Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 8 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. (a) Express $\dfrac{1}{4r^2 - 1}$ in partial fractions.
(1)

(b) Hence prove that

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$$

(3)

(c) Find the exact value of

$$\sum_{r=9}^{25} \frac{5}{4r^2 - 1}$$

(2)

Question Number	Scheme	Notes	Marks
1(a)	$\frac{1}{4r^2-1}$		
	$\frac{1}{2(2r-1)}-\frac{1}{2(2r+1)}$ or $\frac{\frac{1}{2}}{(2r-1)}-\frac{\frac{1}{2}}{(2r+1)}$ or equivalent or $\frac{1}{4r^2-1}\equiv\frac{A}{2r-1}+\frac{B}{2r+1}\Rightarrow A=\frac{1}{2},B=-\frac{1}{2}$	Correct partial fractions or correct values of '*A*' and '*B*'. Isw if possible so if correct values of '*A*' and '*B*' are found, award when seen even if followed by incorrect partial fractions.	B1
			(1)
(b)	$\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{1}{2}\left(1-\frac{1}{3}+......+\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ Attempt at least first and last terms using their partial fractions. May be implied by e.g. $\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$		M1
	$\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$ or $\frac{1}{2}-\frac{1}{2(2n+1)}$ or $\frac{1}{2}-\frac{1}{4n+2}$	Correct expression	A1
	$\frac{n}{2n+1}$ *	Correct completion with no errors	A1*
	Allow a different variable to be used in (a) and (b) but final answer in (b) must be as printed i.e. in terms of *n*.		
			(3)
(c)	$\sum_{r=9}^{25}\frac{5}{4r^2-1}=(5)(\mathrm{f}(25)-\mathrm{f}(8))$	$\mathrm{f}(25)-\mathrm{f}(8)$ where $\mathrm{f}(n)=\frac{n}{2n+1}$	M1
	$=5\left(\frac{25}{51}-\frac{8}{17}\right)=\frac{5}{51}$	cao	A1
	Correct answer with no working in (c) scores both marks.		
			(2)
			Total 6

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2. Use algebra to find the set of values of x for which

$$|x^2 - 9| < |1 - 2x|$$

(6)

Question Number	Scheme	Notes	Marks
2	$\lvert x^2-9 \rvert < \lvert 1-2x \rvert$ **(ignore use of "<" instead of "=" when finding cv's)**		
	$x^2-9=1-2x \Rightarrow x^2+2x-10=0 \Rightarrow x=...$ or $x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	Attempts to solve $x^2-9=1-2x$ **OR** $x^2-9=-1+2x$ to obtain two non-zero values of x	M1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **OR** $x=-2,\ 4$	**One** correct pair of values. Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.32, -4.32 or truncated 2.3, -4.3	A1
	$x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	Attempts to solve $x^2-9=1-2x$ **AND** $x^2-9=-1+2x$ to obtain four non-zero values of x	M1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **AND** $x=-2,\ 4$	**Both** pairs of values correct. Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.32, -4.32 or truncated 2.3, -4.3	A1
	$-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	One correct inequality.	B1
	For $-1+\sqrt{11}$ allow $\frac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\frac{-2-\sqrt{44}}{2}$ but must be exact here. Allow alternative notation e.g. $\left(-1+\sqrt{11},\ 4\right)$, $\left(-1-\sqrt{11},\ -2\right)$ $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$		
	$-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	Both inequalities correct.	B1
	For $-1+\sqrt{11}$ allow $\frac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\frac{-2-\sqrt{44}}{2}$ but must be exact here. Allow alternative notation e.g. $\left(-1+\sqrt{11},\ 4\right)$, $\left(-1-\sqrt{11},\ -2\right)$ $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$		
			(6)
			Total 6

	Q2 Alternative by squaring (ignore use of “<” instead of “=” when finding cv’s)		
	$(x^2-9)^2=(1-2x)^2 \Rightarrow x^4-18x^2+81=1-4x+4x^2$		
	$x^4-22x^2+4x+80=0 \Rightarrow x=...$	Squares and attempts to solve a quartic equation to obtain at least two values of x that are non-zero.	M1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **or** $x=-2,\ 4$	One pair of values correct as defined above	A1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **and** $x=-2,\ 4$	M1: Obtains four non-zero values of x. A1: Both pairs of values correct as defined above	M1A1
	$-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	See notes above	B1
	$-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	See notes above	B1
	In an otherwise fully correct solution, if any extra incorrect regions are given, deduct the final B mark.		

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3. Find, in terms of k, where k is a positive integer, the general solution of the differential equation

$$(1+x)\frac{\mathrm{d}y}{\mathrm{d}x} + ky = x^{\frac{1}{2}}(1+x)^{2-k}, \qquad x > 0$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

	Scheme	Notes	Marks
3	$(1+x)\frac{dy}{dx}+ky=x^{\frac{1}{2}}(1+x)^{2-k}$		
	$\frac{dy}{dx}+\frac{ky}{(1+x)}=\frac{x^{\frac{1}{2}}(1+x)^{2-k}}{(1+x)}$	Divides by $(1 + x)$ including the ky term	M1
	$I=e^{\int\frac{k}{1+x}dx}=(1+x)^k$	dM1: Attempt integrating factor. $I=e^{\int\frac{k}{1+x}dx}$ is sufficient for this mark but must include the k. Condone omission of "dx". A1: $(1+x)^k$	dM1A1
	$y(1+x)^k=\int x^{\frac{1}{2}}(1+x)dx$	Reaches $y\times(\text{their I})=\int x^{\frac{1}{2}}(1+x)^{1-k}\times(\text{their I})dx$	M1
	$\int x^{\frac{1}{2}}(1+x)dx=\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}(+c)$ or by parts $\int x^{\frac{1}{2}}(1+x)dx=\frac{2}{3}x^{\frac{3}{2}}(1+x)-\frac{4}{15}x^{\frac{5}{2}}(+c)$	Correct integration	A1
	$y=\frac{\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}+c}{(1+x)^k}$ or e.g. $y=\frac{\frac{2}{3}x^{\frac{3}{2}}(1+x)-\frac{4}{15}x^{\frac{5}{2}}+c}{(1+x)^k}$ or e.g. $y=\frac{2}{3}x^{\frac{3}{2}}(1+x)^{1-k}-\frac{4}{15}x^{\frac{5}{2}}(1+x)^{-k}+c(1+x)^{-k}$ or e.g. $y=\frac{10x^{\frac{3}{2}}(1+x)-4x^{\frac{5}{2}}+c}{15(1+x)^k}$ or e.g. $y=\frac{2}{3}x^{\frac{3}{2}}(1+x)^{-k}+\frac{2}{5}x^{\frac{5}{2}}(1+x)^{-k}+c(1+x)^{-k}$ **Correct answer with the constant correctly placed.** **Allow any equivalent correct answer.**		A1
			(6)
			Total 6

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4.

$$f(x) = \sin\left(\frac{3}{2}x\right)$$

(a) Find the Taylor series expansion for f(x) about $\frac{\pi}{3}$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^4$

(6)

(b) Hence obtain an estimate of $\sin\frac{1}{2}$, giving your answer to 4 decimal places.

(2)

Question Number	Scheme	Notes	Marks
4(a)	$f(x)=\sin\left(\frac{3}{2}x\right)$ $f'(x)=\frac{3}{2}\cos\left(\frac{3}{2}x\right)$ $f''(x)=-\frac{9}{4}\sin\left(\frac{3}{2}x\right)$ $f'''(x)=-\frac{27}{8}\cos\left(\frac{3}{2}x\right)$ $f''''(x)=\frac{81}{16}\sin\left(\frac{3}{2}x\right)$	M1: Attempt first 4 derivatives. Should be $\sin\to\cos\to\sin\to\cos\to\sin$. I.e. ignore signs and coefficients. A1: $f'=\frac{3}{2}\cos\left(\frac{3}{2}x\right)$ and $f''=-\frac{9}{4}\sin\left(\frac{3}{2}x\right)$ A1: $f'''=-\frac{27}{8}\cos\left(\frac{3}{2}x\right)$ and $f''''=\frac{81}{16}\sin\left(\frac{3}{2}x\right)$ Allow un-simplified e.g. $f''=-\frac{3}{2}\cdot\frac{3}{2}\sin\left(\frac{3}{2}x\right)$	M1A2
	$y\left(\frac{\pi}{3}\right)=1, y'\left(\frac{\pi}{3}\right)=0, y''\left(\frac{\pi}{3}\right)=-\frac{9}{4}, y'''\left(\frac{\pi}{3}\right)=0, y''''\left(\frac{\pi}{3}\right)=\frac{81}{16}$ Attempts at least 1 derivative at $x=\frac{\pi}{3}$		M1
	$f(x)=1-\frac{9}{8}\left(x-\frac{\pi}{3}\right)^2+\frac{27}{128}\left(x-\frac{\pi}{3}\right)^4$	dM1: Correct use of Taylor series. I.e. $f(x)=f\left(\frac{\pi}{3}\right)+\left(x-\frac{\pi}{3}\right)f'\left(\frac{\pi}{3}\right)+\left(x-\frac{\pi}{3}\right)^2\frac{f''\left(\frac{\pi}{3}\right)}{2!}+...$ Evidence of at least one term of the correct structure i.e. $\left(x-\frac{\pi}{3}\right)^n\frac{f^n\left(\frac{\pi}{3}\right)}{n!}$ and not a Maclaurin series. **Dependent on the previous method mark.** A1: Correct expansion. Allow equivalent **single** fractions for $\frac{9}{8}$ and/or $\frac{27}{128}$ and allow decimal equivalents i.e. 1.125 and 0.2109375. Ignore any extra terms.	dM1A1
			(6)
(b)	$f\left(\frac{1}{3}\right)=0.4815$	M1: Attempts $f\left(\frac{1}{3}\right)$ or states $x=\frac{1}{3}$ A1: 0.4815 cao	M1A1
			(2)
			Total 8

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5. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{2z - 1}{z + 3}, \quad z \neq -3$$

The circle in the z-plane with equation $x^2 + y^2 = 1$, where $z = x + \mathrm{i}y$, is mapped by T onto the circle C in the w-plane.

Find the centre and the radius of C.

(7)

Question Number	Scheme	Notes	Marks
5	$z=\frac{3w+1}{2-w}$	M1: Attempt to make z the subject as far as $z = \ldots$ A1: Correct equation	M1A1
	$\lvert z\rvert=1\Rightarrow\left\lvert\frac{3w+1}{2-w}\right\rvert=1\Rightarrow\left\lvert\frac{3(u+\mathrm{i}v)+1}{2-(u+\mathrm{i}v)}\right\rvert=1$	Uses $\lvert z\rvert=1$ and introduces $w = u + \mathrm{i}v$	M1
	$(3u+1)^2+(3v)^2=(u-2)^2+v^2$	M1: Correct use of Pythagoras. Condone missing brackets provided the intention is clear and allow e.g. $(3v)^2 = 3v^2$ but there should be no i's.	M1
	$u^2+v^2+\frac{10}{8}u-\frac{3}{8}=0$		
	$\left(u+\frac{5}{8}\right)^2-\frac{25}{64}+v^2=\frac{3}{8}$	Attempt to complete the square on the equation of a circle. I.e. an equation where the coefficients of u^2 and v^2 are the same and the other terms are in u, v or are constant. (Allow slips in completing the square). **Dependent on all previous M marks.**	ddddM1
	$\left(u+\frac{5}{8}\right)^2+v^2=\frac{49}{64}\Rightarrow\left(-\frac{5}{8},0\right),\frac{7}{8}$	A1: Centre $\left(-\frac{5}{8},0\right)$ A1: Radius $\frac{7}{8}$	A1A1
			(7)
			Total 7
	Alternative for the first 3 marks		
	$z=\frac{3w+1}{2-w}$	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$x+\mathrm{i}y=\frac{3(u+\mathrm{i}v)+1}{2-(u+\mathrm{i}v)}=\frac{(3u+1+3\mathrm{i}v)}{2-u-\mathrm{i}v}\times\frac{2-u+\mathrm{i}v}{2-u+\mathrm{i}v}=\frac{5u+2-3(u^2+v^2)}{(2-u)^2+v^2}+\frac{7v}{(2-u)^2+v^2}\mathrm{i}$ $x^2+y^2=1\Rightarrow\left(\frac{5u+2-3(u^2+v^2)}{(2-u)^2+v^2}\right)^2+\left(\frac{7v}{(2-u)^2+v^2}\right)^2=1$		M1
	Introduces $w = u + \mathrm{i}v$, multiplies numerator and denominator by the complex conjugate of the denominator and uses $x^2+y^2=1$ correctly to obtain an equation in u and v.		

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$$

(9)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

Question Number	Scheme	Notes	Marks
	$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3x^2+2x+1$		
6(a)	$m^2+3m+2=0 \Rightarrow m=-1,-2$	Correct roots (may be implied by their CF)	B1
	$y=Ae^{-2x}+Be^{-x}$	M1: CF of the correct form A1: Correct CF	M1A1
	$y=ax^2+bx+c$	Correct form for PI	B1
	$\frac{dy}{dx}=2ax+b,\ \frac{d^2y}{dx^2}=2a \Rightarrow 2a+3(2ax+b)+2(ax^2+bx+c)=3x^2+2x+1$		M1
	M1: Differentiates twice and substitutes into the lhs of the given differential equation and puts equal to $3x^2+2x+1$ or substitutes into the lhs of the given differential equation and compares coefficients with $3x^2+2x+1$. For the substitution, at least one of y, y' or y'' must be correctly placed.		
	$a=\frac{3}{2}$		A1
	$6a+2b=2 \Rightarrow b=-\frac{7}{2} \Rightarrow c=\frac{17}{4}$	M1: Solves to obtain one of b or c A1: Correct b and c	M1A1
	$y=Ae^{-2x}+Be^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(9)
(b)	$0=A+B+\frac{17}{4}$	Substitutes $x=0$ and $y=0$ into their GS	M1
	$\frac{dy}{dx}=-2Ae^{-2x}-Be^{-x}+3x-\frac{7}{2} \Rightarrow 0=-2A-B-\frac{7}{2}$ Attempts to differentiate and substitutes $x=0$ and $y'=0$		M1
	$0=A+B+\frac{17}{4},\ 0=-2A-B-\frac{7}{2} \Rightarrow A=..,B=..$	Solves simultaneously to obtain values for A and B	M1
	$A=\frac{3}{4},\quad B=-5$	Correct values	A1
	$y=\frac{3}{4}e^{-2x}-5e^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(5)
			Total 14

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7.

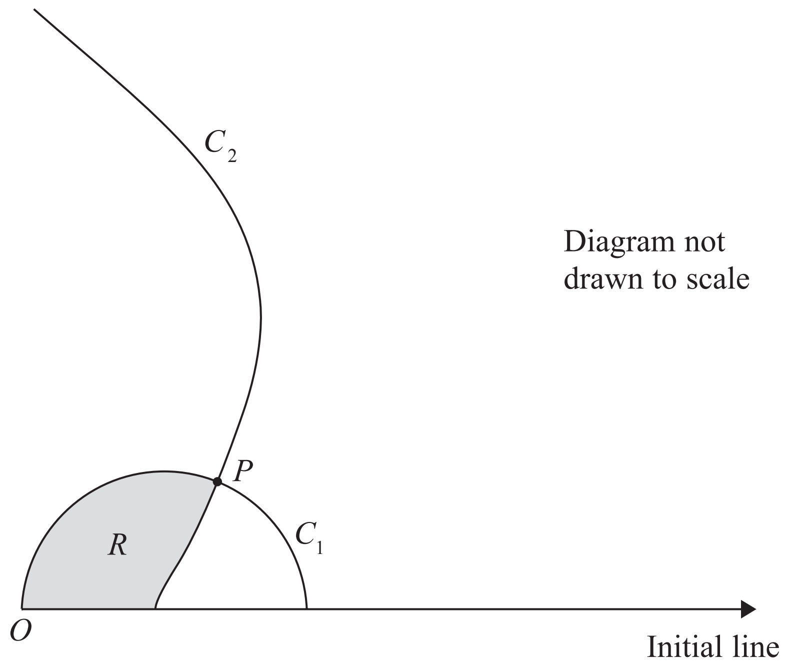

Figure 1

Figure 1 shows a sketch of the curves C_1 and C_2 with polar equations

$$C_1 : r = \frac{3}{2}\cos\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

$$C_2 : r = 3\sqrt{3} - \frac{9}{2}\cos\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The curves intersect at the point P.

(a) Find the polar coordinates of P.

(3)

The region R, shown shaded in Figure 1, is enclosed by the curves C_1 and C_2 and the initial line.

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(8)

Question Number	Scheme	Notes	Marks
7.	$C_1 :\ r = \frac{3}{2}\cos\theta, \quad C_2 :\ r = 3\sqrt{3} - \frac{9}{2}\cos\theta$		
(a)	$\frac{3}{2}\cos\theta = 3\sqrt{3} - \frac{9}{2}\cos\theta \Rightarrow \theta = ...$ or $\cos\theta = \frac{2r}{3} \Rightarrow r = 3\sqrt{3} - 3r \Rightarrow r = ...$	Puts $C_1 = C_2$ and attempt to solve for θ or Eliminates cos θ and solves for r	M1
	$\theta = \frac{\pi}{6}$ or $r = \frac{3\sqrt{3}}{4}$	Correct θ or correct r. Allow θ = awrt 0.524, r = awrt 1.3	A1
	$r = \frac{3\sqrt{3}}{4}$ and $\theta = \frac{\pi}{6}$	Correct r and θ (isw e.g. $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{4}\right)$) Allow θ = awrt 0.524, r = awrt 1.3	A1
			(3)

7(b)	$\frac{1}{2}\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 \mathrm{d}\theta$ or $\frac{1}{2}\int\left(\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta$	M1
	Attempts to use correct formula on either curve. The ½ may be implied by later work.	
	$\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 = 27-27\sqrt{3}\cos\theta+\frac{81}{4}\cos^2\theta = 27-27\sqrt{3}\cos\theta+\frac{81}{4}\frac{(\cos 2\theta+1)}{2}$	M1
	Expands to obtain an expression of the form $a+b\cos\theta+c\cos^2\theta$ and attempts to use $\cos^2\theta=\pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$	
	$\left(\frac{1}{2}\right)\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 \mathrm{d}\theta = \left(\frac{1}{2}\right)\left[\frac{297}{8}\theta-27\sqrt{3}\sin\theta+\frac{81}{16}\sin 2\theta\right]$	M1A1
	M1: Attempts to integrate to obtain at least two terms from $\alpha\theta,\ \beta\sin\theta,\ \gamma\sin 2\theta$ A1: Correct integration with or without the ½ (NB $\frac{297}{8}=27+\frac{81}{8}$)	
	$\left(\frac{1}{2}\right)\left[\frac{297}{8}\theta-27\sqrt{3}\sin\theta+\frac{81}{16}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right)\left\{\left(\frac{297}{8}.\frac{\pi}{6}-27\sqrt{3}.\sin\frac{\pi}{6}+\frac{81}{16}\sin 2.\frac{\pi}{6}\right)(-0)\right\}$	M1
	M1: Uses the limits 0 and their $\frac{\pi}{6}$ If the substitution for $\theta=0$ evaluates to 0 then the substitution for $\theta=0$ does not need to be seen but if it does not evaluate to 0, the substitution for $\theta=0$ needs to be seen.	
	$\frac{1}{2}\int\left(\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta = \frac{9}{16}\int(\cos 2\theta+1)\mathrm{d}\theta = \frac{9}{16}\left[\frac{1}{2}\sin 2\theta+\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{9}{16}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$	M1
	M1: Uses $\cos^2\theta=\pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$, integrates to obtain at least $k\sin 2\theta$ and uses the limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$ to find the other area NB can be done as a segment : $\frac{1}{2}\left(\frac{3}{4}\right)^2\left(\frac{2\pi}{3}\right)-\frac{1}{2}\left(\frac{3}{4}\right)^2\sin\left(\frac{\pi}{3}\right)$ Allow $\frac{1}{2}\left(\frac{3}{4}\right)^2\left(\pi-2\times\text{their}\frac{\pi}{6}\right)-\frac{1}{2}\left(\frac{3}{4}\right)^2\sin\left(\pi-2\times\text{their}\frac{\pi}{6}\right)$	
	$\frac{297}{96}\pi-\frac{351\sqrt{3}}{64}+\frac{9}{16}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)=\frac{105}{32}\pi-\frac{45}{8}\sqrt{3}$	M1A1
	M1: Adds their two areas both of which are of the form $a\pi+b\sqrt{3}$ A1: Correct answer (allow equivalent fractions for $\frac{105}{32}$ and/or $\frac{45}{8}$)	
		(8)
		Total 11

Special Case – Uses $\pm(C_1 - C_2)$

(b)	$\frac{1}{2}\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta-\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta$	M1
	Attempts to use correct formula on $\pm(C_1 - C_2)$. The ½ may be implied by later work.	
	$\left(3\sqrt{3}-6\cos\theta\right)^2 = 27-36\sqrt{3}\cos\theta+36\cos^2\theta = 27-36\sqrt{3}\cos\theta+36\frac{(\cos 2\theta+1)}{2}$	M1
	Expands to obtain an expression of the form $a+b\cos\theta+c\cos^2\theta$ and attempts to use $\cos^2\theta = \pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$	
	$\left(\frac{1}{2}\right)\int\left(3\sqrt{3}-6\cos\theta\right)^2\mathrm{d}\theta = \left(\frac{1}{2}\right)\left[45\theta-36\sqrt{3}\sin\theta+9\sin 2\theta\right]$	M1
	Attempts to integrate to obtain at least two terms from $\alpha\theta,\ \beta\sin\theta,\ \gamma\sin 2\theta$	
	No more marks available	

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8. (a) Use de Moivre's theorem to show that

$$\cos^5\theta \equiv p\cos 5\theta + q\cos 3\theta + r\cos\theta$$

where p, q and r are rational numbers to be found.

(6)

(b) Hence, showing all your working, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, d\theta$$

(4)

Question Number	Scheme	Notes	Marks
8(a) WAY 1	$\left(z+\frac{1}{z}\right)^5 = z^5+5z^3+10z+\frac{10}{z}+\frac{5}{z^3}+\frac{1}{z^5}$	M1: Attempt to expand $\left(z\pm\frac{1}{z}\right)^5$ A1: Correct expansion with correct powers of z.	M1A1
	$z=\cos\theta+\mathrm{i}\sin\theta \Rightarrow z+\frac{1}{z}=2\cos\theta$	May be implied	B1
	$=z^5+\frac{1}{z^5}+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)=2\cos5\theta+10\cos3\theta+20\cos\theta$ Uses at least one of $z^5+\frac{1}{z^5}=2\cos5\theta$ or $z^3+\frac{1}{z^3}=2\cos3\theta$		M1
	$\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos5\theta+\frac{5}{16}\cos3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
			(6)
	WAY 2 (Using $\mathrm{e}^{\mathrm{i}\theta}$)		
	$\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)^5=\mathrm{e}^{5\mathrm{i}\theta}+5\mathrm{e}^{3\mathrm{i}\theta}+10\mathrm{e}^{\mathrm{i}\theta}+10\mathrm{e}^{-\mathrm{i}\theta}+5\mathrm{e}^{-3\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}$	M1: Attempt to expand $\left(\mathrm{e}^{\mathrm{i}\theta}\pm\mathrm{e}^{-\mathrm{i}\theta}\right)^5$ A1: Correct expansion	M1A1
	$2\cos\theta=\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}$	May be implied	B1
	$=\mathrm{e}^{5\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}+5\left(\mathrm{e}^{3\mathrm{i}\theta}+\mathrm{e}^{-3\mathrm{i}\theta}\right)+10\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)=2\cos5\theta+10\cos3\theta+20\cos\theta$ Uses one of $\mathrm{e}^{5\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}=2\cos5\theta$ or $\mathrm{e}^{3\mathrm{i}\theta}+\mathrm{e}^{-3\mathrm{i}\theta}=2\cos3\theta$		M1
	$\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos5\theta+\frac{5}{16}\cos3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
	WAY 3 (Using De Moivre on cos 5θ and identity for cos 3θ)		
	$(\cos\theta+\mathrm{i}\sin\theta)^5=c^5+5\mathrm{i}c^4s+10c^3\mathrm{i}^2s^2+10c^2\mathrm{i}^3s^3+5c\mathrm{i}^4s^4+\mathrm{i}^5s^5$ M1: Attempts to expand. NB may only consider real parts here. A1: Correct real terms (may include i's) (Ignore imaginary parts for this mark)		M1A1
	$\cos5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$	Correct real terms with no i's	B1
	$=\cos^5\theta-10\cos^3\theta\left(1-\cos^2\theta\right)+5\cos\theta\left(1-\cos^2\theta\right)^2$	Uses $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin\theta$	M1
	$16\cos^5\theta=\cos5\theta+20\cos^3\theta-5\cos\theta$		
	$\cos3\theta=4\cos^3\theta-3\cos\theta$	Correct identity for cos 3θ	B1
	$16\cos^5\theta=\cos5\theta+5\cos3\theta+10\cos\theta$		
	$\cos^5\theta=\frac{1}{16}\cos5\theta+\frac{5}{16}\cos3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
			(6)

(b)	$\int\left(\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta\right)\mathrm{d}\theta=\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta$ M1: Attempt to integrate – Evidence of $\cos n\theta \to \pm\frac{1}{n}\sin n\theta$ where $n=5$ or 3 A1ft: Correct integration (ft their p, q, r)		M1A1ft
	$\left[\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\left(\frac{1}{80}\sin\frac{5\pi}{3}+\frac{5}{48}\sin\pi+\frac{5}{8}\sin\frac{\pi}{3}\right)-\left(\frac{1}{80}\sin\frac{5\pi}{6}+\frac{5}{48}\sin\frac{\pi}{2}+\frac{5}{8}\sin\frac{\pi}{6}\right)$ Substitutes the given limits into a changed function and subtracts the right way round. There should be evidence of the substitution of $\frac{\pi}{3}$ and $\frac{\pi}{6}$ into their changed function for at least 2 of their terms and subtraction. If there is no evidence of substitution and the answer is incorrect, score M0 here.		M1
	$=\frac{49\sqrt{3}}{160}-\frac{203}{480}$	Allow exact equivalents e.g. $=\frac{1}{16}\left(4.9\sqrt{3}-\frac{203}{30}\right)$	A1
	If they use the letters *p*, *q* and *r* or their values of *p*, *q* and *r*, even from no working, the M marks are available in (b) but <u>not</u> the A marks.		
			(4)
			Total 10

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9. The complex number z is represented by the point P in an Argand diagram.

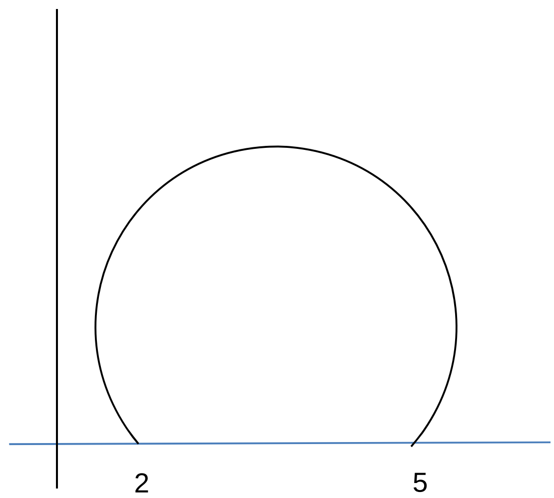
Given that $\arg\left(\dfrac{z-5}{z-2}\right) = \dfrac{\pi}{4}$

(a) sketch the locus of P as z varies,

(3)

(b) find the exact maximum value of $|z|$.

(4)

Question Number	Scheme	Notes	Marks
	$\arg\left(\frac{z-5}{z-2}\right)=\frac{\pi}{4}$		
9(a)		M1 A circle or an arc of a circle anywhere. Allow dotted or dashed. A1 A circle or an arc of a circle (allow dotted or dashed) passing through or touching at 2 and 5 on the positive real axis. (Imaginary axis may be missing) A1 Fully correct diagram with 2 and 5 marked correctly with no part of the circle below the real axis. It must be a major arc and not a semi-circle. The imaginary axis must be present and the arc must not cross or touch the imaginary axis.	
			(3)
(b)	Centre $C(x_c, y_c)$ is at $(3.5, 1.5)$	May be implied and may appear on the diagram. Can score anywhere e.g. from finding the equation of the circle in part (a) or as part of the calculation for *OC*.	B1
	$r=\sqrt{1.5^2+1.5^2}\left(=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}\right)$	$r=\sqrt{2\times y_c^{\ 2}}$ or equivalent work e.g. $r=\frac{1.5}{\cos 45}$, $r=\frac{1.5}{\sin 45}$ or $\frac{3\sqrt{2}}{2}$ seen	M1
	Max $\lvert z\rvert = OC+r=\sqrt{3.5^2+1.5^2}+r$		M1
	$=\frac{\sqrt{58}}{2}+\frac{3}{\sqrt{2}}$	Oe e.g $\sqrt{14.5}+\sqrt{4.5}$, $\frac{\sqrt{58}+3\sqrt{2}}{2}$	A1
			(4)
	Special Case – correct work with arc below the real axis:		
	Centre $C(x_c, y_c)$ is at $(3.5, -1.5)$		B0
	$r=\sqrt{1.5^2+1.5^2}\left(=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}\right)$	$r=\sqrt{2\times y_c^{\ 2}}$ or equivalent work e.g. $r=\frac{1.5}{\cos 45}$, $r=\frac{1.5}{\sin 45}$ or $\frac{3\sqrt{2}}{2}$ seen	M1
	Max $\lvert z\rvert = OC+r=\sqrt{3.5^2+1.5^2}+r$		M1
	$=\frac{\sqrt{58}}{2}+\frac{3}{\sqrt{2}}$	Oe e.g $\sqrt{14.5}+\sqrt{4.5}$, $\frac{\sqrt{58}+3\sqrt{2}}{2}$	A1
			Total 7