Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 22 June 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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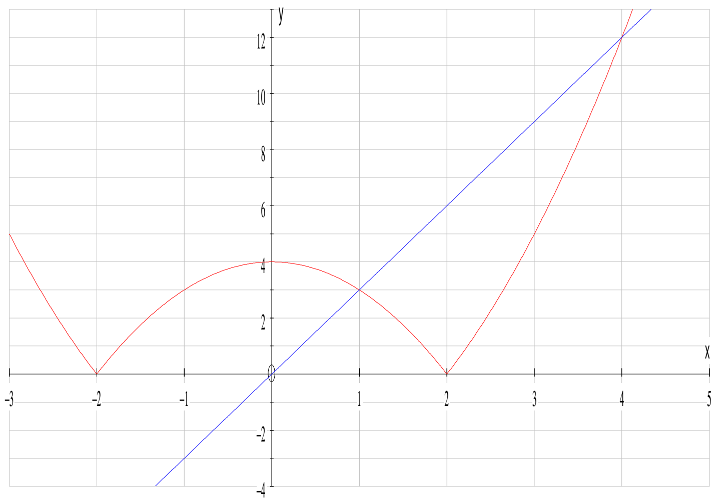
1. Find the set of values of x for which

$$|x^2 - 4| > 3x$$

(5)

Summer 2012
6668 Further Pure Mathematics FP2
Mark Scheme

Question Number	Scheme	Marks
1.	$x^2 - 4 = 3x$ and $x^2 - 4 = -3x$, or graphical method, or squaring both sides, leading to $x = ...$	M1
	$(x = -4,\ x = -1)$ $x = 1,\ x = 4$ **seen anywhere**	B1 B1
	Using only 2 critical values to find an inequality	dM1
	$x < 1$ $x > 4$ **both strict, ignore 'and'**	A1
		(5)
		5
	Notes	
		
	1^{st} M1 accept $\pm(x^2 - 4) > 3x$ or $\pm(x^2 - 4) = 3x$ Require **modulus** of parabola and straight line with positive gradient through origin for graphical method.	
	1^{st} B1 for x=1, 2^{nd} B1 for x=4	
	2^{nd} M1 dependent upon first M1	
	A0 for error in solution of quadratic leading to correct answer.	

Leave blank

2. The curve C has polar equation

$$r = 1 + 2\cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

Question Number	Scheme	Marks
2.	$y = r\sin\theta = \sin\theta + 2\sin\theta\cos\theta$	B1
	$\frac{dy}{d\theta} = \cos\theta + 2\cos 2\theta$	M1
	$4\cos^2\theta + \cos\theta - 2 = 0$	A1oe
	$\cos\theta = \frac{-1 \pm \sqrt{1+32}}{8}$	M1 A1
	$OP = r = 1 + \frac{-1+\sqrt{1+32}}{4} = \frac{3+\sqrt{33}}{4}$	M1 A1
		(7)
		7
	Notes B1 for $sin\theta + 2\sin\theta cos\theta$ or $sin\theta(1 + 2cos\theta)$ 1st M1 for use of Product Rule or Chain Rule (require 2 or condone ½) 1st A1 equation required 2nd M1 Valid attempt at solving 3 term quadratic (usual rules) to give $cos\theta = \cdots$ 2nd A1 for exact or 3 dp or better (-0.843.....and 0.593....) 3rd M1 for 1+2x 'their $cos\theta$' 3rd A1 for any form A0 if negative solution not discounted.	

Leave blank

3. (a) Express the complex number $-2+(2\sqrt{3})\text{i}$ in the form $r(\cos\theta+\text{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.
(3)

(b) Solve the equation

$$z^4=-2+(2\sqrt{3})\text{i}$$

giving the roots in the form $r(\cos\theta+\text{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.
(5)

Question Number	Scheme	Marks
3.		
(a)	$r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$	B1
	$\tan\theta = -\sqrt{3}$ (Also allow M mark for $\tan\theta = \sqrt{3}$)	M1
	M mark can be implied by $\theta = \pm\frac{2\pi}{3}$ or $\theta = \pm\frac{\pi}{3}$	
	$\theta = \frac{2\pi}{3}$	A1
		(3)
(b)	Finding the 4th root of their r: $r = 4^{1/4}$ $(= \sqrt{2})$	M1
	For one root, dividing their θ by 4: $\theta = \frac{2\pi}{3} \div 4 = \frac{\pi}{6}$	M1
	For another root, add or subtract a multiple of 2π to their θ **and** divide by 4 in correct order.	M1
	$\sqrt{2}(\cos\theta + \mathrm{i}\sin\theta)$, where $\theta = -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$	A1 A1
		(5)
		8
	Notes	
(a)	M1 Accept $\pm\sqrt{3}$ *or* $\pm\frac{1}{\sqrt{3}}$	
	A1 Accept awrt 2.1. A0 if in degrees.	
(b)	2nd M1 for awrt 0.52	
	1st A1 for two correct values	
	2nd A1 for all correct values values in correct form and no more	

Leave blank

4. Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2\cos t - \sin t$$

(9)

Question Number	Scheme	Marks
4.	$m^2 + 5m + 6 = 0 \qquad m = -2,\ -3$	M1
	C.F. $\quad (x =) A\mathrm{e}^{-2t} + B\mathrm{e}^{-3t}$	A1
	P.I. $\quad x = P\cos t + Q\sin t$	B1
	$\dot{x} = -P\sin t + Q\cos t$	
	$\ddot{x} = -P\cos t - Q\sin t$	M1
	$(-P\cos t - Q\sin t) + 5(-P\sin t + Q\cos t) + 6(P\cos t + Q\sin t) = 2\cos t - \text{si}$	M1
	$-P + 5Q + 6P = 2$ and $-Q - 5P + 6Q = -1$, and solve for P and Q	M1
	$P = \frac{3}{10}$ and $Q = \frac{1}{10}$	A1 A1
	$x = A\mathrm{e}^{-2t} + B\mathrm{e}^{-3t} + \frac{3}{10}\cos t + \frac{1}{10}\sin t$	B1 ft
		(9)
		9
	Notes 1st M1 form quadratic and attempt to solve (usual rules) 1st B1 Accept negative signs for coefficients. Coefficients must be different. 2nd M1for differentiating their trig PI twice 3rd M1 for substituting x, $\dot{x}$ and $\ddot{x}$ expressions 4th M1 Form 2 equations in two unknowns and attempt to solve 1st A1 for one correct, 2nd A1 for two correct 2nd B1 for x=their CF + their PI as functions of t Condone use of the wrong variable (e.g. x instead of t) for all marks except final B1.	

Leave blank

5. $$x\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + y^2$$

(a) Show that

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (1-2y)\frac{\mathrm{d}y}{\mathrm{d}x} = 3$$

(2)

Given that $y = 1$ at $x = 1$,

(b) find a series solution for y in ascending powers of $(x-1)$, up to and including the term in $(x-1)^3$.

(8)

Question Number	Scheme	Marks
5.		
(a)	$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=3+2y\frac{dy}{dx}$ (Using differentiation of product or quotient **and** also differentiation of implicit function)	M1
	$x\frac{d^2y}{dx^2}+(1-2y)\frac{dy}{dx}=3$ **ag**	A1 cso
		(2)
(b)	$\left(x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+.....$	B1
	$.....\left[(1-2y)\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2\right]=0$	M1 A1
	At x = 1: $\frac{dy}{dx}=4$	B1
	$\frac{d^2y}{dx^2}=7$ $\frac{d^3y}{dx^3}=32$	B1, B1
	$(y=)\text{f}(1)+\text{f}'(1)(x-1)+\frac{\text{f}''(1)(x-1)^2}{2}+\frac{\text{f}'''(1)(x-1)^3}{6}...$	M1
	$y=1+4(x-1)+\frac{7}{2}(x-1)^2+\frac{16}{3}(x-1)^3$ (or equiv.)	A1 ft
		(8) **10**
	Notes	
(a)	Finding second derivative and substituting into given answer acceptable	
(b)	1st M1 for differentiating second term to obtain an expression involving $\frac{d^2y}{dx^2}$ and $\left(\frac{dy}{dx}\right)^2$	
	B1B1B1 for 4,7,32 seen respectively	
	2nd M1 require $\text{f}(1)$ or $1, \text{f}'(1)$ etc and x-1and at least first 3 terms	
	A1 for 4 terms following through their constants	
	Condone f(x)= instead of y=	

Leave blank

6. (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(2)

(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)}$$

where a and b are constants to be found.

(6)

(c) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$$

(3)

Question Number	Scheme	Marks
6.		
(a)	$\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right) = \frac{1}{2r}, -\frac{1}{2r+4}$	B1,B1oe
		(2)
(b)	$r=1: \quad \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$	M1
	$r=2: \quad \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$	
	$r=3: \quad \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$	
	$r=n-1: \quad \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$	
	$r=n: \quad \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$	A1
	Summing: $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$	M1 A1
	$= \frac{1}{2}\left(\frac{3(n+1)(n+2) - 2(n+1) - 2(n+2)}{2(n+1)(n+2)}\right) = \frac{n(3n+5)}{4(n+1)(n+2)}$	M1 A1cao
		(6)
(c)	$\sum_{r=1}^{2n} \frac{1}{r(r+2)} = \frac{2n(6n+5)}{4(2n+1)(2n+2)}$	B1oe
	$S_{2n} - S_n = \frac{2n(6n+5)}{4(2n+1)(2n+2)} - \frac{n(3n+5)}{4(n+1)(n+2)}$	M1
	$= \frac{n(6n+5)(n+2) - n(3n+5)(2n+1)}{4(n+1)(n+2)(2n+1)}$	
	$= \frac{n(6n^2+17n+10-6n^2-13n-5)}{4(n+1)(n+2)(2n+1)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$ (*ag*)	A1 cso
		(3)
		11
(a)	1st and 2nd B1 Any form is acceptable	
(b)	1st M1 must include at least 4 out of 5 of (r=)1,2,3 and n-1, n 1st A1 require all terms that do not cancel to be accurate 2nd M1 Summed expression involving all terms that do not cancel 2nd A1 Correct expression 3rd M1 for attempt to find single fraction	
(c)	1st M1 for expression for $S_{2n} - S_n$	

Leave blank

7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$3xy^2 \frac{dy}{dx} = x^3 + y^3 \qquad \text{(I)}$$

into the differential equation

$$3v^2 x \frac{dv}{dx} = 1 - 2v^3 \qquad \text{(II)}$$

(3)

(b) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = f(x)$.

(6)

Given that $y = 2$ at $x = 1$,

(c) find the value of $\frac{dy}{dx}$ at $x = 1$

(2)

Question Number	Scheme	Marks
7.		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ seen	B1
	$3x^3v^2\left(v + x\frac{\mathrm{d}v}{\mathrm{d}x}\right) = x^3 + v^3x^3 \quad \Rightarrow \quad 3v^2x\frac{\mathrm{d}v}{\mathrm{d}x} = 1 - 2v^3$ (**ag**)	M1 A1 cso
		(3)
(b)	$\int \frac{3v^2}{1-2v^3}\mathrm{d}v = \int \frac{1}{x}\mathrm{d}x$	M1
	$-\frac{1}{2}\ln(1-2v^3) = \ln x \ (+C)$	M1 A1
	$-\ln(1-2v^3) = \ln x^2 + \ln A$	
	$Ax^2 = \frac{1}{1-2v^3}$	M1
	$1 - \frac{2y^3}{x^3} = \frac{1}{Ax^2}$	
	$y = \sqrt[3]{\frac{x^3 - Bx}{2}}$ or equivalent	dM1 A1cso
		(6)
(c)	Using $y = 2$ at $x = 1$: $12\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 8$	M1
	At $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{4}$	A1
		(2)
		11
	Notes	
(a)	M1 for substituting y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ obtaining an expression in v and x only	
(b)	1st M1 for separating variables 2nd M1 for attempting to integrate both sides 1st A1both sides required or equivalent expressions. (Modulus not required.) 3rd M1 Removing logs, dealing correctly with constant 4th M1 dep on 1st M. Substitute $v = \frac{y}{x}$ and rearranging to $y = \mathrm{f}(x)$	
(c)	M1 for finding a numerical value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ A1 for correct numerical answer oe.	

Leave blank

8. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\text{i}| = 2|z - 3|$$

(a) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

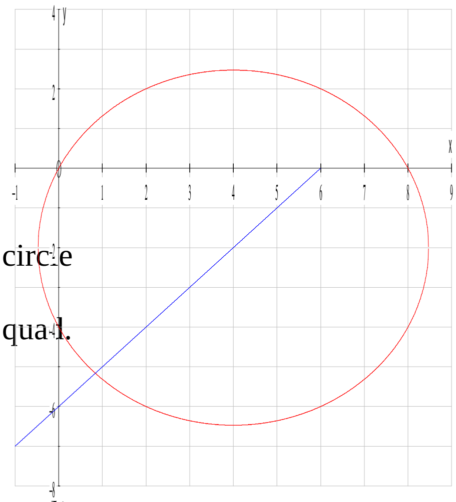
$$\arg(z - 6) = -\frac{3\pi}{4}$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(c) Find the complex number for which both $|z - 6\text{i}| = 2|z - 3|$ and $\arg(z - 6) = -\frac{3\pi}{4}$

(4)

Question Number	Scheme	Marks
8.		
(a)	$\lvert x+\mathrm{i}y-6\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-3\rvert$	M1
	$x^2+(y-6)^2 = 4\left[(x-3)^2+y^2\right]$	M1 A1
	$x^2+y^2-12y+36 = 4x^2-24x+36+4y^2$	
	$3x^2+3y^2-24x+12y=0$	
	$(x-4)^2+(y+2)^2=20$	M1
	Centre $(4,-2)$, Radius $\sqrt{20}=2\sqrt{5}=$ awrt 4.47	A1 A1
		(6)
(b)		
	Centre in correct quad for their circle	M1
	Passes through O centre in 4th quad.	A1cao
	Half line with positive gradient	B1
	Correct position, clearly through (6, 0)	B1
		(4)
(c)	Equation of line $y=x-6$	B1
	Attempting simultaneous solution of $(x-4)^2+(y+2)^2=20$ and $y=x-6$	M1
	$x=4\pm\sqrt{10}$	A1
	$\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	A1cao
		(4)
		14
	Notes	
(a)	1st M Substituting $z=x+iy$ oe	
	2nd M implementing modulus of both sides and squaring. Require Re^2 **plus** Im^2 on both sides & no terms in i. Condone 2 instead of 4 here.	
	3rd M1 for gathering terms and attempting to find centre and / or radius	

Question Number	Scheme	Marks
	2nd A1 for centre, 3rd A1 for radius	
Alt 8(c)	**For geometric approach in this part.**	
	Centre (4,-2) on line, can be implied.	B1
	Use of Pythagoras or trigonometry to find lengths of isosceles triangle	M1
	$x = 4 - \sqrt{10}$	A1
	$\left(4 - \sqrt{10}\right) + \mathrm{i}\left(-2 - \sqrt{10}\right)$	A1cao