Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P43149A
W850/R6668/57570 5/5/5

P 4 3 1 4 9 A 0 1 2 8

Turn over

PEARSON

Leave blank

1. (a) Express $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions.
(2)

(b) Using your answer to (a), find, in terms of n,

$$\sum_{r=1}^{n} \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form.
(3)

Question Number	Scheme	Marks
1.		
(a)	$\frac{2}{(2r+1)(2r+3)} = \frac{A}{2r+1} + \frac{B}{2r+3} =, \quad \frac{1}{2r+1} - \frac{1}{2r+3}$	M1,A1 (2)
(b)	$\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots \frac{1}{2n+1} - \frac{1}{2n+3}$	
	$= \frac{1}{3} - \frac{1}{2n+3} = \frac{2n+3-3}{3(2n+3)}$	M1
	$\sum_{1}^{n} \frac{3}{(2r+1)(2r+3)} = \frac{3}{2} \times \frac{2n}{3(2n+3)} = \frac{n}{2n+3}$	M1depA1 (3)
		[5]

Notes for Question 1

(a)

M1 for any valid attempt to obtain the PFs

A1 for $\frac{1}{2r+1} - \frac{1}{2r+3}$

NB With no working shown award M1A1 if the correct PFs are written down, but M0A0 if either one is incorrect

(b)

M1 for using **their** PFs to split each of the terms of the sum or of $\sum \frac{2}{(2r+1)(2r+3)}$ into 2 PFs.

At least 2 terms at the start and 1 at the end needed to show the diagonal cancellation resulting in two remaining terms.

M1dep for simplifying to a single fraction and multiplying it by the appropriate constant

A1cao for $\sum = \frac{n}{2n+3}$

NB: If r is used instead of n (including for the answer), only M marks are available.

Leave blank

2. $$z = 5\sqrt{3} - 5\text{i}$$

Find

(a) $|z|$,

(1)

(b) $\arg(z)$, in terms of π.

(2)

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$,

(1)

(d) $\arg\left(\frac{w}{z}\right)$, in terms of π.

(2)

Question Number	Scheme	Marks
2		
(a)	$z = 5\sqrt{3} - 5\text{i} = r\left(\cos\theta + \text{i}\sin\theta\right)$	
	$r = \sqrt{\left(5^2 \times 3 + 5^2\right)} = 10$	B1 (1)
(b)	$\arg z = \arctan\left(-\frac{5}{5\sqrt{3}}\right) = -\frac{\pi}{6} \qquad \left(\text{or } -\frac{\pi}{6} \pm 2n\pi\right)$	M1A1 (2)
(c)	$\left\lvert\frac{w}{z}\right\rvert = \frac{2}{10} = \frac{1}{5}$ or 0.2	B1 (1)
(d)	$\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{6}\right), \ = \frac{5\pi}{12} \qquad \left(\text{or } \frac{5\pi}{12} \pm 2n\pi\right)$	M1,A1 (2)
		[6]

Notes for Question 2

(a)

B1 for $|z| = 10$ no working needed

(b)

M1 for $\arg z = \arctan\left(\pm\frac{5}{5\sqrt{3}}\right)$, $\tan(\arg z) = \pm\frac{5}{5\sqrt{3}}$, $\arg z = \arctan\left(\pm\frac{5\sqrt{3}}{5}\right)$ or

$\tan(\arg z) = \pm\frac{5\sqrt{3}}{5}$ OR use their $|z|$ with sin or cos used correctly

A1 for $= -\frac{\pi}{6}$ $\left(\text{or } -\frac{\pi}{6} \pm 2n\pi\right)$ (must be 4th quadrant)

(c)

B1 for $\left|\frac{w}{z}\right| = \frac{2}{10}$ or $\frac{1}{5}$ or 0.2

(d)

M1 for $\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \arg z$ using **their** $\arg z$

A1 for $\frac{5\pi}{12}$ $\left(\text{or } \frac{5\pi}{12} \pm 2n\pi\right)$

Alternative for (d):

Find $\frac{w}{z} = \frac{\left(\sqrt{6}-\sqrt{2}\right)+\left(\sqrt{6}+\sqrt{2}\right)\text{i}}{20}$

$\tan\left(\arg\frac{w}{z}\right) = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ M1 from their $\frac{w}{z}$

$\arg\left(\frac{w}{z}\right) = \frac{5\pi}{12}$ A1 cao

Work for (c) and (d) may be seen together – give B and A marks only if modulus and argument are clearly identified

ie $\frac{1}{5}\left(\cos\frac{5\pi}{12} + \text{i}\sin\frac{5\pi}{12}\right)$ alone scores B0M1A0

Leave blank

3.

$$\frac{d^2y}{dx^2} + 4y - \sin x = 0$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = \frac{1}{8}$ at $x = 0$,

find a series expansion for y in terms of x, up to and including the term in x^3.

(5)

Question Number	Scheme	Marks
3	$(x=0)\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sin 0-4\times\frac{1}{2}=-2$	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\frac{\mathrm{d}y}{\mathrm{d}x}-\cos x\,(=0)$	M1
	$(x=0)\quad \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\cos 0-4\times\frac{1}{8}=\frac{1}{2}$	A1
	$(y=)\,y_0+x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0+\frac{x^2}{2!}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0+\frac{x^3}{3!}\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0+\ldots.$	M1 (2! or 2 and 3! or 6)
	$(y=)\,\frac{1}{2}+x\times\frac{1}{8}+\frac{x^2}{2}\times(-2)+\frac{x^3}{6}\times\frac{1}{2}$	
	$y=\frac{1}{2}+\frac{x}{8}-x^2+\frac{x^3}{12}$	A1 cao **[5]**
	Alt:	
	$y=\frac{1}{2}+\frac{x}{8}+ax^2+bx^3+\ldots$	B1
	$y''=2a+6bx+\ldots$	M1 Diff twice
	$2a+6bx+\ldots=\sin x-\left(\frac{1}{2}+\frac{x}{8}+ax^2+bx^3\ldots\right)$	A1 Correct differentiation and equation used
	$2a+2=0\quad a=-1$	M1
	$6b+\frac{1}{2}=1\quad b=\frac{1}{12}$	
	$y=\frac{1}{2}+\frac{x}{8}-x^2+\frac{x^3}{12}$	A1cao

Notes for Question 3

B1 for $\left(\frac{d^2y}{dx^2}\right)_0 = -2$ wherever seen

M1 for attempting the differentiation of the given equation. To obtain $\frac{d^3y}{dx^3} \pm k\frac{dy}{dx} \pm \cos x (= 0)$ oe

A1 for substituting $x = 0$ to obtain $\left(\frac{d^3y}{dx^3}\right)_0 = \frac{1}{2}$

M1 for using the expansion $\left[y = f(x)\right] = f(0) + xf'(0) + \frac{x^2}{2(!)}f''(0) + \frac{x^3}{3!}f'''(0)$ with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$. Factorial can be omitted in the x^2 term but must be shown explicitly in the x^3 term or implied by further working eg using 6.

A1cao for $y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12}$ (Ignore any higher powers included) Exact decimals allowed. **Must include *y* =….**

Alternative:

B1 for $y = \frac{1}{2} + \frac{x}{8} + ax^2 + bx^3 + ...$

M1 for differentiating this twice to get $y'' = 2a + 6bx + ...$ (may not be completely correct)

A1 for correct differentiation and using the given equation and the expansion of sin *x* to get $2a + 6bx + ... = \left(x - \frac{x^3}{3} + ...\right) - 4\left(\frac{1}{2} + \frac{x}{8} + ...\right)$

M1 for equating coefficients to obtain a value for *a* or *b*

A1 cao for $y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12}$ (Ignore any higher powers included)

Leave blank

4. (a) Given that

$$z = r(\cos\theta + \mathrm{i}\sin\theta), \qquad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \qquad n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5, giving your answer in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(2)

Question Number	Scheme	Marks
4 **(a)**	Assume true for $n=k$: $z^k = r^k(\cos k\theta + \text{i}\sin k\theta)$	
	$n=k+1$: $z^{k+1} = (z^k \times z =) r^k(\cos k\theta + \text{i}\sin k\theta)\times r(\cos\theta + \text{i}\sin\theta)$	M1
	$= r^{k+1}(\cos k\theta\cos\theta - \sin k\theta\sin\theta + \text{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta))$	M1
	$= r^{k+1}(\cos(k+1)\theta + \text{i}\sin(k+1)\theta)$	M1depA1cso
	$\therefore$ <u>if true for $n=k$,</u> also <u>true for $n=k+1$</u>	
	$k=1$ <u>$z^1 = r^1(\cos\theta + \text{i}\sin\theta)$</u>; <u>True for $n=1$</u> $\therefore$ <u>true for all n</u>	A1cso (5)
	Alternative: See notes for use of $r\text{e}^{\text{i}\theta}$ form	
(b)	$w = 3\left(\cos\frac{3\pi}{4} + \text{i}\sin\frac{3\pi}{4}\right)$	
	$w^5 = 3^5\left(\cos\frac{15\pi}{4} + \text{i}\sin\frac{15\pi}{4}\right)$	M1
	$w^5 = 243\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\text{i}\right)$ $\left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}\text{i} \text{ or}\right]$ oe	A1 (2)
		[7]

Notes for Question 4
(a) **NB:** Allow each mark if *n*, *n* + 1 used instead of *k*, *k* + 1 M1 for using the result for $n = k$ to write $z^{k+1}\left(= z^k \times z\right) = r^k\left(\cos k\theta + \mathrm{i}\sin k\theta\right)\times r\left(\cos\theta + \mathrm{i}\sin\theta\right)$ M1 for multiplying out and collecting real and imaginary parts, using $\mathrm{i}^2 = -1$ OR using sum of arguments and product of moduli to get $r^{k+1}\left(\cos\left(k\theta+\theta\right) + \mathrm{i}\sin\left(k\theta+\theta\right)\right)$ M1dep for using the addition formulae to obtain single cos and sin terms OR factorise the argument $r^{k+1}\left(\cos\theta\left(k+1\right) + \mathrm{i}\sin\theta\left(k+1\right)\right)$ Dependent on the second M mark. A1cso for $r^{k+1}\left(\cos\left(k+1\right)\theta + \mathrm{i}\sin\left(k+1\right)\theta\right)$ Only give this mark if all previous steps are fully correct. A1cso All 5 underlined statements must be seen *Alternative:* Using Euler's form

$z = r\left(\cos\theta + i\sin\theta\right) = r\,\mathrm{e}^{\mathrm{i}\theta}$	M1 May not be seen explicitly
$z^{k+1} = z^k \times z = \left(r\,\mathrm{e}^{\mathrm{i}\theta}\right)^k \times r\,\mathrm{e}^{\mathrm{i}\theta} = r^k\,\mathrm{e}^{\mathrm{i}k\theta} \times r\,\mathrm{e}^{\mathrm{i}\theta}$	M1
$= r^{k+1}\,\mathrm{e}^{\mathrm{i}(k+1)\theta}$	M1dep on 2nd M mark
$= r^{k+1}\left(\cos\left(k+1\right)\theta + \mathrm{i}\sin\left(k+1\right)\theta\right)$	A1cso
$k = 1 \quad z^1 = r^1\left(\cos\theta + \mathrm{i}\sin\theta\right)$	
True for $n = 1$ $\therefore$ true for all *n* etc	A1 cso All 5 underlined statements must be seen

(b)

M1 for attempting to apply de Moivre to *w* or attempting to expand w^5 and collecting real and imaginary parts, but no need to simplify these.

A1cao for $243\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{i}\right)$ $\left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}\mathrm{i}\right]$ (oe eg 3^5 instead of 243)

Leave blank

5. (a) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = 4x^2$$

(5)

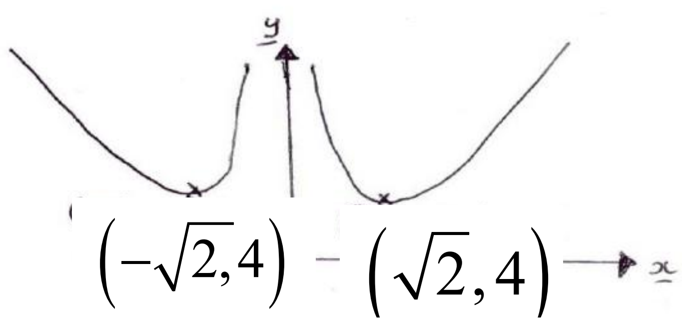
(b) Find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = f(x)$.

(2)

(c) (i) Find the exact values of the coordinates of the turning points of the curve with equation $y = f(x)$, making your method clear.

(ii) Sketch the curve with equation $y = f(x)$, showing the coordinates of the turning points.

(5)

Question Number	Scheme	Marks
5		
(a)	$\frac{dy}{dx}+2\frac{y}{x}=4x$	M1
	I F: $e^{\int \frac{2}{x}dx}=e^{2\ln x}=\left(x^2\right)$	M1
	$x^2\frac{dy}{dx}+2xy=4x^3$	M1dep
	$yx^2=\int 4x^3 dx=x^4\left(+c\right)$	M1dep
	$y=x^2+\frac{c}{x^2}$	A1cso (5)
(b)	$x=1,\ y=5 \Rightarrow c=4$	M1
	$y=x^2+\frac{4}{x^2}$	A1ft (2)
(c)	$\frac{dy}{dx}=2x-\frac{8}{x^3}$	
	$\frac{dy}{dx}=0 \quad x^4=4\ ,\ x=\pm\sqrt{2} \quad \text{or } \pm\sqrt[4]{4}$	M1,A1
	$y=2+\frac{4}{2}=4$	A1cao
	Alt: Complete square on *y* = ... or use the original differential equation	M1
	$x=\pm\sqrt{2},\quad y=4$	A1,A1
		B1 shape B1 turning points shown *somewhere* (5)
		[12]

Notes for Question 5

(a)

M1 for dividing the given equation by x May be implied by subsequent work.

M1 for IF= $e^{\int \frac{2}{x}dx} = e^{2\ln x} = (x^2)$ $\int \frac{2}{x}dx$ must be seen together with an attempt at integrating this. $\ln x$ must be seen in the integrated function.

M1dep for multiplying the equation $\frac{dy}{dx} + 2\frac{y}{x} = 4x$ by their IF dep on 2nd M mark

M1dep for attempting the integration of the resulting equation - constant not needed. Dep on 2nd and 3rd M marks

A1cso for $y = x^2 + \frac{c}{x^2}$ oe eg $yx^2 = x^4 + c$

Alternative: for first three marks: Multiply given equation by x to get straight to the third line. All 3 M marks should be given.

(b)

M1 for using $x = 1,\ y = 5$ in **their** expression for y to obtain a value for c

A1ft for $y = x^2 + \frac{4}{x^2}$ follow through their result from (a)

(c)

M1 for differentiating **their** result from (b), equating to 0 and solving for x

A1 for $x = \pm\sqrt{2}$ (no follow through) or $\pm\sqrt[4]{4}$ No extra real values allowed but ignore any imaginary roots shown.

A1cao for using the particular solution to obtain $y = 4$. No extra values allowed.

Alternatives for these 3 marks:

M1 for making $\frac{dy}{dx} = 0$ in the given differential equation to get $y = 2x^2$ and using this with their particular solution to obtain an equation in one variable

OR complete the square on **their** particular solution to get $y = \left(x + \frac{2}{x}\right)^2 - 4$

A1 for $x = \pm\sqrt{2}$ (no follow through)

A1cao for $y = 4$ No extra values allowed

B1 for the correct shape - must have two minimum points and two branches, both asymptotic to the y-axis

B1 for a fully correct sketch with the coordinates of the minimum points shown somewhere on or beside the sketch. Decimals accepted here.

Leave blank

6. (a) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x$$

(6)

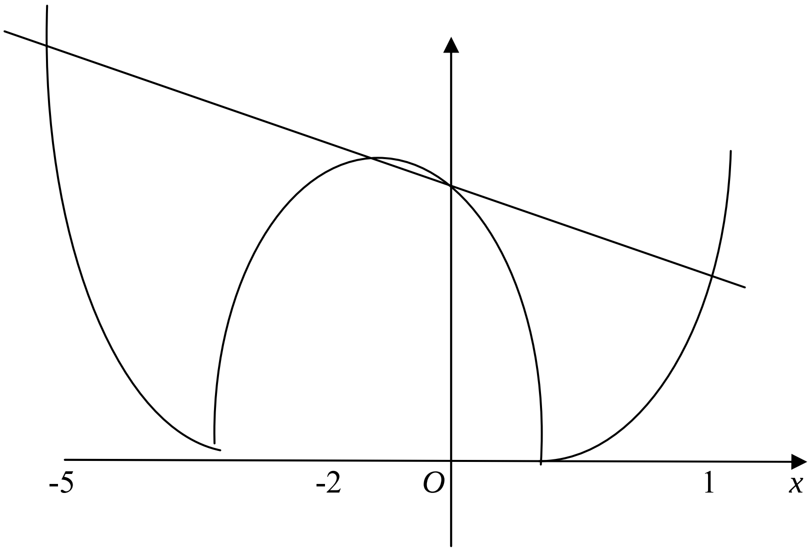
(b) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinates of the points where the line crosses the curve.

(3)

(c) Find the set of values of x for which

$$|2x^2 + 6x - 5| > 5 - 2x$$

(3)

Question Number	Scheme	Marks
6 **(a)**	$2x^2+6x-5=5-2x$	M1
	$2x^2+8x-10=0$	
	$x^2+4x-5=0$	
	$(x+5)(x-1)=0$ or by formula	M1
	$x=-5,\ x=1$	A1
	$-2x^2-6x+5=5-2x$	M1
	$2x^2+4x=0$	A1
	$x=0\ \ x=-2$	A1 (6)
(b)		B1 line B1 quad curve B1ft (on x-coords from (a)) (3)
(c)	$x<-5,\quad -2<x<0,\quad x>1$	B1,B1,B1 (3)
	Special case: Deduct the last B mark earned1 if $\leqslant$ or $\geqslant$ used	**[12]**

Notes for Question 6

(a)**NB: Marks for (a) can only be awarded for work shown in (a):**

M1 for $2x^2+6x-5=5-2x$

M1 for obtaining a 3 term quadratic and attempting to solve by factorising, formula or completing the square

A1 for $x=-5,\ x=1$

M1 for considering the part of the quadratic that needs to be reflected ie for $-2x^2-6x+5=5-2x$ oe

A1 for a correct 2 term quadratic, terms in any order $2x^2+4x=0$ oe

A1 for $x=0\ \ x=-2$

NB: The question demands that algebra is used, so solutions which do not show how the roots have been obtained will score very few if any marks, depending on what is written on the page.

Alternative: Squaring both sides:
M1 Square both sides and simplify to a quartic expression
M1 Take out the common factor *x*
A1 *x*, a correct linear factor and a correct quadratic factor
M1 *x* and 3 linear factors
A1 any two of the required values
A1 all 4 values correct

(b)
B1 for a line drawn, with negative gradient, crossing the positive *y*-axis

B1 for the quadratic curve, with part reflected and the correct shape. It should cross the *y*-axis at the same point as the line and be pointed where it meets the *x*-axis (ie not U-shaped like a turning point)

B1ft for showing the *x* coordinates of the points where the line crosses the curve. They can be shown on the *x*-axis as in the MS (accept *O* for 0) or written alongside the points as long as it is clear the numbers are the *x* coordinates
The line should cross the curve at all the crossing points found *and no others* for this mark to be given.

(c)**NB: No follow through for these marks**
B1 for any one of $x<-5$, $-2<x<0$, $x>1$ correct

B1 for a second one of these correct

B1 for the third one correct

Special case: if $\leqslant$ or $\geqslant$ is used, deduct the last B mark earned.

Leave blank

7. (a) Show that the transformation $y = xv$ transforms the equation

$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4 \qquad \text{(I)}$$

into the equation

$$4\frac{d^2v}{dx^2} + 4v = x \qquad \text{(II)}$$

(6)

(b) Solve the differential equation (II) to find v as a function of x.

(6)

(c) Hence state the general solution of the differential equation (I).

(1)

Question Number	Scheme	Marks
7		
(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = v + x\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}v}{\mathrm{d}x} + \dfrac{\mathrm{d}v}{\mathrm{d}x} + x\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}$	M1A1
	$4x^2\left(2\dfrac{\mathrm{d}v}{\mathrm{d}x} + x\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}\right) - 8x\left(v + x\dfrac{\mathrm{d}v}{\mathrm{d}x}\right) + \left(8 + 4x^2\right)\times xv = x^4$	M1
	$4x^3\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2} + 4x^3v = x^4$	M1
	$4\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2} + 4v = x$ *	A1 (6)
	See end for an alternative for (a)	
(b)	$4\lambda^2 + 4 = 0$	
	$\lambda^2 = -1$ oe	M1A1
	$(v =)C\cos x + D\sin x$ (or $(v =)Ae^{ix} + Be^{-ix}$)	A1
	P.I: Try $v = kx$ $(+l)$	
	$\dfrac{\mathrm{d}v}{\mathrm{d}x} = k$ $\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2} = 0$	M1
	$4\times 0 + 4\left(kx\ (+l)\right) = x$	M1dep
	$k = \dfrac{1}{4}$ $(l = 0)$	
	$v = C\cos x + D\sin x + \dfrac{1}{4}x$ (or $v = Ae^{ix} + Be^{-ix} + \dfrac{1}{4}x$)	A1 (6)
(c)	$y = x\left(C\cos x + D\sin x + \dfrac{1}{4}x\right)$ (or $y = x\left(Ae^{ix} + Be^{-ix} + \dfrac{1}{4}x\right)$)	B1ft (1)

Question 7 continued		
	Alternative for (a):	
	$v = \frac{y}{x}$	
	$\frac{dv}{dx} = \frac{dy}{dx} \times \frac{1}{x} - y \times \frac{1}{x^2}$	M1
	$\frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3}$	M1A1
	$x^3 \frac{d^2v}{dx^2} = x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y$	M1
	$4x^3 \frac{d^2v}{dx^2} + 4x^3v = 4x^2 \frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + 8y + 4x^2y = x^4$	M1
	$4\frac{d^2v}{dx^2} + 4v = x$ *	A1

Leave blank

8.

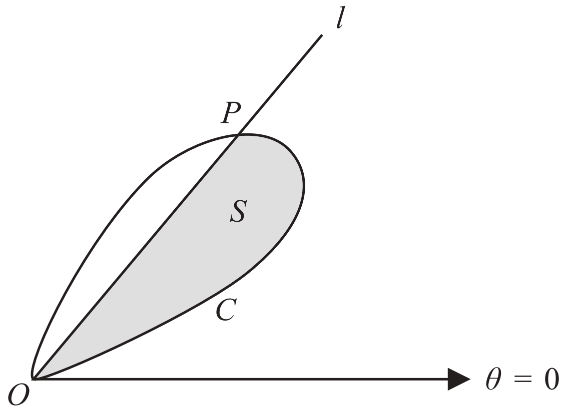


Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta,\ 0 \leqslant \theta \leqslant \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ).

(a) Show that $\cos\phi = \frac{1}{\sqrt{3}}$

(6)

(b) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right) + \sqrt{2}\right)$$

(7)

Question Number	Scheme	Marks
8 (a)	$(y=)\,r\sin\theta = a\sin 2\theta\sin\theta$	M1
	$\left(\frac{dy}{d\theta}=\right)a\left(2\cos 2\theta\sin\theta+\sin 2\theta\cos\theta\right)$	M1depA1
	$\left(\frac{dy}{d\theta}=\right)2a\sin\theta\left(\cos 2\theta+\cos^2\theta\right)$	M1
	At P $\frac{dy}{d\theta}=0 \Rightarrow \sin\theta=0$ (n/a) or $2\cos^2\theta-1+\cos^2\theta=0$ $3\cos^2\theta=1$	M1 $\sin\theta=0$ not needed
	$\cos\theta=\frac{1}{\sqrt{3}}$ *	A1cso (6)
(b)	$r=a\sin 2\theta=2a\sin\theta\cos\theta$	
	$r=2a\sqrt{\left(1-\frac{1}{3}\right)}\sqrt{\frac{1}{3}}=2a\frac{\sqrt{2}}{3}$	M1A1 (2)
(c)	Area = $\int_0^{\phi}\frac{1}{2}r^2\,d\theta=\frac{1}{2}a^2\int_0^{\phi}\sin^2 2\theta\,d\theta$	M1
	$=\frac{1}{2}a^2\int_0^{\phi}\frac{1}{2}\left(1-\cos 4\theta\right)d\theta$	M1
	$=\frac{1}{4}a^2\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\phi}$	M1A1
	$=\frac{1}{4}a^2\left[\phi-\frac{1}{4}\left(4\sin\phi\cos\phi\left(2\cos^2\phi-1\right)\right)\right]$	M1dep on 2nd M mark
	$=\frac{1}{4}a^2\left[\arccos\left(\frac{1}{\sqrt{3}}\right)-\left(\sqrt{\frac{2}{3}}\times\sqrt{\frac{1}{3}}\times\left(\frac{2}{3}-1\right)\right)\right]$	M1 dep (all Ms)
	$\frac{1}{36}a^2\left[9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right]$ *	A1 (7) **[15]**

Notes for Question 8

(a)

M1 for obtaining the y coordinate $y = r\sin\theta = a\sin 2\theta \sin\theta$

M1dep for attempting the differentiation to obtain $\frac{dy}{d\theta}$ Product rule and/or chain rule must be used; sin to become $\pm\cos$ (cos to become $\pm\sin$). The 2 may be omitted. Dependent on the first M mark.

A1 for correct differentiation eg $\frac{dy}{d\theta} = a\left(2\cos 2\theta \sin\theta + \sin 2\theta\cos\theta\right)$ oe

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$ anywhere in their solution to (a)

M1 for setting $\frac{dy}{d\theta} = 0$ and getting a quadratic factor with no $\sin^2\theta$ included.

Alternative: Obtain a quadratic in $\sin\theta$ or $\tan\theta$ and complete to $\cos\theta$ = later.

A1cso for $\cos\theta = \frac{1}{\sqrt{3}}$ or $\cos\phi = \frac{1}{\sqrt{3}}$ *

Question 8 (a) Variations you may see:

y = rsinθ = asin2θsinθ

y = asin2θsinθ	y = 2asin²θcosθ	y = 2a(cosθ – cos³θ)
dy/dθ = a(2cos2θsinθ + sin2θcosθ) = a(2cos2θsinθ + 2sinθcos²θ) = 2asinθ(cos2θ + cos²θ) = 2asinθ(3cos²θ -1) or = 2asinθ(2cos²θ – sin²θ) or = 2asinθ(2 – 3sin²θ)	dy/dθ = 2a(2sinθcos²θ – sin³θ) = 2asinθ(2cos²θ – sin²θ)	dy/dθ = 2a(-sinθ +3sinθcos²θ) = 2asinθ(3cos²θ -1)

At P: dy/dθ = 0 => sin θ = 0 or:		
$2\cos^2\theta - \sin^2\theta = 0$	$3\cos^2\theta - 1 = 0$	$2 - 3\sin^2\theta = 0$
$\tan^2\theta = 2$	$\cos^2\theta = 1/3$	$\sin^2\theta = 2/3$
$\tan\theta = \pm\sqrt{2}$ => $\cos\theta = \pm\frac{1}{\sqrt{3}}$	$\cos\theta = \pm\frac{1}{\sqrt{3}}$	$\sin\theta = \pm\frac{\sqrt{2}}{\sqrt{3}} = \pm\frac{\sqrt{6}}{3}$ => $\cos\theta = \pm\frac{1}{\sqrt{3}}$

(b)

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos^2\theta + \sin^2\theta = 1$ and $\cos\phi = \frac{1}{\sqrt{3}}$ in $r = a\sin 2\theta$ to obtain a numerical multiple of a for R. Need not be simplified.

A1cao for $R = 2a\frac{\sqrt{2}}{3}$

Can be done on a calculator. Completely correct answer with no working scores 2/2; incorrect answer with no working scores 0/2

Notes for Question 8 continued
(c) M1 for using the area formula $\int_0^{\phi}\frac{1}{2}r^2\mathrm{d}\theta=\frac{1}{2}a^2\int_0^{\phi}\sin^2 2\theta\,\mathrm{d}\theta$ Limits not needed M1 for preparing $\int \sin^2 2\theta\,\mathrm{d}\theta$ for integration by using $\cos 2x = 1-2\sin^2 x$ M1 for attempting the integration: $\cos 4\theta$ to become $\pm\sin 4\theta$ - the $\frac{1}{4}$ may be missing but inclusion of 4 implies differentiation - and the constant to become $k\theta$. Limits not needed. A1 for $=\frac{1}{4}a^2\left[\theta-\frac{1}{4}\sin 4\theta\right]$ Limits not needed M1dep for changing **their** integrated function to an expression in $\sin\theta$ and $\cos\theta$ and substituting limits 0 and ϕ. Dependent on the second M mark of (c) M1dep for a numerical multiple of a^2 for the area. Dependent on all previous M marks of (c) A1cso for $\frac{1}{36}a^2\left[9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right]$ * This is a given answer, so check carefully that it can be obtained from the previous step in their working. **Also**: The final 3 marks can only be awarded if the working is **shown** ie $\sin 4\theta$ cannot be obtained by calculator.