Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 6 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. Use algebra to find the set of values of x for which

$$\frac{1}{x-2} > \frac{2}{x}$$

(5)

June 2018
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1	$\frac{1}{x-2} > \frac{2}{x}$		
	$\frac{1}{x-2} - \frac{2}{x} > 0 \Rightarrow \frac{4-x}{x(x-2)} > 0$	Collect to one side and attempt common denominator of $x(x-2)$	M1
	$x = \underline{0,\ 2},\ \underline{\underline{4}}$	B1 for 0 **and** 2, A1 for 4	B1, A1
	$x < 0,\quad 2 < x < 4$ For their critical values α, β and γ in ascending order, attempts $x < \alpha$ and $\beta < x < \gamma$ condoning the use of a mixture of open or closed inequalities or For **one of** $x < 0$ **or** $2 < x < 4$ condoning the use of a mixture of open or closed inequalities		M1
	$x < 0,\quad 2 < x < 4$ $(-\infty, 0)$ or $[-\infty, 0),\ (2, 4)$	Correct inequalities. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$	A1
			(5)
			Total 5
	Alternative 1: $\times x^2(x-2)^2$		
	$x^2(x-2) > 2x(x-2)^2$		
	$x^2(x-2) - 2x(x-2)^2 > 0$		
	$x(x-2)(4-x) > 0$	$\times x^2(x-2)^2$ and attempt to factorise by taking out a factor of $x(x-2)$	M1
	$x = \underline{0,\ 2},\ \underline{\underline{4}}$	B1 for 0 **and** 2, A1 for 4	B1, A1
	Notes: $-x^3 + 6x^2 - 8x > 0$ with no other working is M0 $-x^3 + 6x^2 - 8x > 0 \Rightarrow x = 0,\ 2$ is M1B1 $-x^3 + 6x^2 - 8x > 0 \Rightarrow x = 0,\ 2,\ 4$ is M1B1A1		
	$x < 0,\quad 2 < x < 4$ For their critical values α, β and γ in ascending order, attempts $x < \alpha$ and $\beta < x < \gamma$ condoning the use of a mixture of open or closed inequalities **or** For **one of** $x < 0$ **or** $2 < x < 4$ condoning the use of a mixture of open or closed inequalities		M1
	$x < 0,\quad 2 < x < 4$ $(-\infty, 0)$ or $[-\infty, 0),\ (2, 4)$	Correct inequalities. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$	A1

	Alternative 2 : Considers regions		
	Case 1 $x<0 \Rightarrow x-2<0, x<0 \Rightarrow x(x-2)>0$ $\Rightarrow x>2(x-2) \Rightarrow x<0$		
	Case 2 $0<x<2 \Rightarrow x-2<0, x>0 \Rightarrow x(x-2)<0$ $\Rightarrow x<2(x-2) \Rightarrow x>4 \Rightarrow$ Contradiction		
	Case 3 $x>2 \Rightarrow x-2>0, x>0 \Rightarrow x(x-2)>0$ $\Rightarrow x>2(x-2) \Rightarrow x<4 \Rightarrow 2<x<4$ Contradiction		
	M1: Considers 3 regions as above **B1: *x* = 0 and 2 seen as critical values** **A1: *x* = 4 seen as a critical value**		
	$x<0,\ \ 2<x<4$ For their critical values α, β and γ in ascending order, attempts $x < \alpha$ and $\beta < x < \gamma$ condoning the use of a mixture of open or closed inequalities **or** For **one of** $x < 0$ **or** $2 < x < 4$ condoning the use of a mixture of open or closed inequalities		M1
	$x<0,\ \ 2<x<4$ $(-\infty, 0)$ or $[-\infty, 0)$, $(2, 4)$	Correct inequalities. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$	A1

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2. (a) Find the general solution of the differential equation

$$\left(x^2 + 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + xy - x = 0$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

(b) Find the particular solution for which $y = 2$ when $x = 3$

(2)

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Question Number	Scheme	Notes	Marks
2(a)	$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+xy-x=0$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{xy}{\left(1+x^2\right)}=\frac{x}{\left(1+x^2\right)}$	Correct form.	B1
	$I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{\frac{1}{2}\ln\left(1+x^2\right)}=\left(1+x^2\right)^{\frac{1}{2}}$	M1: $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{k\ln\left(1+x^2\right)}$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integrating factor of $\left(1+x^2\right)^{\frac{1}{2}}$	M1A1
	$y\left(1+x^2\right)^{\frac{1}{2}}=\int\frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}\mathrm{d}x$	Uses their integration factor to reach the form $yI=\int Q\,\mathrm{I}\,\mathrm{d}x$	M1
	$=\left(1+x^2\right)^{\frac{1}{2}}(+c)$	Correct integration (+ c not needed)	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao with the constant correctly placed. (The "$y=$" must appear at some point)	A1
			(6)
Way 2	**Alternative by separation of variables:**		
	$\int\frac{\mathrm{d}y}{1-y}=\int\frac{x}{x^2+1}\,\mathrm{d}x$	Separates variables correctly	B1
	$\int\frac{x}{x^2+1}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+1\right)$	M1: $\int\frac{x}{x^2+1}\,\mathrm{d}x=k\ln\left(x^2+1\right)$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integration $\frac{1}{2}\ln\left(x^2+1\right)$	M1A1
	$\int\frac{\mathrm{d}y}{1-y}=-\ln(1-y)$	$\int\frac{\mathrm{d}y}{1-y}=k\ln(1-y)$ or e.g. $\int\frac{\mathrm{d}y}{y-1}=k\ln(y-1)$	M1
	$-\ln(1-y)=\frac{1}{2}\ln\left(x^2+1\right)(+c)$	Fully correct integration	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao and isw if necessary.	A1
			(6)
(b)	$2=1+c\left(1+3^2\right)^{-\frac{1}{2}}\Rightarrow c=\ldots$	Substitutes $x=3$ and $y=2$ and attempts to find a value for c.	M1
	$(y=)1+\sqrt{10}\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao. ("$y=$" not needed for this mark) and apply isw if necessary.	A1
			(2)
			Total 8

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3.

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 1$$

(a) Show that

$$\frac{d^4y}{dx^4} = \frac{1}{2}\left(a\frac{dy}{dx} + bx\frac{d^2y}{dx^2} + c\frac{d^3y}{dx^3}\right)$$

where a, b and c are constants to be found.

(4)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 2$

(b) find a series solution for y in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^4$. Write each term in its simplest form.

(4)

(c) Use the solution to part (b) to find an approximate value for y when $x = 2.1$, giving your answer to 3 decimal places.

(2)

Question Number	Scheme	Notes	Marks
3	$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}-xy=1$		
(a)	$2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$	B1: $2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or equivalent correct terms if they rearrange the given equation. M1: Attempt product rule on xy. Allow sign errors only so need to see $\pm x\frac{\mathrm{d}y}{\mathrm{d}x}\pm y$	B1M1
	$2\frac{\mathrm{d}^4y}{\mathrm{d}x^4}+\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Differentiates again to obtain an expression that contains the fourth derivative including product rule on $x\frac{\mathrm{d}y}{\mathrm{d}x}$ to give $\pm x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\pm\frac{\mathrm{d}y}{\mathrm{d}x}$. (Allow terms to be "listed")	M1
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=\frac{1}{2}\left(2\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)$	If the "1" is not dealt with correctly e.g. if it "disappears" at the wrong time, this mark should be withheld.	A1
			(4)
(b)	$y''(2)=1,\ y'''(2)=1,\ y''''(2)=\frac{3}{2}$	M1: Attempt $y''(2)$, $y'''(2)$ and $y''''(2)$ A1: Correct values	M1A1
	$y=\mathrm{f}(2)+(x-2)\mathrm{f}'(2)+\frac{(x-2)^2\mathrm{f}''(2)}{2!}+\frac{(x-2)^3\mathrm{f}'''(2)}{3!}+\frac{(x-2)^4\mathrm{f}''''(2)}{4!}$ Attempt correct Taylor expansion **with their values.** Allow the terms to be "listed" for this mark.		M1
	$(y=)1+(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{6}+\frac{(x-2)^4}{16}$	Correct simplified expression.	A1
			(4)
(c)	$x=2.1\Rightarrow y=1+(0.1)+\frac{(0.1)^2}{2}+\frac{(0.1)^3}{6}+\frac{(0.1)^4}{16}$	Substitutes x = 2.1 into an expansion involving (x – 2)	M1
	y = 1.105 **only** Note this is **not** awrt.	Cao (Note that this mark must follow the final A1 in (b) i.e. 1.105 must come from a correct expansion). Incorrect answer with no working scores M0. Correct answer following a correct expansion scores M1A1.	A1
			(2)
			Total 10

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4. A complex number z is represented by the point P in an Argand diagram. Given that

$$|z + \mathrm{i}| = 1$$

(a) sketch the locus of P.

(2)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{3\mathrm{i}z - 2}{z + \mathrm{i}}, \qquad z \neq -\mathrm{i}$$

(b) Given that T maps $|z + \mathrm{i}| = 1$ to a circle C in the w-plane, find a cartesian equation of C.

(7)

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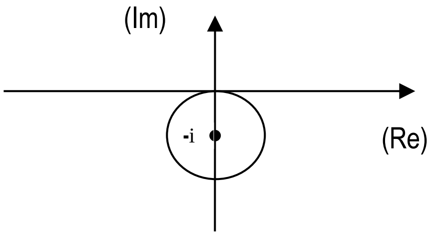
Question Number	Scheme	Notes	Marks
4(a)		M1: A circle anywhere. A1: A circle **correctly positioned** with centre –i or -1 marked in the correct place or (0, -1) or (-1, 0) or (0, -i) or (–i, 0) marked in the **correct place** and passing through (0, 0). The centre may be indicated away from the sketch but the sketch takes precedence. Ignore any shading.	M1A1
			(2)
(b) **Way 1**	$w = \frac{3iz-2}{z+i}$		
	$z = \frac{wi+2}{3i-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$z+i = \frac{wi+2}{3i-w} + i = \frac{wi+2-3-wi}{3i-w}$	Applies z + i and finds common denominator	M1
	$\left\|\frac{wi+2-3-wi}{3i-w}\right\| = 1$	M1: Sets $\|z+i\|=1$ A1: Correct equation, simplified or unsimplified	M1A1
	Note if they work with $w = u + iv$ they should reach $\left\|\frac{2-v+ui-ui-(3-v)}{-u+(3-v)i}\right\| = 1$ *		
	$\left\|\frac{-1}{3i-w}\right\| = 1 \Rightarrow \|w-3i\| = 1 \Rightarrow \|u+iv-3i\| = 1$ $\Rightarrow u^2 + (3-v)^2 = 1$ or equivalent e.g. $u^2 + (v-3)^2 = 1,\ u^2 + v^2 - 6v + 9 = 1$ **d**M1: Introduces u and v or x and y (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form **<u>This mark is dependent on all the previous method marks</u>** A1: Correct equation (allow u, v or x, y or a, b)		**d**M1A1
			(7)

<u>In part (b) apply the scheme that is most beneficial to the candidate.</u>

Way 2	$z=\dfrac{w\text{i}+2}{3\text{i}-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$z=\dfrac{(u+\text{i}v)\text{i}+2}{3\text{i}-(u+\text{i}v)}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}\times\dfrac{-u-(3-v)\text{i}}{-u-(3-v)\text{i}}$ Introduces u + iv and multiplies numerator and denominator by the complex conjugate of the denominator		M1
	$z+\text{i}=\dfrac{u+(5v-6-u^2-v^2)\text{i}+(u^2+v^2+9-6v)\text{i}}{u^2+(3-v)^2}\left(=\dfrac{u+(3-v)\text{i}}{u^2+(3-v)^2}\right)$ M1: Applies z + i and finds a common denominator A1: Correct expression (simplified or unsimplified) but with no i's in the denominator		M1A1
	$\lvert z+\text{i}\rvert=1\Rightarrow\left\lvert\dfrac{u+(3-v)\text{i}}{u^2+(3-v)^2}\right\rvert=1\Rightarrow\dfrac{\sqrt{u^2+(3-v)^2}}{u^2+(3-v)^2}=1$ oe **d**M1: Introduces u and v or x and y (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form which may be unsimplified **This mark is dependent on all the previous method marks** A1: Correct equation (allow u, v or x, y or a, b)		**d**M1A1
			(7)

Way 3	$z=\dfrac{w\text{i}+2}{3\text{i}-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$z=\dfrac{(u+\text{i}v)\text{i}+2}{3\text{i}-(u+\text{i}v)}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}=\dfrac{(2-v)+u\text{i}}{-u+(3-v)\text{i}}\times\dfrac{-u-(3-v)\text{i}}{-u-(3-v)\text{i}}$ Introduces u + iv and multiplies numerator and denominator by the complex conjugate of the denominator		M1
	$z=\dfrac{u+(-u^2-v^2+5v-6)\text{i}}{u^2+(3-v)^2}\Rightarrow x=\dfrac{u}{u^2+(3-v)^2}\quad y=-\dfrac{u^2+(v-3)^2+v-3}{u^2+(3-v)^2}$ M1: Obtains x and y in terms of u and v A1: Correct equations		M1A1
	$x^2+(y+1)^2=1\Rightarrow\dfrac{u^2+(v-3)^2}{\left(u^2+(v-3)^2\right)^2}=1$ oe	**d**M1: Uses $\lvert z+\text{i}\rvert=1$ to find an equation connecting u and v **This mark is dependent on all the previous method marks** A1: Correct equation which may be unsimplified.	**d**M1A1
			(7)

Way 4	$w=\frac{3\mathrm{i}z-2}{z+\mathrm{i}}$		
	$z=\frac{w\mathrm{i}+2}{3\mathrm{i}-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$z+\mathrm{i}=\frac{w\mathrm{i}+2}{3\mathrm{i}-w}+\mathrm{i}=\frac{w\mathrm{i}+2-3-w\mathrm{i}}{3\mathrm{i}-w}$	Applies z + i and finds common denominator	M1
	$z+\mathrm{i}=\frac{-1}{3\mathrm{i}-u-\mathrm{i}v}\times\frac{u-(v-3)\mathrm{i}}{u-(v-3)\mathrm{i}}=\frac{u-(v-3)\mathrm{i}}{u^2+(v-3)^2}$ $\Rightarrow\left\|\frac{u-(v-3)\mathrm{i}}{u^2+(v-3)^2}\right\|=1$ M1: Multiplies numerator and denominator by the complex conjugate of the denominator and sets = 1 A1: Correct equation with no i's in the denominator		M1A1
	$\frac{\sqrt{u^2+(3-v)^2}}{u^2+(3-v)^2}=1$ oe **d**M1: Introduces u and v or x and y (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form which may be unsimplified **<u>This mark is dependent on all the previous method marks</u>** A1: Correct equation (allow u, v or x, y or a, b)		**d**M1A1
			(7)

Way 5	$w=\frac{3\mathrm{i}z-2}{z+\mathrm{i}}$		
	$u+\mathrm{i}v=\frac{3\mathrm{i}(x+\mathrm{i}y)-2}{x+\mathrm{i}y+\mathrm{i}}=\frac{(3\mathrm{i}x-3y-2)(x-(y+1)\mathrm{i})}{x^2+(y+1)^2}$	M1: Substitutes for z and $\times\frac{x-(y+1)\mathrm{i}}{x-(y+1)\mathrm{i}}$ A1: Correct expression	M1A1
	$=\frac{x+\left(3\left(x^2+(y+1)^2\right)-y-1\right)\mathrm{i}}{x^2+(y+1)^2}$	Express rhs in terms of $x^2+(y+1)^2$	M1
	$x^2+(y+1)^2=1\Rightarrow w=x+(2-y)\mathrm{i}$	M1: Use of $\|z+\mathrm{i}\|=1$ A1: $w=x+(2-y)\mathrm{i}$	M1A1
	$x^2+(y+1)^2=1\Rightarrow u^2+(v-3)^2=1$	**d**M1: Attempts equation connecting u and v **<u>This mark is dependent on all the previous method marks</u>** A1: $u^2+(v-3)^2=1$ oe	**d**M1A1
			(7)

Way 6	$w=\frac{3iz-2}{z+i}=\frac{3i(z+i)+1}{z+i}=3i+\frac{1}{z+i}$	M1: Attempt rhs in terms of z + i A1: Correct rearrangement oe	M1A1
	$w-3i=\frac{1}{z+i}$	Isolates z + i	M1
	$\|w-3i\|=\left\|\frac{1}{z+i}\right\|=\frac{1}{\|z+i\|}=1$	M1: Applies $\|z+i\|=1$ A1: Correct equation	M1A1
	$\|w-3i\|=1\Rightarrow u^2+(v-3)^2=1$	**d**M1: Introduces u and v or x and y and uses Pythagoras correctly to find a Cartesian form **<u>This mark is dependent on all the previous method marks</u>** A1: $u^2+(v-3)^2=1$ oe	**d**M1A1
			(7)

Way 7	$z=\frac{wi+2}{3i-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$\|w\|=\left\|\frac{3iz-2}{z+i}\right\|=\|3iz-2\|$	Uses $\|w\|=\left\|\frac{3iz-2}{z+i}\right\|$ and $\|z+i\|=1$	M1
	$\|w\|=\left\|3i\left(\frac{wi+2}{3i-w}\right)-2\right\|=\left\|\frac{-3w+6i-6i+2w}{3i-w}\right\|$	M1: Attempts common denominator A1: Correct equation	M1A1
	$\|w-3i\|=1\Rightarrow u^2+(v-3)^2=1$	**d**M1: Introduces u and v or x and y and uses Pythagoras correctly to find a Cartesian form **<u>This mark is dependent on all the previous method marks</u>** A1: $u^2+(v-3)^2=1$ oe	**d**M1A1
			(7)

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5. (a) Express $\dfrac{4r+2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence, using the method of differences, prove that

$$\sum_{r=1}^{n} \frac{4r+2}{r(r+1)(r+2)} = \frac{n(an+b)}{2(n+1)(n+2)}$$

where a and b are constants to be found.

(5)

Question Number	Scheme	Notes	Marks
5(a)	$\dfrac{4r+2}{r(r+1)(r+2)}$		
	$\dfrac{1}{r}+\dfrac{2}{(r+1)}-\dfrac{3}{(r+2)}$	M1: Correct partial fractions method e.g. substitution or compares coefficients to obtain one of *A*, *B* or *C* for $\dfrac{A}{r}, \dfrac{B}{(r+1)}, \dfrac{C}{(r+2)}$ A1: 2 Correct fractions (or values) A1: All correct (fractions or values)	M1A1 A1
	Correct answer with no working scores full marks in (a)		
			(3)
(b)	**Must have partial fractions of the form** $\dfrac{A}{r}, \dfrac{B}{(r+1)}, \dfrac{C}{(r+2)}$ ***A*, *B*, *C*** $\neq 0$ **to score the <u>first</u> M mark in (b)**		
	$\sum_{r=1}^{n} = \left(\dfrac{1}{1}+\dfrac{2}{2}-\dfrac{3}{3}\right)+\left(\dfrac{1}{2}+\dfrac{2}{3}-\dfrac{3}{4}\right)+\ldots$ $\ldots+\left(\dfrac{1}{n-1}+\dfrac{2}{n}-\dfrac{3}{n+1}\right)+\left(\dfrac{1}{n}+\dfrac{2}{n+1}-\dfrac{3}{n+2}\right)$ Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their fractions identified below**. Allow other letters for *n* (most likely to be *r*) except for the final mark – see below If terms are found beyond the limits of the summation e.g. *r* = 0, *r* = *n* + 1, these can be ignored for this mark as long as at least the terms for *r* = 1, 2, *n* – 1 and *n* are seen		M1
	$=\dfrac{1}{1}+\dfrac{2}{2}+\dfrac{1}{2}-\dfrac{3}{n+1}+\dfrac{2}{n+1}-\dfrac{3}{n+2}$	A1: $\dfrac{1}{1}+\dfrac{2}{2}+\dfrac{1}{2}\left(=\dfrac{5}{2}\right)$ identified as the only constant terms A1: $-\dfrac{3}{n+1}+\dfrac{2}{n+1}-\dfrac{3}{n+2}$ oe e.g $-\dfrac{1}{n+1}-\dfrac{1}{n+2}-\dfrac{2}{n+2}$ identified as the only algebraic terms	A1 A1
	$=\dfrac{5(n^2+3n+2)-2(n+2)-6(n+1)}{2(n+1)(n+2)}$	Attempt common denominator from terms of the form $A, \dfrac{B}{n+1}, \dfrac{C}{n+2}$ only. Must see (*n* + 1)(*n* + 2) in the denominator and an unsimplified polynomial of order 2 in the numerator.	M1
	$\dfrac{n(5n+7)}{2(n+1)(n+2)}$	Must be in terms of *n* for this mark.	A1
			(5)
			Total 8

<table>
<tr><td></td><td colspan="2">Alternative for (b)</td><td></td></tr>
<tr><td></td><td colspan="2">$$\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\left(\frac{1}{r}-\frac{1}{r+2}\right)+2\left(\frac{1}{r+1}-\frac{1}{r+2}\right)$$
$$\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+2}\right)=\frac{1}{1}-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+...+\frac{1}{n-1}-\frac{1}{n+1}+\frac{1}{n}-\frac{1}{n+2}=1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$$
$$2\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)=\frac{1}{2}-\frac{1}{3}+...+\frac{1}{n}-\frac{1}{n+2}=\frac{1}{2}-\frac{1}{n+2}$$
Re-writes their partial fractions correctly and attempts at least 2 groups of terms at start and end for first sum and 1 group at the start and end for the second sum</td><td>M1</td></tr>
<tr><td></td><td rowspan="2">$$\sum_{r=1}^{n}=\frac{5}{2}-\frac{1}{n+1}-\frac{3}{n+2}$$</td><td>A1: $\frac{1}{1}+\frac{2}{2}+\frac{1}{2}\left(=\frac{5}{2}\right)$ identified as the only constant terms</td><td rowspan="2">A1A1</td></tr>
<tr><td></td><td>A1: A1: $-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$
oe e.g $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ identified as the only algebraic terms</td></tr>
<tr><td></td><td>$$=\frac{5\left(n^2+3n+2\right)-2(n+2)-6(n+1)}{2(n+1)(n+2)}$$</td><td>Attempt common denominator from terms of the form $A,\frac{B}{n+1},\frac{C}{n+2}$ only.
Must see $(n + 1)(n + 2)$ in the denominator and an unsimplified polynomial of order 2 in the numerator.</td><td>M1</td></tr>
<tr><td></td><td>$$\frac{n(5n+7)}{2(n+1)(n+2)}$$</td><td>Must be in terms of n for this mark.</td><td>A1</td></tr>
</table>

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6. (a) Show that the transformation $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^{2t} \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II), expressing y as a function of t.

(6)

(c) Hence find the general solution of the differential equation (I).

(1)

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Question Number	Scheme	Notes	Marks
6	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-3x\dfrac{\mathrm{d}y}{\mathrm{d}x}+3y=x^2$		
(a)	$x=\mathrm{e}^t \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\dfrac{\mathrm{d}t}{\mathrm{d}y} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\dfrac{\mathrm{d}y}{\mathrm{d}t}$	M1: Attempt first derivative using the chain rule to obtain $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\dfrac{\mathrm{d}t}{\mathrm{d}y}$	M1A1
		A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\dfrac{\mathrm{d}y}{\mathrm{d}t}$ oe	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=x^{-1}\dfrac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-x^{-2}\dfrac{\mathrm{d}y}{\mathrm{d}t}+x^{-1}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}.\dfrac{\mathrm{d}t}{\mathrm{d}x}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$x^2\left(\dfrac{1}{x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)-3x\left(\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)+3y=\left(\mathrm{e}^t\right)^2$	Substitutes their $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t into the differential equation	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\dfrac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	cso	A1
			(6)
	Alternative		
	$x=\mathrm{e}^t \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^t\dfrac{\mathrm{d}y}{\mathrm{d}x}=x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1: Attempt first derivative using $\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{\mathrm{d}x}{\mathrm{d}t}\times\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1A1
		A1: $\dfrac{\mathrm{d}y}{\mathrm{d}t}=x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ oe	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}=\dfrac{\mathrm{d}x}{\mathrm{d}t}\dfrac{\mathrm{d}y}{\mathrm{d}x}+x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}.\dfrac{\mathrm{d}x}{\mathrm{d}t}=x\dfrac{\mathrm{d}y}{\mathrm{d}x}+x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-x\dfrac{\mathrm{d}y}{\mathrm{d}x}-3x\dfrac{\mathrm{d}y}{\mathrm{d}x}+3y=\mathrm{e}^{2t}$ $=\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-\dfrac{\mathrm{d}y}{\mathrm{d}t}-3\dfrac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	Substitutes their $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t into the differential equation	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\dfrac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	Cso	A1
			(6)

(b)	$m^2-4m+3=0\Rightarrow m=1,\ 3$	Solves (according to the General Guidance) the correct quadratic (so should be m = ±1, ±3)	M1
	$(y=)\,A\mathrm{e}^{3t}+B\mathrm{e}^{t}$	Correct CF in terms of t not x. (May be seen later in their GS)	A1
	$y=k\mathrm{e}^{2t},\ y'=2k\mathrm{e}^{2t},\ y''=4k\mathrm{e}^{2t}$	Correct form for PI and differentiates twice to obtain multiples of e^{2t} each time but do not allow if they are clearly integrating.	M1
	$4k\mathrm{e}^{2t}-8k\mathrm{e}^{2t}+3k\mathrm{e}^{2t}=\mathrm{e}^{2t}\Rightarrow k=...$	Substitutes their $y,\ y',\ y''$ that are of the form $\alpha\mathrm{e}^{2t}$ into the differential equation and sets = e^{2t} and proceeds to find their k	M1
	$(y)=-\mathrm{e}^{2t}$	Correct PI or k = -1	A1
	$y=A\mathrm{e}^{3t}+B\mathrm{e}^{t}-\mathrm{e}^{2t}$	Correct ft GS **in terms of *t*** (their CF + their PI with non-zero PI). **Must be *y* = …**	B1ft
			(6)
(c)	$(y=)\,Ax^3+Bx-x^2$	Allow equivalent expressions in terms of x e.g. $(y=)\,A\mathrm{e}^{3\ln x}+B\mathrm{e}^{\ln x}-\mathrm{e}^{2\ln x}$. **Note that *y* = … is not needed here.**	B1
			(1)
			Total 13

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7. (a) Use de Moivre's theorem to show that

$$\cos 7\theta \equiv 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$$

(6)

(b) Hence find the four distinct roots of the equation

$$64x^7 - 112x^5 + 56x^3 - 7x + 1 = 0$$

giving your answers to 3 decimal places where necessary.

(5)

Question Number	Scheme	Notes	Marks
7(a)	$(\cos\theta + \mathrm{i}\sin\theta)^7 = \cos^7\theta + \binom{7}{1}\cos^6\theta\mathrm{i}\sin\theta + \binom{7}{2}\cos^5\theta(\mathrm{i}\sin\theta)^2 + ...$ Attempts to expand $(\cos\theta + i\sin\theta)^7$ including a recognisable attempt at binomial coefficients (May only see real terms)		M1
	$(\cos 7\theta =) c^7 + {}^7C_2 c^5 \mathrm{i}^2 s^2 + {}^7C_4 c^3 \mathrm{i}^4 s^4 + {}^7C_6 c \mathrm{i}^6 s^6$ Identifies real terms with $\cos 7\theta$		M1
	$= c^7 - 21c^5 s^2 + 35c^3 s^4 - 7cs^6$	Correct expression with coefficients evaluated and i's dealt with correctly	A1
	$= c^7 - 21c^5(1-c^2) + 35c^3(1-c^2)^2 - 7c(1-c^2)^3$	Replaces $\sin^2\theta$ with $1-\cos^2\theta$ used anywhere in their expansion.	M1
	$= 22c^7 - 21c^5 + 35c^3(1-2c^2+c^4) - 7c(1-3c^2+3c^4-c^6)$ Applies the expansions of $(1-\cos^2\theta)^2$ and $(1-\cos^2\theta)^3$ to their expression		M1
	$= 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$*	Correct expression obtained with no errors	A1
	Useful intermediate expression: $= 22c^7 - 21c^5 + 35c^3 - 70c^5 + 35c^7 - 7c + 21c^2 - 21c^5 + 7c^7$		
			(6)

	Alternative 1 for (a):	
	$\left(z+\frac{1}{z}\right)^7 = z^7 + \binom{7}{1}z^6\frac{1}{z} + \binom{7}{2}z^5\frac{1}{z^2} + ...$ Attempts to expand $\left(z+\frac{1}{z}\right)^7$ including binomial coefficients	M1
	$\left(z+\frac{1}{z}\right)^7 = z^7 + \frac{1}{z^7} + 7\left(z^5+\frac{1}{z^5}\right) + 21\left(z^3+\frac{1}{z^3}\right) + 35\left(z+\frac{1}{z}\right)$ $(2\cos\theta)^7 = 2\cos 7\theta + 7(2\cos 5\theta) + 21(2\cos 3\theta) + 35(2\cos\theta)$ M1: Uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ at least once (including n = 1) A1: Correct expression in terms of cos	M1A1
	$128\cos^7\theta = 2\cos 7\theta + 14(16\cos^5\theta - 20\cos^3\theta + 5\cos\theta) + 42(4\cos^3\theta - 3\cos\theta) + 70\cos\theta$ M1: Correct method to find $\cos 5\theta$ in terms of $\cos\theta$ and applies this to their expression M1: Correct method to find $\cos 3\theta$ in terms of $\cos\theta$ and applies this to their expression	M1M1
	$\cos 7\theta = 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$*	A1

	Alternative 2 for (a):	
	$\left(z+\frac{1}{z}\right)^7 = z^7 + \binom{7}{1}z^6\frac{1}{z} + \binom{7}{2}z^5\frac{1}{z^2} + ...$ Attempts to expand $\left(z+\frac{1}{z}\right)^7$ including binomial coefficients	M1
	$\left(z+\frac{1}{z}\right)^7 = z^7 + \frac{1}{z^7} + 7\left(z^5+\frac{1}{z^5}\right) + 21\left(z^3+\frac{1}{z^3}\right) + 35\left(z+\frac{1}{z}\right)$	
	$z^7 + \frac{1}{z^7} = 2\cos 7\theta = \left(z+\frac{1}{z}\right)^7 - 7\left(z^5+\frac{1}{z^5}\right) - 21\left(z^3+\frac{1}{z^3}\right) - 35\left(z+\frac{1}{z}\right)$ M1: Identifies that $z^7 + \frac{1}{z^7} = 2\cos 7\theta$ A1: Correct expression for $2\cos 7\theta$ in terms of z	M1A1
	$2\cos 7\theta = 128\cos^7\theta - 7\left(z^5+\frac{1}{z^5}\right) - 21\left(z^3+\frac{1}{z^3}\right) - 35\left(z+\frac{1}{z}\right)$ Starts the process of replacing $\left(z+\frac{1}{z}\right)^n$ with $(2\cos\theta)^n$	M1
	$= 128\cos^7\theta - 7(2\cos\theta)^5 + 14\left(z^3+\frac{1}{z^3}\right) + 35\left(z+\frac{1}{z}\right)$	
	$= 128\cos^7\theta - 7(2\cos\theta)^5 + 14(2\cos\theta)^3 - 7\left(z+\frac{1}{z}\right)$	
	$= 128\cos^7\theta - 7(2\cos\theta)^5 + 14(2\cos\theta)^3 - 14\cos\theta$ Reaches an expression in terms of cos only	M1
	$\cos 7\theta = 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$	A1

(b)	$\cos 7\theta + 1 = 0 \Rightarrow \cos 7\theta = -1$	$\cos 7\theta = -1$ ($\cos 7x = -1$ is B0)	B1
	$7\theta = \pm 180, \pm 540, \pm 900, \pm 1260, ...$ or $7\theta = \pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, ...$	At least one correct value for 7θ. Condone the use of $7x$ here.	M1
	$\theta = \pm\frac{180}{7}, \pm\frac{540}{7}, \pm\frac{900}{7}, \pm\frac{1260}{7}, ... \Rightarrow \cos\theta = ...$ or $\theta = \pm\frac{\pi}{7}, \pm\frac{3\pi}{7}, \pm\frac{5\pi}{7}, \pm\frac{7\pi}{7}, ... \Rightarrow \cos\theta = ...$	Divides by 7 and attempts at least one value for $\cos\theta$. Condone the use of x for θ here.	M1
	$x = \cos\theta = 0.901, 0.223, -1, -0.623$	A1: Awrt 2 correct values for x A1: Awrt all 4 x values correct and no extras	A1A1
			(5)
			Total 11

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8.

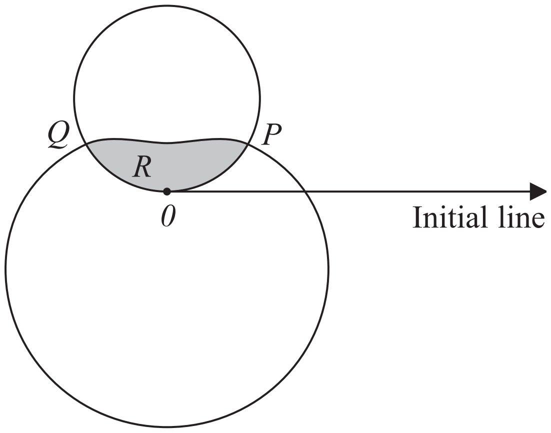

Figure 1

Figure 1 shows a sketch of the curves with polar equations

$$r = 2\sin\theta \qquad 0 \leqslant \theta \leqslant \pi$$
$$r = 1.5 - \sin\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

The curves intersect at the points P and Q.

(a) Find the polar coordinates of the point P and the polar coordinates of the point Q.
(3)

The region R, shown shaded in Figure 1, is enclosed by the two curves.

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers to be found.
(8)

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Question Number	Scheme	Notes	Marks
8(a)	$2\sin\theta = 1.5 - \sin\theta \Rightarrow \theta = ...$ or $\sin\theta = \frac{r}{2} \Rightarrow r = 1.5 - r \Rightarrow r = ...$	Equate and attempt to solve for θ or Eliminates $\sin\theta$ and solves for r	M1
	$P\left(1, \frac{\pi}{6}\right)$	Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 0.524 for $\pi/6$	A1
	$Q\left(1, \frac{5\pi}{6}\right)$	Correct coordinates. Allow the marks as soon as the correct values are seen and allow coordinates the wrong way round and allow awrt 2.62 for $5\pi/6$	A1
			(3)

(b)	$\left(\frac{1}{2}\right)\int(1.5-\sin\theta)^2\mathrm{d}\theta$ or $\left(\frac{1}{2}\right)\int(2\sin\theta)^2\mathrm{d}\theta$ Attempts to use $...\int(\sin\theta)^2\mathrm{d}\theta$ or $...\int(1.5-\sin\theta)^2\mathrm{d}\theta$	M1
	$(1.5-\sin\theta)^2 = 2.25-3\sin\theta+\sin^2\theta = 2.25-3\sin\theta+\frac{(1-\cos 2\theta)}{2}$ Expands (allow poor squaring e.g. $(1.5-\sin\theta)^2 = 2.25+\sin^2\theta$ **and** attempts to use $\sin^2\theta = \pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$	M1
	$\frac{1}{2}\int(1.5-\sin\theta)^2\mathrm{d}\theta = \frac{1}{2}\left[\frac{11}{4}\theta+3\cos\theta-\frac{1}{4}\sin 2\theta\right]$ M1: Attempt to integrate and reaches an expression of the form $\alpha\theta+\beta\cos\theta+\gamma\sin 2\theta$ A1: Correct integration (with or without the ½)	M1A1
	$\frac{1}{2}[\ ]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{1}{2}\left\{\left(\frac{11}{4}.\frac{5\pi}{6}+3.\cos\frac{5\pi}{6}-\frac{1}{4}\sin 2.\frac{5\pi}{6}\right)-\left(\frac{11}{4}.\frac{\pi}{6}+3.\cos\frac{\pi}{6}-\frac{1}{4}\sin 2.\frac{\pi}{6}\right)\right\}$ This is a key step and must be the correct method for this part of the area e.g. uses their $\frac{\pi}{6}$ and their $\frac{5\pi}{6}$ (or twice limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$)	M1
	$\frac{1}{2}\int(2\sin\theta)^2\mathrm{d}\theta = \int(1-\cos 2\theta)\mathrm{d}\theta = \left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)(-0)$ Uses the limits 0 and their $\frac{\pi}{6}$ to find at least one segment. If using integration, must have integrated to obtain $p\theta+q\sin 2\theta$ with correct use of limits NB can be done as: $\frac{1}{2}(1)^2\left(\frac{\pi}{3}\right)-\frac{1}{2}(1)^2\sin\left(\frac{\pi}{3}\right)$ but must be correct work for their angles	M1
	$\frac{11}{12}\pi-\frac{11\sqrt{3}}{8}+2\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right) = \frac{5}{4}\pi-\frac{15}{8}\sqrt{3}$ **dd**M1: Adds their two areas to give a numerical value for the shaded area Dependent on the previous 2 M marks and must be a completely correct strategy so needs to be an attempt at: $\frac{1}{2}\int_{"\frac{\pi}{6}"}^{"\frac{5\pi}{6}"}(1.5-\sin\theta)^2\mathrm{d}\theta$ or $2\times\frac{1}{2}\int_{"\frac{\pi}{6}"}^{\frac{\pi}{2}}(1.5-\sin\theta)^2\mathrm{d}\theta$ + $2\times\frac{1}{2}\int_0^{"\frac{\pi}{6}"}(2\sin\theta)^2\mathrm{d}\theta$ or $\left(\frac{1}{2}\int_0^{"\frac{\pi}{6}"}(2\sin\theta)^2\mathrm{d}\theta+\frac{1}{2}\int_{"\frac{5\pi}{6}"}^{\pi}(2\sin\theta)^2\mathrm{d}\theta\right)$ A1: Correct answer (allow equivalent fractions)	**dd**M1A1
		(8)
		Total 11

	Note that attempts to use $\left(\frac{1}{2}\right)\int(C_1-C_2)^2\mathrm{d}\theta$ e.g. $\left(\frac{1}{2}\right)\int\left(2\sin\theta-(1.5-\sin\theta)\right)^2\mathrm{d}\theta$ Will probably only score a maximum of the first 3 marks i.e. M1 for $\left(\frac{1}{2}\right)\int\left(2\sin\theta-(1.5-\sin\theta)\right)^2\mathrm{d}\theta$ M1 for expanding **and** attempting to use $\sin^2\theta=\pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$ M1 for attempting to integrate and reaching an expression of the form $\alpha\theta+\beta\cos\theta+\gamma\sin 2\theta$	M1