Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 3 June 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference **WFM02/01**

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

PEARSON

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1. Using algebra, find the set of values of x for which

$$\frac{x}{x+2} < \frac{2}{x+5}$$

(7)

June 2015
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\frac{x}{x+2} < \frac{2}{x+5}$		
	Critical Values -2 and -5	Seen anywhere in solution Both correct B1B1; one correct B1B0	B1, B1
	$\frac{x}{x+2} - \frac{2}{x+5} < 0$		
	$\frac{x^2+3x-4}{(x+2)(x+5)} < 0$		
	$\frac{(x+4)(x-1)}{(x+2)(x+5)} < 0$	Attempt single fraction and factorise numerator or use quad formula	M1
	Critical values -4 and 1	Correct critical values May be seen on a graph or number line.	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4) \cup (-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0 Award on basis of the inequalities seen - ignore any and/or between them Set notation answers do not need the union sign.	dM1A1,A1
			(7)
ALT	Critical Values -2 and -5	Seen anywhere in solution	B1, B1
	$\frac{x}{x+2} < \frac{2}{x+5} \Rightarrow x(x+5)^2(x+2) < 2(x+2)^2(x+5)$		
	$\Rightarrow (x+5)(x+2)[x(x+5)-2(x+2)] < 0$		
	$\Rightarrow (x+5)(x+2)[(x-1)(x+4)] < 0$	Multiply by $(x+5)^2(x+2)^2$ and attempt to factorise a quartic or use quad formula	M1
	Critical values -4 and 1	Correct critical values	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4) \cup (-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0	dM1A1,A1
			(7)

Any solutions with no algebra (eg sketch graph followed by critical values with no working) scores max B1B1

Leave blank

2. (a) Express $\dfrac{1}{(r+6)(r+8)}$ in partial fractions.
(1)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+6)(r+8)} = \frac{n(an+b)}{56(n+7)(n+8)}$$

where a and b are integers to be found.
(4)

Question Number	Scheme	Notes	Marks
	$\dfrac{1}{(r+6)(r+8)}$		
2(a)	$\dfrac{1}{2(r+6)}-\dfrac{1}{2(r+8)}$ oe	Correct partial fractions, any equivalent form	B1
			(1)
(b)	$=\left(2\times\dfrac{1}{2}\right)\left(\dfrac{1}{7}-\dfrac{1}{9}+\dfrac{1}{8}-\dfrac{1}{10}+\dfrac{1}{9}-\dfrac{1}{11}...+\dfrac{1}{n+5}-\dfrac{1}{n+7}+\dfrac{1}{n+6}-\dfrac{1}{n+8}\right)$ Expands at least 3 terms at start and 2 at end (may be implied) The partial fractions obtained in (a) can be used without multiplying by 2. Fractions may be $\dfrac{1}{2}\times\dfrac{1}{7}-\dfrac{1}{2}\times\dfrac{1}{9}$ etc These comments apply to both M1 and A1		M1
	$=\dfrac{1}{7}+\dfrac{1}{8}-\dfrac{1}{n+7}-\dfrac{1}{n+8}$	Identifies the terms that do not cancel	A1
	$=\dfrac{15(n+7)(n+8)-56(2n+15)}{56(n+7)(n+8)}$	Attempt common denominator Must have multiplied the fractions from (a) by 2 now	M1
	$=\dfrac{n(15n+113)}{56(n+7)(n+8)}$		A1cso
			(4)
			Total 5

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3. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 2xy = xe^{-x^2}y^3 \quad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4xz = -2xe^{-x^2} \quad \text{(II)}$$

(4)

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$.

(1)

Question Number	Scheme	Notes	Marks
3	$\frac{dy}{dx}+2xy=xe^{-x^2}y^3$		
(a)	$z=y^{-2}\Rightarrow y=z^{-\frac{1}{2}}$		
	$\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1: $\frac{dy}{dx}=kz^{-\frac{3}{2}}\frac{dz}{dx}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}+\frac{2x}{z^{\frac{1}{2}}}=xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1cso
			(4)
	(a) Alternative 1		
	$\frac{dz}{dy}=-2y^{-3}$ oe	M1: $\frac{dz}{dy}=ky^{-3}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	(a) Alternative 2		
	$\frac{dz}{dx}=-2y^{-3}\frac{dy}{dx}$	M1: $\frac{dz}{dx}=ky^{-3}\frac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I=e^{\int -4x\,dx}=e^{-2x^2}$	M1: $I=e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1A1
	$ze^{-2x^2}=\int -2xe^{-3x^2}\,dx$	$z\times I=\int -2xe^{-x^2}I\,dx$	dM1
	$\frac{1}{3}e^{-3x^2}\ (+c)$	$\int xe^{qx^2}\,dx=pe^{qx^2}\ (+c)$	M1
	$z=ce^{2x^2}+\frac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)
(c)	$\frac{1}{y^2}=ce^{2x^2}+\frac{1}{3}e^{-x^2}\Rightarrow y^2=\frac{1}{ce^{2x^2}+\frac{1}{3}e^{-x^2}}$	$y^2=\frac{1}{(b)}\left(=\frac{3e^{x^2}}{1+ke^{3x^2}}\right)$	B1ft
			(1)
			Total 10

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4. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z-1}{z+1}, \quad z \neq -1$$

The line in the z-plane with equation $y = 2x$ is mapped by T onto the curve C in the w-plane.

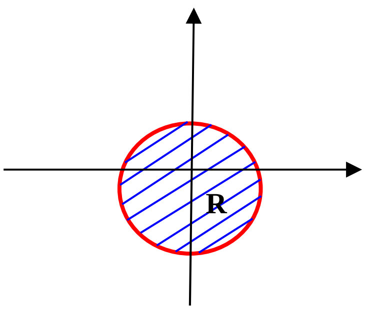
(a) Show that C is a circle and find its centre and radius.

(7)

The region $y < 2x$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Sketch circle C on an Argand diagram and shade and label region R.

(2)

Question Number	Scheme	Notes	Marks
	$w=\dfrac{z-1}{z+1}$		
4(a)	$w=\dfrac{z-1}{z+1}\Rightarrow wz+w=z-1\Rightarrow z=...$	Attempt to make *z* the subject	M1
	$z=\dfrac{w+1}{1-w}$	Correct expression in terms of *w*	A1
	$=\dfrac{u+iv+1}{1-u-iv}\times\dfrac{1-u+iv}{1-u+iv}$	Introduces "*u* + i*v*" and multiplies top and bottom by the complex conjugate of the bottom	M1
	$x=\dfrac{-u^2-v^2+1}{....},\quad y=\dfrac{2v}{....}$		
	$y=2x\Rightarrow 2v=-2u^2-2v^2+2$	Uses real and imaginary parts and $y = 2x$ to obtain an equation connecting "*u*" and "*v*" Can have the 2 on the wrong side.	M1
	$u^2+\left(v+\frac{1}{2}\right)^2-\frac{1}{4}=1$	Processes their equation to a form that is recognisable as a circle ie coefficients of u^2 and v^2 are the same and no *uv* terms	M1
	Centre (0, $-\frac{1}{2}$), radius $\dfrac{\sqrt{5}}{2}$	A1: Correct centre (allow -½i) A1: Correct radius	A1,A1
			(7)
	Special Case:		
	$w=\dfrac{x+\text{i}y-1}{x+\text{i}y+1}=\dfrac{(x-1)+2x\text{i}}{(x+1)+2x\text{i}}\times\dfrac{(x+1)-2x\text{i}}{(x+1)-2x\text{i}}$	M1: rationalise the denominator, may have $2x$ or y	
	$=\dfrac{(x^2-1)+4x^2+2x\text{i}(x+1-(x-1))}{(x+1)^2+4x^2}$	A1: Correct result in terms of *x* only. Must have rational denominator shown, but no other simplification needed	
(b)		B1ft: Their circle correctly positioned provided their equation does give a circle B1: Completely correct sketch and shading	B1ft B1
			(2)
			Total 9

Leave blank

5. Given that $y = \cot x$,

(a) show that

$$\frac{d^2y}{dx^2} = 2\cot x + 2\cot^3 x$$

(3)

(b) Hence show that

$$\frac{d^3y}{dx^3} = p\cot^4 x + q\cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of $\cot x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

Question Number	Scheme	Notes	Marks
5	$y = \cot x$		
(a)	$\frac{dy}{dx} = -\text{cosec}^2 x$		
	$\frac{d^2y}{dx^2} = (-2\text{cosec}\, x)(-\text{cosec}\, x \cot x)$	M1: Differentiates using the chain rule or product/quotient rule A1: Correct derivative	M1A1
	$= 2\text{cosec}^2 x \cot x = 2\cot x + 2\cot^3 x$ *	A1: Correct completion to printed answer $1+\cot^2 x = \text{cosec}^2 x$ or $\cos^2 x + \sin^2 x = 1$ must be used Full working must be shown	A1cso*
			(3)
	Alternative:		
	$y = \frac{\cos x}{\sin x} \to \frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$		
	$\frac{d^2y}{dx^2} = -\left(-2\sin^{-3} x \cos x\right) = ...$		M1A1
	A1: Correct completion to printed answer see above		A1
(b)	$\frac{d^3y}{dx^3} = -2\text{cosec}^2 x - 6\cot^2 x\, \text{cosec}^2 x$	Correct third derivative	B1
	$= -2\left(1+\cot^2 x\right) - 6\cot^2 x\left(1+\cot^2 x\right)$	Uses $1+\cot^2 x = \text{cosec}^2 x$	M1
	$= -6\cot^4 x - 8\cot^2 x - 2$	cso	A1
			(3)
(c)	$f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown		M1
	$(y =)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x-\frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x-\frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x-\frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x-\frac{\pi}{3}\right)^3$ A1: Correct expression Must start $y =$ .... or $\cot x$ f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698.., so accept 0.77) 0.889		M1A1
			(3)
			Total 9

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \quad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

Question Number	Scheme	Notes	Marks
6(a)	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=2\sin x$		
	AE: $m^2-2m-3=0$		
	$m^2-2m-3=0 \Rightarrow m=...(-1,3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)\,Ae^{3x}+Be^{-x}$	Cao	A1
	PI: $(y=)\,p\sin x+q\cos x$	Correct form for PI	B1
	$(y'=)\,p\cos x-q\sin x$ $(y''=)-p\sin x-q\cos x$		
	$-p\sin x-q\cos x-2(p\cos x-q\sin x)-3p\sin x-3q\cos x=2\sin x$ Differentiates twice and substitutes		M1
	$2q-4p=2,\ \ 4q+2p=0$	Correct equations	A1
	$p=-\frac{2}{5},\ \ q=\frac{1}{5}$	A1A1 both correct A1A0 one correct	A1A1
	$y=\frac{1}{5}\cos x-\frac{2}{5}\sin x$		
	$y=Ae^{3x}+Be^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Follow through their *p* and *q* and their CF	B1ft
			(8)
(b)	$y'=3Ae^{3x}-Be^{-x}-\frac{1}{5}\sin x-\frac{2}{5}\cos x$	Differentiates their GS	M1
	$0=A+B+\frac{1}{5},\ \ 1=3A-B-\frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B A1: Correct equations	M1A1
	$A=\frac{3}{10},\ \ B=-\frac{1}{2}$	Solves for *A* and *B* Both correct	A1
	$y=\frac{3}{10}e^{3x}-\frac{1}{2}e^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Sub their values of *A* and *B* in their GS	A1ft
			(5)
			Total 13

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7.

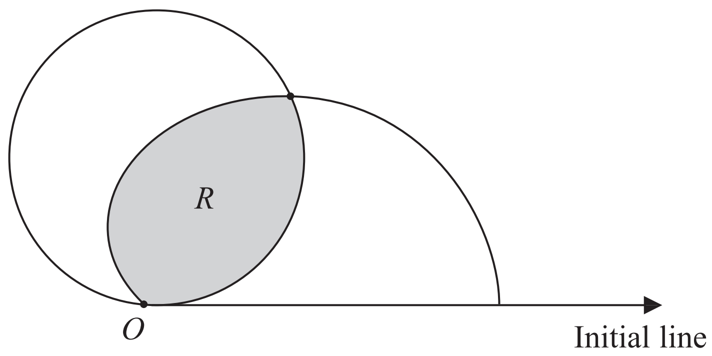

Figure 1

Figure 1 shows the two curves given by the polar equations

$$r = \sqrt{3}\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$$

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \pi$$

(a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$.

(2)

The region R, bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of R, giving your answer in the form $a\left(\pi - \sqrt{3}\right)$, where a is a constant to be found.

(6)

Question Number	Scheme	Notes	Marks
7(a)	$\theta=\frac{\pi}{3}\Rightarrow r=\sqrt{3}\sin\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta=\frac{\pi}{3}\Rightarrow r=1+\cos\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative:		
	Equate rs: $\sqrt{3}\sin\theta=1+\cos\theta$ and verify (by substitution) that $\theta=\frac{\pi}{3}$ is a solution or solve by using $t=\tan\frac{\theta}{2}$ or writing $\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta=\frac{1}{2}$ $\sin\left(\theta-\frac{\pi}{6}\right)=\frac{1}{2}$ $\theta=\frac{\pi}{3}$ Squaring the original equation allowed as θ is known to be between 0 and π		M1
	Use $\theta=\frac{\pi}{3}$ in either equation to obtain $r=\frac{3}{2}$		A1
(b)	$\frac{1}{2}\int(\sqrt{3}\sin\theta)^2\,d\theta,\quad \frac{1}{2}\int(1+\cos\theta)^2\,d\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	M1
	$=\frac{1}{2}\int 3\sin^2\theta\,d\theta,\quad \frac{1}{2}\int(1+2\cos\theta+\cos^2\theta)\,d\theta$		
	$=\left(\frac{1}{2}\right)\int\frac{3}{2}(1-\cos 2\theta)\,d\theta,\quad \left(\frac{1}{2}\right)\int\left(1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)d\theta$ Attempt to use $\sin^2\theta$ or $\cos^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ on either integral **Not** dependent 1/2 may be missing		M1
	$=\frac{3}{4}\left[\theta-\frac{1}{2}\sin 2\theta\right]_{(0)}^{\left(\frac{\pi}{3}\right)},\quad \frac{1}{2}\left[\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin 2\theta\right]_{\left(\frac{\pi}{3}\right)}^{(\pi)}$ Correct integration (ignore limits) A1A1 or A1A0		A1, A1
	$R=\frac{3}{4}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}(-0)\right]+\frac{1}{2}\left[\frac{3\pi}{2}-\left(\frac{\pi}{2}+\sqrt{3}+\frac{\sqrt{3}}{8}\right)\right]$	Correct use of limits for both integrals Integrals must be added. Dep on both previous M marks	**dd**M1
	$=\frac{3}{4}\left(\pi-\sqrt{3}\right)$	Cao No equivalents allowed	A1
			(6)
			Total 8

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8. (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,\mathrm{d}\theta$$

(4)

Question Number	Scheme	Notes	Marks
8(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3$		
	$=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1: Attempt to expand A1: Correct expansion	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors seen	A1
			(3)
(a) ALT	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3}$, $\left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$		M1A1
	M1: Attempt to expand both cubic brackets A1: Correct expansions		
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors	A1
			(3)
(b)(i)(ii)	$z^n=\cos n\theta+i\sin n\theta$	Correct application of de Moivre	B1
	$z^{-n}=\cos(-n\theta)+i\sin(-n\theta)=\pm\cos n\theta\pm\sin n\theta$ but must be different from their z^n	Attempt z^{-n}	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta*$, $z^n-\frac{1}{z^n}=2i\sin n\theta*$	$z^{-n}=\cos n\theta-i\sin n\theta$ must be seen	A1*
			(3)
(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2i\sin 6\theta-6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta=2i\sin 6\theta-6i\sin 2\theta$	Equating right hand sides and simplifying $2^3\times(2i)^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *		A1cso
			(4)
(d)	$\int_0^{\frac{\pi}{8}}\cos^3\theta\sin^3\theta\,d\theta=\int_0^{\frac{\pi}{8}}\frac{1}{32}(3\sin 2\theta-\sin 6\theta)\,d\theta$		
	$=\frac{1}{32}\left[-\frac{3}{2}\cos 2\theta+\frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$	M1: $p\cos 2\theta+q\cos 6\theta$ A1: Correct integration Differentiation scores M0A0	M1A1
	$=\frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)-\left(-\frac{3}{2}+\frac{1}{6}\right)\right]=\frac{1}{32}\left(\frac{4}{3}-\frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark A1: Cao (oe) but must be exact	dM1A1
			(4)
			Total 14