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Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon
Time: 1 hour 30 minutes

Paper Reference

WFM02/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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PEARSON

Question Number	Scheme		Marks
1.(a)	$\frac{1}{2(r+1)(r+2)} - \frac{1}{2(r+2)(r+3)}$ $= \frac{r+3 - (r+1)}{2(r+1)(r+2)(r+3)}$	Attempt common denominator	M1
	$= \frac{2}{2(r+1)(r+2)(r+3)}$ $= \frac{1}{(r+1)(r+2)(r+3)}$	Correct proof	A1
			(2)
(a) Way 2	$\frac{1}{(r+1)(r+2)(r+3)} = \frac{1}{r+2} \left(\frac{1}{(r+1)(r+3)} \right) = \frac{1}{r+2} \left(\frac{1}{2(r+1)} - \frac{1}{2(r+3)} \right)$		M1
	$\frac{1}{2(r+1)(r+2)} - \frac{1}{2(r+2)(r+3)}$		A1
M1: Factor of $\frac{1}{r+2}$ and attempt partial fractions A1: Correct proof			
Other methods: Complete method scores M1 All work correct inc final answer reached A1			
(b)	$\sum_{r=1}^n \frac{1}{(r+1)(r+2)(r+3)} =$ $= \frac{1}{12} - \frac{1}{24} + \dots$ $+ \frac{1}{2(n+1)(n+2)} - \frac{1}{2(n+2)(n+3)}$	Attempt at least the first pair and the last pair of terms as shown. Must start at 1 and end at n	M1
$= \frac{1}{12} - \frac{1}{2(n+2)(n+3)}$			Identifies that the first and last terms do not cancel.
$= \frac{n^2 + 5n + 6 - 6}{12(n+2)(n+3)}$			Correctly combined fractions
$= \frac{n^2 + 5n}{12(n+2)(n+3)} \text{ or } \frac{n(n+5)}{12(n+2)(n+3)} *$			Allow either form isw attempts to multiply out the denominatot
			(4)
			Total 6

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2. Use algebra to find the set of values of x for which

$$\frac{6}{x - 3} \leq x + 2$$

(7)



Question Number	Scheme		Marks
2.	$\frac{6}{x-3} \leq x+2$		
	Way 1		
	$\frac{6}{x-3} \leq x+2 \Rightarrow x+2 - \frac{6}{x-3} \geq 0$		
	$x+2 - \frac{6}{x-3} \geq 0 \Rightarrow \frac{(x+3)(x-4)}{x-3} \geq 0$	Attempt to combine fractions and factorise the numerator	M1
	$x = -3, x = 4$	Correct critical values	A1, A1
	$x \geq 4$	Follow through their 4	A1ft
	$x = 3$	Identifies 3 as a critical value	B1
	$-3 \leq x < 3$	M1: Attempt inside region	M1A1
		A1: Correct inequality	
			(7)
	Way 2		
	$6(x-3) \leq (x+2)(x-3)^2$ $\Rightarrow (x-3)(4-x)(x+3) \geq 0$	Multiplies both sides by $(x-3)^2$ and attempt to factorise	M1
	$x = -3, x = 4$	Correct critical values	A1, A1
	$x \geq 4$	Follow through their 4	A1ft
	$x = 3$	Identifies 3 as a critical value	B1
	$-3 \leq x < 3$	M1: Attempt inside region	M1A1
		A1: Correct inequality	
			(7)
	Way 3		
	$x-3 > 0 \Rightarrow 6 \leq (x+2)(x-3)$ $\Rightarrow (x-4)(x+3) \geq 0$	Multiplies both sides by $(x-3)$ and attempt to factorise Must state $x-3 > 0$	M1
	$x = 4$	Correct critical values	A1
	$x \geq 4$	Follow through their 4	A1ft
	$x = 3$	Identifies 3 as a critical value	B1
	$(x-3 < 0) \Rightarrow 6 \geq (x+2)(x-3)$ $x = -3$	Correct critical value	A1
	$(x+2)(x-3) \leq 6 \Rightarrow (x-4)(x+3) \leq 0$ $\Rightarrow -3 \leq x < 3$	M1: Attempt inside region	M1A1
		A1: Correct inequality	
			(7)

Question Number	Scheme		Marks
3.			
	$r^5 = \sqrt{16^2 + (16\sqrt{3})^2} = 32 \Rightarrow r = 32^{\frac{1}{5}} (= 2)$	Correct value for r	B1
	$\arg(16 - 16i\sqrt{3}) = \frac{5\pi}{3}$	Allow $\frac{5\pi}{3}$ or $-\frac{\pi}{3}$	B1
	$5\theta = \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{23\pi}{3}, \frac{29\pi}{3}$	$\left(\frac{5\pi}{3}\right) + 2n\pi, n = 1, 2, 3, 4$ At least 2 values which must be positive. May be implied by correct final answers.	M1
	$z = \underline{2e^{\frac{\pi i}{3}}}, \underline{2e^{\frac{11\pi i}{15}}}, \underline{2e^{\frac{17\pi i}{15}}}, \underline{2e^{\frac{23\pi i}{15}}}, \underline{2e^{\frac{29\pi i}{15}}}$	2 or $32^{\frac{1}{5}}, e^{\frac{5\pi i}{15}}$ or $e^{\frac{\pi i}{3}}$	<u>B1 A1(all 4 values)</u>
			(5)
			Total 5

Question Number	Scheme	Marks
4	$w = \frac{z}{z+3}$	
	$w = \frac{z}{z+3} \Rightarrow z = \frac{3w}{1-w}$	M1A1 M1: Attempt to make z the subject A1: Correct expression for z
	$ z = 2 \Rightarrow \left \frac{3w}{1-w} \right = 2$ $ 3w = 2 1-w $ $9(u^2 + v^2) = 4(u-1)^2 + 4v^2$	M1A1 M1: Uses $ z = 2$ to obtain an equation in u and v Pythagoras must be used correctly. No i seen A1: Any correct equation in u and v Isw attempts to simplify
	$5u^2 + 5v^2 + 8u - 4 = 0$	
	$\left(u + \frac{4}{5}\right)^2 + v^2 - \frac{16}{25} - \frac{4}{5} = 0$	M1 Rearrange to a suitable form for a circle and attempt centre and/or radius. -16/25 (from completing the square) may be omitted. May be implied by centre and radius correct for their previous equation
	Centre $\left(-\frac{4}{5}, 0\right)$	A1 oe
	Radius $\frac{6}{5}$	A1 oe
		(7)
		Total 7

Question Number	Scheme		Marks
5	$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$		
(a)	$y''' - 2y' - 2xy'' + 2y' (= 0) (y''' = 2xy'')$	M1: Attempt to differentiate including use of the product rule on $2x\frac{dy}{dx}$ Equation may have been re-written as $y'' = \dots$ before differentiating	M1A1
		A1: Correct differentiation	
	$y'''' - 2y'' - 2xy'''' - 2y'' + 2y'' (= 0)$	M1: Second use of product rule. Dependent on first M1.	dM1A1
		A1: Correct differentiation NB A simpler form is obtained if $y''' - 2xy'' = 0$ is used.	
	$y'''' = 2xy'''' + 2y''$ $= 2x(2xy'') + 2y'' = (4x^2 + 2)y''$	Cao and cso	A1
			(5)
(b)	$y_0'' = -2, y_0''' = 0, y_0^{(4)} = -4$	B1: $y_0'' = -2$	B1, M1A1
		M1: Attempts y_0''' and $y_0^{(4)}$	
		A1: All correct and obtained from correct expressions	
	$(y =) 1 + 3x - x^2 - \frac{x^4}{6}$	M1: Correct use of Maclaurin series	M1A1
		A1: Fully correct expansion.	
			(5)
(c)	$(y =) 1 + 3(-0.2) - (-0.2)^2 - \frac{(-0.2)^4}{6}$	Use of the correct Maclaurin series and substitution of $x = -0.2$	M1
		$(y =) 0.3597$	Allow awrt
			(2)
			Total 12

Question Number	Scheme		Marks
6.	$x \frac{dy}{dx} + (1 - 2x)y = x$		
	$\frac{dy}{dx} + \frac{(1 - 2x)}{x}y (= 1)$	Divides by x - may be implied by subsequent working	M1
	Integrating factor $I = e^{\int \frac{1-2x}{x} dx}$	Correct attempt at I , including an attempt at the integration. In must be seen if not fully correct.	dM1
	$= e^{\ln(x)-2x}$	Correct expression	A1
	$= xe^{-2x}$	No errors in working allowed	A1
	$xye^{-2x} = \int xe^{-2x} dx$	Multiply through by their IF and integrate LHS. Can be given if y x their IF is seen	M1
	$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x} dx$	M1: Correct application by parts ie differentiate x and attempt to integrate e^{-2x}	M1A1
		A1: Correct expression	
	$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} (+c)$	A1: Complete the integration to a correct result. Constant not required.	A1
	$y = \frac{ce^{2x}}{x} - \frac{1}{4x} - \frac{1}{2}$	Oe Must have $y = \dots$ Must include a constant. ft their previous line	A1ft
			(9)
			Total 9

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7. The point P represents a complex number z on an Argand diagram, where

$$|z + 1| = |2z - 1|$$

and the point Q represents a complex number w on the Argand diagram, where

$$|w| = |w - 1 + i|$$

Find the exact coordinates of the points where the locus of P intersects the locus of Q . (7)



Question Number	Scheme		Marks
7	$P : z + 1 = 2z - 1 $		
	$z = x + iy \Rightarrow x + iy + 1 = 2(x + iy) - 1 $		
	$(x+1)^2 + y^2 = (2x-1)^2 + (2y)^2$	M1: Correct use of Pythagoras A1: Any correct equation	M1A1
	$Q : w = w - 1 + i $		
	$w = x + iy \Rightarrow x + iy = x + iy - 1 + i $		
	$x^2 + y^2 = (x-1)^2 + (y+1)^2 (\Rightarrow y = x-1)$	M1: Correct use of Pythagoras. Allow with u and v instead of x and y A1: Any correct equation Must have x and y now.	M1A1
	Alternative – M1: identifies perpendicular bisector of (0, 0) and (1, -1) A1: $y = x - 1$		
	$(x-1)^2 + (x-1)^2 = 1$ or $y^2 + y^2 = 1$	Attempt to solve simultaneously tie obtain an equation in one variable and get to $x = \dots$ or $y = \dots$	M1
	$x = 1 + \frac{1}{\sqrt{2}}, 1 - \frac{1}{\sqrt{2}}$ or $y = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$	Both (oe)	A1
	$\left(1 + \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(1 - \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	Both (oe) Pairs must be clearly identifiable but coordinate brackets not needed.	A1
			(7)
			Total 7

Question Number	Scheme	Marks
8.	$x = e^t$	
(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	Attempt to use an appropriate version of the chain rule M1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{e^t} \left(= \frac{1}{x} \frac{dy}{dt} \right)$	Oe A1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2} \frac{dy}{dt} + \frac{1}{x} \frac{d^2y}{dt^2} \frac{dt}{dx}$ or $\frac{d^2y}{dx^2} = -\frac{1}{x^2} \frac{dy}{dt} + \frac{1}{x^2} \frac{d^2y}{dt^2}$	M1: Use of the product rule (penalise chain rule errors by loss of A mark or marks) $\left(\text{Note } t = \ln x \Rightarrow \frac{dt}{dx} = \frac{1}{x} \right)$ M1 A1, A1
	$x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 13y = 0$ $\Rightarrow x^2 \cdot \frac{1}{x^2} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right) + 5x \frac{1}{x} \frac{dy}{dt} + 13y = 0$ $\Rightarrow \frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 13y = 0^*$	M1: Substitutes their first and second derivatives into the given differential equation Depends on both M marks above A1: Correct completion to printed answer ddM1A1
		(7)
(b)	$m^2 + 4m + 13 = 0$ $\Rightarrow (m =) \frac{-4 \pm \sqrt{16 - 52}}{2}$	Attempt to solve the auxiliary equation M1
	$(m =) -2 \pm 3i$	Correct roots May be implied by a correct GS A1
	$y = e^{-2t} (A \cos 3t + B \sin 3t)$ or $y = Ae^{(-2+3i)t} + Be^{(-2-3i)t}$	Correct GS A1
	$t = \ln x$	B1
	$y = \frac{A \cos(3 \ln x) + B \sin(3 \ln x)}{x^2}$ or $y = Ae^{(-2+3i) \ln x} + Be^{(-2-3i) \ln x}$	A1
		(5)
		Total 12

9.

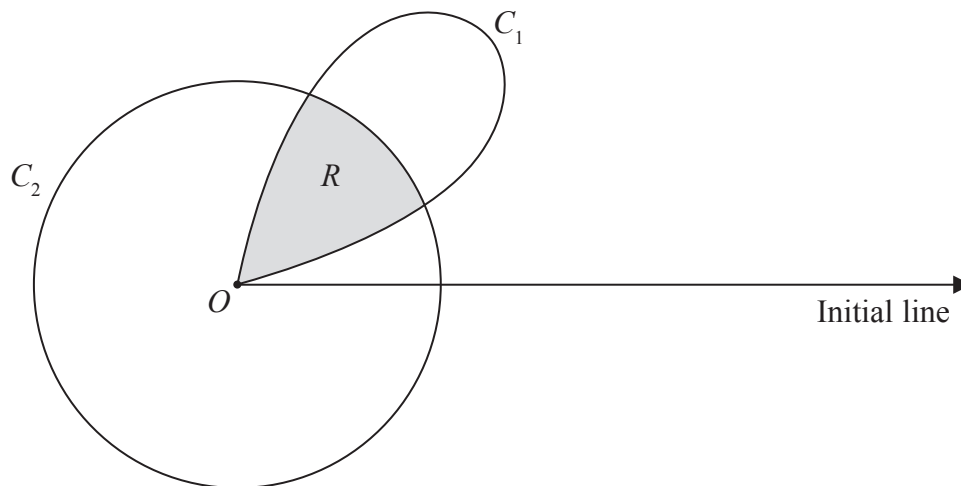


Figure 1

Figure 1 shows the curve C_1 with polar equation $r=2a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and the circle C_2 with polar equation $r = a$, $0 \leq \theta \leq 2\pi$, where a is a positive constant.

(a) Find, in terms of a , the polar coordinates of the points where the curve C_1 meets the circle C_2 (3)

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region R is shaded in Figure 1.

(b) Find the area of the shaded region R , giving your answer in the form $\frac{1}{12} a^2 (p\pi + q\sqrt{3})$, where p and q are integers to be found. (7)



Question Number	Scheme		Marks
9.			
(a)	$a = 2a \sin 2\theta \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \dots$	$C_1 = C_2$ and attempt to solve for 2θ	M1
	$\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	$2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or both Decimals allowed (min 3 sf).	A1
	$\left(a, \frac{\pi}{12}\right), \left(a, \frac{5\pi}{12}\right)$	Both points Can be written $r = a, \theta = \frac{\pi}{12}, \frac{5\pi}{12}$ Decimals allowed (min 3 sf).	A1
			(3)
(b)	$\frac{1}{2} \times a^2 \times \frac{\pi}{3}$ oe	Correct expression for the sector	B1
	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (2a \sin 2\theta)^2 d\theta$	Use of correct formula Limits not needed (ignore any shown)	M1
	$\cos 4\theta = 1 - 2\sin^2 2\theta$ $\Rightarrow \sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$	Uses $\sin^2 2\theta = \frac{\pm 1 \pm \cos 4\theta}{2}$	M1
	$\int (1 - \cos 4\theta) d\theta = \theta - \frac{1}{4} \sin 4\theta$	Correct integration Limits not needed (ignore any shown)	A1
	$I = a^2 \left[\theta - \frac{1}{4} \sin 4\theta \right]_0^{\frac{\pi}{12}}$ $= a^2 \left\{ \left(\frac{\pi}{12} - \frac{1}{4} \sin 4 \cdot \frac{\pi}{12} \right) - (0) \right\}$	An attempt to find one or both of the regions either side of the sector. ie uses limits $0, \frac{\pi}{12}$ and/or $\frac{5\pi}{12}, \frac{\pi}{2}$, limits to be substituted and subtracted (if non-zero after substitution). Limits to be used the correct way round. If two integrals seen award mark if either correct. Both previous method marks must have been scored.	ddM1
	$R = 2I + \frac{a^2 \pi}{6} = 2a^2 \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) + \frac{a^2 \pi}{6}$	Correct strategy for the complete area (sector + $2I$). All areas must be positive.	M1
	$R = \frac{1}{12} a^2 (4\pi - 3\sqrt{3})$	If decimals seen anywhere (either in rt 3 or the limits) this mark is lost.	A1
			(7)
			Total 10