Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1 R	Signature	

Paper Reference(s)

6668/01R

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 2\text{i}}{\text{i}z} \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto a line in the w-plane.

Find an equation of this line.

(4)

Question Number	Scheme	Marks
1.	$z = x \qquad w = \dfrac{x+2\text{i}}{\text{i}x}$	M1A1
	$w = \dfrac{1}{\text{i}} + \dfrac{2\text{i}}{\text{i}x}$	
	$u + \text{i}v = -\text{i} + \dfrac{2}{x}$	
	$\left(u = \dfrac{2}{x}\right) \qquad v = -1$	M1
	$\therefore w$ is on the line $v + 1 = 0$	A1
		4 Marks

NOTES

M1 for replacing at least one z with x to obtain (ie show an appreciation that $y = 0$)

A1 $w = \dfrac{x+2\text{i}}{\text{i}x}$

M1 for writing w as $u + \text{i}v$ and equating real or imaginary parts to obtain either u or v in terms of x or just a number

A1 for giving the equation of the line $v + 1 = 0$ oe must be in terms of v

Question Number	Scheme	Marks
	Q1 - ALTERNATIVE 1:	
	$w=\dfrac{x+iy+2i}{i(x+iy)}$ Replacing z with $x+iy$	
	$w=\dfrac{x+iy+2i}{-y+ix}\times\dfrac{-y-ix}{-y-ix}$	
	$w=\dfrac{(x+i(y+2))(-y-ix)}{y^2+x^2}$	
	$w=\dfrac{2x-i(x^2+y^2+2y)}{y^2+x^2}$	
	$w=\dfrac{2x-ix^2}{x^2}=\dfrac{2}{x}-i$ **Using *y* = 0**. This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real**	M1A1
	$v=-1$ M1, A1 as in main scheme	M1A1
	Q1 - ALTERNATIVE 2:	
	$z=\dfrac{2i}{iw-1}$ Writing the transformation as a function of w	
	$z=\dfrac{2i}{i(u+iv)-1}$	
	$z=\dfrac{2i}{(-v-1)+iu}\times\dfrac{(-v-1)-iu}{(-v-1)-iu}$	
	$z=\dfrac{2u+2i(-v-1)}{(-v-1)^2+u^2}=\dfrac{2u}{(-v-1)^2+u^2}+i\left(\dfrac{2(-v-1)}{(-v-1)^2+u^2}\right)$	
	$\left(\dfrac{2(-v-1)}{(v-1)^2+u^2}\right)=0$ or simply $-2(v+1)=0$ **Using *y* = 0** . This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real**	M1A1
	$v=-1$ M1, A1 as in main mark scheme above	M1A1

Leave blank

2. Use algebra to find the set of values of x for which

$$\frac{6x}{3-x} > \frac{1}{x+1}$$

(7)

Question Number	Scheme	Marks
2.	**NB** Allow the first 5 marks with = instead of inequality	
	$\dfrac{6x}{3-x} > \dfrac{1}{x+1}$	
	$6x(3-x)(x+1)^2 - (3-x)^2(x+1) > 0$	M1
	$(3-x)(x+1)\left(6x^2+6x-3+x\right) > 0$	
	$(3-x)(x+1)(3x-1)(2x+3) > 0$	M1dep
	Critical values $3, -1$	B1
	and $-\dfrac{3}{2}, \dfrac{1}{3}$	A1, A1
	Use critical values to obtain both of $-\dfrac{3}{2} < x < -1$ $\quad \dfrac{1}{3} < x < 3$	M1A1cso
		7 Marks

NOTES

M1 for multiplying through by $(x+1)^2(3-x)^2$
OR: for collecting one side of the inequality and attempting to form a single fraction (see alternative in mark scheme)

M1dep for collecting on one side of the inequality and factorising the result of the above (usual rules for factorising the quadratic)
OR: for factorising the numerator - must be a three term quadratic - usual rules for factorising a quadratic (see alternative in mark scheme)
Dependent on the first M mark

B1 for the critical values $3, -1$

A1 for either $-\dfrac{3}{2}$ or $\dfrac{1}{3}$

A1 for the second of these

NB: the critical values need not be shown explicitly - they may be shown on a sketch or just appear in the ranges or in the working for the ranges.

M1 using **their** 4 critical values to obtain appropriate ranges e.g. use a sketch graph of a quartic, (which must be the correct shape and cross the *x*-axis at the cvs) or a table or number line

A1cso for both of $-\dfrac{3}{2} < x < -1$, $\quad \dfrac{1}{3} < x < 3$

Notes for Question 2 Continued

Set notation acceptable i.e. $\left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{3},3\right)$ All brackets must be round; if square brackets appear anywhere then A0.

If both ranges correct, no working is needed for the last 2 marks, but any working shown must be correct.

Purely graphical methods are unacceptable as the question specifies "Use algebra…".

	Q2 – ALTERNATIVE 1:	
	$\frac{6x}{3-x}-\frac{1}{x+1}>0$	
	$\frac{6x(x+1)-(3-x)}{(3-x)(x+1)}>0$	M1
	$\frac{(3x-1)(2x+3)}{(3-x)(x+1)}>0$	M1dep
	Critical values $3, -1$	B1
	and $-\frac{3}{2}, \frac{1}{3}$	A1A1
	Use critical values to obtain both of $-\frac{3}{2}<x<-1$ $\quad \frac{1}{3}<x<3$	M1A1cso
		7 Marks

Leave blank

3. (a) Express $\dfrac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

(c) Evaluate $\displaystyle\sum_{r=10}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)

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Question Number	Scheme	Marks
3(a)	$\frac{2}{(r+1)(r+3)}=\frac{A}{r+1}+\frac{B}{r+3}$	
	$2=A(r+3)+B(r+1)$	
	$\frac{2}{(r+1)(r+3)}=\frac{1}{r+1}-\frac{1}{r+3}$	M1A1
	N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable.	**(2)**
(b)	$\sum\frac{2}{(r+1)(r+3)}=\sum\frac{1}{r+1}-\frac{1}{r+3}$	
	$=\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+...$	
	$+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$	M1A1ft
	$=\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}$	
	$=\frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{6(n+2)(n+3)}$	M1
	$=\frac{5n^2+25n+30-12n-30}{6(n+2)(n+3)}$	
	$=\frac{n(5n+13)}{6(n+2)(n+3)}$ *	A1
		(4)
(c)	$\sum_{10}^{100}=\sum_{1}^{100}-\sum_{1}^{9}$	M1
	$=\frac{100(500+13)}{6\times102\times103}-\frac{9\times58}{6\times11\times12}=\frac{1425}{1751}-\frac{29}{44}=0.81382\ldots-0.65909\ldots$	
	= 0.1547... = 0.155	A1
		(2)
		8 Marks

Notes for Question 3

Question 3a

M1 for attempting the PFs - any valid method

A1 for correct PFs $\dfrac{2}{(r+1)(r+3)}=\dfrac{1}{r+1}-\dfrac{1}{r+3}$

N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable. Award M1A1 if correct, M0A0 otherwise.

Question 3b

If all work in *r* instead of *n*, penalise last A mark only.

M1 for using **their** PFs to list at least 3 terms at the start and 2 terms at the end so the cancelling can be seen. Must start at $r=1$ and end at $r=n$

A1ft for correct terms follow through their PFs

M1 for picking out the (4) remaining terms and attempting to form a single fraction (unsimplified numerator with at least 2 terms correct)

A1cso for $\dfrac{n(5n+13)}{6(n+2)(n+3)}$ * (Check all steps in the working are correct - in particular 3rd line from end in the mark scheme.)

NB: If final answer reached correctly from $\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{n+2}-\dfrac{1}{n+3}$ (i.e. working shown from this point onwards) give 4/4 (even without individual terms listed)

Notes for Question 3 Continued
Question 3c M1 for attempting $\sum_{1}^{100} - \sum_{1}^{9}$ using the result from (b) (with numbers substituted) Use of $\sum_{1}^{100} - \sum_{1}^{10}$ scores M0 A1cso for sum = 0.155

Leave blank

4. Given that

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = 0$$

(a) find $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y.

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 2$ at $x = 0$

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(5)

Question Number	Scheme	Marks
4(a)	$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5y=0$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{-5\frac{\mathrm{d}y}{\mathrm{d}x}-3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}{y}$	A2,1,0 **(4)**
	Q4a – ALTERNATIVE 1:	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-5y-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{y}=-5-\frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{2}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$	M1M1 A2,1,0
(b)	When $x=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2$ and $y=2$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}(-10-4)=-7$	M1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{-10-3\times2\times-7}{2}=16$	A1
	$y=2+2x-\frac{7}{2(!)}x^2+\frac{16}{3!}x^3+\ldots$	M1
	$y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$	A1 **(5)**
		9 Marks

Question Number	Scheme	Marks
	Alternative: $y = 2+2x+ax^2+bx^3$	M1
	$\left(2+2x+ax^2+bx^3\right)\left(2a+6bx\right)+\left(2+2ax+3bx^2...\right)^2$ $+5\left(2+2x+ax^2+bx^3\right)=0$	M1
	Coeffs x^0 : $4a+4+10=0 \quad a=-\frac{7}{2}$	A1
	Coeffs x: $4a+12b+8a+10=0 \Rightarrow b=\frac{8}{3}$	A1
	$y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$	A1

NOTES

Accept the dash notation in this question

Question 4a

M1 for using the product rule to differentiate $y\frac{d^2y}{dx^2}$.

M1 for differentiating 5*y* and using the product rule or chain rule to differentiate $\left(\frac{dy}{dx}\right)^2$

A2,1,0 for $\frac{d^3y}{dx^3}=\frac{-5\frac{dy}{dx}-3\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}}{y}$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error. If there are two sign errors and no other error then give A1A0.

Do NOT deduct if the two $\frac{d^2y}{dx^2}$ terms are shown separately.

Alternative to Q4a

Can be re-arranged first and then differentiated.

M1M1 for differentiating, product and chain rule both needed (or quotient rule as an alternative to product rule)

A2,1,0 for $\frac{d^3y}{dx^3}=\frac{1}{y^2}\left(\frac{dy}{dx}\right)^3-\frac{2}{y}\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error

Notes for Question 4 Continued

Question 4b

M1 for substituting $\frac{dy}{dx} = 2$ and $y = 2$ in **the equation** to obtain a numerical value for $\frac{d^2y}{dx^2}$

A1 for $\frac{d^2y}{dx^2} = -7$

A1 for obtaining the correct value, 16, for $\frac{d^3y}{dx^3}$

M1 for using the series $y = f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$ (2! or 2, 3! or 6) (The general series may be shown explicitly or implied by their substitution)

A1 for $y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$ oe Must have y = .. and be in ascending powers of

Alternative to Q4b

M1 for setting $y = 2 + 2x + ax^2 + bx^3$

M1 for $(2 + 2x + ax^2 + bx^3)(2a + 6bx) + (2 + 2ax + 3bx^2 \ldots)^2 + 5(2 + 2x + ax^2 + bx^3) = 0$

A1 for equating constant terms to get $a = -\frac{7}{2}$

A1 for equating coeffs of x^2 to get $b = \frac{8}{3}$

A1 for $y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$

Leave blank

5. (a) Find, in the form $y = \mathrm{f}(x)$, the general solution of the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\tan x = \sin 2x, \qquad 0 < x < \frac{\pi}{2}$$

(6)

Given that $y = 2$ at $x = \frac{\pi}{3}$

(b) find the value of y at $x = \frac{\pi}{6}$, giving your answer in the form $a + k\ln b$, where a and b are integers and k is rational.

(4)

Question Number	Scheme	Marks
5(a)	I.F. $= e^{\int 2\tan x\,dx} = e^{2\ln\sec x} = \sec^2 x$	M1A1
	$y\sec^2 x = \int \sec^2 x \sin 2x\,dx$	M1
	$y\sec^2 x = \int \frac{2\sin x\cos x}{\cos^2 x}dx = 2\int \tan x\,dx$	
	$y\sec^2 x = 2\ln\sec x\ (+c)$	M1depA1
	$y = \frac{2\ln\sec x + c}{\sec^2 x}$	A1ft
		(6)
(b)	$y = 2,\ x = \frac{\pi}{3}$	
	$2 = \frac{2\ln\sec\left(\frac{\pi}{3}\right) + c}{\sec^2\left(\frac{\pi}{3}\right)}$	
	$2 = \frac{2\ln(2) + c}{4}$	
	$c = 8 - 2\ln 2$	M1A1
	$x = \frac{\pi}{6} \quad y = \frac{2\ln\sec\left(\frac{\pi}{6}\right) + 8 - 2\ln 2}{\sec^2\left(\frac{\pi}{6}\right)}$	
	$y = \frac{2\ln\frac{2}{\sqrt{3}} + 8 - 2\ln 2}{\frac{4}{3}}$	M1
	$y = \frac{3}{4}\left(8 + 2\ln\frac{1}{\sqrt{3}}\right) = 6 + \frac{3}{2}\ln\frac{1}{\sqrt{3}} = 6 - \frac{3}{4}\ln 3$	A1
		(4)
		10 Marks

Question Number	Scheme	Marks
	Alternative: c may not appear explicitly:	
	$y\sec^2\frac{\pi}{6}-2\sec^2\frac{\pi}{3}=2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$	M1A1
	$\frac{4}{3}y-8=2\ln\frac{1}{\sqrt{3}}$	
	$y=\frac{3}{4}\left(8+2\ln\frac{1}{\sqrt{3}}\right)=6+\frac{3}{2}\ln\frac{1}{\sqrt{3}}=6-\frac{3}{4}\ln 3$	M1A1

NOTES

Question 5a

M1 for the $e^{\int 2\tan x\,dx}$ or $e^{\int \tan x\,dx}$ and attempting the integration - $e^{(2)\ln\sec x}$ should be seen if final result is not $\sec^2 x$

A1 for IF $=\sec^2 x$

M1 for multiplying the equation by **their** IF and attempting to integrate the lhs

M1dep for attempting the integration of the rhs $\sin 2x=2\sin x\cos x$ and $\sec x=\frac{1}{\cos x}$ needed. Dependent on the second M mark

A1cao for all integration correct ie $y\sec^2 x=2\ln\sec x\ (+c)$ constant not needed

A1ft for re-writing **their** answer in the form $y=...$ Accept any equivalent form but the constant must be present. eg $y=\frac{\ln\left(A\sec^2 x\right)}{\sec^2 x}$, $y=\cos^2 x\left[\ln\left(\sec^2 x\right)+c\right]$

Notes for Question 5 Continued

Question 5b

M1 for using the given values $y = 2,\ x = \frac{\pi}{3}$ in **their** general solution to obtain a value for the constant of integration

A1 for eg $c = 8 - 2\ln 2$ or $A = \frac{1}{4}e^8$ (Check the constant is correct for their correct answer for (a)).

Answers to 3 significant figures acceptable here and can include $\cos\frac{\pi}{3}$ or $\sec\frac{\pi}{3}$

M1 for using **their** constant and $x = \frac{\pi}{6}$ in **their** general solution and attempting the simplification to the required form.

A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$

Alternative to 5b

M1 for finding the difference between $y\sec^2\frac{\pi}{6}$ and $2\sec^2\frac{\pi}{3}$ (or equivalent with their general solution)

A1 for $y\sec^2\frac{\pi}{6} - 2\sec^2\frac{\pi}{3} = 2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$

M1 for re-arranging to $y = \ldots$ and attempting the simplification to the required form

A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$

Leave blank

6. The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leqslant \theta < 2\pi$

(4)

Question Number	Scheme	Marks
6(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$	
	$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$	
	$= 2\cos n\theta$ *	M1A1
		(2)
(b)	$\left(z + z^{-1}\right)^5 = 32\cos^5\theta$	B1
	$\left(z + z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1A1
	$32\cos^5\theta = \left(z^5 + z^{-5}\right) + 5\left(z^3 + z^{-3}\right) + 10\left(z + z^{-1}\right)$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1
		(5)
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1
		(4)
		11 Marks

Notes for Question 6

Question 6a

M1 for using de Moivre's theorem to show that either $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ or $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$

A1 for completing to the given result $z^n + z^n = 2\cos n\theta$ *

Question 6b

B1 for using the result in (a) to obtain $\left(z + z^{-1}\right)^5 = 32\cos^5\theta$ Need not be shown explicitly.

M1 for attempting to expand $\left(z + z^{-1}\right)^5$ by binomial, Pascal's triangle or multiplying out the brackets. If ${}^nC_r$ is used do not award marks until changed to numbers

A1 for a correct expansion $\left(z + z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$

M1 for replacing $\left(z^5 + z^{-5}\right), \left(z^3 + z^{-3}\right), \left(z + z^{-1}\right)$ with $2\cos 5\theta,\ 2\cos 3\theta,\ 2\cos\theta$ and equating their revised expression to their result for $\left(z + z^{-1}\right)^5 = 32\cos^5\theta$

A1cso for $\cos^5\theta = \dfrac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *

Question 6c

M1 for attempting re-arrange the equation with one side matching the bracket in the result in (b) Question states "hence", so no other method is allowed.

A1 for using the result in (b) to obtain $16\cos^5\theta = -2\cos\theta$ oe

B1 for stating that there is no solution for $8\cos^4\theta + 1 = 0$ oe eg $8\cos^4\theta + 1 \neq 0$ $8\cos^4\theta + 1 > 0$ or "ignore" but $\cos\theta = \sqrt[4]{-\dfrac{1}{8}}$ without comment gets B0

A1 for $\theta = \dfrac{\pi}{2}$ **and** $\dfrac{3\pi}{2}$ and no more in the range. **Must** be in radians, can be in decimals (1.57..., 4.71... 3 sf or better)

Leave blank

7. (a) Find the value of λ for which $\lambda t^2 e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \qquad t \geqslant 0$$

(5)

(b) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\frac{dy}{dt} = 4$

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)

Question Number	Scheme	Marks
7(a)	$y = \lambda t^2 e^{3t}$	
	$\dfrac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$	M1A1
	$\dfrac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$	A1
	$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$	M1dep
	$\lambda = 3$	A1cso
		(5)
	NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen.	
(b)	$m^2 - 6m + 9 = 0$	
	$(m-3)^2 = 0$	
	C.F. $(y =)\ (A + Bt)e^{3t}$	M1A1
	G.S. $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$	A1ft
		(3)
(c)	$t = 0\ \ y = 5\ \ \Rightarrow A = 5$	B1
	$\dfrac{dy}{dt} = Be^{3t} + 3(A + Bt)e^{3t} + 6te^{3t} + 9t^2 e^{3t}$	M1
	$\dfrac{dy}{dt} = 4 \qquad 4 = B + 15$	M1dep
	$B = -11$	A1
	Solution: $y = (5 - 11t)e^{3t} + 3t^2 e^{3t}$	A1ft
		(5)
		13 Marks

Notes for Question 7

Question 7a

M1 for differentiating $y=\lambda t^2 e^{3t}$ wrt t. Product rule must be used.

A1 for correct differentiation ie $\frac{dy}{dt}=2\lambda t e^{3t}+3\lambda t^2 e^{3t}$

A1 for a correct second differential $\frac{d^2y}{dt^2}=2\lambda e^{3t}+6\lambda t e^{3t}+6\lambda t e^{3t}+9\lambda t^2 e^{3t}$

M1dep for substituting their differentials in the equation and obtaining a numerical value for λ
<u>Dependent on the first M mark.</u>

A1cso for $\lambda=3$ (no incorrect working seen)

NB. Candidates who give $\lambda=3$ without all this working get 5/5 provided no erroneous working is seen. Candidates who attempt the differentiation should be marked on that. If they then go straight to $\lambda=3$ without showing the substitution, give M1A1 if differentiation correct and M1A0 otherwise, as the solution is incorrect. If $\lambda\neq 3$ then the M mark is only available if the substitution is shown.

Question 7b

M1 for solving the 3 term quadratic auxiliary equation to obtain a value or values for *m* (usual rules for solving a quadratic equation)

A1 for the CF $(y=)\ (A+Bt)e^{3t}$

A1ft for using **their** CF and **their numerical** value of λ in the particular integral to obtain the general solution $y=(A+Bt)e^{3t}+3t^2e^{3t}$ Must have *y* = ... and rhs must be a function of *t*.

Question 7c

B1 for deducing that $A=5$

M1 for differentiating **their** GS to obtain $\frac{dy}{dt}=...$ The product rule must be used.

M1dep for using $\frac{dy}{dt}=4$ and **their** value for *A* in **their** $\frac{dy}{dt}$ to obtain an equation for *B* Dependent on the previous M mark (of (c))

A1cao and cso for $B=-11$

A1ft for using **their** numerical values *A* and *B* in **their** GS from (b) to obtain the particular solution. Must have *y* = ... and rhs must be a function of *t*.

Leave blank

8.

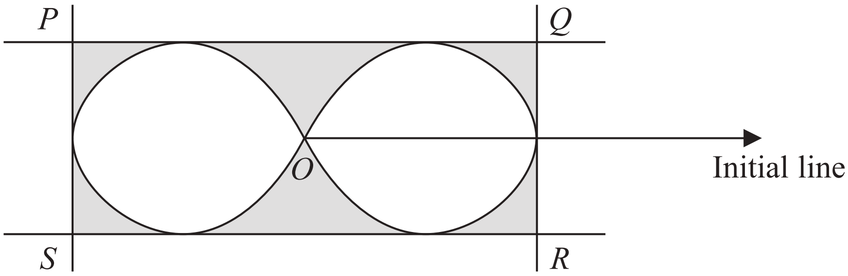

Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \qquad \text{where } -\frac{\pi}{4} < \theta \leqslant \frac{\pi}{4}, \ \frac{3\pi}{4} < \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, SR, PS and QR are tangents to C, where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

(a) Find the total area enclosed by the curve C, shown unshaded inside the rectangle in Figure 1.

(4)

(b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1.

(9)

Question Number	Scheme	Marks
8 (a)	$A=(4\times)\int_0^{\frac{\pi}{4}}\frac{9}{2}\cos 2\theta\,\mathrm{d}\theta$	M1A1(limits for A mark only)
	$=18\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$	M1
	$9\left[\sin\frac{\pi}{2}-0\right]=9$	A1
		(4)
(b)	$r=3(\cos 2\theta)^{\frac{1}{2}}$	
	$r\sin\theta=3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	M1
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta+3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$	M1depA1
	At max/min: $\frac{-3\sin 2\theta\sin\theta}{(\cos 2\theta)^{\frac{1}{2}}}+3(\cos 2\theta)^{\frac{1}{2}}\cos\theta=0$	M1
	$\sin 2\theta\sin\theta=\cos 2\theta\cos\theta$	
	$2\sin^2\theta\cos\theta=(1-2\sin^2\theta)\cos\theta$	
	$\cos\theta(1-4\sin^2\theta)=0$	
	$(\cos\theta=0)\quad \sin^2\theta=\frac{1}{4}$	
	$\sin\theta=\pm\frac{1}{2}\qquad \theta=\pm\frac{\pi}{6}$	M1A1
	$r\sin\frac{\pi}{6}=3\left(\cos\frac{\pi}{3}\right)^{\frac{1}{2}}\times\frac{1}{2}=\frac{3\sqrt{2}}{4}$	B1
	$\therefore$ length $PS=\frac{3\sqrt{2}}{2}$, (length PQ = 6)	

Question Number	Scheme	Marks
	Shaded area = $6\times\frac{3\sqrt{2}}{2}-9, =9\sqrt{2}-9$ oe	M1,A1 **(9)** **13 Marks**

NOTES

Question 8a

M1 for $A=\frac{1}{2}\int r^2\,\mathrm{d}\theta=\frac{1}{2}\int \alpha\cos 2\theta\,\mathrm{d}\theta$ with $\alpha=3$ or 9 4 or 2 and limits not needed for this mark - ignore any shown.

A1 for $A=(4\times)\int_0^{\frac{\pi}{4}}\frac{9}{2}\cos 2\theta\,\mathrm{d}\theta$ Correct limits $\left(0,\frac{\pi}{4}\right)$ with multiple 4 or $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ with multiple 2 needed. 4 or 2 may be omitted here, provided it is used later.

M1 for the integration $\cos 2\theta$ to become $\pm\left(\frac{1}{2}\right)\sin 2\theta$ Give M0 for $\pm 2\sin 2\theta$. Limits and 4 or 2 not needed

A1cso for using the limits and 4 or 2 as appropriate to obtain 9

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Notes for Question 8 Continued

Question 8b

M1 for $r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$ or $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$ 3 or 9 allowed

M1dep for differentiating the rhs of the above wrt θ. Product and chain rule must be used.

A1 for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$ or correct differentiation of $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$

M1 for equating their expression for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)$ to 0

M1dep for solving the resulting equation to $\sin k\theta = ...$ or $\cos k\theta = ...$ incuding the use of the appropriate trig formulae (must be correct formulae)

A1 for $\sin\theta = \frac{1}{2}$ or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = (\pm)\frac{\pi}{6}$ oe ignore extra answers

B1 for the length of $\frac{1}{2}PS = \frac{3\sqrt{2}}{4}$ (1.0606...) or of PS May not be shown explicitly. Give this mark if the correct area of the rectangle is shown. Length of PQ is not needed for this mark.

M1 for attempting the shaded area by their $PS \times 6 -$ **their** answer to (a). There must be evidence of PS being obtained using their θ

A1 for $9\sqrt{2} - 9$ oe 3.7279....or awrt 3.73

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