Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Wednesday 3 June 2015 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Use algebra to find the set of values of x for which

$$x + 2 > \frac{12}{x + 3}$$

(6)

(b) Hence, or otherwise, find the set of values of x for which

$$x + 2 > \frac{12}{|x + 3|}$$

(1)

June 2015 Home
6668 FP2
Mark Scheme

Question Number	Scheme	Marks
1		
(a)	$(x+2)(x+3)^2-12(x+3)=0$ OR $\dfrac{(x+3)(x+2)-12}{(x+3)}>0$	M1
	$(x+3)(x^2+5x-6)=0 \qquad (x+3)(x+6)(x-1)=0$	
	CVs: $-3,\ -6,\ 1$	B1,A1,A1
	$-6<x<-3, \quad x>1$	dM1A1
	OR: $x\in(-6,-3)\cup(1,\infty)$	(6)
(b)	$x>1$	B1 (1)
		[7]

(a)

M1 Mult through by $(x+3)^2$ and collect on one side or use any other valid method (NOT calculator)

Eg work from $\dfrac{(x+3)(x+2)-12}{(x+3)}>0$

NB: Multiplying by $(x+3)$ is **not** a valid method unless the two cases $x>3$ and $x<3$ are considered separately or -3 stated to be a cv

B1 for -3 seen anywhere

A1A1 other cvs (A1A0 if only one correct)

dM1 obtaining inequalities using their critical values and no other numbers. Award if one correct inequality seen or any valid method eg sketch graph or number line seen

A1 correct inequalities and no extras. Use of … or „ scores A0. May be written in set notation.

No marks for candidates who draw a sketch graph and follow with the cvs without any algebra shown. **Those who use some algebra** after their graph may gain marks as earned (possibly all)

(b) B1 correct answer only shown. Allow $x \ldots 1$ if already penalised in (a)

Leave blank

2.

$$z = -2 + \left(2\sqrt{3}\right)\text{i}$$

(a) Find the modulus and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^6, simplifying your answer,

(2)

(c) find the values of w such that $w^4 = z^3$, giving your answers in the form $a + \text{i}b$ where $a, b \in \mathbb{R}$.

(4)

Question Number	Scheme	Marks
2 (a)	$\lvert z\rvert = 4$	B1
	$\arg z = \arctan\left(\frac{-2\sqrt{3}}{2}\right) = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3}$ or 120°	M1A1 (3)
(b)	$z^6 = \left(4\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)\right)^6 = 4^6\left(\cos 4\pi + \mathrm{i}\sin 4\pi\right)$ or $z^6 = \left(4\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}\right)^6$	M1
	$= 4096$ or 4^6 or 2^{12}	A1 cso (2)
	(a) and (b) can be marked together	
(c)	$z^{\frac{3}{4}} = 4^{\frac{3}{4}}\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)^{\frac{3}{4}} = 4^{\frac{3}{4}}\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$	
	$w = \mathrm{i}2\sqrt{2}$ oe or any other correct root	B1
	$4^{\frac{3}{4}}\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + \mathrm{i}\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)^{\frac{3}{4}}$	M1
	$(n = 0$ see above$)$	
	$n = 1$ $w = 2\sqrt{2}$ oe	
	$n = 2$ $w = -\mathrm{i}2\sqrt{2}$ oe	
	$n = 3$ $w = -2\sqrt{2}$ oe	A1A1 (4)
		[9]

(a) B1	Correct modulus seen **Must** be 4
M1	Attempt arg using arctan, nos either way up. Must include minus sign or other consideration of quadrant. (arg = $\frac{\pi}{3}$ scores M0)
A1	$\frac{2\pi}{3}$ or 120^{0} Correct answer only seen, award M1A1
(b) M1	apply de Moivre
A1cso	4096 or 4^6 Must have been obtained with the correct argument for z
(c) B1	For $w = \mathrm{i}2\sqrt{2}$ or any single correct root (0 or 0i may be included in all roots) in any Form including polar
M1	Applying de Moivre and use a correct method to attempt 2 or 3 further roots
A1A1	For the other roots (3 correct scores A1A1; 2 correct scores A1)
	Accept eg $2\sqrt{2}, \sqrt{8}$, 2.83, $64^{\frac{1}{4}}$, $4^{\frac{3}{4}}$, $4096^{\frac{1}{8}}$ Decimals must be 3 sf min.
ALT 1 for (c):	$z^3 = 64 = w^4 \Rightarrow w = (\pm)2\sqrt{2}$ ($\pm$ not needed) B1
	Use rotational symmetry to find other 2/3 roots M1
	Remaining roots as above A1A1
ALT 2:	$z^4 = 64$ $z^2 = \pm 8$
	$z = \pm 2\sqrt{2}$ $z = \pm\sqrt{-8} = \pm \mathrm{i}2\sqrt{2}$
	B1 any one correct, M1 attempt remaining 2/3 roots; A1A1 as above

Leave blank

3. Find, in the form $y = \mathrm{f}(x)$, the general solution of the differential equation

$$\tan x \frac{\mathrm{d}y}{\mathrm{d}x} + y = 3\cos 2x \tan x, \qquad 0 < x < \frac{\pi}{2}$$

(6)

Question Number	Scheme		Marks
3	$\frac{dy}{dx}+\frac{y}{\tan x}=3\cos 2x$		
	$\int \cot x\,dx=\ln\lvert\sin x\rvert$, IF $=\sin x$		M1
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$		
	$y\sin x=\int 3\cos 2x\sin x\,dx$		M1A1
	$y\sin x=\int 3\left(2\cos^2 x-1\right)\sin x\,dx$	$y\sin x=\frac{3}{2}\int\left(\sin 3x-\sin x\right)dx$	
	$y\sin x=3\left[-\frac{2}{3}\cos^3 x+\cos x\right](+c)$	$y\sin x=\frac{3}{2}\left[-\frac{1}{3}\cos 3x+\cos x\right](+c)$	dM1A1
	$y=\frac{3\cos x-2\cos^3 x+c'}{\sin x}$ oe	$y=\frac{-3\cos 3x+3\cos x+c'}{2\sin x}$	B1ft [6] (A1 on e-PEN)

M1 Divide by tan and attempt IF $e^{\int \cot x\,dx}$ or equivalent needed
M1 Multiply through by IF and integrate LHS
A1 correct so far
dM1 dep (on previous M mark) integrate RHS using double angle or factor formula

$$k\cos^2 x\sin x\to\pm\cos^3 x,\ k\sin^2 x\cos x\to k\sin^3 x,\ \cos 3x\to\pm\frac{1}{3}\sin 3x,\ \sin 3x\to\pm\frac{1}{3}\cos 3x$$

A1 All correct so far constant not needed
B1ft obtain answer in form $y=$ **...** any equivalent form Constant must be included and dealt with correctly. Award if correctly obtained from the previous line

Alternatives for integrating the RHS:
(i) By parts: Needs 2 applications of parts or one application followed by a trig method. Give M1 only if method is complete and A1 for a correct result.
(ii) $y\sin x=\int 3\left(1-2\sin^2 x\right)\sin x\,dx=\int 3\sin x-6\sin^3 x\,dx$

Then use $\sin 3x=3\sin x-4\sin^3 x$ to get $y\sin x=\int \frac{3}{2}\left(\sin 3x-\sin x\right)dx$ and integration shown above - both steps needed for M1

	ALTERNATIVE: Mult through by $\cos x$	
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$	M1
	$y\sin x=\int 3\cos 2x\sin x\,dx$	M1A1
	Rest as main scheme	

Leave blank

4. (a) Show that

$$r^2(r+1)^2 - (r-1)^2\, r^2 \equiv 4r^3$$

(3)

Given that $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

(b) use the identity in (a) and the method of differences to show that

$$(1^3 + 2^3 + 3^3 + \ldots + n^3) = (1 + 2 + 3 + \ldots + n)^2$$

(4)

Question Number	Scheme	Marks
4		
(a)	$r^2\left(r^2+2r+1\right)-\left(r^2-2r+1\right)r^2$	M1 A1
	$\equiv r^4+2r^3+r^2-r^4+2r^3-r^2$ or $r^2\left(r^2+2r+1-r^2+2r-1\right)$	
	$\equiv 4r^3$ *	A1 (3)
(b)	$\left(\sum_1^n 4r^3=\right)$ $\left(1\times 2^2-0\right)+\left(2^2\times 3^2-1^2\times 2^2\right)+\left(3^2\times 4^2-2^2\times 3^2\right)...$	M1
	$+\left(n^2\times(n+1)^2-(n-1)^2\times n^2\right)$	
	$=n^2\left(n+1\right)^2$	A1
	$\sum_1^n r^3=\frac{1}{4}n^2\left(n+1\right)^2$	A1
	$\therefore \sum_1^n r^3=\left(\frac{1}{2}n\left(n+1\right)\right)^2=\left(\sum_1^n r\right)^2$	
	So $\left(1^3+2^3+3^3+....+n^3\right)=\left(1+2+3...+n\right)^2$ *	A1cso (4) [7] (B1 on e-PEN)

(a) M1 Multiply out brackets May remove common factor r^2 first
A1 a correct statement
A1 fully correct solution which must include at least one intermediate line
ALT: Use difference of 2 squares:
M1 remove common factor and apply diff of 2 squares to rest
A1 $r^2\left(r+1+r-1\right)\left(r+1-\left(r-1\right)\right)$
$=r^2\left(2r\times 2\right)$
A1 $=4r^3$

(b) M1 Use result to write out a list of terms; sufficient to show cancelling needed
Minimum 2 at start and 1 at end $\sum_1^n 4r^3$ or $\sum_1^n r^3$ need not be shown here or for next mark
A1 Correctly extracting $n^2\left(n+1\right)^2$ as the only remaining non-zero term.
A1 Obtaining $\sum_1^n r^3=\frac{1}{4}n^2\left(n+1\right)^2$
A1cso (Shown B1 on e-PEN) for deducing the required result.

Working from **either** side can gain full marks
Working from **both** sides can gain full marks provided the working joins correctly in the middle.
If ***r* used instead of *n***, penalise the final A mark.

Leave blank

5. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + 3\mathrm{i}}, \quad z \neq -3\mathrm{i}$$

The circle with equation $|z| = 2$ is mapped by T onto the curve C.

(a) (i) Show that C is a circle.

(ii) Find the centre and radius of C.

(8)

The region $|z| \leqslant 2$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Shade the region R on an Argand diagram.

(2)

Question Number	Scheme	Marks
5 (a)	$w=\dfrac{z}{z+3\text{i}}$	
	$w(z+3\text{i})=z \quad z=\dfrac{3\text{i}w}{1-w}$ or $\dfrac{-3\text{i}w}{w-1}$	M1A1
	$\lvert z\rvert=2 \quad \left\lvert\dfrac{3\text{i}w}{1-w}\right\rvert=2$	dM1
	$\lvert 3\text{i}w\rvert=2\lvert 1-w\rvert$	
	$w=u+\text{i}v \quad 9(u^2+v^2)=4((1-u)^2+v^2)$	ddM1A1
	$9u^2+9v^2=4(1-2u+u^2+v^2)$	
(i)	$5u^2+5v^2+8u-4=0$	dddM1
	$\left(u+\dfrac{4}{5}\right)^2+v^2=\dfrac{36}{25}$	
(ii)	So a circle, Centre $\left(-\dfrac{4}{5},0\right)$ Radius $\dfrac{6}{5}$ (oe fractions or decimals)	A1A1 (8)
(b)	Circle drawn on an Argand diagram in correct position ft their centre and radius	B1ft
	Region inside correct circle shaded no ft	B1 (2) **[10]**

(a) M1	re-arrange to $z=$ ....
A1	correct result
dM1	dep (on first M1) using $\lvert z\rvert=2$ with their previous result
ddM1	dep (on both previous M marks) use $w=u+\text{i}v$ (or eg $w=x+\text{i}y$) and find the moduli. Moduli to contain no is and must be +. Allow 9 or 3 and 4 or 2
A1	for a correct equation in u and v or any other pair of variables
dddM1	dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms
A1A1	deduce circle and give correct centre and radius. Completion of square may not be shown. Deduct 1 for each error or omission. (Enter A1A0 on e-PEN)
	Special Case: If $z=\dfrac{3\text{i}w}{w-1}$ obtained, give M1A0 but all other marks can be awarded.
(b)	Mark diagram only - ignore any working shown.
B1ft	No numbers needed but circle must be in the correct region (or on the correct axis) for *their* centre and the centre and radius must be consistent (ie check how the circle crosses the axes) B0 if the equation in (a) is not an equation of a circle.
B1	Region inside the **correct** circle shaded. (no ft here)

Question Number	Scheme	Marks
	ALTERNATIVE for 5(a):	
	Let $z = x + \mathrm{i}y$	
	$w = \dfrac{x+\mathrm{i}y}{x+\mathrm{i}(y+3)}$	
	$= \dfrac{(x+\mathrm{i}y)(x-\mathrm{i}(y+3))}{(x+\mathrm{i}(y+3))(x-\mathrm{i}(y+3))}$	M1
	$= \dfrac{x^2+y^2+3y-3\mathrm{i}x}{x^2+y^2+6y+9}$	A1
	$\dfrac{3y+4-3\mathrm{i}x}{6y+13}$ as $\lvert z\rvert = 2 \Rightarrow x^2+y^2=4$	dM1
	$w = u+\mathrm{i}v$ so $u = \dfrac{3y+4}{6y+13}$ $v = \dfrac{-3x}{6y+13}$	ddM1 A1
	Using $u = \dfrac{\frac{1}{2}(6y+13)}{6y+13} - \dfrac{\frac{5}{2}}{6y+13}$	
	$u^2+v^2 = \dfrac{9y^2+24y+16+9x^2}{(6y+13)^2} = \dfrac{24y+52}{(6y+13)^2} = \dfrac{4}{6y+13}$	
	$= \dfrac{8}{5}\left(\dfrac{1}{2}-u\right)$	dddM1
	$\therefore 5u^2+5v^2+8u=4$ Then as main scheme: Circle, centre, radius	A1A1 (8)

M1 Rationalise the denominator - must use conjugate of the denominator
A1 Expand brackets and obtain correct numerator and denominator
dM1 Use $x^2+y^2=4$ in their expression to remove the squares
ddM1 Equating real and imaginary parts
A1 Correct expressions for u and v in terms of x and y
dddM1 Uses $u^2+v^2=\ldots$ to eliminate x and y and obtain an equation of the circle
A1A1 As main scheme

Leave blank

6.

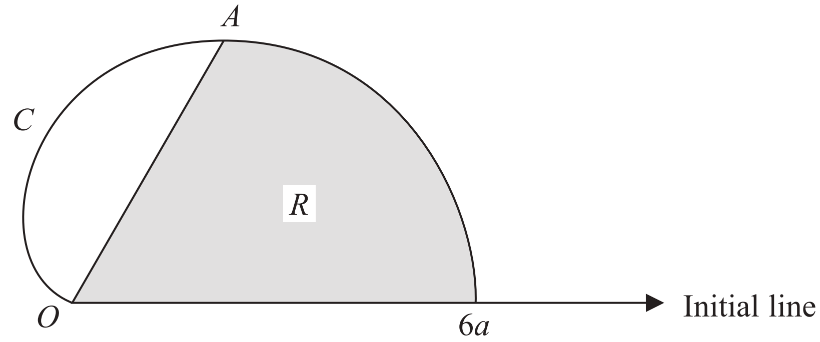

Figure 1

The curve C, shown in Figure 1, has polar equation

$$r = 3a\,(1 + \cos\theta), \qquad 0 \leqslant \theta < \pi$$

The tangent to C at the point A is parallel to the initial line.

(a) Find the polar coordinates of A.

(6)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line OA.

(b) Use calculus to find the area of the shaded region R, giving your answer in the form $a^2\left(p\pi + q\sqrt{3}\right)$, where p and q are rational numbers.

(5)

Question Number	Scheme	Marks
6 (a)	$r\sin\theta = 3a\sin\theta + 3a\sin\theta\cos\theta$ OR $3a\sin\theta + \frac{3}{2}a\sin 2\theta$	M1
	$\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 3a\cos\theta + 3a\cos^2\theta - 3a\sin^2\theta$ | $3a\cos\theta + 3a\cos 2\theta$	dM1
	$2\cos^2\theta + \cos\theta - 1 = 0$ terms in any order	A1
	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	
	$\cos\theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ $(\theta = \pi$ need not be seen$)$	ddM1A1
	$r = 3a \times \frac{3}{2} = \frac{9}{2}a$	A1 (6)
(b)	Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{3}} 9a^2(1+\cos\theta)^2\,\mathrm{d}\theta$	
	$= \frac{9a^2}{2}\int_0^{\frac{\pi}{3}}(1 + 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$	M1
	$= \frac{9a^2}{2}\int_0^{\frac{\pi}{3}}\left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right)\mathrm{d}\theta$	M1
	$= \frac{9a^2}{2}\left[\theta + 2\sin\theta + \frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$\frac{9a^2}{2}\left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4}\times\frac{\sqrt{3}}{2} + \frac{\pi}{6} \quad (-0)\right]$	
	$\frac{9a^2}{2}\left[\frac{\pi}{2} + \frac{9\sqrt{3}}{8}\right] = \left(\frac{9\pi}{4} + \frac{81\sqrt{3}}{16}\right)a^2$	A1 (5) [11]

(a)M1 using $r\sin\theta$ $r\cos\theta$ scores M0

dM1 Attempt the differentiation of $r\sin\theta$, inc use of product rule or $\sin 2\theta = 2\sin\theta\cos\theta$

A1 Correct 3 term quadratic in $\cos\theta$

ddM1 dep on both M marks. Solve their quadratic (usual rules) giving one or two roots

A1 Correct quadratic solved to give $\theta = \frac{\pi}{3}$

A1 Correct *r* obtained No need to see coordinates together in brackets
Special Case: If $r\cos\theta$ used, score M0M1A0M0A0A0to

(b)M1 Use of correct area formula, $\frac{1}{2}$ may be seen later, inc squaring the bracket to obtain 3 terms - limits need not be shown.

M1 Use double angle formula (formula to be of form $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$) to obtain an integrable function - limits need not be shown, $\frac{1}{2}$ from area formula may be missing,

dM1 attempt the integration - limits not needed – dep on 2nd M mark but not the first

A1 correct integration – substitution of limits not required

A1 correct final answer any equivalent provided in the demanded form.

Leave blank

7.

$$y = \tan^2 x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Show that $\dfrac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$

(4)

(b) Hence show that $\dfrac{d^3y}{dx^3} = 8\sec^2 x \tan x\,(A\sec^2 x + B)$, where A and B are constants to be found.

(3)

(c) Find the Taylor series expansion of $\tan^2 x$, in ascending powers of $\left(x - \dfrac{\pi}{3}\right)$, up to and including the term in $\left(x - \dfrac{\pi}{3}\right)^3$

(4)

Question Number	Scheme	Marks
7 (a)	$\frac{dy}{dx}=2\tan x\sec^2 x$	B1
	OR $\frac{dy}{dx}=2\tan x\left(1+\tan^2 x\right)$	
	$\frac{d^2y}{dx^2}=2\sec^4 x+4\tan^2 x\sec^2 x$ | OR $\frac{d^2y}{dx^2}=2\sec^2 x+2\times3\tan^2 x\sec^2 x$	M1 A1
	$=2\sec^4 x+4\left(\sec^2 x-1\right)\sec^2 x$ | $=2\sec^2 x+6\left(\sec^2 x-1\right)\sec^2 x$	
	$=6\sec^4 x-4\sec^2 x$ *	A1cso (4)
(b)	$\frac{d^3y}{dx^3}=24\sec^3 x\sec x\tan x-8\sec^2 x\tan x$	M1A1
	$=8\sec^2 x\tan x\left(3\sec^2 x-1\right)$	A1cso (3)
(c)	$y_{\frac{\pi}{3}}=\left(\sqrt{3}\right)^2\ (=3)\quad \left(\frac{dy}{dx}\right)_{\frac{\pi}{3}}=2\sqrt{3}\times\left(\frac{2}{1}\right)^2\ \left(=8\sqrt{3}\right)$	B1(both)
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{3}}=6\times2^4-4\times2^2=80$	
	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{3}}=8\times4\times\sqrt{3}\left(3\times2^2-1\right)=352\sqrt{3}$	M1(attempt both)
	$\tan^2 x=y_{\frac{\pi}{3}}+\left(x-\frac{\pi}{3}\right)\left(\frac{dy}{dx}\right)_{\frac{\pi}{3}}+\frac{1}{2!}\left(x-\frac{\pi}{3}\right)^2\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{3}}+\frac{1}{3!}\left(x-\frac{\pi}{3}\right)^3\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{3}}$	
	$=3+8\sqrt{3}\left(x-\frac{\pi}{3}\right)+40\left(x-\frac{\pi}{3}\right)^2+\frac{176}{3}\sqrt{3}\left(x-\frac{\pi}{3}\right)^3$	M1A1 (4)[11]

(a)B1 $\frac{dy}{dx}=2\tan x\sec^2 x$

M1 attempting the second derivative, inc using the product rule or $\sec^2\theta=\tan^2\theta+1$ **Must** start from the result given in (a)

A1 a correct second derivative in any form

A1cso for a correct result following completely correct working $\sec^2\theta=\tan^2\theta+1$ must be d seen or used

Question Number	Scheme	Marks
(b) M1	attempting the third derivative, inc using the chain rule	
A1	a correct derivative	
A1	a completely correct final result	
(c) B1	$y_{\frac{\pi}{3}} = \left(\sqrt{3}\right)^2$ or 3 **and** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\frac{\pi}{3}} = 2\sqrt{3} \times \left(\frac{2}{1}\right)^2$ or $8\sqrt{3}$	
M1	obtaining values for second and third derivatives at $\frac{\pi}{3}$ (need not be correct but must be obtained from their derivatives)	
M1	using a correct Taylor's expansion using $\left(x - \frac{\pi}{3}\right)$ and their derivatives. (2! or 2, 3! or 6 must be seen or implied by the work shown) This mark is not dependent.	
A1	for a correct final answer **Must** start $\tan^2 x = ...$ or y = ... f(x) scores A0 unless defined as $\tan^2 x$or y here or earlier. Accept equivalents eg awrt 610 (609.6…) $\sqrt{371712}$ But no factorials in this final answer.	

Leave blank

8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2 y}{dx^2} - 7x \frac{dy}{dx} + 16y = 2 \ln x, \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2 y}{du^2} - 8 \frac{dy}{du} + 16y = 2u \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of the differential equation (I).

(1)

Question Number	Scheme	Marks
8 (a)	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=x$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-u}\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}u}+\mathrm{e}^{-u}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-7x\frac{\mathrm{d}y}{\mathrm{d}x}+16y=2\ln x$	
	$\mathrm{e}^{2u}\times\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)-7\mathrm{e}^u\times\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2\ln\left(\mathrm{e}^u\right)$	dM1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-8\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2u$ *	A1cso (6)

(a) B1 for $\frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ oe as shown seen explicitly or used

M1 obtaining $\frac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later

M1 obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)

A1 a correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form

dM1 substituting in the equation to eliminate x **Only** u and y now Depends on the 2nd M mark

A1cso obtaining the given result from completely correct work

	ALTERNATIVE 1	
	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u=x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}u}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}=1\frac{\mathrm{d}x}{\mathrm{d}u}\times\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}\right)-7x\times\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2\ln\left(\mathrm{e}^u\right)$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-8\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2u$ *	dM1A1cso (6)

B1 As above

M1 obtaining $\frac{\mathrm{d}y}{\mathrm{d}u}$ using chain rule here or seen later

M1 obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ using product rule (penalise lack of chain rule by the A mark)

Question Number	Scheme	Marks
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1A1cso	As main scheme	
	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}\right) - 7x \times \frac{1}{x}\frac{dy}{du} + 16y = 2u$	
	$\frac{d^2y}{du^2} - 8\frac{dy}{du} + 16y = 2u$ *	dM1A1cso

See the notes for the main scheme.

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2 - 8m + 16 = 0$	
	$(m-4)^2 = 0 \quad m = 4$	M1A1
	(CF =) $(A + Bu)\mathrm{e}^{4u}$	A1
	PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, c = 0)	
	$\frac{\mathrm{d}y}{\mathrm{d}u} = a \quad \frac{\mathrm{d}^2 y}{\mathrm{d}u^2} = 0$	M1
	$0 - 8a + 16(au + b) = 2u$	
	$a = \frac{1}{8} \quad b = \frac{1}{16}$ oe (decimals must be 0.125 and 0.0625)	dM1A1
	$\therefore y = (A + Bu)\mathrm{e}^{4u} + \frac{1}{8}u + \frac{1}{16}$	B1ft (7)
(c)	$y = (A + B\ln x)x^4 + \frac{1}{8}\ln x + \frac{1}{16}$	B1 (1)
		[14]

(b) M1 writing down the correct aux equation and solving to m = … (usual rules)
A1 the correct solution ($m = 4$)
A1 the correct CF – can use any (single) variable
M1 using an appropriate PI and finding $\frac{\mathrm{d}y}{\mathrm{d}u}$ **and** $\frac{\mathrm{d}^2 y}{\mathrm{d}u^2}$ Use of $y = \lambda u$ scores M0
dM1 substitute in the equation to obtain values for the unknowns Dependent on the second M1
A1 correct unknowns two or three (c = 0)
B1ft a complete solution, follow through their CF and PI. Must have y = a function of u
Allow recovery of incorrect variables.

(c) B1 reverse the substitution to obtain a correct expression for y in terms of x No ft here
x^4 or $\mathrm{e}^{4\ln x}$ allowed. Must start y = …