmatrix}$ multiplied by $\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$ **(either way round)**	M1
	$= \begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a(a^k - b^k)}{a-b} + b^k & b^{k+1} \end{pmatrix}$ or $\begin{pmatrix} a^{k+1} & 0 \\ a^k + \dfrac{b(a^k - b^k)}{a-b} & b^{k+1} \end{pmatrix}$ or e.g. $\begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a(a^k - b^k)}{a-b} + \dfrac{b^k(a-b)}{(a-b)} & b^{k+1} \end{pmatrix}$	Multiplies out to give a correct un-simplified matrix	A1
	$= \begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a^{k+1} - b^{k+1}}{a-b} & b^{k+1} \end{pmatrix}$	**dependent on the previous A mark** Achieves this result with no algebraic errors	A1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 **cso**
			(5)
			5
	Question 8 Notes		
8.	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.	
	Note	Give B0 for stating LHS = RHS by itself with no reference to LHS = RHS $= \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$	
	Note	Give B0 for just stating $\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^1 = \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$	
	Note	E.g. $\begin{pmatrix} a^k & 0 \\ \dfrac{a^k - b^k}{a-b} & b^k \end{pmatrix}\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} = \begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a^{k+1} - b^{k+1}}{a-b} & b^{k+1} \end{pmatrix}$ with no intermediate working is M1A0A0A0	
	Note	Writing $\begin{pmatrix} a^k & 0 \\ \dfrac{a^k - b^k}{a-b} & b^k \end{pmatrix}\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} = \begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a(a^k - b^k)}{a-b} + b^k & b^{k+1} \end{pmatrix} = \begin{pmatrix} a^{k+1} & 0 \\ \dfrac{a^{k+1} - b^{k+1}}{a-b} & b^{k+1} \end{pmatrix}$ is M1A1A1	

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9. Given that

$$\frac{z - k\mathrm{i}}{z + 3\mathrm{i}} = \mathrm{i}, \text{ where } k \text{ is a positive real constant}$$

(a) show that $z = -\frac{(k+3)}{2} + \frac{(k-3)}{2}\mathrm{i}$ **(4)**

(b) Using the printed answer in part (a),

(i) find an exact simplified value for the modulus of z when $k = 4$

(ii) find the argument of z when $k = 1$. Give your answer in radians to 3 decimal places, where $-\pi < \arg z < \pi$ **(4)**

Question Number	Scheme	Notes	Marks
9.	(a) $\frac{z-k\text{i}}{z+3\text{i}}=\text{i}$ (b)(i) $k=4$ (ii) $k=1$		
(a) **Way 1**	$z-k\text{i}=\text{i}(z+3\text{i}) \Rightarrow z-k\text{i}=\text{i}z-3$ $\Rightarrow z-\text{i}z=-3+k\text{i} \Rightarrow z(1-\text{i})=-3+k\text{i}$	Complete method of making z the subject	M1
	$\Rightarrow z=\frac{-3+k\text{i}}{(1-\text{i})}$	Correct expression for $z=...$	A1
	$z=\frac{(-3+k\text{i})}{(1-\text{i})}\frac{(1+\text{i})}{(1+\text{i})}$ $\left\{=\frac{(-3+k\text{i})(1+\text{i})}{2}\right\}$	**dependent on the previous M mark** Multiplies numerator and denominator by the conjugate of the denominator	dM1
	$z=-\frac{(k+3)}{2}+\frac{(k-3)}{2}\text{i}$ *	Achieves the correct answer with no errors seen	A1* **cso**
			(4)
(a) **Way 2**	$z-k\text{i}=\text{i}(z+3\text{i})$ $(x+y\text{i})-k\text{i}=\text{i}(x+y\text{i}+3\text{i})$ $x+(y-k)\text{i}=-y-3+x\text{i}$	Multiplies both sides by $(z+3\text{i})$, applies $z=x+y\text{i}$, o.e., multiplies out and attempts to equate **both** the real part **and** the imaginary part of the resulting equation	M1
	$\{\text{Real} \Rightarrow\}$ $x=-y-3$ $\{\text{Imaginary} \Rightarrow\}$ $y-k=x$	Both correct equations which can be simplified or un-simplified	A1
	$\begin{cases} x+y=-3 \\ x-y=-k \end{cases} \Rightarrow x=\frac{-k-3}{2},\ y=\frac{k-3}{2}$	**dependent on the previous M mark** Obtains two equations both in terms of x and y and solves them simultaneously to give at least one of $x=...$ or $y=...$	dM1
	$\Rightarrow z=-\frac{(k+3)}{2}+\frac{(k-3)}{2}\text{i}$ *	Finds $x=\frac{-k-3}{2},\ y=\frac{k-3}{2}$ and writes down the given result	A1* **cso**
			(4)
(b)(i)	$\{k=4 \Rightarrow\}$ $z=-\frac{(4+3)}{2}+\frac{(4-3)}{2}\text{i}$ $\left\{=-\frac{7}{2}+\frac{1}{2}\text{i}\right\}$ $\{\lvert z\rvert=\}$ $\sqrt{\left(-\frac{7}{2}\right)^2+\left(\frac{1}{2}\right)^2}$	Some evidence of substituting $z=4$ into the given expression for z **and** a full attempt at applying Pythagoras to find $\lvert z\rvert$	M1
	$=\sqrt{\frac{50}{4}},\ \sqrt{12.5},\ \frac{\sqrt{50}}{2},\ \frac{5}{2}\sqrt{2}$ or $\frac{5}{\sqrt{2}}$ or $\sqrt{\frac{25}{2}}$	Correct **exact** answer	A1
(ii)	$\{k=1 \Rightarrow\}$ $z=-\frac{(1+3)}{2}+\frac{(1-3)}{2}\text{i}$ $\{=-2-\text{i}\}$ $\arg z=-\pi+\tan^{-1}\left(\frac{1}{2}\right)$	Some evidence of substituting $z=1$ into the given expression for z **and** uses trigonometry to find an expression for $\arg z$ in the range $(-3.14..., -1.57...)$ or $(-180°, -90°)$ or $(3.14..., 4.71...)$ or $(180°, 270°)$	M1
	$\{\arg z=-\pi+0.463647... \Rightarrow\}$ $\arg z=-2.677945...$ $\{=-2.678\ (3\text{ dp})\}$	awrt -2.678	A1
			(4)
			8

Question Number	Scheme	Notes	Marks
9.	(a) $\frac{z-k\text{i}}{z+3\text{i}}=\text{i}$ (b)(i) $k=4$ (ii) $k=1$		
(a) **Way 3**	$\frac{z-k\text{i}}{\text{i}}=z+3\text{i} \Rightarrow \frac{\text{i}z+k}{(-1)}=z+3\text{i}$	Complete method of making z the subject	M1
	$\Rightarrow -\text{i}z-k=z+3\text{i} \Rightarrow -k-3\text{i}=z+\text{i}z$ $\Rightarrow -k-3\text{i}=z(1+\text{i})$ $\Rightarrow z=\frac{-k-3\text{i}}{(1+\text{i})}$	Correct expression for $z=...$	A1
	$z=\frac{(-k-3\text{i})}{(1+\text{i})}\frac{(1-\text{i})}{(1-\text{i})}$	**dependent on the previous M mark** Multiplies numerator and denominator by the conjugate of the denominator	dM1
	$z=-\frac{(k+3)}{2}+\frac{(k-3)}{2}\text{i}$ *	Achieves the correct answer with no errors seen	A1* **cso**
			(4)

	Question 9 Notes	
9. (a)	**Note**	Condone any of e.g. $z=-\frac{k+3}{2}+\frac{k-3}{2}\text{i}$ or $z=-\frac{(3+k)}{2}+\frac{(-3+k)}{2}\text{i}$ for the final A mark
(b)(i)	**Note**	M1 can be implied by awrt 3.54 or truncated 3.53
	Note	Give A0 for 3.5355… without reference to $\sqrt{\frac{50}{4}}$, $\sqrt{12.5}$, $\frac{\sqrt{50}}{2}$, $\frac{5}{2}\sqrt{2}$ or $\frac{5}{\sqrt{2}}$ or $\sqrt{\frac{25}{2}}$
(b)(ii)	**Note**	Allow M1 (implied) for awrt -2.7, truncated -2.6, awrt $-153°$ or awrt $207°$ or awrt 3.6

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10. The rectangular hyperbola H has equation $xy = 144$. The point P, on H, has coordinates $\left(12p, \frac{12}{p}\right)$, where p is a non-zero constant.

(a) Show, by using calculus, that the normal to H at the point P has equation

$$y = p^2x + \frac{12}{p} - 12p^3$$

(5)

Given that the normal through P crosses the positive x-axis at the point Q and the negative y-axis at the point R,

(b) find the coordinates of Q and the coordinates of R, giving your answers in terms of p.
(3)

(c) Given also that the area of triangle OQR is 512, find the possible values of p.
(5)

Question Number	Scheme	Notes	Marks
10.	$H: xy = 144;\ P\left(12p, \frac{12}{p}\right),\ p \neq 0,$ lies on H. Normal to H at P crosses positive x-axis at Q and negative y-axis at R		
(a)	$y = \frac{144}{x} = 144x^{-1} \Rightarrow \frac{dy}{dx} = -144x^{-2}$ or $-\frac{144}{x^2}$	$\frac{dy}{dx} = \pm k x^{-2};\ k \neq 0$	M1
	$xy = 144 \Rightarrow x\frac{dy}{dx} + y = 0$	Uses product rule to give $\pm x\frac{dy}{dx} \pm y$	
	$x = 12t,\ y = \frac{12}{t} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\left(\frac{12}{t^2}\right)\left(\frac{1}{12}\right)$	their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dy}{dt}}$; **Condone** $t \equiv p$	
	So at P, $m_T = -\frac{1}{p^2}$	Correct calculus work leading to $m_T = -\frac{1}{p^2}$	A1
	So, $m_N = p^2$	Applies $m_N = \frac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \frac{12}{p} = "p^2"(x - 12p)$ **or** • $\frac{12}{p} = "p^2"(12p) + c \Rightarrow y = "p^2"x +$ their c	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus.	M1
	Correct algebra leading to $y = p^2x + \frac{12}{p} - 12p^3$ *	Correct solution only	A1 *
	Note: m_N must be a function of p for the 2nd M1 and 3rd M1 mark		**(5)**
(b)	$y = 0 \Rightarrow x_Q = 12p - \frac{12}{p^3}$	Puts $y = 0$ and finds x **or** puts $x = 0$ and finds y	M1
	$x = 0 \Rightarrow y_R = \frac{12}{p} - 12p^3$	At least one of x_Q or y_R correct, o.e.	A1
	$\left(12p - \frac{12}{p^3}, 0\right)$ and $\left(0, \frac{12}{p} - 12p^3\right)$	Both sets of coordinates correct. {Ignore labelling of coordinates}	A1
			(3)
(c)	Area $OQR = \frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left\|\left(\frac{12}{p} - 12p^3\right)\right\| = 512$	$\frac{1}{2} \times (\pm \text{ their } x_Q)(\pm \text{ their } y_R) = 512$	M1
		Correct equation which can be un-simplified or simplified	A1
	$144p^4 - 1312 + \frac{144}{p^4} = 0$		
	$144p^8 - 1312p^4 + 144 = 0$ $\{\Rightarrow 9p^8 - 82p^4 + 9 = 0\}$	Correct 3 term quadratic in p^4 **Note:** $144p^8 + 144 = 1312p^4$ is acceptable for this mark	A1
	$(9p^4 - 1)(p^4 - 9) = 0 \Rightarrow p^4 = ...$	**dependent on the previous M mark** Uses a 3TQ in p^4 (or an implied 3TQ in p^4) to find at least one value of $p^4 = ...$	dM1
	$p = \sqrt{3}$ and $p = -\frac{1}{\sqrt{3}}$	Obtains both $p = \sqrt{3}$ and $p = -\frac{1}{\sqrt{3}}$ ***only*** **Note:** **Allow** $p = -\frac{\sqrt{3}}{3}$ in place of $p = -\frac{1}{\sqrt{3}}$	A1
			(5)
			13

Question Number	Scheme	Notes	Marks
10. (c)	Area $OQR = \frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left\lvert\left(\frac{12}{p} - 12p^3\right)\right\rvert = 512$	$\frac{1}{2}\times(\pm \text{ their } x_Q)(\pm \text{ their } y_R) = 512$	M1
		Correct equation which can be un-simplified or simplified	A1
	$144\left(p - \frac{1}{p^3}\right)\left(p^3 - \frac{1}{p}\right) = 1024 \Rightarrow p^4 - 2 + \frac{1}{p^4} = \frac{1024}{144}$		
	$\left(p^2 - \frac{1}{p^2}\right)^2 = \frac{64}{9} \Rightarrow p^2 - \frac{1}{p^2} = \pm\frac{8}{3}$		
	$3p^4 - 8p^2 - 3 = 0$ **and** $3p^4 + 8p^2 - 3 = 0$	Both correct 3 term quadratics in p^2 **Note:** Both $p^4 - 1 = \frac{8}{3}p^2$ and $3p^4 + 8p^2 = 3$ is acceptable for this mark	A1
	$(3p^2+1)(p^2-3) = 0 \Rightarrow p^2 = ...$ **or** $(3p^2-1)(p^2+3) = 0 \Rightarrow p^2 = ...$	**dependent on the previous M mark** Uses a 3TQ in p^2 (or an implied 3TQ in p^2) to find at least one value of $p^2 = ...$	dM1
	$p = \sqrt{3}$ and $p = -\frac{1}{\sqrt{3}}$	Obtains both $p = \sqrt{3}$ and $p = -\frac{1}{\sqrt{3}}$ ***only***	A1
			(5)

		Question 10 Notes
10. (a)	**Note**	Allow $y = p^2x - 12p^3 + \frac{12}{p}$ {order of terms interchanged in $y = ...$} for final A1
(b)	**Note**	For the accuracy marks in part (b) allow equivalents such as • $x = 12p - \frac{12}{p^3}$ or $x = \frac{12p^4 - 12}{p^3}$ or $x = \frac{12(p^2-1)(p^2+1)}{p^3}$ • $y = \frac{12}{p} - 12p^3$ or $y = \frac{12 - 12p^4}{p}$
(c)	**Note**	Give 1st M1, 1st A1 for • $\frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left\lvert\left(\frac{12}{p} - 12p^3\right)\right\rvert = 512$ {correct use of modulus} • $\frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left(12p^3 - \frac{12}{p}\right) = 512$ {modulus has been applied here} • $-\frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left(\frac{12}{p} - 12p^3\right) = 512$ {modulus has been applied here}
	Note	Give 1st M1, 1st A0 for $\frac{1}{2}\left(12p - \frac{12}{p^3}\right)\left(\frac{12}{p} - 12p^3\right) = 512$ {modulus has not been applied on y_R }
	Note	Writing a correct $144p^4 - 1312 + \frac{144}{p^4} = 0$ o.e. followed by a correct e.g. $p^4 = 9$ with no intermediate working is 2nd A0, 2nd M1
	Note	Writing a correct $144p^4 - 1312 + \frac{144}{p^4} = 0$ o.e. followed by $p^4 = 9$ **and** $p^4 = \frac{1}{9}$ with no intermediate working is 2nd A1 (implied), 2nd M1