Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 7 June 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. Solve the equation

$$z^5 = 32$$

Give your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$

(5)

Question Number	Scheme	Notes	Marks
1	$2(\cos 0 + \mathrm{i}\sin 0)$ or 2	$(z =)\ 2$ or $(z =)\ 2(\cos 0 + \mathrm{i}\sin 0)$ or $2\cos 0 + \mathrm{i}\sin 0$ or $2 + 0\mathrm{i}$. Allow $2(\cos 0\pi + \mathrm{i}\sin 0\pi)$	B1
	$2\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$	This answer in this form. Do not allow e.g. $2\mathrm{e}^{\frac{2\pi}{5}\mathrm{i}}$ but allow $2\cos\frac{2\pi}{5} + 2\mathrm{i}\sin\frac{2\pi}{5}$	B1
	$2\left(\cos\frac{2k\pi}{5} + \mathrm{i}\sin\frac{2k\pi}{5}\right), (k = 2, 3, 4)$	Attempts at least 2 more solutions whose arguments differ by $\frac{2\pi}{5}$. Allow this mark if the arguments are out of range. May be implied by their answers.	M1
	Note that this answer in general solution form can score full marks if correct i.e. the A marks below can be implied. E.g. $z = 2\left(\cos\frac{2k\pi}{5} + \mathrm{i}\sin\frac{2k\pi}{5}\right), (k = 0, 1, 2, 3, 4)$ scores full marks		
	$2\left(\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5}\right)$ $2\left(\cos\frac{6\pi}{5} + \mathrm{i}\sin\frac{6\pi}{5}\right)$ $2\left(\cos\frac{8\pi}{5} + \mathrm{i}\sin\frac{8\pi}{5}\right)$	A1: One further correct answer, allow the brackets to be expanded. A1: All correct, allow the brackets to be expanded.	A1 A1
Do not allow $2\left(\cos\frac{4\pi}{5} - \mathrm{i}\sin\frac{4\pi}{5}\right)$ or $2\left(\cos\left(-\frac{4\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{4\pi}{5}\right)\right)$ for $2\left(\cos\frac{6\pi}{5} + \mathrm{i}\sin\frac{6\pi}{5}\right)$ Do not allow $2\left(\cos\frac{2\pi}{5} - \mathrm{i}\sin\frac{2\pi}{5}\right)$ or $2\left(\cos\left(-\frac{2\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{5}\right)\right)$ for $2\left(\cos\frac{8\pi}{5} + \mathrm{i}\sin\frac{8\pi}{5}\right)$			
Ignore answers outside the range. For a fully correct solution that has extra solutions in range, deduct the final A mark.			
Answers in **degrees**: Penalise once the first time it occurs. Answers in degrees are: 0, 72, 144, 216, 288			
			(5)
			Total 5

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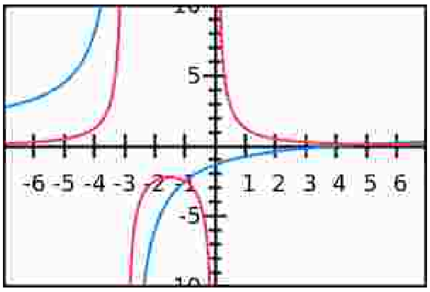
2. Use algebra to find the set of values of x for which

$$\frac{x-4}{(x+3)} \leqslant \frac{5}{x(x+3)}$$

(9)

Question Number	Scheme	Notes	Marks
2.	$\frac{x-4}{(x+3)} \leq \frac{5}{x(x+3)}$		
Way 1	$\frac{x-4}{(x+3)} - \frac{5}{x(x+3)} (\leq 0)$	Collects expressions to one side	M1
	$\frac{x^2-4x-5}{x(x+3)} (\leq 0)$	M1: Attempt common denominator A1: Correct single fraction	M1A1
	$x = 0, -3$	Correct critical values	B1
	$x^2-4x-5 \Rightarrow (x-5)(x+1)=0$ $\Rightarrow x = ...$	Attempt to solve their quadratic as far as $x = \ldots$ to obtain the **other** 2 critical values	M1
	$x = -1, 5$	Correct critical values	A1
	$-3 < x \leq -1,\ 0 < x \leq 5$ or e.g. $(-3, -1] \cup (0, 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b,\ c * x * d$ where * is "<" or "≤" and $a < b < c < d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	dM1A1A1
Notes Intervals may be separated by commas, written separately, $\cup$ or "or" or "and" may be used but not $\cap$ All marks are available for correct work if "=" is used instead of "$\leq$" for the first 6 marks			
			(9)
			Total 9
Way 2	Multiplies by $x^2(x+3)^2$		
	$x^2(x+3)(x-4) \leq 5x(x+3)$	Multiplies both sides by $x^2(x+3)^2$. May multiply by more terms but must be a positive multiplier containing $x^2(x+3)^2$	M1
	$x^3(x+3)-4x^2(x+3)-5x(x+3) \leq 0$	M1: Collects expressions to one side A1: Correct inequality (or equation)	M1A1
	$x = 0, -3$	Correct critical values	B1
	$x(x+3)(x-5)(x+1)=0 \Rightarrow x = ...$	Attempt to solve their quartic as far as $x = \ldots$ to obtain the **other** 2 critical values. Allow the x and $x + 3$ to be "cancelled" to obtain the other critical values so may end up solving a cubic or even a quadratic.	M1
	$x = -1, 5$	Correct critical values	A1
	$-3 < x \leq -1,\ 0 < x \leq 5$ or e.g. $(-3, -1] \cup (0, 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b,\ c * x * d$ where * is "<" or "≤" and $a < b < c < d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	dM1A1A1
			(9)

Way 3		Draws a sketch of graphs $y=\frac{x-4}{x+3}$ and $y=\frac{5}{x(x+3)}$	
	$x=0,-3$	Correct critical values (vertical asymptotes)	B1
	$\frac{x-4}{(x+3)}=\frac{5}{x(x+3)}$	Eliminate y	M1
	$x(x-4)=5$	M1: Obtains quadratic equation A1: Correct quadratic equation	M1A1
	$x^2-4x-5=0 \Rightarrow x=-1,5$	M1: Solves their quadratic equation as far as $x=\ldots$ A1: Correct critical values	M1A1
	$-3<x\le -1,\ \ 0<x\le 5$ or e.g. $(-3,-1]\cup(0,\ 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b,\ c * x * d$ where * is "<" or "≤" and $a<b<c<d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	M1A1A1
If the candidate takes the above approach and there is no sketch e.g. just cross multiplies to obtain the critical values -1 and 5 then no marks are available i.e. the cv's 0 and -3 must be stated somewhere to give access to subsequent marks in this case.			
	Considers Regions:		
Way 4	Considers $x<-3 \Rightarrow x(x+3)>0$ $x(x-4)\le 5 \Rightarrow -1\le x\le 5$ But $x<-3$ so no solution	Can be marked as: B1: Critical values 0 and -3 M1: Considers 3 regions M1: Obtains quadratic equation A1: Correct quadratic M1: Solves quadratic A1: cv's -1 and 5 Final 3 marks as already defined.	
	Considers $-3<x<0 \Rightarrow x(x+3)<0$ $x(x-4)\ge 5 \Rightarrow x\ge 5$ or $x\le -1$ But $-3<x<0$ so $-3<x\le -1$		
	Considers $x>0 \Rightarrow x(x+3)>0$ $x(x-4)\le 5 \Rightarrow -1\le x\le 5$ But $x>0$ so $0<x\le 5$		

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3. (a) Show that $r^3 - (r-1)^3 \equiv 3r^2 - 3r + 1$

(1)

(b) Hence prove by the method of differences that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

[*You may use* $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ *without proof.*]

(5)

Question Number	Scheme	Notes	Marks
3.(a)	$r^3-(r-1)^3 \equiv r^3-(r^3-3r^2+3r-1)$ or $r^3-\left(r^3+\binom{3}{1}r^2(-1)+\binom{3}{2}r(-1)^2+(-1)^3\right)$ $\equiv 3r^2-3r+1$ * or $r^3-(r-1)^3 \equiv (r^2+r(r-1)+(r-1)^2)$ $\equiv 3r^2-3r+1$ *	Shows a correct expansion of $(r-1)^3$ or uses $a^3-b^3=(a-b)(a^2+ab+b^2)$ and achieves the printed answer with no errors.	B1*
			(1)
(b)	$n^3-(n-1)^3$ $(n-1)^3-(n-2)^3$ $(n-2)^3-(n-3)^3$ … … 3^3-2^3 2^3-1^3 1^3-0^3	Uses the method of differences. Must include at least $r=1,2,....,n$ or $r=1,....,(n-1),\ n.$ But may implied by sight of $\sum r^3-(r-1)^3=n^3$ if insufficient terms shown. If method is clearly other than differences (see note below), then score M0.The final A mark can be witheld if differences not shown i.e. just writes down n^3.	M1
	$n^3=\sum_{r=1}^{n}(3r^2-3r+1)=\sum_{r=1}^{n}3r^2-\sum_{r=1}^{n}3r+\sum_{r=1}^{n}1$ Sets $n^3=\sum_{r=1}^{n}(3r^2-3r+1)$ and attempts to expand RHS		M1
	$\sum_{r=1}^{n}1=n$	$\sum_{r=1}^{n}1=n$ seen or implied	B1
	$3\sum_{r=1}^{n}r^2=n(n-1)(n+1)+\frac{3}{2}n(n+1)$	Rearranges to make $k\sum_{r=1}^{n}r^2$ the subject and substitutes for $\sum_{r=1}^{n}r$. **Dependent on the first method mark.**	dM1
	$\sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1)$ **	Completely correct solution with no errors seen.	A1*
	Allow e.g. $\frac{2n^3+3n^2+n}{6}=\frac{1}{6}n(n+1)(2n+1)$		
			(5)
	Note: May be seen in (b): $\sum_{r=1}^{n}r^3-(r-1)^3=\frac{1}{4}n^2(n+1)^2-\frac{1}{4}n^2(n-1)^2=n^3$ etc. Scores a maximum M0M1B1dM0A0 (not using differences)		
	Generally, there are no marks for proof by induction		
			Total 6

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4. $$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$$

is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 40e^{-x}\sin 3x$$

where A is a constant.

(a) Find the value of A.

(5)

(b) Hence find the general solution of this differential equation.

(4)

(c) Find the particular solution of this differential equation for which both $y = 3$ and $\frac{dy}{dx} = 3$ at $x = 0$

(4)

Question Number	Scheme	Notes	Marks
4(a)	$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$ $\Rightarrow \frac{dy}{dx} = -3e^{-x}\cos 3x - 9e^{-x}\sin 3x - Ae^{-x}\sin 3x + 3Ae^{-x}\cos 3x$ $\left(= (-3+3A)e^{-x}\cos 3x + (-9-A)e^{-x}\sin 3x\right)$ Attempts to differentiate the **given expression** by using the product rule on $3e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$ or by using the product rule on $Ae^{-x}\sin 3x$ to give $\alpha Ae^{-x}\cos 3x + \beta Ae^{-x}\sin 3x$		M1
	$\frac{d^2y}{dx^2} = (-24-6A)e^{-x}\cos 3x + (18-8A)e^{-x}\sin 3x$ (Terms may be uncollected) Uses the product rule again on an expression of the form $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$. **Dependent on the first method mark.**		**d**M1
	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = (12-12A)e^{-x}\cos 3x + (36+4A)e^{-x}\sin 3x$ Substitute their results into the differential equation. (May be implied)		M1
	$12-12A=0$ or $36+4A=40 \Rightarrow A = ...$	Compares coefficients of $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ and attempts to find *A*. **Dependent on the previous method mark.**	**d**M1
	$\Rightarrow A = 1$	cao	A1
			(5)
(b) **Marks for (b) can score anywhere in their answer.**	$m^2 - 2m + 10 = 0 \Rightarrow m = 1 \pm 3i$	M1: Forms and attempts to solve the Auxiliary Equation. See General Principles. A1: Correct solution for the AE	M1 A1
	$(y=)e^{x}(C\cos 3x + D\sin 3x)$ or $(y=)Ce^{(1+3i)x} + De^{(1-3i)x}$	Correct form for CF using their complex roots from the AE	M1
	$y = e^{x}(C\cos 3x + D\sin 3x) + 3e^{-x}\cos 3x + e^{-x}\sin 3x$ GS = their CF + their PI (**Allow ft on their CF and PI**) Must start *y* = … and depends on at least one the M's being scored and must have been using a PI of the form given.		A1ft
			(4)
(c)	$x=0, y=3 \Rightarrow 3 = C + 3 (\Rightarrow C = 0)$	Attempts to substitute *x* = 0 and *y* = 3 into their answer to (b)	M1
	$\frac{dy}{dx} = (C+3D)e^{x}\cos 3x + (-3C+D)e^{x}\sin 3x - 10e^{-x}\sin 3x$ Attempt to differentiate their GS with or without their *C*		M1
	$3 = C + 3D$	Attempt to substitute *x* = 0 and $\frac{dy}{dx} = 3$ into their $\frac{dy}{dx}$	M1
	$y = e^{x}\sin 3x + 3e^{-x}\cos 3x + e^{-x}\sin 3x$	Correct answer. **Must start *y* = …**	A1cao
			(4)
			Total 13

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5. $$y = e^{\cos^2 x}$$

(a) Show that

$$\frac{d^2 y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$$

(4)

(b) Hence find the Maclaurin series expansion of $e^{\cos^2 x}$ up to and including the term in x^2

(3)

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Question Number	Scheme	Notes	Marks
5	$y = e^{\cos^2 x}$		
(a)	$\frac{dy}{dx} = -2\sin x \cos x e^{\cos^2 x} = -\sin 2x e^{\cos^2 x}$ M1: Differentiates using the chain rule to obtain an expression of the form $\alpha \sin x \cos x e^{\cos^2 x}$ or $\beta \sin 2x e^{\cos^2 x}$ A1: Correct derivative Note that candidates may use $\frac{1}{2}(1+\cos 2x)$ instead of $\cos^2 x$		M1A1
	$\frac{d^2y}{dx^2} = -\sin 2x(-2\sin x \cos x e^{\cos^2 x}) - 2\cos 2x e^{\cos^2 x}$ Correct use of the Product Rule on their first derivative **Dependent on the first method mark.**		**d**M1
	$\frac{d^2y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$ *	Achieves the printed answer with no errors.	A1*
	Alternative using logs: $y = e^{\cos^2 x} \Rightarrow \ln y = \cos^2 x \Rightarrow \frac{1}{y}\frac{dy}{dx} = -2\sin x \cos x$ M1: $\frac{1}{y}\frac{dy}{dx} = k\sin x \cos x$ or $k \sin 2x$ A1: $\frac{1}{y}\frac{dy}{dx} = -2\sin x \cos x$ $\frac{dy}{dx} = -y\sin 2x \Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx}\sin 2x - 2y\cos 2x$ M1: Correct use of product rule $\frac{d^2y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$ * A1: Achieves the printed answer with no errors.		
			(4)
(b)	$\frac{dy}{dx} = 0,\ \frac{d^2y}{dx^2} = -2e$	Both seen, can be implied by subsequent work.	B1
	$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \ldots$ $= e^{\cos^2 0} - \sin 0 e^{\cos^2 0} x + \frac{1}{2}e^{\cos^2 0}(\sin^2 0 - 2\cos 0)x^2 + \ldots$ Applies the **correct** Maclaurin expansion, the "$\frac{1}{2}$" is required and there must be no x's in the derivatives. This can be implied by their expansion but if the expansion in incorrect for their values and the formula is not quoted, score M0.		M1
	$\left(e^{\cos^2 x}\right) = e(1 - x^2 + \ldots)$	Or exact equivalent e.g. $e - ex^2$ (i.e. all trig. evaluated)	A1
			(3)
			Total 7

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6. Find the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = (\cos^2 x) \ln x, \qquad 0 < x < \frac{\pi}{2}$$

Give your answer in the form $y = f(x)$.

(8)

Question Number	Scheme	Notes	Marks
6.	$\cos x\dfrac{dy}{dx}+y\sin x=(\cos^2 x)\ln x$		
	$\dfrac{dy}{dx}+y\dfrac{\sin x}{\cos x}=\cos x\ln x$	Attempt to divide through by $\cos x$. If the intention is not clear, must see at least 2 terms divided by $\cos x$.	M1
	$I=e^{\int\frac{\sin x}{\cos x}dx}=e^{-\ln\cos x}$	M1: $e^{\int \pm \text{their } P(x)\,(dx)}$. Dependent on the first method mark. A1: $e^{-\ln\cos x}$ or $e^{\ln\sec x}$	dM1A1
	$=\dfrac{1}{\cos x}$	$\dfrac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$	A1
	$\dfrac{y}{\cos x}=\int \ln x\,dx$ or $\dfrac{d}{dx}\left(\dfrac{y}{\cos x}\right)=\ln x$	M1: $y\times\text{their } I=\int Q(x)\times\text{their } I\,dx$ or $\dfrac{d}{dx}(y\times\text{their } I)=Q(x)\times\text{their } I$ A1: $\dfrac{y}{\cos x}=\int\ln x\,dx$ or $\dfrac{d}{dx}\left(\dfrac{y}{\cos x}\right)=\ln x$	M1A1
	$\dfrac{y}{\cos x}=x\ln x-x+C$	Attempts $\int\ln x\,dx$ by parts correctly (correct sign needed unless correct formula quoted and used).	M1
	$y=(x\ln x-x+C)\cos x$	Any equivalent with the constant correctly placed and "$y=\ldots$" must appear at some stage.	A1
			Total 8
	Note: Failure to divide by cos *x* at the start would mean that only the 3rd Method mark is available.		

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7.

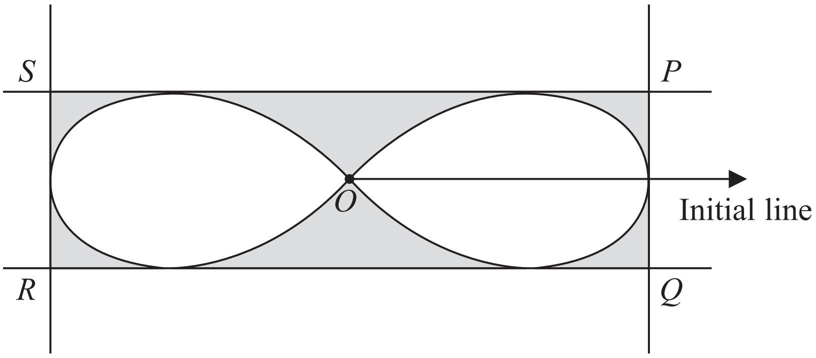


Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 4\cos 2\theta, \quad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4} \quad \text{and} \quad \frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, QR, RS and SP are tangents to C, where QR and SP are parallel to the initial line and PQ and RS are perpendicular to the initial line.

(a) Find the polar coordinates of the points where the tangent SP touches the curve. Give the values of θ to 3 significant figures.

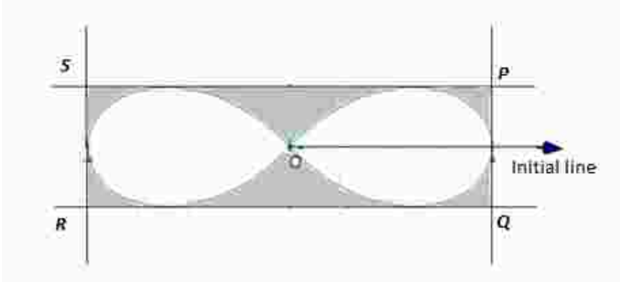
(5)

(b) Find the exact area of the finite region bounded by the curve C, shown unshaded in Figure 1.

(5)

(c) Find the area enclosed by the rectangle $PQRS$ but outside the curve C, shown shaded in Figure 1.

(5)

Question Number	Scheme	Notes	Marks
7			
(a)	$y = r\sin\theta = 4\cos 2\theta \sin\theta$	Attempts to use $r\sin\theta$	M1
	$\frac{dy}{d\theta} = 4\cos 2\theta\cos\theta - 8\sin 2\theta\sin\theta$ or $y = 4(1-2\sin^2\theta)\sin\theta = 4\sin\theta - 8\sin^3\theta \Rightarrow \frac{dy}{d\theta} = 4\cos\theta - 24\sin^2\theta\cos\theta$ A correct expression for $\frac{dy}{d\theta}$ **or any multiple of** $\frac{dy}{d\theta}$		B1
	$\frac{dy}{d\theta} = 0 \Rightarrow \theta = ...$	Set their $\frac{dy}{d\theta} = 0$ and attempt to solve to obtain a value for θ	M1
	$r = \frac{8}{3},\ \theta = 0.421,\ \theta = 2.72$	Any one of: $r = \frac{8}{3}$ (or awrt 2.7) or $\theta = 0.421...$ or $\theta = 2.72...$	A1
	$r = \frac{8}{3}$ $\theta = 0.421,\ 2.72$	Correct value for r and both angles correct. May be seen as $\left(\frac{8}{3}, 0.421\right), \left(\frac{8}{3}, 2.72\right)$. Allow $\left(0.421, \frac{8}{3}\right), \left(2.72, \frac{8}{3}\right)$ but coordinates do not have to be paired and accept awrt 0.421, 2.72 and allow awrt 2.7 for $\frac{8}{3}$. Ignore any other coordinates given once the correct values have been seen.	A1
			(5)

(b)	$A = ...\int (4\cos 2\theta)^2\,\mathrm{d}\theta$	Indication that the integration of $(4\cos 2\theta)^2$ is required. Ignore any limits and ignore any constant factors at this stage.	M1
	$\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)$	A correct identity seen or implied.	A1
	$A = ...[\alpha\theta + \beta\sin 4\theta]$	Integrates to obtain an expression of the form $\alpha\theta + \beta\sin 4\theta$. Ignore any limits and ignore any constant factors. Dependent on the first method mark.	**d**M1
	$= 16\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{4}}$	A fully correct method that if evaluated correctly would give the answer 4π. Note that the correct "constant factor" may only be applied at the very last stage of their working and this method mark would only be awarded at that point. **Dependent on all previous method marks.**	**dd**M1
	Examples that could score the final M1 (following correct work): $16\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{4}}$, $8\left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$, $8\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$, $16\left[\theta + \frac{1}{4}\sin 4\theta\right]_{\frac{3\pi}{4}}^{\pi}$		
	$= 4\pi$	cao	A1
			(5)

(c)	$PQ = 2r\sin\theta = \frac{16}{3\sqrt{6}}$	Correct expression or value for *PQ* or *PQ*/2. E.g. $2\left(\frac{8}{3}\right)\frac{1}{\sqrt{6}}$, $2\left(\frac{8}{3}\right)\sin 0.421$, $2\left(\frac{8}{3}\right)\sin 2.72$, $\frac{8\sqrt{6}}{9}$ or half of these. May be implied by awrt 2.2 or awrt 1.1	B1
	$SP = 8$ or $\frac{SP}{2} = 4$	Correct value for *SP* or *SP*/2	B1
	Area $PQRS = \frac{16}{3\sqrt{6}} \times 8 \left(= \frac{64\sqrt{6}}{9}\right)$	Their $PQ \times SP$. Must be the complete rectangle here.	M1
	Required area $= \frac{128}{3\sqrt{6}} - 4\pi$	M1: Their rectangle area – their answer to part (b) A1: Correct exact answer or equivalent exact form e.g. $\frac{64\sqrt{6}}{9} - 4\pi$ or allow awrt 4.8 or 4.9	M1A1
			(5)
			Total 15

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8. (a) Use de Moivre's theorem to

(i) show that

$$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

(ii) find an expression for $\sin 5\theta$ in terms of $\cos\theta$ and $\sin\theta$

(4)

(b) Hence show that

$$\tan 5\theta = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$$

where $t = \tan\theta$ and $\cos 5\theta \neq 0$

(2)

(c) Hence find a quadratic equation whose roots are $\tan^2\frac{\pi}{5}$ and $\tan^2\frac{2\pi}{5}$

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers to be found.

(4)

(d) Deduce that $\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = \sqrt{5}$

(2)

DO NOT WRITE IN THIS AREA

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DO NOT WRITE IN THIS AREA

Question Number	Scheme	Notes	Marks
8(a)(i)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (c+\mathrm{i}s)^5 = c^5 + 5c^4\mathrm{i}s + 10c^3\mathrm{i}^2s^2 + 10c^2\mathrm{i}^3s^3 + 5c\mathrm{i}^4s^4 + \mathrm{i}^5s^5$ Attempts to expand $(c+\mathrm{i}s)^5$ including binomial coefficients (NB may only see real terms here)		M1
	$\cos 5\theta = \mathrm{Re}(c+\mathrm{i}s)^5 = c^5 + 10c^3\mathrm{i}^2s^2 + 5c\mathrm{i}^4s^4 = c^5 - 10c^3s^2 + 5cs^4$ Extracts real terms and uses $\mathrm{i}^2 = -1$ to eliminate i.		M1
	$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$* Achieves the printed result with no errors seen.		A1*
	Alternative: $\left(z = \cos\theta + \mathrm{i}\sin\theta,\ z^{-1} = \cos\theta - \mathrm{i}\sin\theta,\ z^n = \cos n\theta + \mathrm{i}\sin n\theta\right)$ $\left(z+\frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z^2}\right) + 10z^2\left(\frac{1}{z^3}\right) + 5z\left(\frac{1}{z^4}\right) + \frac{1}{z^5}$ $(2\cos\theta)^5 = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ **M1:** Expands $\left(z+\frac{1}{z}\right)^5$ including binomial coefficients and uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ at least once to obtain an equation in cos $\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta \Rightarrow \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 15\cos\theta - 10\cos\theta$ **M1:** Uses **correct** identity for cos3θ to obtain $\cos 5\theta$ in terms of single angles $= \cos^5\theta + 15\cos\theta\left(1-\sin^2\theta\right)^2 - 20\cos^3\theta + 5\cos\theta$ $= \cos^5\theta - 10\cos\theta\sin^2\theta + 15\cos\theta\sin^4\theta$ $= \cos^5\theta + 5\cos\theta\sin^4\theta + 10\cos\theta\sin^2\theta\left(\sin^2\theta - 1\right)$ $\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$* **A1:** Achieves the printed result with no errors seen (may need careful checking)		
(ii)	$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ This expression (or equivalent) with no i's seen. **Note that some candidates may re-start and expand here as above.**		B1
			(4)
(b)	$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$ Uses $\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta}$ and substitutes the results from part (a)		M1
	$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta} = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$* Achieves the printed result with no errors seen.		A1*
	Note that a minimum could be: $\tan 5\theta = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$* **Note that some candidates may work backwards which is acceptable:** E.g. $\frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1} = \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}$ $= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} = \tan 5\theta$		
			(2)

(c)	$\tan 5\theta = 0$ or $\dfrac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1} = 0$ or $\dfrac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1} = 0$	Considers $\tan 5\theta = 0$. This may be implied by $\tan^5\theta - 10\tan^3\theta + 5\tan\theta = 0$ or $t^5 - 10t^3 + 5t = 0$ or $\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	M1
	$\tan^5\theta - 10\tan^3\theta + 5\tan\theta = 0$ or $t^5 - 10t^3 + 5t = 0$	Equate numerator to 0. This may be implied by $\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	M1
	$\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	Correct quartic	A1
	$x^2 - 10x + 5 = 0$	$x^2 - 10x + 5 = 0$ or equivalent	A1
			(4)
(d)	Product of roots: $\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5$ Or solves "$x^2 - 10x + 5 = 0$" and attempts to multiply roots together e.g. $x = \dfrac{10 \pm \sqrt{100-20}}{2} = 5 \pm 2\sqrt{5}$ and $\left(5+2\sqrt{5}\right)\left(5-2\sqrt{5}\right) = ...$	Must clearly state product of roots or e.g. $\alpha\beta = 5$ or $x_1x_2 = 5$ and uses their constant in (c) or solves their quadratic and attempts product of roots.	M1
	$\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5 \Rightarrow \tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = \sqrt{5}$ *	Shows the given result with no errors.	A1
			(2)
			Total 12