Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	8	/	0	1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Thursday 23 June 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x}$$

(7)

June 2011
Further Pure Mathematics FP 26668
Mark Scheme

Question Number	Scheme	Marks
1.	$3x=(x-4)(x+3) \qquad x^2-4x-12=0$ $x=-2,\ x=6$ both Other critical values are $x=-3,\ x=0$ $-3<x<-2, \qquad 0<x<6$	M1 A1 B1, B1 M1 A1 A1 (7) **7**
	1st M1 for $\pm(x^2-4x-12)$ – '=0' not required. B marks can be awarded for values appearing in solution e.g. on sketch of graph or in final answer. 2nd M1 for attempt at method using graph sketch or +/- If cvs correct but correct inequalities are not strict award A1A0.	

Leave blank

2.

$$\frac{d^2y}{dx^2} = e^x\left(2y\frac{dy}{dx} + y^2 + 1\right)$$

(a) Show that

$$\frac{d^3y}{dx^3} = e^x\left[2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + ky\frac{dy}{dx} + y^2 + 1\right],$$

where k is a constant to be found.

(3)

Given that, at $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$,

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(4)

Question Number	Scheme	Marks
2.		
(a)	$\dfrac{d^3y}{dx^3} = e^x\left(2y\dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 + 2y\dfrac{dy}{dx}\right) + e^x\left(2y\dfrac{dy}{dx} + y^2 + 1\right)$	M1 A1
	$\dfrac{d^3y}{dx^3} = e^x\left(2y\dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 + 4y\dfrac{dy}{dx} + y^2 + 1\right)$ $(k = 4)$	A1
		(3)
(b)	$\left(\dfrac{d^2y}{dx^2}\right)_0 = e^0(4+1+1) = 6$	B1
	$\left(\dfrac{d^3y}{dx^3}\right)_0 = e^0(12+8+8+1+1) = 30$	B1
	$y = 1 + 2x + \dfrac{6x^2}{2} + \dfrac{30x^3}{6} = 1 + 2x + 3x^2 + 5x^3$	M1 A1ft
		(4) **7**
(a) **(b)**	1st M1 for evidence of Product Rule 1st A1 for completely correct expression or equivalent 2nd A1 for correct expression or $k = 4$ stated 2nd M1 require four terms and denominators of 2 and 6 (might be implied) A1 follow through from their values in the final answer.	

Leave blank

3. Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+5y=\frac{\ln x}{x}, \qquad x>0$$

giving your answer in the form $y=\mathrm{f}(x)$.

(8)

Question Number	Scheme	Marks
3.	$\frac{dy}{dx}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Integrating factor $\mathrm{e}^{\int \frac{5}{x}}$	M1
	$e^{\int \frac{5}{x}}=e^{5\ln x}=x^5$	A1
	$\int x^3 \ln x dx=\frac{x^4 \ln x}{4}-\int \frac{x^3}{4}dx$	M1 M1 A1
	$=\frac{x^4 \ln x}{4}-\frac{x^4}{16}$ $(+C)$	A1
	$x^5 y=\frac{x^4 \ln x}{4}-\frac{x^4}{16}+C$ $y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$	M1 A1
		(8) **8**
	1st M1 for attempt at correct Integrating Factor 1st A1 for simplified IF 2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their '$x^3 \ln x$' 3rd M1 for attempt at correct Integration by Parts 2nd A1 for both terms correct 3rd A1 constant not required 4th M1 $x^5 y=$ their answer + C	

Leave blank

4. Given that

$$(2r+1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(a) find the values of the constants A, B and C.

(2)

(b) Show that

$$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$$

(2)

(c) Using the result in part (b) and the method of differences, show that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(5)

Question Number	Scheme	Marks
4. (a)	$(2r+1)^3 = (2r)^3 + 3(2r)^2 + 3(2r) + 1$	M1
	$A = 8,\ B = 12,\ C = 6$	A1 (2)
(b)	$(2r-1)^3 = (2r)^3 - 3(2r)^2 + 3(2r) - 1$	M1
	$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$ (*)	A1cso (2)
(c)	$r=1:\quad 3^3 - 1^3 = 24\times 1^2 + 2$	
	$r=2:\quad 5^3 - 3^3 = 24\times 2^2 + 2$	
	$\vdots \qquad \vdots$	M1 A1
	$r=n:\quad (2n+1)^3 - (2n-1)^3 = 24\times n^2 + 2$	
	Summing: $(2n+1)^3 - 1 = 24\sum r^2 + \left(\sum\ \right)2$	M1
	$\left(\sum 2\right) = 2n$	B1
	Proceeding to $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1cso (5) **9**
(a) (b) (c)	1st M1 require coefficients of 1,3,3,1 or equivalent 1st M1 require 1,-3,3,-1 or equivalent 1st M1 for attempt with at least 1,2 and n if summing expression incorrect. RHS of display not required at this stage. 1st A1 for 1,2 and n correct. 2nd M1 require cancelling and use of $24r^2 + 2$ Award B1 for correct kn for their approach 2nd A1 is for correct solution only	

Leave blank

5. The point P represents the complex number z on an Argand diagram, where

$$|z - \mathrm{i}| = 2$$

The locus of P as z varies is the curve C.

(a) Find a cartesian equation of C.

(2)

(b) Sketch the curve C.

(2)

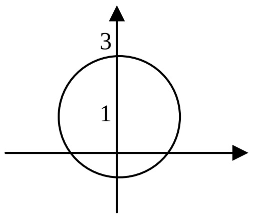
A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + \mathrm{i}}{3 + \mathrm{i}z}, \quad z \neq 3\mathrm{i}$$

The point Q is mapped by T onto the point R. Given that R lies on the real axis,

(c) show that Q lies on C.

(5)

Question Number	Scheme	Marks
5. (a)	$x^2+(y-1)^2=4$	M1 A1 (2)
(b)	 M1: Sketch of circle A1: Evidence of correct centre and radius	M1 A1 (2)
(c)	$w=\frac{(x+\mathrm{i}y)+\mathrm{i}}{3+\mathrm{i}(x+\mathrm{i}y)}=\frac{x+\mathrm{i}(y+1)}{(3-y)+\mathrm{i}x}$ $=\frac{\left[x+\mathrm{i}(y+1)\right]\left[(3-y)-\mathrm{i}x\right]}{\left[(3-y)+\mathrm{i}x\right]\left[(3-y)-\mathrm{i}x\right]}$ On x-axis, so imaginary part = 0: $(y+1)(3-y)-x^2=0$ $(y+1)(3-y)-x^2=0 \Rightarrow x^2+(y-1)^2=4$, so Q is on C	M1 M1 M1 A1 A1cso (5) **9**
Alt. (c)	Let $w=u+\mathrm{i}v$: $u=\frac{z+\mathrm{i}}{3+\mathrm{i}z}$ (since v = 0) $z=\frac{3u-\mathrm{i}}{1-u\mathrm{i}}$ $z-\mathrm{i}=\frac{3u-\mathrm{i}-\mathrm{i}-u}{1-u\mathrm{i}}=\frac{2(u-\mathrm{i})}{1-u\mathrm{i}}$ $\lvert z-\mathrm{i}\rvert=\frac{2\sqrt{u^2+1}}{\sqrt{u^2+1}}=2$, so Q is on C	M1 dM1 M1 A1 A1cso
(a) (b) (c)	M1 Use of $z=x+\mathrm{i}y$ and find modulus Award A0 if circle doesn't intersect x - axis twice 1st M for subbing $z=x+\mathrm{i}y$ and collecting real and imaginary parts 2nd M for multiply numerator and denominator by their complex conjugate 3rd M for equating imaginary parts of numerator to 0 Award A1 for equation matching part (a), statement not required.	

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6.

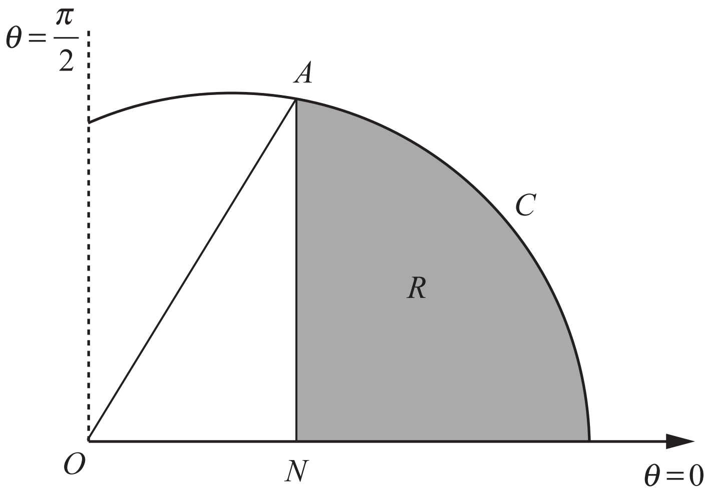

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R.

(9)

Question Number	Scheme	Marks
6.	$2+\cos\theta=\frac{5}{2}\Rightarrow\theta=\frac{\pi}{3}$	B1
	$\frac{1}{2}\int(2+\cos\theta)^2 d\theta=\frac{1}{2}\int(4+4\cos\theta+\cos^2\theta)d\theta$	M1
	$=\frac{1}{2}\left[4\theta+4\sin\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right]$	M1 A1
	Substituting limits $\left(\frac{1}{2}\left[\frac{9\pi}{6}+4\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{8}\right]=\frac{1}{2}\left(\frac{3\pi}{2}+\frac{17\sqrt{3}}{8}\right)\right)$	M1
	Area of triangle = $\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{25}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}\left(=\frac{25\sqrt{3}}{32}\right)$	M1 A1
	Area of R = $\frac{3\pi}{4}+\frac{17\sqrt{3}}{16}-\frac{25\sqrt{3}}{32}=\frac{3\pi}{4}+\frac{9\sqrt{3}}{32}$	M1 A1
		(9) **9**
	1st M1 for use of $\frac{1}{2}\int r^2\,d\theta$ and correct attempt to expand 2nd M1 for use of double angle formula - $\sin 2\theta$ required in square brackets 3rd M1 for substituting their limits 4th M1 for use of $\frac{1}{2}$ base x height 5th M1 area of sector – area of triangle Please note there are no follow through marks on accuracy.	

Leave blank

7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(b) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval $0 \leqslant \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

Question Number	Scheme	Marks
7. (a)	$\sin 5\theta = \text{Im}(\cos\theta + \text{i}\sin\theta)^5$	B1
	$5\cos^4\theta(\text{i}\sin\theta) + 10\cos^2\theta(\text{i}^3\sin^3\theta) + \text{i}^5\sin^5\theta$	M1
	$= \text{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$	A1
	$\left(\text{Im}(\cos\theta + \text{i}\sin\theta)^5\right) = 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$	M1
	$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ (*)	A1cso
		(5)
(b)	$16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 5\left(3\sin\theta - 4\sin^3\theta\right)$	M1
	$16\sin^5\theta - 10\sin\theta = 0$	M1
	$\sin^4\theta = \dfrac{5}{8}$ $\quad\theta = 1.095$	A1
	Inclusion of solutions from $\sin\theta = -\sqrt[4]{\dfrac{5}{8}}$	M1
	Other solutions: $\theta = 2.046,\ 4.237,\ 5.188$	A1
	$\sin\theta = 0 \Rightarrow \theta = 0,\ \theta = \pi\ (3.142)$	B1
		(6)
		11
(a) (b)	Award B if solution considers Imaginary parts and equates to $\sin 5\theta$ 1st M1 for correct attempt at expansion and collection of imaginary parts 2nd M1 for substitution powers of $\cos\theta$ 1st M for substituting correct expressions 2nd M for attempting to form equation Imply 3rd M if 4.237 or 5.188 seen. Award for their negative root. Ignore 2π but 2nd A0 if other extra solutions given.	

Leave blank

8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \quad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Find the general solution of this differential equation.

(8)

(b) Find the particular solution of this differential equation for which, at $t = 0$,

$x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$.

(5)

On the graph of the particular solution defined in part (b), the first turning point for $t > 30$ is the point A.

(c) Find approximate values for the coordinates of A.

(2)

Question Number	Scheme	Marks
8.		
(a)	$m^2+6m+9=0 \qquad m=-3$	M1
	C.F. $\quad x=(A+Bt)e^{-3t}$	A1
	P.I. $\quad x=P\cos 3t+Q\sin 3t$	B1
	$\dot{x}=-3P\sin 3t+3Q\cos 3t$ $\ddot{x}=-9P\cos 3t-9Q\sin 3t$	M1
	$(-9P\cos 3t-9Q\sin 3t)+6(-3P\sin 3t+3Q\cos 3t)+9(P\cos 3t+Q\sin 3t)=\cos$	M1
	$-9P+18Q+9P=1$ and $-9Q-18P+9Q=0$	M1
	$P=0$ and $Q=\dfrac{1}{18}$	A1
	$x=(A+Bt)e^{-3t}+\dfrac{1}{18}\sin 3t$	A1ft
		(8)
(b)	$t=0: \quad x=A=\dfrac{1}{2}$	B1
	$\dot{x}=-3(A+Bt)\mathrm{e}^{-3t}+B\mathrm{e}^{-3t}+\dfrac{3}{18}\cos 3t$	M1
	$t=0: \quad \dot{x}=-3A+B+\dfrac{1}{6}=0 \qquad B=\dfrac{4}{3}$	M1 A1
	$x=\left(\dfrac{1}{2}+\dfrac{4t}{3}\right)\mathrm{e}^{-3t}+\dfrac{1}{18}\sin 3t$	A1
		(5)
(c)	$t\approx\dfrac{59\pi}{6} \quad (\approx 30.9)$	B1
	$x\approx-\dfrac{1}{18}$	B1ft
		(2) **15**
(a)	1st M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential. 2nd M1 for attempt to differentiate PI twice 3rd M1 for substituting their expression into differential equation 4th M1 for substitution of both boundary values	
(b)	1st M1 for correct attempt to differentiate their answer to part (a) 2nd M1 for substituting boundary value	