Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	8	/	0	1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 19 June 2009 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this question paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(1)

(b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$.

(5)

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June 2009
6668 Further Pure Mathematics FP2 (new)
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$	$\frac{1}{2r} - \frac{1}{2(r+2)}$	B1 **aef**
			(1)
(b)	$\sum_{r=1}^{n} \frac{4}{r(r+2)} = \sum_{r=1}^{n} \left(\frac{2}{r} - \frac{2}{r+2}\right)$		
	$= \left(\frac{2}{1} - \frac{2}{3}\right) + \left(\frac{2}{2} - \frac{2}{4}\right) + \;......$ $.............+ \left(\frac{2}{n-1} - \frac{2}{n+1}\right) + \left(\frac{2}{n} - \frac{2}{n+2}\right)$	List the first two terms and the last two terms	M1
	$= \frac{2}{1} + \frac{2}{2} ; - \frac{2}{n+1} - \frac{2}{n+2}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		$\frac{2}{1} + \frac{2}{2} - \frac{2}{n+1} - \frac{2}{n+2}$	A1
	$= 3 - \frac{2}{n+1} - \frac{2}{n+2}$		
	$= \frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{(n+1)(n+2)}$ $= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \frac{3n^2 + 5n}{(n+1)(n+2)}$		
	$= \frac{n(3n+5)}{(n+1)(n+2)}$	Correct Result	A1 **cso AG**
			(5)
			[6]

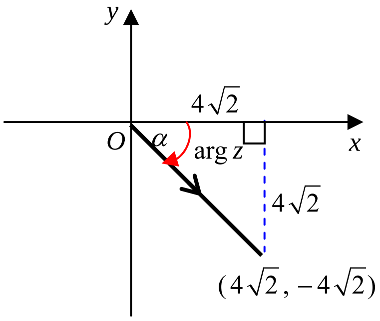
Leave blank

2. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$.

(6)

Question Number	Scheme		Marks
Q2 (a)	$z^3 = 4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}\,,\ -\pi < \theta \leqslant \pi$		
			
	$r = \sqrt{(4\sqrt{2})^2 + (-4\sqrt{2})^2} = \sqrt{32+32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$.	M1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$		
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4}}{3}\right) + \mathrm{i}\sin\left(\frac{-\frac{\pi}{4}}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3.	M1
	$\Rightarrow z = 2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	A1
	Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + \mathrm{i}\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{9\pi}{4}\right)\right)$	Adding or subtracting 2π to the argument for z^3 in order to find other roots.	M1
	$\Rightarrow z = 2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$	Any one of the final two roots	A1
	and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$	Both of the final two roots.	A1
			[6]
	Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$. **Special Case 2:** If *r* is incorrect (and not equal to 8) and candidate states the brackets ( ) correctly then give the first accuracy mark ONLY where this is applicable.		

Leave blank

3. Find the general solution of the differential equation

$$\sin x \frac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \sin 2x \sin x,$$

giving your answer in the form $y = \mathrm{f}(x)$.

(8)

Question Number	Scheme		Marks
Q3	$\sin x \dfrac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \sin 2x \sin x$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x \sin x}{\sin x}$	An attempt to divide every term in the differential equation by $\sin x$. Can be implied.	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin x} = \sin 2x$		
	Integrating factor $= e^{\int -\frac{\cos x}{\sin x}\,\mathrm{d}x} = e^{-\ln \sin x}$	$e^{\int \pm \frac{\cos x}{\sin x}(\mathrm{d}x)}$ or $e^{\int \pm \text{their } P(x)(\mathrm{d}x)}$	dM1
		$e^{-\ln \sin x}$ or $e^{\ln \operatorname{cosec} x}$	A1 aef
	$= \dfrac{1}{\sin x}$	$\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$	A1 aef
	$\left(\dfrac{1}{\sin x}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$		
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	$\dfrac{\mathrm{d}}{\mathrm{d}x}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$	M1
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = 2\cos x$ or $\dfrac{y}{\sin x} = \int 2\cos x\,(\mathrm{d}x)$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\,\mathrm{d}x$		
	$\dfrac{y}{\sin x} = 2\sin x + K$	A credible attempt to integrate the RHS with/without $+K$	dddM1
	$y = 2\sin^2 x + K\sin x$	$y = 2\sin^2 x + K\sin x$	A1 **cao**
			[8]

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4.

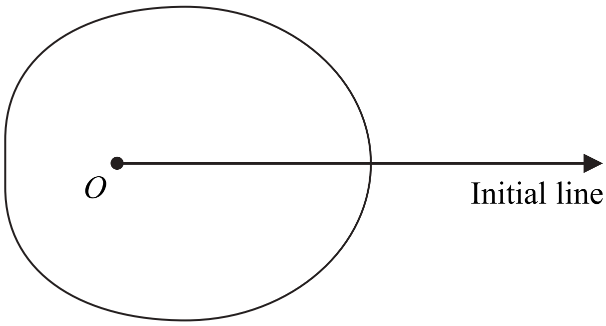

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve is $\frac{107}{2}\pi$.

Find the value of a.

(8)

Question Number	Scheme		Marks
Q4	$A = \frac{1}{2}\int_0^{2\pi} (a + 3\cos\theta)^2 \, d\theta$	Applies $\frac{1}{2}\int_0^{2\pi} r^2 \, (d\theta)$ with correct limits. Ignore $d\theta$.	B1
	$(a + 3\cos\theta)^2 = a^2 + 6a\cos\theta + 9\cos^2\theta$		
	$= \underline{a^2 + 6a\cos\theta + 9\left(\frac{1+\cos 2\theta}{2}\right)}$	$\cos^2\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$ Correct underlined expression.	M1 A1
	$A = \frac{1}{2}\int_0^{2\pi} \left(a^2 + 6a\cos\theta + \frac{9}{2} + \frac{9}{2}\cos 2\theta\right) d\theta$		
	$= \left(\frac{1}{2}\right)\left[a^2\theta + 6a\sin\theta + \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]_0^{2\pi}$	Integrated expression with at least 3 out of 4 terms of the form $\pm A\theta \pm B\sin\theta \pm C\theta \pm D\sin 2\theta$. Ignore the $\frac{1}{2}$. Ignore limits. $a^2\theta + 6a\sin\theta +$ correct ft integration. Ignore the $\frac{1}{2}$. Ignore limits.	M1* A1 ft
	$= \frac{1}{2}\left[\left(2\pi a^2 + 0 + 9\pi + 0\right) - (0)\right]$		
	$= \pi a^2 + \frac{9\pi}{2}$	$\pi a^2 + \frac{9\pi}{2}$	A1
	Hence, $\pi a^2 + \frac{9\pi}{2} = \frac{107}{2}\pi$	Integrated expression equal to $\frac{107}{2}\pi$.	dM1*
	$a^2 + \frac{9}{2} = \frac{107}{2}$		
	$a^2 = 49$		
	As $a > 0$, $a = 7$	$a = 7$	A1 cso **[8]**
	Some candidates may achieve $a = 7$ from incorrect working. Such candidates will not get full marks		

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5.

$$y = \sec^2 x$$

(a) Show that $\dfrac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$.

(4)

(b) Find a Taylor series expansion of $\sec^2 x$ in ascending powers of $\left(x - \dfrac{\pi}{4}\right)$, up to and including the term in $\left(x - \dfrac{\pi}{4}\right)^3$.

(6)

Question Number	Scheme		Marks
Q5	$y = \sec^2 x = (\sec x)^2$		
(a)	$\frac{dy}{dx} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$	Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$	B1 aef
	Apply product rule: $\left\{\begin{matrix} u = 2\sec^2 x & v = \tan x \\ \frac{du}{dx} = 4\sec^2 x \tan x & \frac{dv}{dx} = \sec^2 x \end{matrix}\right\}$		
	$\frac{d^2y}{dx^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$	Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.	M1
		Correct differentiation	A1
	$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$		
	Hence, $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$	Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.	A1 **AG**
			(4)
(b)	$y_{\frac{\pi}{4}} = \left(\sqrt{2}\right)^2 = \underline{2}$, $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2\left(\sqrt{2}\right)^2(1) = \underline{4}$	Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \underline{4}$	B1
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = 24 - 8 = 16$	Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{d^2y}{dx^2}$.	M1
	$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$	Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct	M1
	$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$		
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 24\left(\sqrt{2}\right)^4(1) - 8\left(\sqrt{2}\right)^2(1) = 96 - 16 = 80$	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = \underline{80}$	B1
	$\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + ...$	Applies a Taylor expansion with at least 3 out of 4 terms ft correctly.	M1
		Correct Taylor series expansion.	A1
			(6)
	$\left\{\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + 8\left(x - \frac{\pi}{4}\right)^2 + \frac{40}{3}\left(x - \frac{\pi}{4}\right)^3 + ...\right\}$		
			[10]

Leave blank

6. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + \mathrm{i}}, \quad z \neq -\mathrm{i}$$

The circle with equation $|z| = 3$ is mapped by T onto the curve C.

(a) Show that C is a circle and find its centre and radius.

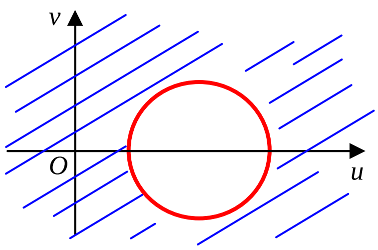
(8)

The region $|z| < 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Shade the region R on an Argand diagram.

(2)

M 3 5 1 4 4 A 0 1 6 2 8

Question Number	Scheme		Marks
Q6	$w = \frac{z}{z + i}, \quad z = -i$		
(a)	$w(z + i) = z \Rightarrow wz + iw = z \Rightarrow iw = z - wz$	Complete method of rearranging to make z the subject.	M1
	$\Rightarrow iw = z(1 - w) \Rightarrow z = \frac{iw}{(1 - w)}$	$z = \frac{iw}{(1 - w)}$	A1 **aef**
	$\lvert z\rvert = 3 \Rightarrow \left\lvert \frac{iw}{1 - w} \right\rvert = 3$	Putting $\lvert z$ in terms of their $w\rvert = 3$	dM1
	$\left\{ \lvert iw\rvert = 3\lvert 1 - w\rvert \Rightarrow \lvert w\rvert = 3\lvert w - 1\rvert \Rightarrow \lvert w\rvert^2 = 9\lvert w - 1\rvert^2 \Rightarrow \lvert u + iv\rvert^2 = 9\lvert u + iv - 1\rvert^2 \right\}$		
	$\Rightarrow u^2 + v^2 = 9\left[(u - 1)^2 + v^2\right]$	Applies $w = u + iv$, and uses Pythagoras correctly to get an equation in terms of u and v without any i's.	ddM1
		Correct equation.	A1
	$\left\{ \Rightarrow u^2 + v^2 = 9u^2 - 18u + 9 + 9v^2 \Rightarrow 0 = 8u^2 - 18u + 8v^2 + 9 \right\}$		
	$\Rightarrow 0 = u^2 - \frac{9}{4}u + v^2 + \frac{9}{8}$	Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0$.	dddM1
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 - \frac{81}{64} + v^2 + \frac{9}{8} = 0$		
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64}$		
	{Circle} centre $\left(\frac{9}{8}, 0\right)$, radius $\frac{3}{8}$	One of centre or radius correct.	A1
		Both centre and radius correct.	A1
			(8)
(b)		Circle indicated on the Argand diagram in the correct position in follow through quadrants. Ignore plotted coordinates.	B1ft
		Region outside a circle indicated only.	B1
			(2)
			[10]

Leave blank

7. (a) Sketch the graph of $y = |x^2 - a^2|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes.

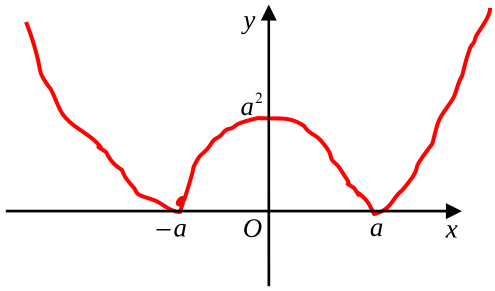
(2)

(b) Solve $|x^2 - a^2| = a^2 - x$, $a > 1$.

(6)

(c) Find the set of values of x for which $|x^2 - a^2| > a^2 - x$, $a > 1$.

(4)

Question Number	Scheme		Marks
Q7	$y = \left\|x^2 - a^2\right\|,\ a > 1$		
(a)		Correct Shape. Ignore cusps. Correct coordinates.	B1 B1
			(2)
(b)	$\left\|x^2 - a^2\right\| = a^2 - x\ ,\ a > 1$		
	$\{\|x\| > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow x^2 + x - 2a^2 = 0$		
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\|x\| < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow x = 0\ ,1$	$x = 0$ $x = 1$	B1 A1
			(6)
(c)	$\left\|x^2 - a^2\right\| > a^2 - x\ ,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}$ $\{\text{or}\}$ $x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value x is greater than their maximum value	B1 ft B1 ft
	$\{\text{or}\}\quad 0 < x < 1$	For $\{\|x\| < a\}$, Lowest $< x <$ Highest $0 < x < 1$	M1 A1
			(4)
			[12]

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8.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

(a) find x in terms of t.

(8)

The solution to part (a) is used to represent the motion of a particle P on the x-axis. At time t seconds, where $t > 0$, P is x metres from the origin O.

(b) Show that the maximum distance between O and P is $\frac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

(7)

Question Number	Scheme		Marks
Q8	$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x = 2e^{-t}$, $x=0, \frac{dx}{dt}=2$ at $t=0$.		
(a)	AE, $m^2+5m+6=0 \Rightarrow (m+3)(m+2)=0$ $\Rightarrow m=-3,-2.$		
	So, $x_{CF}=Ae^{-3t}+Be^{-2t}$	$Ae^{m_1 t}+Be^{m_2 t}$, where $m_1 \neq m_2$. $Ae^{-3t}+Be^{-2t}$	M1 A1
	$\left\{ x=ke^{-t} \Rightarrow \frac{dx}{dt}=-ke^{-t} \Rightarrow \frac{d^2x}{dt^2}=ke^{-t} \right\}$		
	$\Rightarrow ke^{-t}+5(-ke^{-t})+6ke^{-t}=2e^{-t} \Rightarrow 2ke^{-t}=2e^{-t}$ $\Rightarrow k=1$	Substitutes ke^{-t} into the differential equation given in the question. Finds $k=1$.	M1 A1
	$\left\{ \text{So, } x_{PI}=e^{-t} \right\}$		
	So, $x=Ae^{-3t}+Be^{-2t}+e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\frac{dx}{dt}==-3Ae^{-3t}-2Be^{-2t}-e^{-t}$	Finds $\frac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t=0,\ x=0 \Rightarrow \quad 0=A+B+1$ $t=0, \frac{dx}{dt}=2 \Rightarrow \quad 2=-3A-2B-1$	Applies $t=0,\ x=0$ to x and $t=0, \frac{dx}{dt}=2$ to $\frac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\left\{ \begin{array}{l} 2A+2B=-2 \\ -3A-2B=3 \end{array} \right\}$		
	$\Rightarrow A=-1, B=0$		
	So, $x=-e^{-3t}+e^{-t}$	$x=-e^{-3t}+e^{-t}$	A1 **cao** (8)

Question Number	Scheme		Marks
	$x = -\mathrm{e}^{-3t} + \mathrm{e}^{-t}$		
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3\mathrm{e}^{-3t} - \mathrm{e}^{-t} = 0$	Differentiates their x to give $\frac{\mathrm{d}x}{\mathrm{d}t}$ and puts $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to 0.	M1
	$3 - \mathrm{e}^{2t} = 0$	A credible attempt to solve.	dM1*
	$\Rightarrow t = \frac{1}{2}\ln 3$	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55	A1
	So, $x = -\mathrm{e}^{-\frac{3}{2}\ln 3} + \mathrm{e}^{-\frac{1}{2}\ln 3} = -\mathrm{e}^{\ln 3^{-\frac{3}{2}}} + \mathrm{e}^{\ln 3^{-\frac{1}{2}}}$ $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$	Substitutes their t back into x and an attempt to eliminate out the ln's.	ddM1
	$= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$	uses exact values to give $\frac{2\sqrt{3}}{9}$	A1 **AG**
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -9\mathrm{e}^{-3t} + \mathrm{e}^{-t}$ At $t = \frac{1}{2}\ln 3$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -9\mathrm{e}^{-\frac{3}{2}\ln 3} + \mathrm{e}^{-\frac{1}{2}\ln 3}$	Finds $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ and substitutes their t into $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	dM1*
	$= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$		
	As $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \left\{-\frac{2}{\sqrt{3}}\right\} < 0$ then x is maximum.	$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} < 0$ and maximum conclusion.	A1
			(7)
			[15]