Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 4 4 5 1 7 A 0 1 3 2

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1. (a) Show that

$$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{1}{2(r+1)(r+2)} - \frac{1}{2(r+2)(r+3)}$$

(2)

(b) Hence, or otherwise, find

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)(r+3)}$$

giving your answer as a single fraction in its simplest form.

(4)

Question Number	Scheme		Marks
1.(a)	$\frac{1}{2(r+1)(r+2)}-\frac{1}{2(r+2)(r+3)}$ $=\frac{r+3-(r+1)}{2(r+1)(r+2)(r+3)}$	Attempt common denominator	M1
	$=\frac{2}{2(r+1)(r+2)(r+3)}$ $=\frac{1}{(r+1)(r+2)(r+3)}$	Correct proof	A1
			(2)
(a) Way 2	$\frac{1}{(r+1)(r+2)(r+3)}=\frac{1}{r+2}\left(\frac{1}{(r+1)(r+3)}\right)=\frac{1}{r+2}\left(\frac{1}{2(r+1)}-\frac{1}{2(r+3)}\right)$ $\frac{1}{2(r+1)(r+2)}-\frac{1}{2(r+2)(r+3)}$		M1 A1
	M1: Factor of $\frac{1}{r+2}$ and attempt partial fractions A1: Correct proof		
	Other methods**: Complete** method scores M1 All work correct inc final answer reached A1		
(b)	$\sum_{r=1}^{n}\frac{1}{(r+1)(r+2)(r+3)}=$ $=\frac{1}{12}-\frac{1}{24}+........$ $+\frac{1}{2(n+1)(n+2)}-\frac{1}{2(n+2)(n+3)}$	Attempt at least the first pair and the last pair of terms as shown. Must start at 1 and end at *n*	M1
	$=\frac{1}{12}-\frac{1}{2(n+2)(n+3)}$	Identifies that the first and last terms do not cancel.	M1
	$=\frac{n^2+5n+6-6}{12(n+2)(n+3)}$	Correctly combined fractions	A1
	$=\frac{n^2+5n}{12(n+2)(n+3)}$ or $\frac{n(n+5)}{12(n+2)(n+3)}$ *	Allow either form isw attempts to multiply out the denominatot	A1
			(4)
			Total 6

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2. Use algebra to find the set of values of x for which

$$\frac{6}{x-3} \leqslant x+2$$

(7)

Question Number	Scheme		Marks
2.	$\frac{6}{x-3} \le x+2$		
	Way 1		
	$\frac{6}{x-3} \le x+2 \Rightarrow x+2-\frac{6}{x-3} \ge 0$		
	$x+2-\frac{6}{x-3} \ge 0 \Rightarrow \frac{(x+3)(x-4)}{x-3} \ge 0$	Attempt to combine fractions and factorise the numerator	M1
	$x=-3, \quad x=4$	Correct critical values	A1, A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$-3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)
	Way 2		
	$6(x-3) \le (x+2)(x-3)^2$ $\Rightarrow (x-3)(4-x)(x+3) \ge 0$	Multiplies both sides by $(x-3)^2$ and attempt to factorise	M1
	$x=-3, \quad x=4$	Correct critical values	A1, A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$-3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)
	Way 3		
	$x-3>0 \Rightarrow 6 \le (x+2)(x-3)$ $\Rightarrow (x-4)(x+3) \ge 0$	Multiplies both sides by $(x-3)$ and attempt to factorise Must state $x-3>0$	M1
	$x=4$	Correct critical values	A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$(x-3<0) \Rightarrow 6 \ge (x+2)(x-3)$ $x=-3$	Correct critical value	A1
	$(x+2)(x-3) \le 6 \Rightarrow (x-4)(x+3) \le 0$ $\Rightarrow -3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)

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3. Solve the equation

$$z^5 = 16 - 16\text{i}\sqrt{3}$$

giving your answers in the form $r\text{e}^{\text{i}\theta}$ where θ is in terms of π and $0 \leqslant \theta < 2\pi$.

(5)

Question Number	Scheme		Marks
3.			
	$r^5 = \sqrt{16^2 + (16\sqrt{3})^2} = 32 \Rightarrow r = 32^{\frac{1}{5}} \left(= 2\right)$	Correct value for r	B1
	$\arg(16 - 16i\sqrt{3}) = \frac{5\pi}{3}$	Allow $\frac{5\pi}{3}$ or $-\frac{\pi}{3}$	B1
	$5\theta = \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{23\pi}{3}, \frac{29\pi}{3}$	$\left(\frac{5\pi}{3}\right) + 2n\pi, n = 1, 2, 3, 4$ At least 2 values which must be positive. May be implied by correct final answers.	M1
	$z = \underline{2e^{\frac{\pi}{3}i}}, \underline{\underline{2e^{\frac{11\pi}{15}i}, 2e^{\frac{17\pi}{15}i}, 2e^{\frac{23\pi}{15}i}, 2e^{\frac{29\pi}{15}i}}}$	2 or $32^{\frac{1}{5}}$, $e^{\frac{5\pi}{15}i}$ or $e^{\frac{\pi}{3}i}$	B1 A1(all 4 values)
			(5)
			Total 5

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4. A transformation from the z-plane to the w-plane is given by

$$w = \frac{z}{z+3}, \qquad z \neq -3$$

Under this transformation, the circle $|z| = 2$ in the z-plane is mapped onto a circle C in the w-plane.

Determine the centre and the radius of the circle C.

(7)

Question Number	Scheme		Marks
4	$w=\frac{z}{z+3}$		
	$w=\frac{z}{z+3}\Rightarrow z=\frac{3w}{1-w}$	M1: Attempt to make *z* the subject A1: Correct expression for *z*	M1A1
	$\lvert z\rvert=2\Rightarrow\left\lvert\frac{3w}{1-w}\right\rvert=2$ $\lvert 3w\rvert=2\lvert 1-w\rvert$ $9\left(u^2+v^2\right)=4\left(u-1\right)^2+4v^2$	M1: Uses $\lvert z\rvert=2$ to obtain an equation in *u* and *v* Pythagoras must be used correctly. No i seen A1: Any correct equation in *u* and *v* Isw attempts to simplify	M1A1
	$5u^2+5v^2+8u-4=0$		
	$\left(u+\frac{4}{5}\right)^2+v^2-\frac{16}{25}-\frac{4}{5}=0$	Rearrange to a suitable form for a circle and attempt centre and/or radius. -16/25 (from completing the square) may be omitted. May be implied by centre and radius correct for their previous equation	M1
	Centre $\left(-\frac{4}{5},0\right)$	oe	A1
	Radius $\frac{6}{5}$	oe	A1
			(7)
			Total 7

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5.

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(a) Show that

$$\frac{d^4y}{dx^4} = \left(ax^2 + b\right)\frac{d^2y}{dx^2}$$

where a and b are constants to be found.

(5)

Given that $y = 1$ and $\frac{dy}{dx} = 3$ at $x = 0$

(b) find a series solution for y in ascending powers of x up to and including the term in x^4

(5)

(c) use your series to estimate the value of y at $x = -0.2$, giving your answer to four decimal places.

(2)

Question Number	Scheme		Marks
5	$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$		
(a)	$y''' - 2y' - 2xy'' + 2y'(=0)\left(y''' = 2xy''\right)$	M1: Attempt to differentiate including use of the product rule on $2x\frac{dy}{dx}$ Equation may have been re-written as $y'' = \ldots$ before differentiating A1: Correct differentiation	M1A1
	$y'''' - 2y'' - 2xy''' - 2y'' + 2y''(=0)$	M1: Second use of product rule. Dependent on first M1. A1: Correct differentiation NB A simpler form is obtained if $y''' - 2xy'' = 0$ is used.	dM1A1
	$y'''' = 2xy''' + 2y''$ $= 2x\left(2xy''\right) + 2y'' = \left(4x^2 + 2\right)y''$	Cao and cso	A1
			(5)
(b)	$y_0'' = -2,\ \ y_0''' = 0\ \ \ y_0^{iv} = -4$	B1: $y_0'' = -2$ M1: Attempts y_0''' and y_0^{iv} A1: All correct and obtained from correct expressions	B1, M1A1
	$(y =)1 + 3x - x^2 - \frac{x^4}{6}$	M1: Correct use of Maclaurin series A1: Fully correct expansion.	M1A1
			(5)
(c)	$(y =)1 + 3(-0.2) - (-0.2)^2 - \frac{(-0.2)^4}{6}$	Use of the correct Maclaurin series and substitution of $x = -0.2$	M1
	$(y =)\ 0.3597$	Allow awrt	A1
			(2)
			Total 12

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6.

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + (1-2x)y = x, \qquad x > 0$$

Find the general solution of the differential equation, giving your answer in the form $y = \mathrm{f}(x)$.

(9)

Question Number	Scheme		Marks
6.	$x\frac{dy}{dx}+(1-2x)y=x$		
	$\frac{dy}{dx}+\frac{(1-2x)}{x}y(=1)$	Divides by x - may be implied by subsequent working	M1
	Integrating factor $I=e^{\int\frac{1-2x}{x}dx}$	Correct attempt at I, including an attempt at the integration. ln must be seen if not fully correct.	dM1
	$=e^{\ln(x)-2x}$	Correct expression	A1
	$=xe^{-2x}$	No errors in working allowed	A1
	$xye^{-2x}=\int xe^{-2x}\,dx$	Multiply through by their IF and integrate LHS. Can be given if y x their IF is seen	M1
	$=-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}\,dx$	M1: Correct application by parts ie differentiate x and attempt to integrate e^{-2x} A1: Correct expression	M1A1
	$=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}\ (+c)$	A1: Complete the integration to a correct result. Constant not required.	A1
	$y=\frac{ce^{2x}}{x}-\frac{1}{4x}-\frac{1}{2}$	Oe Must have $y=$ ... Must include a constant. ft their previous line	A1ft
			(9)
			Total 9

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7. The point P represents a complex number z on an Argand diagram, where

$$|z+1| = |2z-1|$$

and the point Q represents a complex number w on the Argand diagram, where

$$|w| = |w-1+\mathrm{i}|$$

Find the exact coordinates of the points where the locus of P intersects the locus of Q.
(7)

Question Number	Scheme		Marks
7	$P: \lvert z+1 \rvert = \lvert 2z-1 \rvert$		
	$z = x+\mathrm{i}y \Rightarrow \lvert x+\mathrm{i}y+1 \rvert = \lvert 2(x+\mathrm{i}y)-1 \rvert$		
	$(x+1)^2 + y^2 = (2x-1)^2 + (2y)^2$	M1: Correct use of Pythagoras A1: Any correct equation	M1A1
	$Q: \lvert w \rvert = \lvert w-1+\mathrm{i} \rvert$		
	$w = x+\mathrm{i}y \Rightarrow \lvert x+\mathrm{i}y \rvert = \lvert x+\mathrm{i}y-1+\mathrm{i} \rvert$		
	$x^2 + y^2 = (x-1)^2 + (y+1)^2 \; (\Rightarrow y = x-1)$	M1: Correct use of Pythagoras. Allow with *u* and *v* instead of *x* and *y* A1: Any correct equation Must have *x* and *y* now.	M1A1
	Alternative – M1: identifies perpendicular bisector of (0, 0) and (1, -1) A1: $y = x - 1$		
	$(x-1)^2 + (x-1)^2 = 1$ or $y^2 + y^2 = 1$	Attempt to solve simultaneously tie obtain an equation in one variable and get to $x = ...$ or $y = ...$	M1
	$x = 1+\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}$ or $y = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$	Both (oe)	A1
	$\left(1+\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(1-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	Both (oe) Pairs must be clearly identifiable but coordinate brackets not needed.	A1
			(7)
			Total 7

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8. (a) Show that the substitution $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = 0, \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$$

(7)

(b) Hence find the general solution of the differential equation (I).

(5)

P 4 4 5 1 7 A 0 2 4 3 2

Question Number	Scheme		Marks
8.	$x = \mathrm{e}^t$		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$	Attempt to use an appropriate version of the chain rule	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\mathrm{e}^t} \left(= \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t} \right)$	Oe	A1
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}, + \frac{1}{x}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ or $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}, + \frac{1}{x^2}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$	M1: Use of the product rule (penalise chain rule errors by loss of A mark or marks) $\left(\text{Note } \; t = \ln x \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{x} \right)$	M1 A1, A1
	$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5x\frac{\mathrm{d}y}{\mathrm{d}x} + 13y = 0$ $\Rightarrow x^2 . \frac{1}{x^2}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t} \right) + 5x\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 0$ $\Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 0*$	M1: Substitutes their first and second derivatives into the given differential equation Depends on both M marks above A1: Correct completion to printed answer	ddM1A1
			(7)
(b)	$m^2 + 4m + 13 = 0$ $\Rightarrow (m =) \frac{-4 \pm \sqrt{16 - 52}}{2}$	Attempt to solve the auxiliary equation	M1
	$(m =) -2 \pm 3i$	Correct roots May be implied by a correct GS	A1
	$y = \mathrm{e}^{-2t}\left(A\cos 3t + B\sin 3t \right)$ or $y = A\mathrm{e}^{(-2+3\mathrm{i})t} + B\mathrm{e}^{(-2-3\mathrm{i})t}$	Correct GS	A1
	$t = \ln x$		B1
	$y = \frac{A\cos(3\ln x) + B\sin(3\ln x)}{x^2}$ or $y = A\mathrm{e}^{(-2+3\mathrm{i})\ln x} + B\mathrm{e}^{(-2-3\mathrm{i})\ln x}$		A1
			(5)
			Total 12

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9.

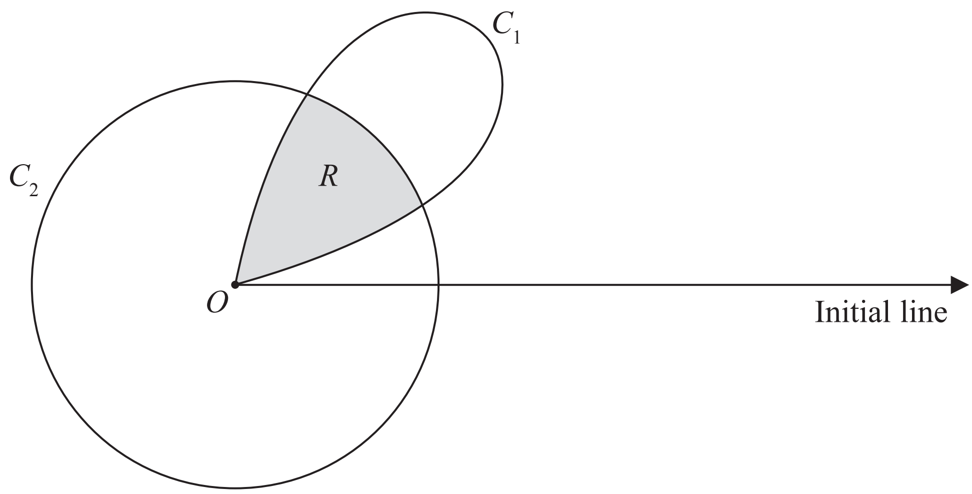

Figure 1

Figure 1 shows the curve C_1 with polar equation $r = 2a\sin 2\theta,\ 0 \leqslant \theta \leqslant \dfrac{\pi}{2}$, and the circle C_2 with polar equation $r = a$, $0 \leqslant \theta \leqslant 2\pi$, where a is a positive constant.

(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2

(3)

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region R is shaded in Figure 1.

(b) Find the area of the shaded region R, giving your answer in the form $\dfrac{1}{12}a^2\left(p\pi + q\sqrt{3}\right)$, where p and q are integers to be found.

(7)

Question Number	Scheme		Marks
9.			
(a)	$a = 2a\sin 2\theta \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = ...$	$C_1 = C_2$ and attempt to solve for 2θ	M1
	$\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	$2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or both Decimals allowed (min 3 sf).	A1
	$\left(a, \frac{\pi}{12}\right), \left(a, \frac{5\pi}{12}\right)$	Both points Can be written $r = a$, $\theta = \frac{\pi}{12}, \frac{5\pi}{12}$ Decimals allowed (min 3 sf).	A1
			(3)
(b)	$\frac{1}{2} \times a^2 \times \frac{\pi}{3}$ oe	Correct expression for the sector	B1
	$\frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int (2a\sin 2\theta)^2 \mathrm{d}\theta$	Use of correct formula Limits not needed (ignore any shown)	M1
	$\cos 4\theta = 1 - 2\sin^2 2\theta$ $\Rightarrow \sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$	Uses $\sin^2 2\theta = \frac{\pm 1 \pm \cos 4\theta}{2}$	M1
	$\int (1 - \cos 4\theta)\mathrm{d}\theta = \theta - \frac{1}{4}\sin 4\theta$	Correct integration Limits not needed (ignore any shown)	A1
	$I = a^2\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{12}}$ $= a^2\left\{\left(\frac{\pi}{12} - \frac{1}{4}\sin 4.\frac{\pi}{12}\right) - (0)\right\}$	An attempt to find one or both of the regions either side of the sector. ie uses limits $0, \frac{\pi}{12}$ and/or $\frac{5\pi}{12}, \frac{\pi}{2}$, limits to be substituted and subtracted (if non-zero after substitution). Limits to be used the correct way round. If two integrals seen award mark if either correct. Both previous method marks must have been scored.	ddM1
	$R = 2I + \frac{a^2\pi}{6} = 2a^2\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) + \frac{a^2\pi}{6}$	Correct strategy for the complete area (sector + 2*I*). All areas must be positive.	M1
	$R = \frac{1}{12}a^2\left(4\pi - 3\sqrt{3}\right)$	If decimals seen anywhere (either in rt 3 or the limits) this mark is lost.	A1
			(7)
			Total 10