Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1 R	Signature	

Paper Reference(s)

6668/01R

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Express $\frac{2}{4r^2 - 1}$ in partial fractions.

(2)

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$$

(3)

Question Number	Scheme	Marks
1.		
(a)	$\frac{2}{4r^2-1}=\frac{A}{2r+1}+\frac{B}{2r-1}$	
	$2=A(2r-1)+B(2r+1) \Rightarrow A=-1,\ \ B=1$	
	$\frac{2}{4r^2-1}=\frac{1}{2r-1}-\frac{1}{2r+1}$	M1A1 (2)
(b)	$(2)\sum_{r=1}^{n}\frac{1}{4r^2-1}=\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$	
	$=1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+...+\frac{1}{2n-1}-\frac{1}{2n+1}=1-\frac{1}{2n+1}$	M1A1ft
	$=\frac{2n+1-1}{2n+1}$	
	$\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{n}{2n+1}$ *	A1 (3)
		[5]
Notes for Question 1		
(a)	M1 complete method for finding PFs A1 both PFs correct Award M1A1 for **both** PFs seen correct w/o working. M0A0 otherwise	
(b)	M1 showing fractions with their PFs. Min 2 at start and 1 at end. Must start at 1 and end at *n*. Required sum may be used or 2 x sum A1ft Identify 2 non-cancelling fractions, follow through their PFs - sum or 2 x sum A1cso correct final answer	

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2. Using algebra, find the set of values of x for which

$$3x - 5 < \frac{2}{x}$$

(5)

Question Number	Scheme	Marks
2	$3x-5-\frac{2}{x}=0$ (or <) or mult through by x^2	
	$\frac{3x^2-5x-2}{x}=0$ (or <)	
	$\frac{(3x+1)(x-2)}{x}=0$ or $x(3x+1)(x-2)=0$	M1
	CVs $x=-\frac{1}{3}, 2$	A1
	$x=0$	B1
	$x<-\frac{1}{3},\ 0<x<2$ or in set language (with curved brackets for A1)	M1A1 (5)
	Special case If $\leqslant$ used deduct final mark only.	**[5]**
	Notes for Question 2	
	M1 obtaining two non-zero cvs by any valid method (not calculator) A1 non-zero cvs correct B1 $x=0$ M1 deducing one appropriate range from their cvs A1 both ranges correct First 3 marks – award with inequalities or = M1A0 if strict inequality **not** used **ALT: If multiplied through by *x*:**	
	$x>0$: $3x^2-5x<2$ $(3x+1)(x-2)<0$	
	cvs $x=-\frac{1}{3},\ x=2$	M1(solve quad)
	$\therefore\ 0,<x<2$	B1,A1
	$x<0$ $3x^2-5x-2>0$	M1
	cvs $x=-\frac{1}{3},\ x=2$	
	$\therefore\ x<-\frac{1}{3}$	A1

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3. (a) Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y\tan x = e^{4x}\cos^2 x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

giving your answer in the form $y = f(x)$.

(6)

(b) Find the particular solution for which $y = 1$ at $x = 0$

(2)

Question Number	Scheme	Marks
3. **(a)**	$\frac{dy}{dx}+2y\tan x=e^{4x}\cos^2 x$	
	$e^{2\int \tan x\,dx}=e^{2\ln\sec x}=\sec^2 x$ or $\frac{1}{\cos^2 x}$	M1A1
	$\sec^2 x\frac{dy}{dx}+2y\tan x\sec^2 x=e^{4x}\cos^2 x\sec^2 x$	dM1
	$\frac{d}{dx}\left(y\sec^2 x\right)=e^{4x}$	B1ft($y\sec^2 x$)
	$y\sec^2 x=\frac{1}{4}e^{4x}\quad(+c)$	M1
	$y=\left(\frac{1}{4}e^{4x}+c\right)\cos^2 x$ oe	A1
		(6)
(b)	$y=1,\quad x=0\quad 1=\left(\frac{1}{4}+c\right)$	M1
	$c=\frac{3}{4}$	
	$y=\frac{1}{4}\left(e^{4x}+3\right)\cos^2 x$ oe	A1
		(2) **[8]**

Notes for Question 3

(a)	M1 attempting the integrating factor, including integration of (2)tan x $\ln\cos$ or $\ln\sec$ seen A1 correct integrating factor $\sec^2 x$ or $\frac{1}{\cos^2 x}$ M1 multiplying the equation by the integrating factor – may be implied by the next line. B1ft $y\times$ their IF M1 attempting a complete integration of rhs Must include ke^{4x} but $4e^{4x}$ would imply differentiation. Constant not needed (Incorrect IF may lead to integration by parts, so integration must be complete) A1 correct solution in form $y=...$ constant must be included	
(b)	M1 using given initial conditions to obtain a value for c A1 fully correct final answer May be in the form $y\sec^2 x=...$ or $4y\sec^2 x=...$	

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4.

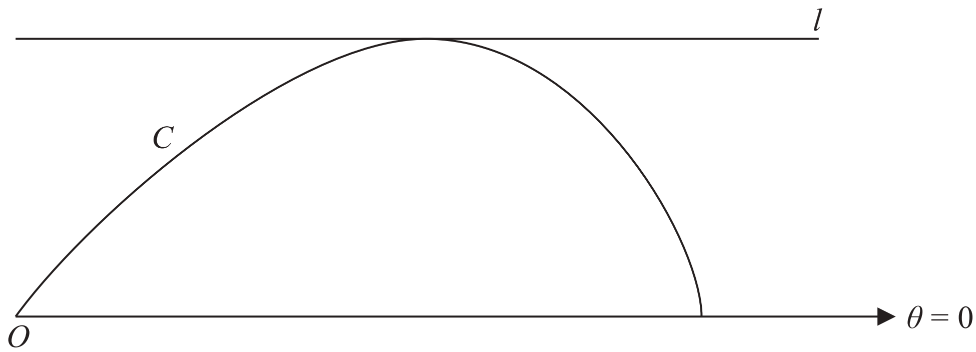

Figure 1

Figure 1 shows the curve C with polar equation

$$r = 2\cos 2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{4}$$

The line l is parallel to the initial line and is a tangent to C.

Find an equation of l, giving your answer in the form $r = \mathrm{f}(\theta)$.

(9)

Question Number	Scheme	Marks
4.	$(y=)r\sin\theta = 2\cos 2\theta\sin\theta$	M1
	$\frac{dy}{d\theta} = -4\sin 2\theta\sin\theta + 2\cos 2\theta\cos\theta$	M1A1
	$2\sin 2\theta\sin\theta - \cos 2\theta\cos\theta = 0$	
	$4\sin^2\cos\theta - (1-2\sin^2\theta)\cos\theta = 0$	dM1
	$(6\sin^2\theta - 1)\cos\theta = 0$	
	$(\cos\theta = 0 \quad \text{no solutions in range})$	
	$\therefore \sin\theta = \frac{1}{\sqrt{6}}$	ddM1A1
	ALT for last 3 marks above: $\cos 2\theta\cos\theta = 2\sin 2\theta\sin\theta \Rightarrow \tan 2\theta\tan\theta = 1/2$	
	$\frac{2\tan^2\theta}{1-\tan\theta} = \frac{1}{2}$	dM1(double angle formula)
	$5\tan^2\theta = 1 \quad \tan\theta = 1/\sqrt{5}$	ddM1A1
	$(\sin\theta = 1/\sqrt{6} \quad \cos\theta = \sqrt{5/6}$	
	$r\sin\theta = 2\cos 2\theta\sin\theta$	
	$\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \cos 2\theta = 1-2\sin^2\theta = 1-2\times\frac{1}{6} = \frac{2}{3}$	M1
	Eqn. l: $r\sin\theta = 2\times\frac{2}{3}\times\frac{1}{\sqrt{6}} = \frac{4}{3\sqrt{6}}$	M1
	$r = \frac{2\sqrt{6}}{9}\text{cosec}\,\theta$ oe $(0<\theta<\pi)$ Must be seen in exact form	A1
		[9]
	Notes for Question 4	
	M1 Using $y = r\sin\theta = 2\cos 2\theta\sin\theta$ M1 differentiate $r\sin\theta$ or $r\cos\theta$ using product rule or $\cos 2\theta = 1-2\sin^2\theta$ and chain rule A1 correct differentiation of $r\sin\theta$ dM1 equate their derivative to 0 and use $\cos 2\theta = 1-2\sin^2\theta$ if not used prior to differentiation, or an appropriate double angle formula for their derivative. Depends on second M mark ddM1 solve the resulting equation. Depends on second and third M mark A1 correct value for $\sin\theta$ or $\tan\theta$ or $\cos\theta$ depending on the equation solved M1 use their value for a trig function to obtain an exact value for $\cos 2\theta$ and $\sin\theta$ if needed now. May be implied by the next stage. M1 use their values for $\sin\theta$ and $\cos 2\theta$ in $r\sin\theta = 2\cos 2\theta\sin\theta$ **NB:** These two M marks require $0 \le \sin\theta \le 1/\sqrt{2},\quad 1/\sqrt{2} \le \cos\theta \le 1,\quad 0 \le \tan\theta \le 1$ A1 correct equation in form $r = ..(0<\theta<\pi \quad \text{not needed})$	

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5.

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y = 0$$

(a) Find an expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ in terms of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and y.

(4)

Given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.5$ at $x = 0$,

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(5)

Question Number	Scheme	Marks
5.		
(a)	$\frac{d^2y}{dx^2} = -\frac{2}{y}\left(\frac{dy}{dx}\right)^2 - 2$	
	$\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$ seen	B1
	$\frac{d^3y}{dx^3} = -\frac{4}{y}\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + \frac{2}{y^2}\left(\frac{dy}{dx}\right)^3$	M1 (÷ and diff) A1A1
	Alt: $\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$	B1
	$\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right) + y\frac{d^3y}{dx^3} + 4\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + 2\frac{dy}{dx} = 0$	M1
	$\frac{d^3y}{dx^3} = \frac{1}{y}\left(-5\frac{d^2y}{dx^2} - 2\right)\frac{dy}{dx}$	A1 A1
		(4)
(b)	At $x = 0$ $\quad \frac{d^2y}{dx^2} = \frac{1}{2}\left(-2\times\left(\frac{1}{2}\right)^2 - 4\right) = -\frac{9}{4}$ $\quad$ (or -2.25)	M1A1
	$\frac{d^3y}{dx^3} = \frac{1}{2}\left(-5\times\frac{1}{2}\times-\frac{9}{4} - 2\times\frac{1}{2}\right) = \frac{37}{16}$ $\quad$ (or 2.325)	A1
	$y = 2 + \frac{1}{2}x + \left(-\frac{9}{4}\right)\frac{x^2}{2!} + \left(\frac{37}{16}\right)\frac{x^3}{3!} + \ldots$	M1(2! or 2, 3! or 6)
	$y = 2 + \frac{1}{2}x - \frac{9}{8}x^2 + \frac{37}{96}x^3 + \ldots$	A1
		(5)
	0.5 $\quad$ 1.125 $\quad$ 0.3854 $\quad$ 3 sf or better	**[9]**

Notes for Question 5		
(a)	B1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ seen in the differentiation M1 divide equation by *y* and differentiate wrt *x* chain and product/quotient rules needed A1A1 -1 for each error. Ignore any simplification following the differentiation and obtaining $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=...$ ALT: B1 as above M1 differentiating before dividing A1A1 rearrange to a correct expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$, -1 each error	
(b)	M1 using values for *x* and $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain a value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ A1 correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ A1 correct value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ M1 Taylor's series formed using their values for the differentials, accept 2! or 2 and 3! or 6. A1 correct series, must start *y* = (or end =*y*)	

Leave blank

6. The transformation T maps points from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$.

The transformation T is given by

$$w = \frac{z}{\mathrm{i}z + 1}, \quad z \neq \mathrm{i}$$

The transformation T maps the line l in the z-plane onto the line with equation $v = -1$ in the w-plane.

(a) Find a cartesian equation of l in terms of x and y.

(5)

The transformation T maps the line with equation $y = \frac{1}{2}$ in the z-plane onto the curve C in the w-plane.

(b) (i) Show that C is a circle with centre the origin.

(ii) Write down a cartesian equation of C in terms of u and v.

(6)

Question Number	Scheme	Marks
6.		
(a)	$w=(u-\mathrm{i}=)\dfrac{x+\mathrm{i}y}{\mathrm{i}x-y+1}$	M1
	$w=(u-\mathrm{i}=)\dfrac{x+\mathrm{i}y}{\mathrm{i}x-y+1}\times\dfrac{(1-y-\mathrm{i}x)}{(1-y-\mathrm{i}x)}$	M1
	$w=(u-\mathrm{i}=)\dfrac{x-xy+xy+\mathrm{i}\left(y-y^2-x^2\right)}{(1-y)^2+x^2}$	A1
	$-1=\dfrac{y-y^2-x^2}{(1-y)^2+x^2}$	M1
	$-1+2y-y^2-x^2=y-y^2-x^2$	
	$y=1$	A1 (5)
	Alternative 1: $z=\dfrac{w}{1-w\mathrm{i}}$	M1
	$x+\mathrm{i}y=\dfrac{u-\mathrm{i}}{1-(u-\mathrm{i})\mathrm{i}}$	M1
	$=\dfrac{u-\mathrm{i}}{-u\mathrm{i}}$	A1
	$=\mathrm{i}+\dfrac{1}{u}$	M1
	$y=1$	A1 (5)
	Alternative 2:	
	$\lvert w+2\mathrm{i}\rvert=\lvert w\rvert$	M1
	$\left\lvert\dfrac{z}{\mathrm{i}z+1}+2\mathrm{i}\right\rvert=\left\lvert\dfrac{z}{\mathrm{i}z+1}\right\rvert$	M1
	$\left\lvert\dfrac{z-2z+2\mathrm{i}}{\mathrm{i}z+1}\right\rvert=\left\lvert\dfrac{z}{\mathrm{i}z+1}\right\rvert$	A1
	$\lvert z-2\mathrm{i}\rvert=\lvert z\rvert$	M1
	$\Rightarrow y=1$	A1 (5)

Question Number	Scheme	Marks
(b)	$u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}\left(x+\frac{1}{2}\mathrm{i}\right)+1}=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}}$	M1
	$u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}}\times\dfrac{\frac{1}{2}-x\mathrm{i}}{\frac{1}{2}-x\mathrm{i}}$	M1
	$u+\mathrm{i}v=\dfrac{x+\mathrm{i}\left(\frac{1}{4}-x^2\right)}{\frac{1}{4}+x^2}$	A1
	$u=\dfrac{x}{\frac{1}{4}+x^2}\qquad v=\dfrac{\frac{1}{4}-x^2}{\frac{1}{4}+x^2}$	M1
	$u^2+v^2=\dfrac{x^2+\left(\frac{1}{4}-x^2\right)^2}{\left(\frac{1}{4}+x^2\right)^2}=\dfrac{\frac{1}{16}-\frac{1}{2}x^2+x^2+x^4}{\left(\frac{1}{4}+x^2\right)^2}=1$	M1
	$u^2+v^2=1$, Centre O *	M1,A1 (6)
	Alternative 1:	
	$w=u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}},\ \left(=r\mathrm{e}^{\mathrm{i}\theta}\right)$ or $\dfrac{x+\mathrm{i}y}{\mathrm{i}x+y}$	M1
	$\lvert w\rvert=\dfrac{\sqrt{x^2+\frac{1}{4}}}{\sqrt{x^2+\frac{1}{4}}},\ =1$	M1M1,A1
	$u^2+v^2=1\ \therefore$	M1
	Centre O *	A1
		[11]

Question Number	Scheme	Marks
	Alternative 2:	
	$\lvert z-\mathrm{i}\rvert = \lvert z\rvert$	M1
	$\left\lvert \frac{w}{1-\mathrm{i}w} - \frac{\mathrm{i}(1-\mathrm{i}w)}{1-\mathrm{i}w} \right\rvert = \left\lvert \frac{w}{1-\mathrm{i}w} \right\rvert$	M1
	$\lvert w-\mathrm{i}+\mathrm{i}^2 w\rvert = \lvert w\rvert$	A1
	$\Rightarrow \lvert w\rvert = 1$	M1
	$u^2+v^2=1$	M1
	Centre O	A1 (6)
	Alternative 3:	
	$w=\frac{z}{\mathrm{i}z+1}$	
	$z=\frac{w}{1-\mathrm{i}w}=\frac{u+\mathrm{i}v}{1-(u+\mathrm{i}v)\mathrm{i}}$	M1
	Realise the denominator Correct result	M1 A1
	Set imaginary part $=\frac{1}{2}$ and simplify expression	M1
	$u^2+v^2=1$	M1
	Centre O	A1 (6)

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7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(b) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary.

(5)

(c) Use the identity given in (a) to find

$$\int_0^{\frac{\pi}{4}} (4\sin^5\theta - 5\sin^3\theta)\,d\theta$$

expressing your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers.

(4)

Question Number	Scheme	Marks
7. (a)	$(\cos\theta+\mathrm{i}\sin\theta)^5=\cos5\theta+\mathrm{i}\sin5\theta$	B1
	$=\cos^5\theta+5\cos^4(\mathrm{i}\sin\theta)+\frac{5\times4}{2!}\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+\frac{5\times4\times3}{3!}\cos^2\theta(\mathrm{i}\sin\theta)^3+\frac{5\times4\times3\times2}{4!}\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$	M1
	$=\cos^5\theta+5\mathrm{i}\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta$ $-10\mathrm{i}\cos^2\theta\sin^3\theta+5\cos\theta\sin^4\theta+\mathrm{i}\sin^5\theta$	A1
	$\sin5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$	
	$=5(1-\sin^2\theta)^2\sin\theta-10(1-\sin^2\theta)\sin^3\theta+\sin^5\theta$	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1 (5)
(b)	Let $x=\sin\theta$ $16x^5-20x^3+5x=-\frac{1}{2}\Rightarrow\sin5\theta=-\frac{1}{2}$	M1
	$5\theta=210,\ 330,\ 570,\ 690,\ 930,\ 1050,\ 1290$ (or in radians) Or $210,\ 570,\ 930,\ 1290,\ 1650$	A1, A1
	$\theta=42,\ 66,\ (114),\ (138),\ 186,\ 210,\ 258$ (or in radians) Or $42,\ 114,\ 186,\ 258,\ 330$	dM1(at least 2 values)
	$\sin\theta=0.669,\ 0.914,\ -0.105,\ -0.5,\ -0.978$	A1 (5)
(c)	$\int_0^{\frac{\pi}{4}}(4\sin^5\theta-5\sin^3\theta)\,\mathrm{d}\theta=\frac{1}{4}\int_0^{\frac{\pi}{4}}(\sin5\theta-5\sin\theta)\,\mathrm{d}\theta$	M1
	$=\frac{1}{4}\left[-\frac{1}{5}\cos5\theta+5\cos\theta\right]_0^{\frac{\pi}{4}}$	A1
	$\frac{1}{4}\left[-\frac{1}{5}\cos\frac{5\pi}{4}+5\cos\frac{\pi}{4}-\left(-\frac{1}{5}+5\right)\right]$	
	$=\frac{1}{4}\left[\frac{1}{5}\times\frac{1}{\sqrt{2}}+\frac{5}{\sqrt{2}}-4\frac{4}{5}\right]$	M1
	$=\frac{13\sqrt{2}}{20}-\frac{6}{5}$	A1 (4)
		[14]

Notes for Question 7		
(a)	B1 applies de Moivre correctly M1 uses binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$ May only show imaginary parts - ignore errors in real part A1 simplifies coefficients to obtain a simplified result with all imaginary terms correct M1 equates imaginary parts and obtains an expression for $\sin 5\theta$ in terms of powers of $\sin\theta$ A1 cso correct result	
(b)	M1 uses substitution $x = \sin\theta$ deduces that $\sin 5\theta = \pm\frac{1}{2}$ A1A1 gives a set of results for 5θ - A1 for 3 useable results A1 for the remaining 2 useable results (no repeats in the set of 5) M1 at least 2 values for θ A1 for the 5 **different** values of x	
(c)	M1 uses previous work to change the integrand A1 correct result after integrating - limits can be ignored M1 substitute given limits and use numerical values for trig functions A1 final answer correct (oe provided in the given form)	

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8. (a) Show that the substitution $x = e^z$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x, \quad x > 0 \qquad \text{(I)}$$

into the equation

$$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z \qquad \text{(II)}$$

(7)

(b) Find the general solution of the differential equation (II).

(6)

(c) Hence obtain the general solution of the differential equation (I) giving your answer in the form $y = f(x)$.

(1)

Question Number	Scheme	Marks
8. **(a)**	$x = \mathrm{e}^z$	
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^z \dfrac{\mathrm{d}z}{\mathrm{d}y}$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-z} \dfrac{\mathrm{d}y}{\mathrm{d}z}$	A1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\mathrm{e}^{-z}\dfrac{\mathrm{d}z}{\mathrm{d}x}\times\dfrac{\mathrm{d}y}{\mathrm{d}z} + \mathrm{e}^{-z}\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}\times\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{1}{x^2}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}z} + \dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}\right)$	M1A1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y = 3\ln x$	
	$x^2\left(-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}z} + \dfrac{1}{x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}\right) + 2x\times\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}z} - 2y = 3z$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2} + \dfrac{\mathrm{d}y}{\mathrm{d}z} - 2y = 3z$	A1 (7)
	Alt: $z = \ln x$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}z}\times\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}z}$	M1A1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}z} + \dfrac{1}{x}\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}\times\dfrac{\mathrm{d}z}{\mathrm{d}x} = -\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}z} + \dfrac{1}{x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}$	M1A1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y = 3\ln x$	
	$x^2\left(-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}z} + \dfrac{1}{x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2}\right) + 2x\times\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}z} - 2y = 3z$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}z^2} + \dfrac{\mathrm{d}y}{\mathrm{d}z} - 2y = 3z$	A1 (7)

Question Number	Scheme	Marks
(b)	Aux eqn: $m^2+m-2=0$	
	$(m+2)(m-1)=0$	
	$m=-2,\ 1$	M1A1
	CF: $y=Ae^{-2z}+Be^{z}$	A1
	PI: Try $y=az+b$	
	$\frac{dy}{dz}=a \quad \frac{d^2y}{dz^2}=0$	
	$a-2(az+b)=3z$	
	$a=-\frac{3}{2},\quad b=-\frac{3}{4}$	M1
	Complete soln: $y=Ae^{-2z}+Be^{z}-\frac{3}{2}z-\frac{3}{4}$	A1A1 (6)
(c)	$y=Ax^{-2}+Bx-\frac{3}{2}\ln x-\frac{3}{4}$	B1 ft (1)
		[14]
	Notes for Question 8	
(a)	M1 differentiates $x=e^{z}$ wrt y; chain rule must be used A1 correct differentiation M1 differentiates again to obtain $\frac{d^2y}{dx^2}$ A1A1 one mark for each correct term M1 substitutes in the given equation A1cso obtains the required equation ALT: Works with $z=\ln x$; marks awarded as above	
(b)	M1 forms and solves the auxiliary equation A1 both values for *m* correct - may be implied by their CF A1 correct CF M1 tries a suitable expression for the PF and obtains values for the constants in the PF A1A1 shows the complete solution; one mark for each correct term in the PF	
(c)	B1ft reverses the substitution to obtain the solution in the form *y* = ... Follow through their complete solution from (b)	