Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Wednesday 7 June 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
6668/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. (a) Show that, for $r > 0$

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$$

(1)

(b) Hence prove that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{n(n+2)}{(n+1)^2}$$

(3)

(c) Show that, for $n \in \mathbb{N}$, $n > 1$

$$\sum_{r=n}^{3n} \frac{6r+3}{r^2(r+1)^2} = \frac{an^2 + bn + c}{n^2(3n+1)^2}$$

where a, b and c are constants to be found.

(3)

Question Number	Scheme	Notes	Marks
1(a)	$\frac{1}{r^2}-\frac{1}{(r+1)^2}=\frac{(r+1)^2-r^2}{r^2(r+1)^2}=\frac{2r+1}{r^2(r+1)^2}$	Correct proof (minimum as shown) ($(r+1)^2$ or r^2+2r+1. Can be worked in either direction.	B1
			(1)
(b)	$\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right) = 1-\frac{1}{4}+\frac{1}{4}-\frac{1}{9}......+\left(\frac{1}{n^2}\right)-\frac{1}{(n+1)^2}$ Terms of the series with $r=1$, $r=n$ and one of $r=2, r=n-1$ should be shown.		M1
	$1-\frac{1}{(n+1)^2}$	Extracts correct terms that do not cancel	A1
	$\frac{(n+1)^2-1}{(n+1)^2}=\frac{n(n+2)}{(n+1)^2}$ *	Correct completion with no errors	A1*cso
			(3)
(c)	$\sum_{r=n}^{3n}\frac{6r+3}{r^2(r+1)^2}=3\left(\frac{3n(3n+2)}{(3n+1)^2}-\frac{(n-1)(n+1)}{n^2}\right)$	Attempts to use $\mathrm{f}(3n)-(\mathrm{f}(n-1)\text{ or }\mathrm{f}(n))$ 3 may be missing	M1
	$=3\left(\frac{3n^3(3n+2)-(3n+1)^2(n^2-1)}{n^2(3n+1)^2}\right)$	Attempt at common denominator, Denom to be $n^2(3n+1)^2$ or $(n+1)^2(3n+1)^2$ Numerator to be difference of 2 quartics. 3 may be missing	dM1
	$=\frac{24n^2+18n+3}{n^2(3n+1)^2}$	cao	A1cao
			(3)
			Total 7
	Alternative for part (c)		
	$\sum_{r=n}^{3n}\frac{6r+3}{r^2(r+1)^2}=3\left(\frac{1}{n^2}-\frac{1}{(3n+1)^2}\right)$ OR: $3\left(\frac{1}{(n+1)^2}-\frac{1}{(3n+1)^2}\right)$	Attempts the difference of 2 terms (either difference accepted) 3 may be missing	M1
	$=3\left(\frac{(3n+1)^2-n^2}{n^2(3n+1)^2}\right)$	Valid attempt at common denominator for their fractions 3 may be missing	dM1
	$=\frac{24n^2+18n+3}{n^2(3n+1)^2}$	cao	A1
	If (b) and/or (c) are worked with *r* instead of *n* do **NOT** award the final A mark for the parts affected. This applies even if *r* is changed to *n* at the end.		

	Alternative for (b) - by induction. **NB:** No marks available if result in (a) is not used.		
	Assume true for $n=k$		
	$\sum_{r=1}^{k+1}\frac{2r+1}{r^2(r+1)^2}=\frac{k(k+2)}{(k+1)^2}+\frac{1}{(k+1)^2}-\frac{1}{(k+2)^2}$	Uses $\sum_{r=1}^{k}$ together with the $(k + 1)$th term as 2 fractions (see (a))	M1
	$.=\frac{k^2+2k+1}{(k+1)^2}-\frac{1}{(k+2)^2}$.		
	$1-\frac{1}{(k+2)^2}=\frac{k^2+4k+3}{(k+2)^2}=\frac{(k+1)(k+3)}{(k+2)^2}$	Combines the 3 fractions to obtain a single fraction. Must be correct but numerator need not be factorised.	A1
	Show true for $n=1$	This must be seen somewhere	
	Hence proved by induction	Complete proof with no errors and a concluding statement.	A1

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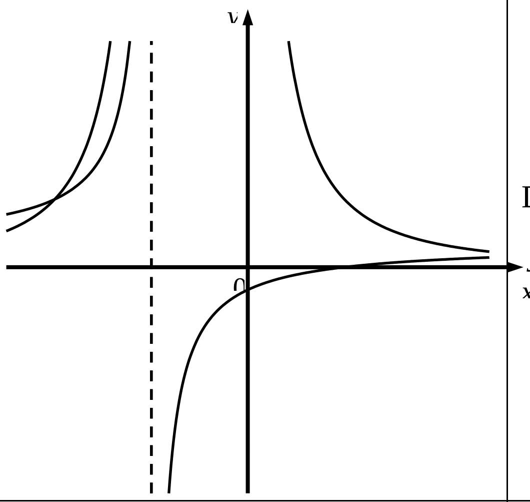
2. Use algebra to find the set of values of x for which

$$\frac{x-2}{2(x+2)} \leqslant \frac{12}{x(x+2)}$$

(9)

Question Number	Scheme	Notes	Marks
2.	$\frac{x-2}{2(x+2)} \le \frac{12}{x(x+2)}$		
NB	Question states "Use algebra..." so purely graphical solutions score max 1/9 (the B1). A sketch and some algebra to find CVs or intersection points can score according to the method used.		
	Can use $\le, <$ or $=$ for the first 6 marks **in all methods**		
	$\frac{x-2}{2(x+2)} - \frac{12}{x(x+2)} (\le 0)$	Collects expressions to one side.	M1
	$\frac{x^2-2x-24}{2x(x+2)} (\le 0)$	M1: Attempt common denominator A1: Correct single fraction	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^2-2x-24 \Rightarrow (x+4)(x-6)(=0) \Rightarrow x = ...$	Attempt to solve their quadratic as far as x = ...	M1
	$x = -4, 6$	Correct critical values. May be seen on a sketch.	A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ $[-4,-2) \cup (0,6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao **(9)**
			Total 9
	Alternative 1: Multiplies both sides by $x^2(x+2)^2$		
	$x^2(x-2)(x+2) \le 24x(x+2)$ $x^3(x+2) - 2x^2(x+2) \le 24x(x+2)$	Both sides $\times x^2(x+2)^2$ May multiply by more terms but must be a positive multiplier containing $x^2(x+2)^2$	M1
	$x^3(x+2) - 2x^2(x+2) - 24x(x+2)(\le 0)$	M1: Collects expressions to one side A1: Correct inequality	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^4 - 28x^2 - 48x(=0)$ $x(x+2)(x-6)(x+4)(=0) \Rightarrow x = ...$	Attempt to solve their quartic as far as x = ... to obtain the **other** critical values Can cancel x and solve a cubic or x and $(x+2)$ and solve a quadratic.	M1
	$x = -4, 6$	Correct critical values	A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2,\ \ 0 < x \le 6$ $[-4,-2) \cup (0,6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao **(9)**
			Total 9

	Alternative 2: using a sketch graph (probably from calculator)		
		Draw graphs of $y=\frac{x-2}{2(x+2)}$ and $y=\frac{12}{x(x+2)}$	
	CVs $x=0,-2$	(Vertical asymptotes of graphs.)	B1
	$\frac{x-2}{2(x+2)}=\frac{12}{x(x+2)}$	Eliminate y	M1
	$x(x-2)=24$	M1: Obtains a quadratic equation A1: Correct equation	M1A1
	$x^2-2x-24\Rightarrow(x+4)(x-6)=0\Rightarrow x=..$	Attempt to solve their quadratic as far as $x=...$	M1
	CVs $x=-4,6$	Correct critical values	A1
	$-4\le x<-2,\ \ 0<x\le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark)	dM1
	$-4\le x<-2,\ \ 0<x\le 6$	A1: All 4 CVs in the inequalities correct	A1
		A1: All inequality signs correct	A1cao **(9)**
NB	As above, but with no sketch graph shown: CVs $x=0,-2$ **must** be stated somewhere.		B1
	Otherwise no marks available.		

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3. Solve the equation

$$z^3 + 32 + 32\mathrm{i}\sqrt{3} = 0$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$

(6)

	Scheme	Notes	Marks
3.	$z^3 + 32 + 32\mathrm{i}\sqrt{3} = 0$		
	$\arg\left(z^3\right) = \frac{4\pi}{3}$ or $-\frac{2\pi}{3}$	M1: Uses tan to find arg z^3 $\arctan\sqrt{3}$, $\arctan\frac{1}{\sqrt{3}}$, $\frac{\pi}{3}$ or $\frac{\pi}{6}$ seen. Allow equivalent angles A1: Either of values shown	M1A1
	$\lvert z\rvert = r = 4$	Correct r seen anywhere (eg only in answers)	B1
	$3\theta = \frac{4\pi}{3},\ -\frac{2\pi}{3},\ -\frac{8\pi}{3}$		
	$\theta = \frac{4\pi}{9},\ -\frac{2\pi}{9},\ -\frac{8\pi}{9}$	Divides by 3 to obtain at least 2 values of θ which differ by $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$.	M1
	$\theta = \frac{4\pi}{9},\ -\frac{2\pi}{9}$ or $\frac{16\pi}{9},\ -\frac{8\pi}{9}$ or $\frac{10\pi}{9}$	At least 2 correct (and distinct) values from list shown	A1
	$z = 4\mathrm{e}^{\frac{4\pi}{9}\mathrm{i}},\ 4\mathrm{e}^{-\frac{2\pi}{9}\mathrm{i}},\ 4\mathrm{e}^{-\frac{8\pi}{9}\mathrm{i}}$ or $4\mathrm{e}^{\mathrm{i}\theta}$ where $\theta = ...$	A1: All correct and in either of the forms shown Ignore extra answers outside the range	A1 **(6)**
			Total 6

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4.

$$y = \ln\left(\frac{1}{1-2x}\right), \qquad |x| < \frac{1}{2}$$

(a) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

(4)

(b) Hence, or otherwise, find the series expansion of $\ln\left(\frac{1}{1-2x}\right)$ about $x = 0$, in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(3)

(c) Use your expansion to find an approximate value for $\ln\left(\frac{3}{2}\right)$, giving your answer to 3 decimal places.

(3)

Question Number	Scheme	Notes	Marks
4.	$y=\ln\left(\frac{1}{1-2x}\right)$		
(a)	$y=\ln(1-2x)^{-1}=(\ln 1)-\ln(1-2x)$ $\frac{dy}{dx}=-\frac{1}{1-2x}\times -2\left(=\frac{2}{1-2x}\right)$	M1: $\frac{dy}{dx}=\frac{-1}{(1-2x)}\times\frac{d(1-2x)}{dx}$ Must use chain rule ie $\frac{k}{1-2x}$ with $k\neq\pm1$ needed. Minus sign may be missing. A1: Correct derivative	M1A1
OR	$\frac{dy}{dx}=(1-2x)\times-(1-2x)^{-2}\times -2$ $\left(=\frac{2}{1-2x}\right)$	M1: $\frac{dy}{dx}=\frac{1}{(1-2x)^{-1}}\times\frac{d(1-2x)^{-1}}{dx}$ Must use chain rule. Minus sign may be missing. A1: Correct derivative	M1A1
	$\frac{d^2y}{dx^2}=-2\times(1-2x)^{-2}\times -2$ $\left(=\frac{4}{(1-2x)^2}\right)$	Correct second derivative obtained from a correct first derivative.	A1
	$\frac{d^3y}{dx^3}=-8\times(1-2x)^{-3}\times -2$ $\left(=\frac{16}{(1-2x)^3}\right)$	Correct third derivative obtained from correct first and second derivatives	A1
			(4)
	Alternative by use of exponentials and implicit differentiation		
(a)	$y=\ln\left(\frac{1}{1-2x}\right)\Rightarrow e^y=\frac{1}{1-2x}=(1-2x)^{-1}$		
	$e^y\frac{dy}{dx}=2(1-2x)^{-2}$	Differentiates using implicit differentiation and chain rule.	M1
	$\frac{dy}{dx}=2e^{-y}(1-2x)^{-2}$ or $\frac{2}{(1-2x)}$	Correct derivative in either form. Equivalents accepted.	A1
	If $\frac{dy}{dx}=\frac{2}{(1-2x)}$ has been used from here, see main scheme for second and third derivatives		

(b)	$(y_0 = 0), y'_0 = 2, y''_0 = 4, y'''_0 = 16$	Attempt values at $x = 0$ using their derivatives from (a) $y_0 = 0$ need not be seen but other 3 values must be attempted.	M1
	$(y =)(0) + 2x + \frac{4x^2}{2!} + \frac{16x^3}{3!}$	Uses their values in the correct Maclaurin series. Must see x^3 term Can be implied by a final series which is correct for their values. 2!,3! or 2 and 6	M1
	$y = 2x + 2x^2 + \frac{8}{3}x^3$	Correct expression. Must start y = ... or $\ln\left(\frac{1}{1-2x}\right) = ...$ f(x) = ... allowed **only** if f(x) is defined to be one of these.	A1cao
			(3)
	Alternative (b)		
	$y = \ln\left(\frac{1}{1-2x}\right) = -\ln(1-2x)$	Log power law applied correctly	M1
	$= -\left((-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3}\right)$	Replaces x with -2x in the expansion for ln(1 + x) (in formula book)	M1
	$y = 2x + 2x^2 + \frac{8}{3}x^3$	Correct expression	A1cao
(c)	$\frac{1}{1-2x} = \frac{3}{2} \Rightarrow x = \frac{1}{6}$	Correct value for x, seen explicitly or substituted in their expansion	B1
	$\ln\left(\frac{3}{2}\right) \approx 2\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right)^2 + \frac{8}{3}\left(\frac{1}{6}\right)^3$	Substitute their value of x into their expansion. May need to check this is correct for their expansion and their x. (Calculator value for $\ln\left(\frac{3}{2}\right)$ is 0.405)	M1
	= 0.401	Must come from correct work	A1cso
NB:	$\ln 3 - \ln 2$ or $\ln 3 + \ln\left(\frac{1}{2}\right)$ scores 0/3 as $\lvert x \rvert$ must be $< \frac{1}{2}$		
	Answer with no working scores 0/3		**(3)**
			Total 10

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5. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 26\sin 3x$$

(8)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

Question Number	Scheme	Notes	Marks
5.	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 26\sin 3x$		
(a)	$m^2 - 2m = 0 \Rightarrow m = 0,\ 2$	Solves AE	M1
	$(\text{CF or } y =)\ A + Be^{2x}$ or $Ae^0 + Be^{2x}$ oe	Correct CF (CF or $y =$ not needed)	A1
	$(\text{PI or } y =)\ a\cos 3x + b\sin 3x$	Correct form for PI (PI or y = not needed)	B1
	$\frac{dy}{dx} = -3a\sin 3x + 3b\cos 3x,\ \frac{d^2y}{dx^2} = -9a\cos 3x - 9b\sin 3x$		M1A1
	M1: Differentiates twice; change of trig functions needed, ± 1 or ± 3 for coeffs for first derivative, ± 1, ± 3 or ± 9 for second derivative (1/3 etc indicates integration) A1: Correct derivatives		
	$-9a\cos 3x - 9b\sin 3x + 6a\sin 3x - 6b\cos 3x = 26\sin 3x$		
	$\therefore -9a - 6b = 0,\ \ -9b + 6a = 26 \Rightarrow a = ..., b = ...$	Substitutes and forms simultaneous equations (by equating coeffs) and attempts to solve for a and b Depends on the second M mark	dM1
	$a = \frac{4}{3},\ b = -2$	Correct a and b	A1
	$y = A + Be^{2x} + \frac{4}{3}\cos 3x - 2\sin 3x$	Forms the GS (ft their CF and PI) Must start y = ...	A1ft **(8)**
(b)	$0 = A + B + \frac{4}{3}$	Substitutes $x = 0$ and $y = 0$ into their GS	M1
	$\left(\frac{dy}{dx}\right) = 2Be^{2x} - 4\sin 3x - 6\cos 3x \Rightarrow 0 = 2B - 6$ Differentiates and substitutes $x = 0$ and $y' = 0$ (change of trig functions needed, ± 1 or ± 3 for coeffs)		M1
	$0 = A + B + \frac{4}{3},\ 0 = 2B - 6 \Rightarrow A = .., B = ..$	Solves simultaneously to obtain values for A and B Depends on the second M mark	dM1
	$A = \frac{-13}{3},\quad B = 3$	Correct values	A1
	$y = 3e^{2x} - \frac{13}{3} + \frac{4}{3}\cos 3x - 2\sin 3x$	Follow through their GS and A and B Must start y = ...	A1ft **(5)**
			Total 13
ALT for (a)	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 26\sin 3x \Rightarrow \frac{dy}{dx} - 2y = -\frac{26}{3}\cos 3x + c$	M1: Integrates both sides wrt x A1: Correct expression	M1A1
	$I = e^{\int -2dx} = e^{-2x}$	Correct integrating factor	B1
	$ye^{-2x} = \int e^{-2x}\left(-\frac{26}{3}\cos 3x + c\right)dx$	M1: Uses $yI = \int I\left(-\frac{26}{3}\cos 3x + c\right)dx$ A1: Correct expression	M1A1
	$= \frac{4}{3}e^{-2x}\cos 3x - 2e^{-2x}\sin 3x - \frac{1}{2}ce^{-2x} + B$	M1: Integration by parts twice A1: Correct expression	M1A1
	$y = -\frac{1}{2}c + Be^{2x} + \frac{4}{3}\cos 3x - 2\sin 3x$	Must start y = ...	

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6.

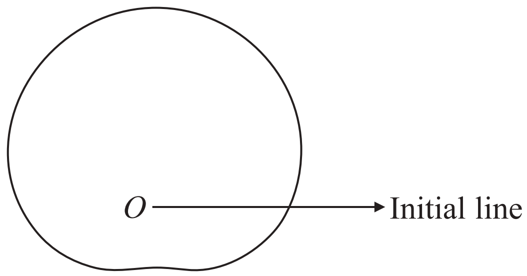

Figure 1

Figure 1 shows a sketch of a curve with polar equation

$$r = 6 + a\sin\theta$$

where $0 < a < 6$ and $0 \leqslant \theta < 2\pi$

The area enclosed by the curve is $\dfrac{97\pi}{2}$

Find the value of the constant a.

(8)

Question Number	Scheme	Notes	Marks
6.	$r = 6 + a\sin\theta$		
	$A = \frac{1}{2}\int (6 + a\sin\theta)^2 \, d\theta$	Use of $\frac{1}{2}\int r^2 (d\theta)$ Limits not needed. Can be gained if $\frac{1}{2}$ appears later	B1
	$(6 + a\sin\theta)^2 = 36 + 12a\sin\theta + a^2\sin^2\theta$		
	$(6 + a\sin\theta)^2 = 36 + 12a\sin\theta + a^2\left(\frac{1-\cos 2\theta}{2}\right)$	M1: Squares ($36 + k\sin^2\theta$, where $k = a^2$ or a as min) and attempts to change : $\sin^2\theta$ to an expression in $\cos 2\theta$ A1: Correct expression	M1A1
	$\left(\frac{1}{2}\right)\left[36\theta - 12a\cos\theta + \frac{a^2}{2}\theta - \frac{a^2}{4}\sin 2\theta\right]$	dM1: Attempt to integrate $\cos 2\theta \to \pm\frac{1}{2}\sin 2\theta$ Limits not needed A1: Correct integration limits not needed	dM1A1
	$= 36\pi + \frac{\pi a^2}{2}$	Correct area obtained from correct integration and correct limits. No need to simplify but trig functions must be evaluated.	A1
	$36\pi + \frac{\pi a^2}{2} = \frac{97\pi}{2} \Rightarrow a = ...$	Set their area = $\frac{97\pi}{2}$ and attempt to solve for a (depends on both M marks above) If $\frac{1}{2}$ omitted from the initial formula and area set = 97π, give the B1 by implication as well as this mark.	ddM1
	$a = 5$	cao and cso $a = \pm 5$ or $a = -5$ scores A0	A1cso
			Total 8
	Alternatives: Splitting the area and so using 2 integrals with different limits.		
	Marks the same as the main scheme.		
1	Limits 0 to π (area above initial line) and limits π to 2π (area below initial line) and add the two results.		
2	Limits 0 to $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ to 2π Twice the sum of the results needed.		

Leave blank

7. (a) Find, in the form $y = \mathrm{f}(x)$, the general solution of the equation

$$\cos x\frac{\mathrm{d}y}{\mathrm{d}x} + y\sin x = 2\cos^3 x\sin x + 1, \quad 0 < x < \frac{\pi}{2}$$

(8)

Given that $y = 5\sqrt{2}$ when $x = \frac{\pi}{4}$

(b) find the value of y when $x = \frac{\pi}{6}$, giving your answer in the form $a + b\sqrt{3}$, where a and b are rational numbers to be found.

(3)

Question Number	Scheme	Notes	Marks
7.	$\cos x\frac{dy}{dx}+y\sin x=2\cos^3 x\sin x+1$		
(a)	$\frac{dy}{dx}+y\tan x=2\cos^2 x\sin x+\frac{1}{\cos x}$	Divides by cos x LHS both terms divided RHS min 1 term divided	M1
	$I=e^{\int\tan x\,dx}=e^{\ln\sec x}=\sec x$	M1: Attempt integrating factor $e^{\int\tan x\,dx}$ needed A1: Correct integrating factor, $\sec x$ or $\frac{1}{\cos x}$	dM1A1
	$y\sec x=\int\left(2\sin x\cos x+\sec^2 x\right)dx$	Multiply through by their IF and integrate LHS (integration may be done later) $yI=\int(\text{their RHS})I\,dx$	M1
	$y\sec x=-\frac{1}{2}\cos 2x+\tan x(+c)$	M1: Attempt integration of at least one term on RHS (provided both sides have been multiplied by their IF.) OR $\sec^2 x\to K\tan x$ A1: $-\frac{1}{2}\cos 2x$ or equivalent integration of $2\sin x\cos x$ ($\sin^2 x$ or $-\cos^2 x$) A1: $\tan x$ constant not needed.	M1A1A1
	$y=\left(-\frac{1}{2}\cos 2x+\tan x+c\right)\cos x$ $y=\left(-\cos^2 x+\tan x+c\right)\cos x$ $y=\left(\sin^2 x+\tan x+c\right)\cos x$	Include the constant and deal with it correctly. Must start y = ... Or equivalent eg $y=-\frac{1}{2}\cos 2x\cos x+\sin x+c\cos x$ Follow through from the line above	A1ft
			(8)
(b)	$x=\frac{\pi}{4}\Rightarrow 5\sqrt{2}=.....\Rightarrow c=.....$	Substitutes for x and y and solves for c (If substitution not shown award for at least one term evaluated correctly.)	M1
	$x=\frac{\pi}{6}\Rightarrow y=.........$	Substitutes $x=\frac{\pi}{6}$ to find a value for y	M1
	$y=\frac{1}{2}+\frac{35}{8}\sqrt{3}$ or $y=0.5+4.375\sqrt{3}$	Must be in given form. Equivalent fractions allowed. ...	A1cao
			(3)
NB	(b) There may be no working shown due to use of calculator. In such cases: Final answer correct (and in required form with no decimals instead of $\sqrt{3}$ seen), score 3/3. Final answer incorrect (or decimals instead of $\sqrt{3}$ seen), score 0/3. This applies whether (a) is correct or not.		
			Total 11

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8. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 3\text{i}}{1 + \text{i}z}, \quad z \neq \text{i}$$

The transformation T maps the circle $|z| = 1$ in the z-plane onto the line l in the w-plane.

(a) Find a cartesian equation of the line l.

(5)

The circle $|z - a - b\text{i}| = c$ in the z-plane is mapped by T onto the circle $|w| = 5$ in the w-plane.

(b) Find the exact values of the real constants a, b and c.

(6)

Question Number	Scheme	Notes	Marks
8.	$w = \dfrac{z+3\text{i}}{1+\text{i}z}$		
(a)	$z = \dfrac{w-3\text{i}}{1-\text{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert = 1 \Rightarrow \left\lvert\dfrac{w-3\text{i}}{1-\text{i}w}\right\rvert = 1 \Rightarrow \lvert w-3\text{i}\rvert = \lvert 1-w\text{i}\rvert$ $\therefore\ \lvert u+\text{i}v-3\text{i}\rvert = \lvert(u+\text{i}v)\text{i}-1\rvert$	Uses $\lvert z\rvert = 1$ and introduce "u + iv" (or $x+\text{i}y$) for w	M1
	$u^2+(v-3)^2 = u^2+(v+1)^2$	Correct use of Pythagoras on either side.	M1
	$v = 1$ oe	$v=1$ or $y=1$	A1
			(5)
	Alternative 1 for (a)		
	eg $w(1) = \dfrac{1+3\text{i}}{1+\text{i}} = 2+\text{i}$	M1: Maps one point on the circle using the given transformation A1:Correct mapping	M1A1
	eg $w(-\text{i}) = \dfrac{2\text{i}}{2} = \text{i}$	Maps a second point on the circle	M1
	$v = 1$ oe	M1: Forms Cartesian equation using their 2 points A1: $v=1$ or $y=1$	M1A1
	Alternative 2 for (a)		
	$z = \dfrac{w-3\text{i}}{1-\text{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert = 1 \Rightarrow \left\lvert\dfrac{w-3\text{i}}{1-\text{i}w}\right\rvert = 1 \Rightarrow \lvert w-3\text{i}\rvert = \lvert 1-w\text{i}\rvert$ $\lvert w-3\text{i}\rvert = \lvert w+\text{i}\rvert = \lvert w-(-\text{i})\rvert$	Uses $\lvert z\rvert = 1$ and changes to form $\lvert w-...\rvert = \lvert w-...\rvert$ or draws a diagram	M1
	Perpendicular bisector of points $(0,3)$ and $(0,-1)$	Uses a correct geometrical approach	M1
	$v = 1$ oe	$v=1$ or $y=1$	A1

	Alternative 3 for (a)		
	Let $z = x + \mathrm{i}y,\ \lvert z\rvert = 1 \Rightarrow x^2 + y^2 = 1$		
	$w = \frac{z+3\mathrm{i}}{1+\mathrm{i}z} = \frac{x+\mathrm{i}y+3\mathrm{i}}{1+\mathrm{i}(x+\mathrm{i}y)} = \frac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x}$		
	$w = \frac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x} \times \frac{(1-y)-\mathrm{i}x}{(1-y)-\mathrm{i}x}$	Substitute $z = x + \mathrm{i}y$ and multiply numerator and denominator by complex conjugate of their denominator	M1
	$w = \frac{x(1-y)-\mathrm{i}x^2+\mathrm{i}(y+3)(1-y)-\mathrm{i}^2x(y+3)}{(1-y)^2-\mathrm{i}x(1-y)+\mathrm{i}x(1-y)-\mathrm{i}^2x^2}$		
	$w = \frac{[x(1-y)+x(y+3)]+\mathrm{i}[-x^2+(y+3)(1-y)]}{(1-y)^2+x^2}$	M1: Multiply out and collect real and imaginary parts in numerator. Denominator must be real. A1: all correct	M1 A1
	$w = \frac{[x-xy+xy+3x]+\mathrm{i}[-x^2+y-y^2+3-3y]}{1-2y+y^2+x^2}$		
	$w = \frac{[4x]+\mathrm{i}[-1+3-2y]}{2-2y}$	Applies $x^2 + y^2 = 1$	M1
	$w = \frac{4x+\mathrm{i}[2-2y]}{2-2y} = \frac{4x}{2-2y} + \mathrm{i}$		
	$y = 1$	$y = 1$ or $v = 1$	A1
(b)	$\lvert w\rvert = 5 \Rightarrow \left\lvert\frac{z+3\mathrm{i}}{1+\mathrm{i}z}\right\rvert = 5 \Rightarrow \lvert z+3\mathrm{i}\rvert = 5\lvert 1+\mathrm{i}z\rvert$ $\therefore\ \lvert x+\mathrm{i}y+3\mathrm{i}\rvert = 5\lvert(x+\mathrm{i}y)\mathrm{i}+1\rvert$	Uses $\lvert w\rvert = 5$ and introduce "$x + \mathrm{i}y$"	M1
	$x^2+(y+3)^2 = 25\left(x^2+(1-y)^2\right)$	M1: Correct use of Pythagoras Allow 25 or 5 A1: Correct equation	M1A1
	$x^2+y^2-\frac{7}{3}y+\frac{2}{3}=0$		
	$x^2+\left(y-\frac{7}{6}\right)^2 = \frac{25}{36}$	Attempt circle form or attempt r^2 from the line above.	M1
	$a = 0,\ b = \frac{7}{6},\ c = \frac{5}{6}$	A1: 2 correct A1: All correct	A1, A1
			(6)
			Total 11
	Or, for the last 3 marks:		
	$\left\lvert z-0-\frac{7}{6}\mathrm{i}\right\rvert = \frac{5}{6}$		M1A1A1
	If 0 not shown score M1A1A0		
	No need to list *a*, *b*, *c* separately if answer in this form.		