Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions.

(2)

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}$$

(3)

(c) Evaluate $\displaystyle\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures.

(2)

June 2010
Further Pure Mathematics FP2 6668
Mark Scheme

Question Number	Scheme	Marks
1(a)	$\frac{1}{3r-1}-\frac{1}{3r+2}$	M1 A1 (2)
(b)	$\sum_{r=1}^{n}\frac{3}{(3r-1)(3r+2)}=\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\ldots\frac{1}{3n-1}-\frac{1}{3n+2}$	M1 A1ft
	$=\frac{1}{2}-\frac{1}{3n+2}=\frac{3n}{2(3n+2)}$ *	A1 (3)
(c)	Sum = f(1000) – f(99)	
	$\frac{3000}{6004}-\frac{297}{598}=0.00301$ or 3.01×10^{-3}	M1 A1 (2)
		7

Leave blank

2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

(5)

Question Number	Scheme	Marks
2	$\text{f}''(t) = -x - \cos x, \quad \text{f}''(0) = -1$	B1
	$\text{f}'''(t) = (-1 + \sin x)\dfrac{\text{d}x}{\text{d}t}, \quad \text{f}'''(0) = -0.5$	M1A1
	$\text{f}(t) = \text{f}(0) + t\text{f}'(0) + \dfrac{t^2}{2}\text{f}''(0) + \dfrac{t^3}{3!}\text{f}'''(0) + \ldots$	
	$= 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \ldots$	M1 A1
		5

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3. (a) Find the set of values of x for which

$$x+4>\frac{2}{x+3}$$

(6)

(b) Deduce, or otherwise find, the values of x for which

$$x+4>\frac{2}{|x+3|}$$

(1)

Question Number	Scheme	Marks
3(a)	$(x+4)(x+3)^2-2(x+3)=0$, $(x+3)(x^2+7x+10)=0$ so $(x+2)(x+3)(x+5)=0$ or alternative method including calculator	M1
	Finds critical values –2 and -5	A1 A1
	Establishes $x > -2$	A1ft
	Finds and uses critical value –3 to give $-5 < x < -3$	M1A1 (6)
(b)	$x > -2$	B1ft (1)
		7

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4. $$z = -8 + (8\sqrt{3})\text{i}$$

(a) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^3,

(2)

(c) find the values of w such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

(5)

Question Number	Scheme	Marks
4(a)	Modulus = 16 Argument = $\arctan(-\sqrt{3})=\frac{2\pi}{3}$	B1 M1A1 (3)
(b)	$z^3=16^3(\cos(\frac{2\pi}{3})+\mathrm{i}\sin(\frac{2\pi}{3}))^3=16^3(\cos 2\pi+\mathrm{i}\sin 2\pi)$ =4096 or 16^3	M1 A1 (2)
(c)	$w=16^{\frac{1}{4}}(\cos(\frac{2\pi}{3})+\mathrm{i}\sin(\frac{2\pi}{3}))^{\frac{1}{4}}=2(\cos\left(\frac{\pi}{6}\right)+\mathrm{i}\sin\left(\frac{\pi}{6}\right))\ \left(=\sqrt{3}+\mathrm{i}\right)$ OR $-1+\sqrt{3}\mathrm{i}$ OR $-\sqrt{3}-\mathrm{i}$ OR $1-\sqrt{3}\mathrm{i}$	M1 A1ft M1A2(1,0) (5) **10**

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5.

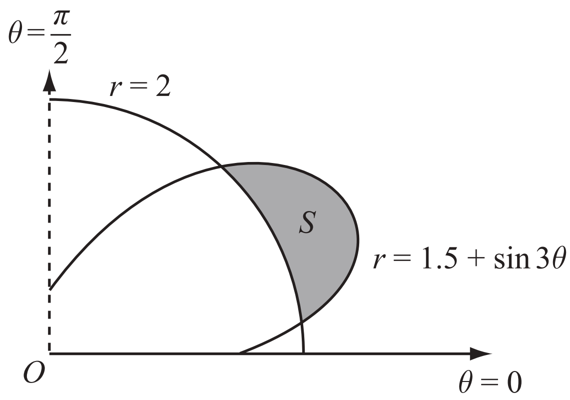

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

$$\text{and} \qquad r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

Question Number	Scheme	Marks
5(a)	$1.5+\sin 3\theta = 2 \quad \rightarrow \quad \sin 3\theta = 0.5 \quad \therefore 3\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6}\right),$	M1 A1,
	and $\therefore \theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$	A1 (3)
(b)	Area $= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5+\sin 3\theta)^2 \mathrm{d}\theta\right], \; -\frac{1}{9}\pi \times 2^2$	M1, M1
	$= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25+3\sin 3\theta+\frac{1}{2}(1-\cos 6\theta))\mathrm{d}\theta\right] - \frac{1}{9}\pi \times 2^2$	M1
	$= \frac{1}{2}\left[(2.25\theta - \cos 3\theta + \frac{1}{2}(\theta - \frac{1}{6}\sin 6\theta))\right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{1}{9}\pi \times 2^2$	M1 A1
	$= \frac{13\sqrt{3}}{24} - \frac{5\pi}{36}$	M1 A1 (7)
		10

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6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z-6|=|z|$, sketch the locus of P.

(2)

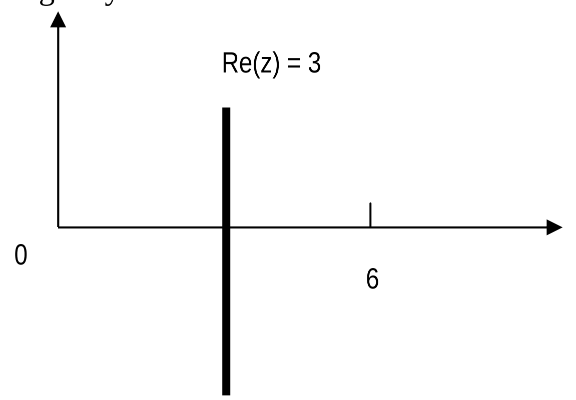
(b) Find the complex numbers z which satisfy both $|z-6|=|z|$ and $|z-3-4\text{i}|=5$.

(3)

The transformation T from the z-plane to the w-plane is given by $w=\dfrac{30}{z}$.

(c) Show that T maps $|z-6|=|z|$ onto a circle in the w-plane and give the cartesian equation of this circle.

(5)

Question Number	Scheme	Marks
6(a)	 Vertical Straight line Through 3 on real axis	B1 B1 (2)
(b)	These are points where line x = 3 meets the circle centre (3, 4) with radius 5. The complex numbers are 3 + 9i and 3 – i.	M1 A1 A1 (3)
(c)	$\|z-6\|=\|z\| \Rightarrow \|\frac{30}{w}-6\|=\|\frac{30}{w}\|$ $\therefore \|30-6w\|=\|30\| \;\; \Rightarrow \;\; \therefore \|5-w\|=\|5\|$ This is a circle with Cartesian equation $(u-5)^2+v^2=25$	M1 M1 A1 M1 A1 (5) **10**

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7. (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 4y\tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 2z\tan x = 1 \qquad \text{(II)}$$

(5)

(b) Solve the differential equation (II) to find z as a function of x.

(6)

(c) Hence obtain the general solution of the differential equation (I).

(1)

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{dy}{dz}.\frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z.\frac{dz}{dx}$	M1 M1 A1
	Substituting to get $2z.\frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x dx} = e^{2\ln\cos x} = \cos^2 x$	M1 A1
	$\therefore \frac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \quad \therefore z\cos^2 x = \int \cos^2 x dx$	M1
	$\therefore z\cos^2 x = \int \frac{1}{2}(\cos 2x + 1)\,dx = \frac{1}{4}\sin 2x + \frac{1}{2}x + c$	M1 A1
	$\therefore z = \frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x$	A1 (6)
(c)	$\therefore y = (\frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1) **12**

Leave blank

8. (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 25y = 3\cos 5x$$

(4)

(b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 25y = 3\cos 5x$$

(3)

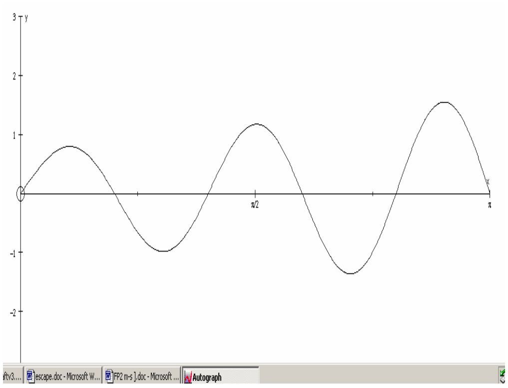
Given that at $x = 0$, $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = \mathrm{f}(x)$.

(5)

(d) Sketch the curve with equation $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant \pi$.

(2)

Question Number	Scheme	Marks
8(a)	Differentiate twice and obtaining $\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \sin 5x + 5\lambda x \cos 5x$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$	M1 A1
	Substitute to give $\lambda = \frac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A\cos 5x + B\sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$	M1 A1
	So general solution is $y = A\cos 5x + B\sin 5x + \frac{3}{10} x \sin 5x$ or in exponential form	A1ft (3)
(c)	y= 0 when x = 0 means A = 0	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5B\cos 5x + \frac{3}{10}\sin 5x + \frac{3}{2} x \cos 5x$ and at x = 0 $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$ and so 5 = 5A	M1 M1
	So B = 1	A1
	So $y = \sin 5x + \frac{3}{10} x \sin 5x$	A1 (5)
(d)	 "Sinusoidal" through O amplitude becoming larger Crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$	B1 B1 (2) **14**