Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Wednesday 6 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
6668/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. (a) Express $\frac{1}{(r+3)(r+4)}$ in partial fractions.

(1)

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{1}{(r+3)(r+4)} = \frac{n}{a(n+a)}$$

where a is a constant to be found.

(5)

(c) Find the exact value of $\sum_{r=15}^{30} \frac{1}{(r+3)(r+4)}$

(2)

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Question Number	Scheme	Notes	Marks
	Mark (a) and (b) together – ignore labels		
1(a)	$\frac{1}{(r+3)(r+4)} \equiv \frac{1}{(r+3)} - \frac{1}{(r+4)}$	Cao No working needed – ignore any shown	B1
			(1)
(b)	$r=1$: $\frac{1}{4}-\frac{1}{5}$		
	$r=2$: $\frac{1}{5}-\frac{1}{6}$		
	$...r=n-1$: $\frac{1}{(n+2)}-\frac{1}{(n+3)}$		
	$r=n$: $\frac{1}{(n+3)}-\frac{1}{(n+4)}$	First 2 and last term or first and last 2 terms required. Must start at $r=1$ (First term complete, 2nd and last may be partial or last term complete 1st and penultimate partial.)	M1
	$\sum_{r=1}^{n}\frac{1}{(r+2)(r+3)} = \frac{1}{4}-\frac{1}{(n+4)}$	Cancel terms.	M1A1
	$\sum_{r=1}^{n}\frac{1}{(r+2)(r+3)} = \frac{n}{4(n+4)}$	Find common denominator, dep on second M mark Cso (All M marks required)	dM1 A1cso
	$(a=4)$	Need not be shown explicitly	(5)
NB: 1	All marks can be awarded if work done with values 1,2,...*r* and then *r* replaced with *n*; if no replacement made, deduct final A mark.		
2	$\frac{1}{4}-\frac{1}{(n+4)}$ with **NO** other working gets M0M1A1M1A0 max		
(c)	$\sum_{r=15}^{30}\frac{1}{(r+3)(r+4)} = \frac{30}{4(30+4)} - \frac{14}{4(14+4)}$	Accept $n=30$ and $n=14$ only in their answer to (b) Must be subtracted	M1
	$=\frac{4}{153}$ oe (exact)	Exact answer $\frac{4}{153}$ implies method provided no incorrect work seen in (c).	A1
			(2)
ALT	Use the method of differences again, starting at $r=15$ and ending at $r=30$	Complete method	M1
	$=\frac{4}{153}$ oe (exact)	Correct answer	A1
			Total 8

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2. A transformation from the z-plane to the w-plane is given by

$$w = \frac{1 - \mathrm{i}z}{z}, \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto the line l in the w-plane.
Find an equation of the line l.

(4)

Question Number	Scheme	Notes	Marks
2	$z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$ used. Candidates may use any suitable letters.		
	$z = x \Rightarrow w = \dfrac{1-\mathrm{i}x}{x}$	Replaces at least one z with x ie indicate that $y = 0$ (may be done later)	M1
	$w = \dfrac{1}{x} - \mathrm{i}$ or $w = \dfrac{1-\mathrm{i}x}{x}$ oe	Reach this statement somewhere	A1
	$u + \mathrm{i}v = \dfrac{1}{x} - \mathrm{i}$	$w = u + \mathrm{i}v$ and equating real or imaginary parts to obtain either u or v in terms of x or just a (real) number	M1
	$v = -1$ oe $\left(u = \dfrac{1}{x}\right.$ need not be shown$\left.\right)$	$v = -1$ or $v + 1 = 0$ oe ie equation of the line	A1
NB	If $x + \mathrm{i}y$ has been used for z and then also for w allow M1A1M1A0 max.		
			(4)
ALT 1	$z = \dfrac{1}{w+\mathrm{i}} = \dfrac{1}{u+\mathrm{i}v+\mathrm{i}} = \dfrac{u-\mathrm{i}(v+1)}{u^2+(v+1)^2}$	Multiplies numerator and denominator by complex conjugate.	M1
		$\dfrac{u-\mathrm{i}(v+1)}{u^2+(v+1)^2}$	A1
	$(y = 0 \Rightarrow)\dfrac{(v+1)}{u^2+(v+1)^2} = 0 \Rightarrow v + 1 = 0$	Uses $y = 0$ and equates real or imaginary parts to obtain either u or v in terms of x or just a number	M1
		$v = -1$ or $v + 1 = 0$ oe	A1
NB 1	If $x + \mathrm{i}y$ has been used for z and then also for w allow M1A1M1A0 max.		
2.	M1A0M1A1 is possible		(4)
ALT 2	$\lvert z+\mathrm{i}\rvert = \lvert z-\mathrm{i}\rvert$		
	$\left\lvert\dfrac{1}{w+\mathrm{i}}+\mathrm{i}\right\rvert = \left\lvert\dfrac{1}{w+\mathrm{i}}-\mathrm{i}\right\rvert$	M1: Use of real line and attempt to substitute A1: Correct substitution	M1 A1
	$\left\lvert\dfrac{1+w\mathrm{i}\text{-}1}{w+\mathrm{i}}\right\rvert = \left\lvert\dfrac{1-w\mathrm{i}+1}{w+\mathrm{i}}\right\rvert$		
	$\lvert w\mathrm{i}\rvert = \lvert 2-w\mathrm{i}\rvert$	Common denominator and equate numerators	M1
	$\lvert w\rvert = \lvert w+2\mathrm{i}\rvert$	Equation of the line – any form accepted	A1
			(4)
ALT 3	$z = \dfrac{1}{w+\mathrm{i}}$		
	z lies on real axis $\Rightarrow \dfrac{1}{w+\mathrm{i}}$ is real	Re-arrange equation and state that $\dfrac{1}{w+\mathrm{i}}$ is real	M1
	$\Rightarrow w + \mathrm{i}$ is real	Deduce that $w + \mathrm{i}$ is real	A1
	$w = u + iv,\quad u + \mathrm{i}(v+1)$ is real	Replace w with $u + \mathrm{i}v$ (any letters inc $x + \mathrm{i}y$ allowed here)	M1

	$v+1=0$	Deduce equation of the line	A1
			(4)
ALT 4	Choose any 2 points on the real axis in the z-plane:		
	$z=a: w_a = \dfrac{1-\mathrm{i}a}{a}$	Any one point	M1
	$z=b: w_a = \dfrac{1-\mathrm{i}b}{b}$	Any two points	A1
	$w_a = \dfrac{1}{a}-\mathrm{i} \quad w_b = \dfrac{1}{b}-\mathrm{i}$	Simplify both	M1
	$v=-1$ oe	Any letter (inc y) allowed here	A1
NB	The work can be done using arguments to find the equation. If seen, send to review.		
			Total 4

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3. (a) By writing $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, show that

(i) $\sin\left(\frac{\pi}{12}\right) = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$

(ii) $\cos\left(\frac{\pi}{12}\right) = \frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right)$

(4)

(b) Hence find the exact values of z for which

$$z^4 = 4\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$$

Give your answers in the form $z = a + \mathrm{i}b$ where $a, b \in \mathbb{R}$

(5)

Question Number	Scheme	Notes	Marks
3(a)	$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ $\sin\frac{\pi}{12}=\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}-\frac{1}{2}\frac{\sqrt{2}}{2}$	Correct expansion for sine, including surd values for all 4 trig functions. $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ accepted	M1
(i)	$\sin\frac{\pi}{12}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ **	Completion to **given answer**: No errors seen, cso	A1cso
	$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$ $\cos\frac{\pi}{12}=\frac{1}{2}\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$	Correct expansion for cosine, including surd values for all 4 trig functions. $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ accepted OR other complete method eg using $\sin^2\theta+\cos^2\theta=1$	M1 NB A1 on e-PEN
(ii)	$\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$ **	Completion to **given answer**: No errors seen, cso	A1cso
			(4)
(b)	**Allow all marks using EXACT calculator values for the trig functions. Decimal answers qualify for M marks only.**		
	$z^4=4\left(\cos\left(2k\pi+\frac{\pi}{3}\right)+\mathrm{i}\sin\left(2k\pi+\frac{\pi}{3}\right)\right)$ OR $z^4=4\mathrm{e}^{\mathrm{i}\left(2k\pi+\frac{\pi}{3}\right)}$	Use a valid method to generate at least 2 roots (eg use of $2k\pi$ or rotate through $\frac{\pi}{2}$, multiply by I, symmetry)	M1
	$z=4^{\frac{1}{4}}\left(\cos\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)+\mathrm{i}\sin\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)\right)$ OR $z=4^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)}$	Application of de Moivre's theorem resulting in at least 1 root being found. ($4\to\sqrt{2}$ and arg divided by 4) $4^{\frac{1}{4}}$ or $\sqrt{2}$ accepted	M1
	$(k=0\to)$ $z=\sqrt{2}\left(\cos\frac{\pi}{12}+\mathrm{i}\sin\frac{\pi}{12}\right)$ or $\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{12}\right)}$ or $\frac{\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)$ or $\frac{1+\sqrt{3}}{2}+\mathrm{i}\frac{-1+\sqrt{3}}{2}$ oe	Any correct root (this is the most likely one if only one found) Can be in any exact form ($4^{\frac{1}{4}}$ or $\sqrt{2}$ oe) Can be unsimplified using results from (a) ie $\frac{\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)$ with $\frac{\sqrt{2}}{4}$ or $\frac{1}{2\sqrt{2}}$ or $4^{-\frac{3}{4}}$ oe Or simplified/calculator values ie $\frac{1+\sqrt{3}}{2}+\mathrm{i}\frac{-1+\sqrt{3}}{2}$	B1

	$\left\{(k=1\to)\, z=\sqrt{2}\left(\cos\frac{7\pi}{12}+\mathrm{i}\sin\frac{7\pi}{12}\right) \text{ or } \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{7\pi}{12}\right)}\right\}$ $=\frac{-\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right) \text{ or } \frac{1}{2}\left(1-\sqrt{3}\right)+\frac{\mathrm{i}}{2}\left(1+\sqrt{3}\right)$ $\left\{(k=2\to)\, z=\sqrt{2}\left(\cos\frac{13\pi}{12}+\mathrm{i}\sin\frac{13\pi}{12}\right) \text{ or } \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{13\pi}{12}\right)}\right\}$ $=\frac{-\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)-\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right) \text{ or } -\frac{1}{2}\left(1+\sqrt{3}\right)+\frac{\mathrm{i}}{2}\left(1-\sqrt{3}\right)$ $\left\{(k=3\to)\, z=\sqrt{2}\left(\cos\frac{19\pi}{12}+\mathrm{i}\sin\frac{19\pi}{12}\right) \text{ or } \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{19\pi}{12}\right)}\right\}$ $=\frac{\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)-\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right) \text{ or } \frac{1}{2}\left(-1+\sqrt{3}\right)-\frac{\mathrm{i}}{2}\left(1+\sqrt{3}\right)$	
	Two correct roots in form a + ib unsimplified or calculator values, must be exact surd form	A1
	All 4 correct roots in form a + ib unsimplified or calculator values must be exact surd form.	A1
		(5)
		Total 9

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4. Use algebra to find the set of values of x for which

$$|x^2 - 2| > 4x$$

(7)

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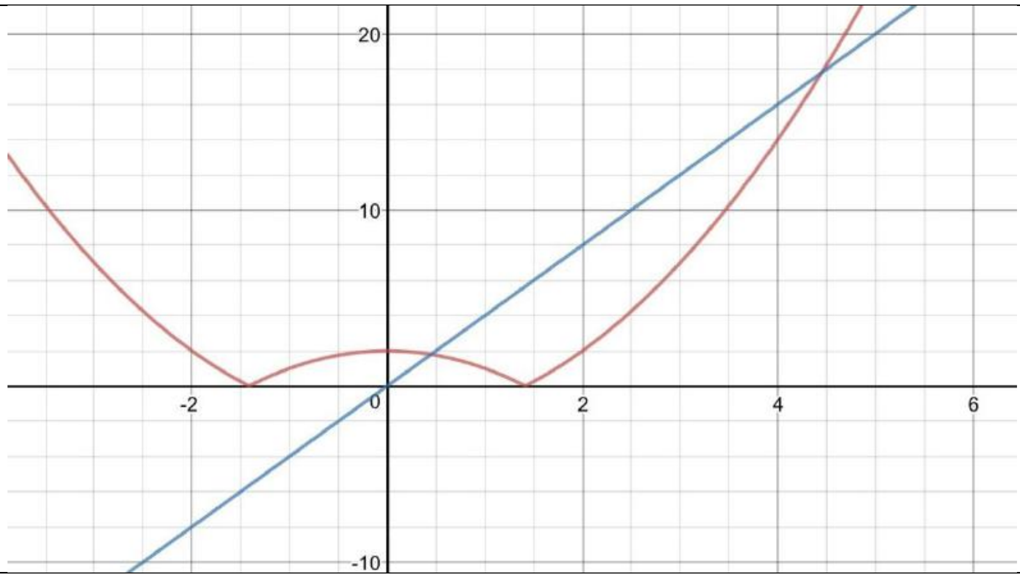
Question Number	Scheme	Notes	Marks
4.	$\lvert x^2 - 2 \rvert > 4x$		
			
	Note: Candidates may include a sketch such as the one shown at some point in their working. Please be aware that this sketch without algebra used to find the critical values merits **0 marks**. Marks may be only be awarded for the algebra used		
NB	**First 4 marks are available with =, > or < used**		
	$x^2 - 2 = 4x$ ①	Form 3 TQ **and** attempt to solve - may be implied by correct value(s) (allow decimals 4.449 ,-0.449..)	M1
	$x = 2 \pm \sqrt{6}$ or $2 + \sqrt{6}$	Correct exact values or value (NB: Corresponding 3TQ must have been seen)	A1
	$x^2 - 2 = -4x$ ②	Form 3 TQ **and** attempt to solve - may be implied by correct value(s) (allow decimals -4.449 ,0.449..))	M1
	$x = -2 \pm \sqrt{6}$ or $x = -2 + \sqrt{6}$	Correct exact values or value (NB: Corresponding 3TQ must have been seen)	A1
	$x >$ larger root of ① or $x <$ larger root of ②	Forms at least one of the required inequalities using their exact values Must be a strict inequality Depends on **either** previous M mark	dM1
	One of $x < -2 + \sqrt{6}$ or $x > 2 + \sqrt{6}$	Or exact equivalent	A1
	Both of $x < -2 + \sqrt{6}$ or $x > 2 + \sqrt{6}$	No others seen. Exact equivalents allowed Allow "or" or "and" but not $\cap$ if set notation used	A1

ALT	$(x^2-2)^2=16x^2$	Square both sides **and** attempt to solve quadratic in x^2 may be implied by correct value(s) (allow decimals 19.79... -0.202..)	M1
	$x^2=10\pm\sqrt{96}$	$x^2=10\pm4\sqrt{6}$ oe	A1
	$x=2\pm\sqrt{6}$ and $x=-2\pm\sqrt{6}$ $\left(x=2+\sqrt{6} \text{ and } x=-2+\sqrt{6} \text{ sufficient}\right)$	Valid attempt required to find exact form for x e.g. $(a+\sqrt{b})^2=10\pm\sqrt{96}$	M1A1
	$x >$ largest root or $x <$ 2nd largest root	As main scheme	dM1
	As main scheme	As main scheme	A1,A1
			Total 7

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5.

$$y\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y^2 = 0$$

Given that at $x = 0, y = 2$ and $\frac{dy}{dx} = 1$

(a) show that, at $x = 0$, $\frac{d^3y}{dx^3} = \frac{3}{2}$ **(6)**

(b) Find a series solution for y up to and including the term in x^3 **(3)**

Question Number	Scheme	Notes	Marks
5.	$y\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-3y^2=0$		
(a)	$y\frac{d^3y}{dx^3}+\frac{dy}{dx}\frac{d^2y}{dx^2}$	M1: Use of Product Rule on $y\frac{d^2y}{dx^2}$, 2 terms added with at least one term correct. A1: Fully correct derivative of $y\frac{d^2y}{dx^2}$	M1,A1
	$+3x\frac{d^2y}{dx^2}+3\frac{dy}{dx}$	Correct derivative of $3x\frac{dy}{dx}$	B1
	$-6y\frac{dy}{dx}$	oe.	B1
	At $x=0$, $2\frac{d^2y}{dx^2}+3(0)(1)-3(4)=0 \Rightarrow \frac{d^2y}{dx^2}=...$ **and** $2\frac{d^3y}{dx^3}+(1)(6)+3(1)-6(2)(1)=0 \Rightarrow \frac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\frac{dy}{dx}=1$ (must use these values) leading to numerical values for $\frac{d^2y}{dx^2}$ **and** $\frac{d^3y}{dx^3}$	M1
	$\frac{d^3y}{dx^3}=\frac{3}{2}$ **	**Given answer** cso	A1cso(6)
ALT 1	**Divide by *y* before differentiating:**		
(a)	$\frac{d^2y}{dx^2}+\frac{3x}{y}.\frac{dy}{dx}-3y=0$		
	$\left(\frac{3y-3x\frac{dy}{dx}}{y^2}\right)\frac{dy}{dx}+\frac{3x}{y}\times\frac{d^2y}{dx^2}$ oe	M1 Use of Product Rule on $\frac{3x}{y}\times\frac{dy}{dx}$, 2 terms added with at least one term correct A1 Correct derivative	M1A1
	$\frac{d^3y}{dx^3}$	oe	B1
	$-3\frac{dy}{dx}$	oe	B1
	$\frac{d^3y}{dx^3}+\left(\frac{3\times2-0}{2^2}\right)\times1+0\times\frac{d^2y}{dx^2}-3\times1 \to \frac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\frac{dy}{dx}=1$ (must use these values) leading to numerical value for $\frac{d^3y}{dx^3}$ (value for $\frac{d^2y}{dx^2}$ not needed)	M1
	$\frac{d^3y}{dx^3}=\frac{3}{2}$ **	**Given answer** cso	A1cso

ALT 2	**Re-arrange and divide by *y* before differentiating:**		
	$\frac{d^2y}{dx^2}=\frac{1}{y}\left(3y^2-3\frac{dy}{dx}x\right)$		
	$\frac{d^3y}{dx^3},=+\frac{1}{y}\left(6y\frac{dy}{dx}-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}\right),$ $-\frac{1}{y^2}\frac{dy}{dx}\left(3y^2-3x\frac{dy}{dx}\right)$	B1 $\frac{d^3y}{dx^3}$, M1 Differentiate using product rule. 2 terms added with at least one term correct A1: $+\frac{1}{y}\left(6y\frac{dy}{dx}-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}\right)$ B1 $-\frac{1}{y^2}\frac{dy}{dx}\left(3y^2-3x\frac{dy}{dx}\right)$	B1, M1A1, B1
	$\frac{d^3y}{dx^3}=\frac{1}{2}(6\times2\times1-3\times1-3\times0\times6)$ $-\frac{1}{4}\times1(3\times4-3\times0\times1)\Rightarrow\frac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\frac{dy}{dx}=1$ (must use these values) leading to numerical value for $\frac{d^3y}{dx^3}$ (value for $\frac{d^2y}{dx^2}$ not needed)	M1
	$\frac{d^3y}{dx^3}=\frac{3}{2}$ **	**Given answer** cso	A1cso
(b)	$(y=)\ 2+x$	Use the given values to form the first 2 terms of the series	B1
	$(y=)\ 2+x+\frac{6}{2!}x^2+\frac{\frac{3}{2}}{3!}x^3\ (+...)$	Find a numerical value for $\frac{d^2y}{dx^2}$ (may be seen in (a)) and use with the **given** value of $\frac{d^3y}{dx^3}$ to form the x^2 and x^3 terms of the series expansion	M1
	$y=\ 2+x+3x^2+\frac{1}{4}x^3\ (+...)$	Follow through their value of $\frac{d^2y}{dx^2}$ used correctly. Must start *y* = ... Allow f(*x*) **only** if this has been defined anywhere in the question to be equal to *y*	A1ft
			(3)
			Total 9

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6. (a) Find the general solution of the differential equation

$$6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = x - 6x^2$$

(8)

(b) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = \frac{3}{2}$ when $x = 0$

(5)

Question Number	Scheme	Notes	Marks
6.(a)	$6\frac{d^2y}{dx^2}+5\frac{dy}{dx}-6y=x-6x^2$		
	$6m^2+5m-6=0\Rightarrow(3m-2)(2m+3)=0$ $m=\frac{2}{3},\frac{-3}{2}$	M1 Forms and solves auxiliary equation A1 Correct roots	M1A1
	Complementary Function $Ae^{\frac{2}{3}x}+Be^{-\frac{3}{2}x}$	CF of the form shown formed using their 2 real roots Can be awarded if seen in gen solution	B1ft NB A1 on e-PEN
	Particular Integral $(y=)\ Cx^2+Dx+E$	May include higher powers	B1
	$\frac{dy}{dx}=2Cx+D,\frac{d^2y}{dx^2}=2C$	Differentiates their PI twice All powers of x to decrease by 1	M1
	$6(2C)+5(2Cx+D)-6(Cx^2+Dx+E)\equiv-6x^2+x$		
	$-6C=-6$ $10C-6D=1$ $12C+5D-6E=0$	Substitutes their derivatives into the equation and equates at least one pair of coefficients	M1
	$C=1$ $10-6D=1\Rightarrow D=\frac{3}{2}$ $12+5\left(\frac{3}{2}\right)-6E=0\Rightarrow E=\frac{13}{4}$	Attempt to solve 3 equations. Must reach a numerical value for all 3 coefficients	M1
	General Solution $y=Ae^{\frac{2}{3}x}+Be^{-\frac{3}{2}x}+x^2+\frac{3}{2}x+\frac{13}{4}$	Must start y = ... cao	A1
			(8)
(b)	$\frac{dy}{dx}=\frac{2}{3}Ae^{\frac{2}{3}x}-\frac{3}{2}Be^{-\frac{3}{2}x}+2x+\frac{3}{2}$	Differentiates their GS – min 4 terms in their GS	M1
	$y=0,\ \frac{dy}{dx}=\frac{3}{2},x=0$ $\quad 0=A+B+\frac{13}{4}$ $\frac{3}{2}=\frac{2}{3}A-\frac{3}{2}B+\frac{3}{2}$	Forms 2 simultaneous equations using given boundary values	M1
	$4A+4B=-13,4A-9B=0$,	Attempt to solve Must reach A = ... or B = ...	M1
	$A=-\frac{9}{4},B=-1$	Both correct	A1
	$y=x^2+\frac{3}{2}x+\frac{13}{4}-\frac{9}{4}e^{\frac{2}{3}x}-e^{-\frac{3}{2}x}$	Must start y = ...	A1 (5)
			Total 13

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7.

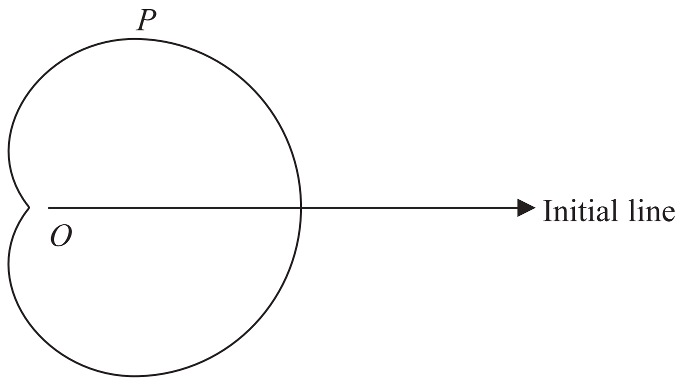

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \sqrt{3}\cos\theta, \qquad 0 \leqslant \theta < 2\pi$$

The tangent to C at the point P is parallel to the initial line.

(a) Show that $OP = \frac{1}{2}(3 + \sqrt{7})$

(6)

(b) Find the exact area enclosed by the curve C.

(6)

Question Number	Scheme	Notes	Marks
7. (a)	$r = 2+\sqrt{3}\cos\theta$		
Way 1	$y = r\sin\theta = 2\sin\theta + \sqrt{3}\cos\theta\sin\theta$	Multiplies r by $\sin\theta$	B1
	$\left(\frac{dy}{d\theta}\right) = 2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\sin^2\theta$	M1 Differentiates using product rule A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\left(1-\cos^2\theta\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = ...$	M1
	$\cos\theta = \frac{-2\pm\sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso
			(6)
Way 2	$y = r\sin\theta = (2+\sqrt{3}\cos\theta)\sin\theta$	Leaves y as a product	B1
	$\left(\frac{dy}{d\theta}\right) = (2+\sqrt{3}\cos\theta)\cos\theta - \sqrt{3}\sin\theta\sin\theta$	M1 Differentiates using product rule A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\left(1-\cos^2\theta\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = ...$	M1
	$\cos\theta = \frac{-2\pm\sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso
			(6)
Way 3	$y = r\sin\theta = 2\sin\theta + \frac{\sqrt{3}}{2}\sin 2\theta$	Uses a double angle formula	B1
	$\left(\frac{dy}{d\theta}\right) = 2\cos\theta + \sqrt{3}\cos 2\theta$	M1 Differentiates A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\left(2\cos^2\theta - 1\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use a double angle identity to form a 3TQ in $\cos\theta$. $\cos 2\theta = \left(2\cos^2\theta - 1\right)$ Attempt to solve their 3TQ. Reach $\cos\theta = ...$	M1
	$\cos\theta = \frac{-2\pm\sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1

	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso(6)
Way 4	$y = r\sin\theta$		
	$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -\sqrt{3}\sin\theta$	Correct derivative	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = \frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta$	Differentiate using product rule	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = -\sqrt{3}\sin^2\theta + \left(2+\sqrt{3}\cos\theta\right)\cos\theta$	Correct derivative as a function of θ	A1
	$-\sqrt{3}\left(1-\cos^2\theta\right) + 2\cos\theta + \sqrt{3}\cos^2\theta = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = ...$	M1
	$\cos\theta = \frac{-2\pm\sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or + Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso
			(6)
Special Case	$y = r\cos\theta$	**NOT** $x = r\cos\theta$	
	$r\cos\theta = 2\cos\theta + \sqrt{3}\cos^2\theta$		B0
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta} =\right) -2\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$	Differentiates Cannot obtain correct derivative	M1 A0
	No further marks available		

(b)	$\left(2+\sqrt{3}\cos\theta\right)^2 = 4+4\sqrt{3}\cos\theta+3\cos^2\theta$	Attempt to find r^2 as a 3 term quadratic and use a double angle formula $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$	M1
	$= 4+4\sqrt{3}\cos\theta+\frac{3}{2}(\cos 2\theta+1)$	Correct result	A1
	$\int r^2 \mathrm{d}\theta = 4\theta+4\sqrt{3}\sin\theta+3\left(\frac{1}{4}\sin 2\theta+\frac{1}{2}\theta\right)$ oe	dM1 Attempts to integrate their r^2 Depends on first M of (b) $\cos\theta \to \pm\sin\theta$ $\cos 2\theta \to \pm k\sin 2\theta$ $k=1$ or $\frac{1}{2}$ A1 Correct integral	dM1A1
	Check the integration carefully as the sine terms become 0 when limits substituted.		
	$\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta \quad = \frac{1}{2}(8\pi+3\pi-0)$	Substitutes correct limits in $\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta$ or $\left(2\times\frac{1}{2}\right)\int_0^{\pi} r^2 \mathrm{d}\theta$ or $\frac{1}{2}\int_{-\pi}^{\pi} r^2 \mathrm{d}\theta$	ddM1
	$= \frac{11\pi}{2}$	Correct answer must be exact Accept 5.5π No errors in the working	A1cso
			(6)
			Total 12
NB:	$\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{2\pi}\left(2+\sqrt{3}\cos\theta\right)^2 \mathrm{d}\theta = \frac{11}{2}\pi$	Integral evaluated on a calculator. Correct answer – send to review. Incorrect answer – 0/6	

Leave blank

8. (a) Using the substitution $t = x^2$, or otherwise, find

$$\int 2x^5 e^{-x^2}\,dx$$

(6)

(b) Hence find the general solution of the differential equation

$$x\frac{dy}{dx} + 4y = 2x^2 e^{-x^2}$$

giving your answer in the form $y = f(x)$.

(4)

Given that $y = 0$ when $x = 1$

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question Number	Scheme	Notes	Marks
8.	$\int 2x^5 e^{-x^2}\,dx$		
(a)	$t = x^2 \Rightarrow dt = 2x\,dx$ or $dx = \frac{1}{2}t^{-\frac{1}{2}}dt$ oe	May be implied by subsequent work	M1
	$\int 2x^5 e^{-x^2}\,dx = \int t^2 e^{-t}\,dt$	Integral in terms of t only required. dt may be implied. Must have attempted to change dx to dt (ie not just used dx = dt)	M1
	$= -t^2e^{-t} + 2\int te^{-t}\,dt$	Use of integration by parts. Reduce the power of t. Sign errors are allowed. $\int kt^p e^{-t} \to \pm kt^p e^{-t} \pm A\int t^{p-1}e^{-t}\,dt$	M1
	$= -t^2e^{-t} - 2te^{-t} + 2\int e^{-t}\,dt$	Use of integration by parts again in the same direction	dM1
	$= -t^2e^{-t} - 2te^{-t} - 2e^{-t}(+C)$ oe	Correct integration, constant not needed	A1
	$= -x^4e^{-x^2} - 2x^2e^{-x^2} - 2e^{-x^2}(+C)$ oe	Reverse substitution, constant not needed. **This mark cannot be recovered in (b)**	A1
			(6)
ALTs	Attempts without substitution which may merit part marks – send to review.		
	$x\frac{dy}{dx} + 4y = 2x^2e^{-x^2}$		
(b)	Integrating Factor $e^{\int \frac{4}{x}dx} = x^4$	Use of x^4 seen	B1
	$\frac{d}{dx}(x^4y) = 2x^5e^{-x^2}$ or $x^4y = \int 2x^5e^{-x^2}\,dx$	Multiply through by their IF	M1
	$x^4y = -x^4e^{-x^2} - 2x^2e^{-x^2} - 2e^{-x^2}(+C)$	Use their answer for (a), which must be a function of x, to integrate RHS	A1ft
	$y = -e^{-x^2} - \frac{2e^{-x^2}}{x^2} - \frac{2e^{-x^2}}{x^4} + \frac{C}{x^4}$	Complete to y = ... Include the constant and deal with it correctly **Not follow through**	A1
			(4)

Question Number	Scheme	Notes	Marks
ALT:	Use the same substitution as in (a) Following work uses the work shown in (a) rather than just the final answer. No marks until a first order exact equation in *y* and *t* reached and an attempt is made to solve this.		
	$t = x^2 \Rightarrow \mathrm{d}t = 2x\mathrm{d}x$ or $\mathrm{d}x = \frac{1}{2}t^{-\frac{1}{2}}\mathrm{d}t$,		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$ $x\frac{\mathrm{d}y}{\mathrm{d}x} = 2t\frac{\mathrm{d}y}{\mathrm{d}t}$ Equation becomes $2t\frac{\mathrm{d}t}{\mathrm{d}x} + 4y = 2t\mathrm{e}^{-t}$		
	Integrating Factor $\mathrm{e}^{\int \frac{2}{t}\mathrm{d}x} = t^2$	Use of t^2 seen	B1
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(t^2 y\right) = t^2\mathrm{e}^{-t}$ or $t^2 y = \int t^2\mathrm{e}^{-t}\mathrm{d}t$	Multiply through by IF	M1
	$t^2 y = -t^2\mathrm{e}^{-t} - 2t\mathrm{e}^{-t} - 2\mathrm{e}^{-t}(+C)$ oe	Use their work in (a) to integrate RHS	A1ft
	$y = -\mathrm{e}^{-x^2} - \frac{2\mathrm{e}^{-x^2}}{x^2} - \frac{2\mathrm{e}^{-x^2}}{x^4} + \frac{C}{x^4}$	Reverse the substitution Complete to *y* = ... Include the constant and deal with it correctly **Not follow through**	A1
			(4)
(c)	$0 = -\mathrm{e}^{-1} - 2\mathrm{e}^{-1} - 2\mathrm{e}^{-1} + C$	Attempt to substitute $x = 1, y = 0$ into their *y* provided it includes a constant	M1
	$\Rightarrow C = 5\mathrm{e}^{-1}$ oe	NB: **Not ft** so must have been obtained using a correct expression for *y*	A1
	$y = -\mathrm{e}^{-x^2} - \frac{2\mathrm{e}^{-x^2}}{x^2} - \frac{2\mathrm{e}^{-x^2}}{x^4} + \frac{5\mathrm{e}^{-1}}{x^4}$	Must start *y* = ... Follow through their *C* and expression for *y*	A1ft
			(3)
			Total 13
	Some common alternative forms for the answers: **NB:** *This list is not exhaustive.*		
(a)	1) $-x^4\mathrm{e}^{-x^2} - 2x^2\mathrm{e}^{-x^2} - 2\mathrm{e}^{-x^2}(+C)$ 2) $\mathrm{e}^{-x^2}\left(-x^4 - 2x^2 - 2\right)$ $(+C)$ 3) $-\mathrm{e}^{-x^2}\left(x^4 + 2x^2 + 2\right)$ $(+C)$ 4) $\frac{-\left(x^4 + 2x^2 + 2\right)}{\mathrm{e}^{x^2}}$ $(+C)$		

Question Number	Scheme	Notes	Marks
(b)	1) $y = -e^{-x^2} - \frac{2e^{-x^2}}{x^2} - \frac{2e^{-x^2}}{x^4} + \frac{C}{x^4}$ 2) $y = e^{-x^2}\left(-1 - \frac{2}{x^2} - \frac{2}{x^4}\right) + \frac{C}{x^4}$ 3) $y = -e^{-x^2}\left(1 + \frac{2}{x^2} + \frac{2}{x^4}\right) + \frac{C}{x^4}$ 4) $y = \frac{-\left(x^4 + 2x^2 + 2\right)}{x^4 e^{x^2}} + \frac{C}{x^4}$		
(c)	1) $y = -e^{-x^2} - \frac{2e^{-x^2}}{x^2} - \frac{2e^{-x^2}}{x^4} + \frac{5e^{-1}}{x^4}$ 2) $y = e^{-x^2}\left(-1 - \frac{2}{x^2} - \frac{2}{x^4}\right) + \frac{5e^{-1}}{x^4}$ 3) $y = -e^{-x^2}\left(1 + \frac{2}{x^2} + \frac{2}{x^4}\right) + \frac{5e^{-1}}{x^4}$ 4) $y = \frac{-\left(x^4 + 2x^2 + 2\right)}{x^4 e^{x^2}} + \frac{5e^{-1}}{x^4}$		