Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C12
Advanced Subsidiary

Wednesday 23 May 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. The table below shows corresponding values of x and y for $y = \dfrac{1}{\sqrt{(x+1)}}$, with the values for y rounded to 3 decimal places where necessary.

x	0	3	6	9	12	15
y	1	0.5	0.378	0.316	0.277	

(a) Complete the table by giving the value of y corresponding to $x = 15$

(1)

(b) Use the trapezium rule with all the values of y from the completed table to find an approximate value for

$$\int_0^{15} \frac{1}{\sqrt{(x+1)}}\,dx$$

giving your answer to 2 decimal places.

(4)

June 2018
International A Level WMA01/01 Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1.(a)	x: 0, 3, 6, 9, 12, 15; y: 1, 0.5, 0.378, 0.316, 0.277, **0.25**	B1 **[1]**
(b)	State h = 3, or use of $\frac{1}{2}\times 3$;	B1
	For sight of the expression $1+0.25+2(\ 0.5+0.378+0.316+0.277\ )$	M1
	Area= $\frac{1}{2}\times 3\times\{1+0.25+2(\ 0.5+0.378+0.316+0.277\ )\}$, = awrt 6.29	A1ft, A1 **[4]**
		5 marks

x	0	3	6	9	12	15
y	1	0.5	0.378	0.316	0.277	**0.25**

Notes

(a)

B1: 0.25 or exact equivalent. Allow 0.250.
It may not be in the table. Award even if it is within the trapezium rule.

(b)

B1: for using $\frac{1}{2}\times 3$ or h = 3 or sight of $1.5\{........(...)\}$ Award for the expression $\frac{15-0}{5}$ or similar.

M1: For an attempt at the correct $\{......\}$ bracket structure.
Just look for the correct sequence of terms within an expression. $1+0.25+2(\ 0.5+0.378+0.316+0.277\ )$
Condone this bracketing error $1.5\times(1+0.25)+2(0.5+0.378+0.316+0.277)$ for the M mark
If there is a copying error or one value is omitted from the 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if **all** values used in brackets are x values instead of y values
A1ft: for a correct un simplified expression with correct bracketing following through on their h and their 0.25.
Note that candidates may recover following a bracketing error if the final answer is correct
The bracketing error (with h =3) usually results in an answer of 4.817. This scores B1M1A0A0 in (b)
A1: for answer which rounds to 6.29

NB: Separate trapezia may be used: B1 for 3, M1 for 1/2 $h(a + b)$ used 4 or 5 times (and A1 if it is all correct) Then A1 as before.

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2. $$f(x) = ax^3 + 2x^2 + bx - 3$$

where a and b are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 1

(a) Show that

$$a + 4b = 28$$

(2)

When $f(x)$ is divided by $(x + 1)$ the remainder is –17

(b) Find the value of a and the value of b.

(4)

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Question Number	Scheme	Marks
2.	$\mathrm{f}(x) = ax^3 + 2x^2 + bx - 3$	
(a)	Attempts $\mathrm{f}(\pm\frac{1}{2})$ and puts expression equal to 1 **or** Use long division and sets remainder =1	M1
	$\mathrm{f}(\frac{1}{2}) = a\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) + b\left(\frac{1}{2}\right) - 3 = 1$ so $a + 4b = 28$* **or** $-3 + b\left(\frac{1}{2}\right) + \frac{1}{2} + \frac{a}{8} = 1$ so $a + 4b = 28$*	A1 **[2]**
(b)	Attempts $\mathrm{f}(\pm 1)$ and puts expression equal to -17 Or Use long division and sets remainder equal to -17	M1
	$\Rightarrow -3 - b - a + 2 = -17$ { so $a + b = 16$ }	A1
	Solve simultaneous equations to give values for a **and** b	dM1
	$a = 12$ **and** $b = 4$	A1 **[4]**
		6 marks

Notes

(a)

M1: Puts $\mathrm{f}(\pm\frac{1}{2}) = 1$ or $\mathrm{f}(\pm\frac{1}{2}) - 1 = 0$

Alternatively uses long division and produces a remainder in a and b that is set equal to 1

A1: cao Note that answer is printed so some working, which needs to be correct, (see scheme) needs to be seen.

Accept $1a + 4b = 28$ as well as $28 = 1a + 4b$

(b)

M1: Attempts $\mathrm{f}(\pm 1) = \pm 17$ or may attempt $\mathrm{f}(\pm 1) + 17 = 0$

Alternatively uses long division and produces a remainder in a and b that is set equal to -17

A1: A correct un-simplified equation but the powers of -1 must have been processed correctly– does not need to be simplified to $a + b$=16

dM1: Solves their equation with a+4b =28 to arrive at values for a and b. Do not be worried by the processing of this Allow the answer(s) to appear from two equations. It is dependent upon the previous M.

A1: Correct values

Leave blank

3. The line l_1 passes through the points $A(-1, 4)$ and $B(5, -8)$

(a) Find the gradient of l_1

(2)

The line l_2 is perpendicular to the line l_1 and passes through the point $B(5, -8)$

(b) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Question Number	Scheme	Marks
3. (a)	Gradient = $\frac{4-(-8)}{-1-5}=-2$	M1 A1 **[2]**
(b)	Perpendicular line has gradient $\frac{-1}{m}\left(=\frac{1}{2}\right)$	M1
	Line has equation $y-(-8)=\frac{1}{2}(x-5)$ or $y=\frac{1}{2}x+c,\ with\ c=-8-\frac{1}{2}\times 5\ so\ y=\frac{1}{2}x-10\frac{1}{2}$	M1 A1
	So $x-2y-21=0$	A1 **[4]**
		6 marks
	Notes	

(a)

M1: Attempts to use gradient formula $\frac{\Delta y}{\Delta x}$

You may condone only one sign slip even after a correct formula is quoted.

It may be implied by $\frac{-12}{4},\frac{12}{-4},\frac{-4}{-6},\frac{4}{6},\frac{12}{6},2$

The correct answer of -2 implies both marks.

Alternatively solves simultaneous equations $-8=5m+c$ and $4=-1m+c$ and proceeds to find *m*. Allow one sign slip here.

A1: cao. Do not allow fractions

(b)

M1: Uses or states negative reciprocal of their gradient

M1: Uses line equation with point $(5,-8)$ and a **changed** gradient. Condone one sign slip Eg $y-8=\frac{1}{2}(x-5)$

If the form $y=mx+c$ is used they must proceed as far as *c* = ...

A1: Any un-simplified form of correct line equation

A1: cao – accept $k(x-2y-21)=0$ where *k* is an integer ≠ 0 and accept any order of the terms = 0.

Allow $1x-2y-21=0$

Allow the candidate to state $a=1,b=-2,c=-21$ but do not penalise after a correct answer.

Leave blank

4. Given that

$$y = \frac{64x^6}{25}, \quad x > 0$$

express each of the following in the form kx^n where k and n are constants.

(a) $y^{-\frac{1}{2}}$

(3)

(b) $(25y)^{\frac{2}{3}}$

(2)

Question Number	Scheme	Marks
4. (a)	$y^{-\frac{1}{2}} = \left(\frac{64x^6}{25}\right)^{-\frac{1}{2}} = \frac{5}{8}x^{-3}$	M1 A1 A1 **[3]**
(b)	$(25y)^{\frac{2}{3}} = 16x^4$	B1, B1 **[2]**
		5 marks
	Notes	

(a)

M1: Sight of 5 or 0.2, 8 or 0.125, x^3 or x^{-3}

Do not award if the 5 is 5^2 or the x^3 is $\left(x^3\right)^{-2}$

A1: For achieving the correct coefficient $\frac{5}{8}x^p, \frac{5}{8x^p}, \frac{1}{1.6}x^p, 0.625\,x^p$ in their final answer

or the correct index qx^{-3} in their final answer.

A1: $\frac{5}{8}x^{-3}$ cao final answer . Accept $0.625\,x^{-3}$

Note that $\frac{0.625}{x^3}$ is not in the correct form. See the demand of the question

Do not withhold the mark if $\frac{5}{8}x^{-3}$ is followed by $\frac{5}{8x^3}$ in the candidate's response.

(b)

B1: 16 or x^4 correct, in the final answer

B1: 16 x^4 cao final answer. Allow $16 \times x^4$

Leave blank

5. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1+\frac{x}{3}\right)^{18}$$

giving each term in its simplest form.

(4)

(b) Use the answer to part (a) to find an estimated value for $\left(\frac{31}{30}\right)^{18}$, stating the value of x that you have used and showing your working. Give your estimate to 4 decimal places.

(3)

Question Number	Scheme	Marks
5.	(a) $\left(1+\frac{x}{3}\right)^{18}=1+\binom{18}{1}\cdot\left(\frac{x}{3}\right)+\binom{18}{2}\cdot\left(\frac{x}{3}\right)^2+\binom{18}{3}\cdot\left(\frac{x}{3}\right)^3...$	M1
	$=1+6x,+17x^2,+\frac{272}{9}x^3...$	B1, A1, A1
		[4]
	(b) Use $x = 0.1$	B1
	$=1+6\times0.1+17\times(0.1)^2+\frac{272}{9}\times(0.1)^3...$ or equivalent	M1
	$= 1.8002$	A1cao **[3]**
		7 marks
	Notes	

(a)

M1: The **method** mark is awarded for an attempt at Binomial to get the second third or fourth term – need to see $\frac{x}{3}$ used with a **correct** power of x and **a correct** binomial coefficient. Eg $\binom{18}{3}\cdot\left(\frac{x^3}{3}\right)$ is fine for M1

Accept any notation for ${}^{18}C_1$, ${}^{18}C_2$ and ${}^{18}C_3$, e.g. $\binom{18}{1}$, $\binom{18}{2}$ and $\binom{18}{3}$ (un-simplified) or 18, 153 and 816 from Pascal's triangle. This mark may be given if no working is shown, but either or both of the terms including x correct.

B1: For the first two terms. The coefficient of x may be un simplified, but the 1^{18} must become 1.

Accept $1+18\left(\frac{x}{3}\right)$ or listed as $1, 6x$

A1: For either $+17x^2$, or $+\frac{272}{9}x^3...$ which must be in the simplified form

A1: is cao and is for all of the terms correct and simplified –(ignore extra terms). Accept $30\frac{2}{9}x^3$ or $30.\dot{2}x^3$

It is OK to write as a list $1, 6x, 17x^2, \frac{272}{9}x^3$

Remember to isw after the correct answer. (Some students will go on to multiply by 9)

(b)

B1: States or uses $x = 0.1$ or equivalent such as $\frac{1}{10}$ or $\frac{3}{30}$ This must be seen, it is a demand of the question

M1: This is for fully substituting their value into their series expansion with at least 4 terms.

Accept sight of a value of x substituted into their expression as evidence of the M1 but do not allow $x=\frac{31}{30}$ or $\frac{31}{30}^{18}$

A1: This is for 1.8002 and is cao.

Note: The calculator answer to $\left(\frac{31}{30}\right)^{18}$ is 1.8044 This scores B0 M0 A0 ....unless $x = 0.1$ is stated and then scores B1 M0 A0

Note: A candidate just writing $\left(\frac{31}{30}\right)^{18}=1.8002$, the correct answer, scores SC B0 M1 A1

Leave blank

6. Find the exact values of x for which

$$2\log_5(x + 5) - \log_5(2x + 2) = 2$$

Give your answers as simplified surds.

(7)

Question Number	Scheme	Marks
6.	Use or state $2\log_5(x+5)=\log_5(x+5)^2$	M1
	Use or states $\log_5(x+5)^2-\log_5(2x+2)=\log_5\dfrac{(x+5)^2}{(2x+2)}$ or $\log_5(2x+2)+\log_5 5^2=\log_5 5^2(2x+2)$ etc	M1
	Use or state $\log_5 25=2$	M1
	$(x+5)^2=25(2x+2)$ or equivalent	A1
	$x^2-40x-25=0$	A1
	Solves their quadratic to give $x=$ (use formula, calculator or completing the square)	M1
	$x=20\pm5\sqrt{17}$	A1
		[7]
		7 marks

Notes

M1: Uses or states $2\log_5(x+5)=\log_5(x+5)^2$ Can be scored without sight of the base 5 of the log

M1: Uses addition (or subtraction) law correctly at least once. Can be scored without sight of the base 5 on the log

This may follow an incorrect line. Eg. $\log_5 2(x+5)-\log_5(2x+2)=\log_5\dfrac{2(x+5)}{(2x+2)}$ would be fine for this mark as would

$\log_5 10+\log_5(2x+2)=\log_5 10(2x+2)$ but $2\log_5(x+5)-\log_5(2x+2)=2\log_5\dfrac{(x+5)}{(2x+2)}$ would not score this mark as it is incorrect subtraction law. If the lhs is going to score this mark, the coefficient of "2" must have been dealt with.

M1: Connects 2 with 25 OR 5^2 correctly

A1: Correct equation, not involving logs, in any form (un-simplified). **Dependent upon all 3 M's being awarded**.

A1: Obtains correct 3TQ **Dependent upon all 3 M's being awarded.**

M1: Solves a 3TQ by formula, calculator or completing the square to give a surd answer.

A1: CSO $x=20\pm5\sqrt{17}$

If they reject one of the solutions, usually $x=20-5\sqrt{17}$ then withhold the final mark.

....

There are students who make two or more errors and fortuitously manage to form the correct equation.

Eg $2\log_5(x+5)-\log_5(2x+2)=2\Rightarrow\dfrac{2\log_5(x+5)}{\log_5(2x+2)}=2\Rightarrow\dfrac{\log_5(x+5)^2}{\log_5(2x+2)}=2\Rightarrow\dfrac{(x+5)^2}{(2x+2)}=5^2$

This student scores M1 (shown) M0 (incorrect subtraction law), M1 (shown).

As they have not scored the 3 M marks they only have access to the final M for a total 3 out of 7

Students who start $2\log_5(x+5)=2\log_5 2+2\log_5 5$ will only have access to M3

Leave blank

7. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = u_n - 5, \qquad n \geqslant 1$$

Find the values of

(a) u_2, u_3 and u_4 **(2)**

(b) u_{100} **(3)**

(c) $\sum_{i=1}^{100} u_i$ **(3)**

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Question Number	Scheme	Marks
7. (a)	$u_2 = -2,\ u_3 = -7$ and $u_4 = -12$	M1, A1 **[2]**
(b)	$d = -5$ and arithmetic	B1
	Uses $a + (n - 1)\, d$ with $a = 3$ and $n = 100$, to give -492	M1, A1 **[3]**
(c)	$S_{100} = \frac{n}{2}(2a + (n-1)d)$ *or* $\frac{n}{2}(a + l)$.	M1
	$S_{100} = \frac{100}{2}(6 + 99 \times -5)$ *or* $\frac{100}{2}(3 + -492)$	dM1
	$= -24\,450$	A1 **[3]**
		8 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. ("Subtract 5") Follow through on an incorrect u_2 or u_3
A1: three correct answers

(b)

B1: Assumes AP and uses or states that $d = -5.$ Hence B0 if you see for example $d = -5,$ followed by $3 \times (-5)^{99}$
You may assume an AP if you see any AP formula.
M1: Correct formula used and processed correctly. Look for $3 + 99 \times "d"$ or $-2 + 98 \times "d'$ with their *d*.
The (*n* -1) must be multiplied by *d*.
So, students that write $S_{100} = a + (n-1)d = 3 + (100-1) - 5 = 97$ score B1 M0 A0 for incorrect processing
A1: -492 (cao)

(c)
M1: States or uses a correct sum formula for an AP with *n* = 100 with any values for *a*, *d* and *l*
dM1: Uses and processes a correct sum formula for an AP with $a = 3$ *or* $-2, d = \pm 5$ and ft on their l

Note that students who write $S_{100} = \frac{n}{2}(2a + (n-1)d) = \frac{100}{2}(6 + (100-1) - 5) = 5000$ score M1 dM0 A0

A1: Obtains $-24\,450$

Leave blank

8. The equation $(k - 4)x^2 - 4x + k - 2 = 0$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$k^2 - 6k + 4 > 0$$

(3)

(b) Find the exact range of possible values for k.

(4)

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Question Number	Scheme	Marks
8.		
(a)	$(k-4)x^2 - 4x + k - 2 = 0$	
	Uses $b^2 - 4ac$ with $a = k-4$, $b = -4$ and $c = k-2$	M1
	Uses $b^2 - 4ac < 0$ or $b^2 < 4ac \Rightarrow$ *Eg.* $16 - 4(k-4)(k-2) < 0$, $16 < 4k^2 - 24k + 32$ oe	dM1
	proceeds correctly to $k^2 - 6k + 4 > 0$ *	A1*
		[3]
(b)	Attempts to solve $k^2 - 6k + 4 = 0$ to give $k =$	M1
	$\Rightarrow$ Critical values, $k = 3 \pm \sqrt{5}$	A1
	$k^2 - 6k + 4 > 0$ gives $k > 3 + \sqrt{5}$ (or) $k < 3 - \sqrt{5}$	M1 A1cao
		[4]
		7 marks
	Notes	

You may mark (a) and (b) as one whole question

(a)

M1: Attempts $b^2 - 4ac$ with $a = k-4$, $b = -4$ and $c = k-2$ condoning one slip. Eg $a = k+4$
or uses the quadratic formula to solve equation
or uses the discriminant on two sides of an equation or inequation e.g. $b^2 < 4ac$ or $b^2 = 4ac$ or $b^2 > 4ac$
This cannot be awarded with $a = (k-4)x^2$, $b = -4x$ and $c = k-2$

dM1: Uses $b^2 - 4ac < 0$ or $b^2 < 4ac$ with correct *a*, *b* and *c* and forms a correct inequality, in any form, seen at least once before the given answer.

A1*: CSO. Uses discriminant condition and proceeds to given answer with no errors. The inequality cannot just appear on the last line. Condone missing bracket on $-4^2 = 16$

(b)

M1: Uses formula, or completion of square method to find two answers to the given quadratic.
They may use their calculator to find the answers, implied by awrt 5.24 and 0.76

A1: Obtains the correct critical values $3 \pm \sqrt{5}$ (which may be un simplified $\frac{6 \pm \sqrt{20}}{2}$ and may be within an inequality)

M1: Chooses outside region **(** $k <$ Their Lower Limit $k >$ Their Upper Limit) for their critical values.
Do not award simply for diagram or table. Condone (for this mark) the inclusion of the boundary, or the outside region expressed as *x* not *k*.

A1: $k > 3 + \sqrt{5}$ (or) $k < 3 - \sqrt{5}$ must be exact. Allow $k > 3 + \sqrt{5} \cup k < 3 - \sqrt{5}$
Withhold if this is given in terms of *x*, has "**and" or "&"** between the two inequalities or is just one inequality.
$3 + \sqrt{5} < k < 3 - \sqrt{5}$ scores M1 A0

Leave blank

9. A cyclist aims to travel a total of 1200 km over a number of days.
He cycles 12 km on day 1
He increases the distance that he cycles each day by 6% of the distance cycled on the previous day, until he reaches the total of 1200 km.

(a) Show that on day 8 he cycles approximately 18 km.
(3)

He reaches his total of 1200 km on day N, where N is a positive integer.

(b) Find the value of N.
(4)

The cyclist stops when he reaches 1200 km.

(c) Find the distance that he cycles on day N. Give your answer to the nearest km.
(2)

Question Number	Scheme	Marks
9. (a)	Assumes GP and uses or states r = 1.06	B1
	$u_8 = ar^7 = 12\times(1.06)^7 =$	M1
	$=12\times(1.06)^7$ = 18.04 so approximately 18* .	A1* **[3]**
(b)	$\frac{12(1.06^N-1)}{1.06-1}=1200 \Rightarrow r^N = k, \quad k>0$	M1
	$(1.06)^N = 7$	A1
	$N=\frac{\log(7)}{\log 1.06}=33.395 \Rightarrow N=34$	M1 A1 **[4]**
(c)	Distance on day N is $1200-\frac{12((1.06)^{N-1}-1)}{1.06-1}=$	M1
	= awrt 32km	A1 **[2]**
		9 marks
	Notes	

(a)
B1: Assumes GP (implied by any GP formula or term by term increase ×1.06) and uses or states r = 1.06 or 106% or 1+6%
M1: Uses **correct** formula with **correct** a, r and n
A1: Obtains awrt 18.0 or awrt 18.04 before writing = / ≈ 18(km)
You may see just terms. 12, 12.72, 13.48, 14.29, 15.15, 16.06, 17.02, 18.04
Three values correct to 1 dp scores B1. Eight values correct to 1dp scores M1, with conclusion scores all three marks
(b) Now marked M1 A1 M1 A1
M1: Uses correct sum formula with their r and 1200 and proceeds to $r^N = k, \quad k>0$.
$\frac{12(1-1.06^N)}{1-1.06}=1200$ is also a correct starting point
A1: $(1.06)^N = 7$
M1: Uses a correct method using logs to solve power equation.
May be scored from $\frac{12(1.06^{N-1}-1)}{1.06-1}=1200$ or even a term equation
A1: N=34 cao It cannot be scored via incorrect inequality work.

It is possible you may see a trial and improvement solution. It is possible to use a tighter interval.
Score M1: Uses GP sum formula with a =13, r =1.06 and a value of n, $33\le n\le 34$
A1: Finds an accurate value for the sum. Note: S (33)= 1168 and S (34) =1250 respectively
M1: Substitutes both 33 and 34 (or a tighter interval spanning 33.395) into the formula and achieves awrt 1168 and awrt 1250 respectively
A1**:** N=34 cao
(c)
M1: For a correct expression or for using 1200 – (sum of their first (N-1) days) or 1200 – '1168' via trial and improvement.
A1: 31.88 km or awrt 32km (they do not need to state km)
Please note that there are alternative ways of finding this:Eg.Finds $12\times1.06^{33}-\left(\frac{12(1.06^{34}-1)}{1.06-1}-1200\right)$

Leave blank

10.

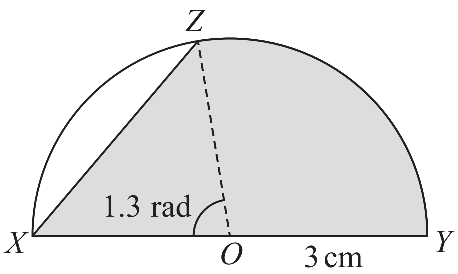


Diagram not drawn to scale

Figure 1

Figure 1 shows a semicircle with centre O and radius 3 cm. XY is the diameter of this semicircle. The point Z is on the circumference such that angle $XOZ = 1.3$ radians. The shaded region enclosed by the chord XZ, the arc ZY and the diameter XY is a template for a badge.

Find, giving each answer to 3 significant figures,

(a) the length of the chord XZ,

(2)

(b) the perimeter of the template $XZYX$,

(4)

(c) the area of the template.

(4)

Question Number	Scheme	Marks
10.(a)	$XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3,$ or $\sin0.65 = \frac{x}{3}$ *so* $XZ = 2\times x$	M1
	$XZ = 3.63$	A1 **[2]**
(b)	Arc length $ZY = 3\times\theta$,= 3 × (π – 1.3) (= 5.52 / 5.53)	M1, A1
	Perimeter = 3 + 3 + arc *ZY* + chord *XZ* = 15.2 (cm)	dM1 A1 **[4]**
(c)	Area of triangle $OXZ = \frac{1}{2}\times3\times3\times\sin1.3$ (=4.34)	M1
	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times3^2\times(\pi-1.3)$ (= 8.28 / 8.29)	M1
	Total area is $\frac{1}{2}\times3^2\times(\pi-1.3)+\frac{1}{2}\times3\times3\times\sin1.3$	dM1
	= 12.6 (cm^2)	A1 **[4]**
		10 marks
	Notes	

(a)

M1: Uses cosine rule – must be correct. Allow $XZ^2 = 3^2 + 3^2 - 2\times3\times3\cos1.3,$ for the M1
Or splits into right angled triangles correctly, uses sin 0.65 and then doubles the result
Uses angles in a triangle rule with the sine rule to find the required side. Eg $\frac{x}{\sin1.3} = \frac{3}{\sin0.92}$

A1: awrt 3.63

(b)

M1: Arc length formula *r θ* with r = 3 and $\theta = 1.3, (\pi - 1.3)$ or $(2\pi - 1.3)$ If decimals are seen accept 1.8 or 5.0
If the degree formula is being used look for $\frac{\theta}{360}\times2\pi r$ with $\theta = 74^\circ - 75^\circ$ or $\theta = 105^\circ - 106^\circ$

A1: Uses arc length formula with a correct angle. It does not need to be processed
Allow $3(\pi-1.3), 3\times1.84,$ awrt $5.52/5.53$ In degrees look for the minimum accuracy of $\frac{105.5}{360}\times2\pi\times3$

dM1: Complete method for perimeter. It is dependent upon the previous M. Look for $6+(a)+\text{arc length}$

A1: awrt15.2 (cm) – you do not need to see units

(c)

M1: Uses area formula for triangle correctly. If $\frac{1}{2}bh$ is used it must be the correct combinations found using a correct method.

M1: Uses the formula $\frac{1}{2}r^2\theta$ to find the area of the correct sector. There must be some valid attempt to use the correct angle. Allow as a minimum awrt 1.8 radians $(3.1-1.3)$

dM1: Adds two correct area formulae together. Both M's must have been awarded

A1: Accept awrt 12.6 (do not need units)

Alt (c)

M1: Attempts to find the area of the segment $\frac{1}{2}\times3^2(1.3-\sin1.3)$

M1: Attempts area of semi circle **along with** the area of segment

dM1: Finds area of the semi circle - segment $\frac{\pi\times3^2}{2}-\frac{1}{2}\times3^2(1.3-\sin1.3)$

A1: awrt 12.6

.

Leave blank

11. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = \frac{5x^2 + 4}{2\sqrt{x}} - 5$$

It is given that the point $P(4, 14)$ lies on C.

(a) Find $\mathrm{f}(x)$, writing each term in a simplified form.
(6)

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are constants.
(4)

Question Number	Scheme	Marks
11. (a)	$\frac{5x^2+4}{2\sqrt{x}} = \frac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$	B1
	$\text{f}(x) = \frac{\frac{5}{2}x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} - 5x \quad (+c)$	M1 A1A1
	Uses f(4) = 14 to find c =	
	$c = -6$ and so $\text{f}(x) = x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 5x - 6$ o.e. e.g $x^{\frac{1}{2}}\left(x^2+4\right) - 5x - 6$	dM1A1
		[6]
(b)	Gradient of curve at (4, 14) is $\text{f}'(4) = \frac{84}{4} - 5 = 16$	M1 A1
	So $(y - 14) =$ '16' $(x - 4)$ and $y = 16x - 50$	dM1 A1 **[4]**
		10 marks

(a)

B1: $\frac{5x^2+4}{2\sqrt{x}} = \frac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$ which may have un simplified coefficients. Allow decimal indices. This B mark may be implied by later work.

M1: Attempt to integrate - one power, even if incorrect, increased by one. Usually scored for $-5 \to -5x$

Allow for $x^{\frac{3}{2}} \to x^{\frac{3}{2}+1}$

Do not award if the candidate integrates the numerator and denominator without first attempting division.

A1: **Two** of the three terms in x correct un-simplified or simplified– (ignore no constant here).

The indices must now be simplified / calculated.

A1: All **three terms** correct **un-simplified. There is no need to have + *c***

dM1: Uses x = 4 when f(x) = 14 to find numerical value for c (may make slips). They must have attempted to integrate.

A1cao: All four terms **correct simplified** with **-6 included.** You may condone the omission of f (x) =

(b)

M1: For an attempt to substitute x = 4 into $\text{f}'(x) = \frac{5x^2+4}{2\sqrt{x}} - 5$ or their 'simplified' function from (a). Also allow a candidate to differentiate their answer to part (a) and substitute x = 4 in the result.

Look for evidence but allow f ' (4) =... Condone slips (eg. forgetting to subtract 5) BUT do not allow this if an incorrect value just appears from nowhere.

A1: Get f '(4) = 16

dM1: Linear equation with their gradient through (4,14). It must be their $\text{f}'(4)$ and not a "normal"

If they use the form $y = mx + c$ they must proceed as far as c = …

A1: cao: $y = 16x - 50$

Leave blank

12. [*In this question solutions based entirely on graphical or numerical methods are not acceptable.*]

(i) Solve for $0 \leqslant x < 360°$,

$$5\sin(x + 65°) + 2 = 0$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 2\pi$, all the solutions of

$$12\sin^2\theta + \cos\theta = 6$$

giving your answers in radians to 3 significant figures.

(6)

Question Number	Scheme	Marks
12 (i)	$\sin(...) = -\frac{2}{5}$	M1
	.... = − 23.6° (or 203.6° or 336.4°)	A1
	So x = 138.6° or 271.4° (allow awrt)	dM1 A1
		[4]
(ii)	$12(1-\cos^2\theta)+\cos\theta=6$	M1
	Solves their three term quadratic "$12\cos^2\theta-\cos\theta-6=0$" to give roots	dM1
	So $(\cos\theta=)$ $-\frac{2}{3}$ or $\frac{3}{4}$	A1
	$\theta = 2.30,\ 3.98,\ 0.723$ or 5.56	M1 A1 A1
		[6]
		10 marks
	Notes	

(i)

M1: As in scheme. Allow for $\text{invsin}\left(\frac{2}{5}\right)=23.6°$ or one of the given angles.

(In radians allow for awrt 0.41, 2.73)

A1: Requires one of the answers given in the scheme. This is implied by a correct final answer

dM1: For subtracting 65 from any of **their** answers. Dependent upon the first M. Allow for -88.6
This may be implied by one of the final answers

A1: cao (Work in radians gets first M mark only unless all converted inc 65). Withhold this mark if there are any extra solutions within the range 0 to 360 degrees.

(ii)

M1: Attempts to use $\sin^2\theta=(1-\cos^2\theta)$

dM1: Solves three term quadratic and proceeds to find roots
This is implied by the sight of ± the correct values.

A1: For $-\frac{2}{3}$ or $\frac{3}{4}$

M1: Uses inverse cosine to obtain a correct value of θ for their $\cos\theta$ Allow in degrees (nearest degree) or radians (to 1 dp) Do not allow this on trivial values such as $\cos\theta = 0$ *or* ± 1

A1: Two angles correct **in degrees or radians.** Degree answers are awrt 41.4, 319, 132, 228

A1: All four correct (awrt) and no extra's. Condone 2.3 for 2.30
Allow multiples of π So allow awrt $0.732\pi, 1.267\pi, 0.230\pi, 1.770\pi$

Leave blank

13. The point $A(9, -13)$ lies on a circle C with centre the origin and radius r.

(a) Find the exact value of r.
(2)

(b) Find an equation of the circle C.
(1)

A straight line through point A has equation $2y + 3x = k$, where k is a constant.

(c) Find the value of k.
(1)

This straight line cuts the circle again at the point B.

(d) Find the exact coordinates of point B.
(6)

Question number	Scheme	Marks
13 (a)	See $(9)^2 + (\pm 13)^2 = r^2$	M1
	$r = \sqrt{250} = 5\sqrt{10}$	A1 **[2]**
(b)	$x^2 + y^2 = 250$	B1 **[1]**
(c)	Substitute $x = 9$ when $y = -13$ to give $k = 1$	B1 **[1]**
(d)	Attempts to combine $2y + 3x = k$ and their $x^2 + y^2 = r^2$	M1
	i.e. $13x^2 - 6x - 999 = 0$ **or** when $13y^2 - 4y - 2249 = 0$	A1
	Solve to give $x =$ (**or** $y =$)	M1
	Substitute to give $y =$ (**or** $x =$)	M1
	$\left(-\frac{111}{13}, \frac{173}{13}\right)$	A1 A1 **[6]**
		(10 marks)

Notes

(a)
M1 : Allow for a correct expression for r^2 or r Implied by awrt 15.8
The method is scored for the distance from $(9,-13)$ to the origin. Some candidates are finding the formula for a circle centre $(9,-13)$ passing through the origin.
A1: For $\sqrt{250}$ *or* $5\sqrt{10}$ Either value implies the previous M. (See above).
(b)
B1: Accept any multiple of $x^2 + y^2 = 250$
Even accept $(x \pm 0)^2 + (y \pm 0)^2 = 250$, $x^2 + y^2 = \left(\sqrt{250}\right)^2$, $x^2 + y^2 = \left(5\sqrt{10}\right)^2$ and $x^2 + y^2 = \sqrt{250}^2$
(c)
B1: $k = 1$ stated or implied by $2y + 3x = 1$
(d)
M1: Eliminates x or y from their two equations to get an equation in just one variable. Allow with a numerical or algebraic k
The two equations must be of the form $2y + 3x = k$ and $(x \pm a)^2 + (y \pm b)^2 = r^2$
Do not allow the circle equation to be incorrectly simplified to $y = 5\sqrt{10} - x$
A1: Correct 3TQ equation in x or in y. The three terms need not be on the same side of the equation, just look for the correct 3 terms. You may see " $\frac{13}{4}y^2 - y = \frac{2249}{4}$ " for instance
M1: Solve a 3TQ, using a correct method, to give at least one value of x or y.
If a calculator is used 2sf is OK. You will have to use a calculator to check.
M1: Substitute x or y (in either equation) to give a value for y or x that is not -13 or 9.
It is dependent upon having started with allowable equations and having solved a 2 or 3 term quadratic equation by a correct method.
A1: One correct coordinate $x = -\frac{111}{13}$ *or* $-8\frac{7}{13}$, $y = \frac{173}{13}$ *or* $13\frac{4}{13}$
A1: Both correct answers. See above. Allow separately (not in coordinate form)

NOTE: It is possible to solve this question by geometry where *M*, the mid- point of the chord *AB* is found by solving $2y + 3x = 1$ and $y = \frac{2}{3}x$ simultaneously.

Leave blank

14.

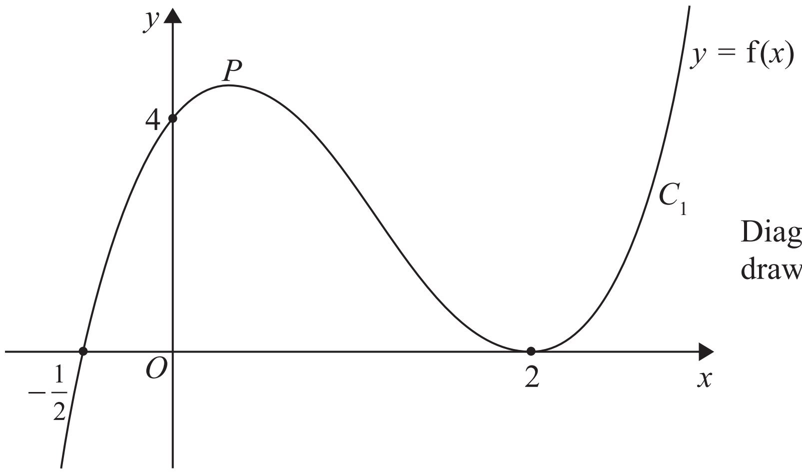


Diagram not drawn to scale

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x - 2)^2(2x + 1), \quad x \in \mathbb{R}$$

The curve crosses the x-axis at $\left(-\frac{1}{2}, 0\right)$, touches it at (2, 0) and crosses the y-axis at (0, 4). There is a maximum turning point at the point marked P.

(a) Use $\mathrm{f}'(x)$ to find the exact coordinates of the turning point P.
(7)

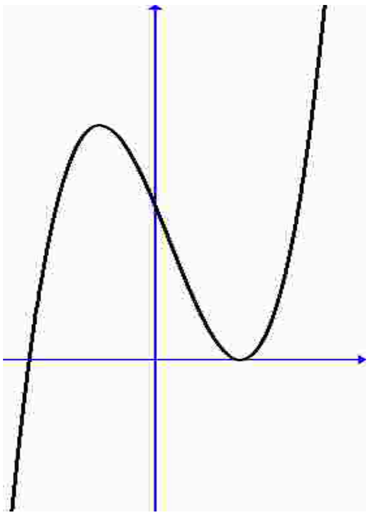
A second curve C_2 has equation $y = \mathrm{f}(x + 1)$.

(b) Write down an equation of the curve C_2
You may leave your equation in a factorised form.
(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C_2 meets the y-axis.
(2)

(d) Write down the coordinates of the two turning points on the curve C_2
(2)

(e) Sketch the curve C_2, with equation $y = \mathrm{f}(x + 1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.
(3)

Question Number	Scheme	Marks
14. (a)	$\text{f}(x)=(x-2)^2(2x+1)=2x^3-7x^2+4x+4$	M1
	So $\text{f}'(x)=6x^2-14x\ +4$	M1 A1
	Puts $\text{f}'(x)$ =0 and solves three term quadratic to obtain for example 2(3*x* – 1)(*x* – 2) = 0 so *x* =	M1
	$x=\frac{1}{3}$ (with *x* = 2)	A1
	Calculates f(their *x*) and find *y* $\Rightarrow\left(\frac{1}{3},\frac{125}{27}\right)$ Allow $x=\frac{1}{3}, y=4\frac{17}{27}$	dM1 A1 **[7]**
(b)	$y=(x-1)^2(2x+3)$	B1 **[1]**
(c)	When *x* = 0, *y* = 3	M1 A1 **[2]**
(d)	(1, 0) and $\left(-\frac{2}{3}, "\frac{125}{27}"\right)$	M1 A1ft **[2]**
(e)	 M1: Shape same as before, +ve cubic, but moved. Don't be overly concerned about the position of the maximum point.	M1
	A1: Shape same as before but moved to the **left** (maximum must be in second quadrant and minimum on +ve *x* - axis) and graph lies in three quadrants	A1
	A1: (1,0) and (-1.5,0) or marked on the x axis as 1 and -1.5	A1
		[3]
		15 marks
	Notes	

(a)

M1: Expand brackets, must have a four term cubic with or without collected terms.
M1: Differentiates to a quadratic– reduction of a power by one seen at least once
A1: Completely correct $f'(x) = 6x^2 - 14x + 4$
M1: Puts their derivative = 0 and solves to find the other root to '2'. The derivative must be a 3TQ expression.
A1: Allow exact equivalences including recurring decimals. May include $x = 2$
dM1: Substitutes their 1/3 into f(x) to find the y coordinates. Implied by y = awrt 4.63 Dependent upon previous M
A1: $x = \frac{1}{3}, y = \frac{125}{27}$ must be exact. Allow mixed numbers, allow recurring decimals

....

The first 3 marks could be done by the product rule
M1: For $f'(x) = A(x-2)^2 + B(2x+1)(x-2)$
M1 A1: For $f'(x) = 2(x-2)^2 + 2(2x+1)(x-2)$

....

(b)
B1: cao. Must be in the form y =....or f (x) = or f (x+1) =
Allow $y = 2(x+1)^3 - 7(x+1)^2 + 4(x+1) + 4$ You may isw after seeing this.
Do not allow the mark if the function is left in the form $y = (x+1-2)^2(2(x+1)+1)$

(c)
M1: Puts x = 0 into their new function. Allow embedded values or correct ft.
A1: y = 3 The function must have been correct, but not necessarily simplified, to score this mark,
Condone lack of y = if the candidates work implies that y is being found at x = 0

(d)
M1: Either coordinate pair correct. Follow through their point P.
So (1,0) or (a – 1, b) where P had coordinates (a, b)
A1ft: Both pairs correct, follow through **only** on the ***y* coordinate** of P
You may condone a decimal approximation such as 0.33
So if $P = \left(\frac{1}{3}, 2\right)$ the answer of $(1,0)$ *and* $\left(-\frac{2}{3}, 2\right)$ would score M1 A1ft

Note: If they do differentiate again they only score the marks as above. They cannot be awarded from the sketch in (e)

(e)
M1: Curve moved in any way. Evidence could be, for example, the maximum to the left of the y axis or the minimum not on the x axis or a point adapted. Be tolerant on slips in shape.
A1: Shape same as before but translated to the **left** (maximum must be in second quadrant and minimum on +ve x - axis) and graph lies in three quadrants. If the maximum looks on the y - axis, do not allow.
A1: For the new curve having a minimum point on the x axis at (1,0) and passing through the x axis at -1.5. Allow this mark if it just stops at the x axis at -1.5. (It would lose the earlier A1 for not appearing in quadrant 3)

Watch for the curve been superimposed on Figure 2. If it appears twice, on blank page and on Figure 2, the blank page takes precedence. Be tolerant of slips on shape especially for the M1. Also do not penalise changes in height as we need to mark this attempt in exactly the same way as an attempt on its own.

Leave blank

15.

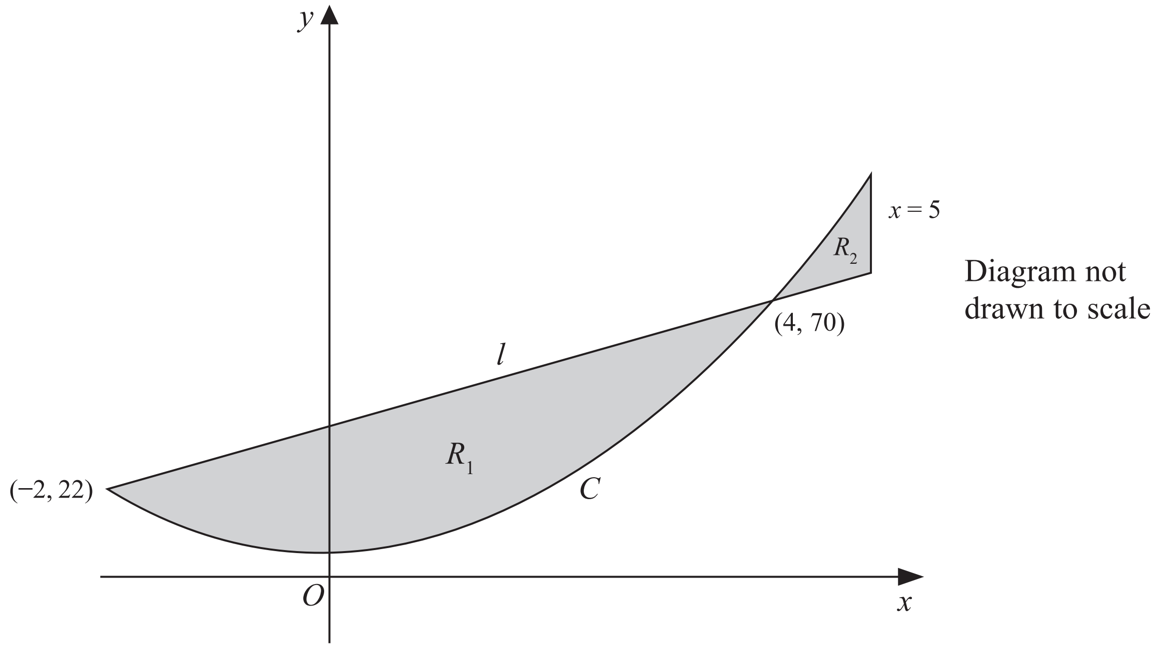


Figure 3

A design for a logo consists of two finite regions R_1 and R_2, shown shaded in Figure 3.

The region R_1 is bounded by the straight line l and the curve C.

The region R_2 is bounded by the straight line l, the curve C and the line with equation $x = 5$

The line l has equation $y = 8x + 38$

The curve C has equation $y = 4x^2 + 6$

Given that the line l meets the curve C at the points (–2, 22) and (4, 70), use integration to find

(a) the area of the larger lower region, labelled R_1

(6)

(b) the exact value of the total area of the two shaded regions.

(3)

Given that

$$\frac{\text{Area of } R_1}{\text{Area of } R_2} = k$$

(c) find the value of k.

(1)

Question Number	Scheme	Marks
15.	Line $y = 8x + 38$ and curve $y = 4x^2 + 6, \ -2 < x < 5$	
	Way 1: Integrates separately	
(a)	Attempts integration $\int (4x^2 + 6)\mathrm{d}x = \frac{4x^3}{3} + 6x$	M1 A1
	Uses limits and finds area under curve $\left[\frac{4x^3}{3} + 6x\right]_{-2}^{4} = \left((109\tfrac{1}{3}) - (-22\tfrac{2}{3}) = 132\right)$	dM1 A1
	Full method: Area under trapezium $\frac{1}{2}\times(4+2)(22+70) - "132"$ or $\left[4x^2 + 38x\right]_{-2}^{4} - "132"$	M1
	So area = $276 - 132 = 144$	A1
		[6]
(b)	Attempts to find area of R_2 *i.e.* $\left[\frac{4x^3}{3} + 6x\right]_{4}^{5} - \frac{(5-4)(70+78)}{2}$	M1
	So area of R_2 = $196\tfrac{2}{3} - (109\tfrac{1}{3}) - 74 = 13\tfrac{1}{3}$	A1
	Total Area shaded = $144 + 13\tfrac{1}{3} = 157\tfrac{1}{3}$	A1
		[3]
(c)	(k)= 10.8 oe	B1 **[1]**
	Way 2: Integrates line - curve	
(a)	Subtracts, and integrates: $\pm\int\left\{(8x+38) - (4x^2+6)\right\}\mathrm{d}x = \pm\left\{8\frac{x^2}{2} + 38x - \frac{4x^3}{3} - 6x\right\}$	M1 A1
	Uses correct limits $\pm\left[+4x^2 + 32x - \frac{4x^3}{3}\right]_{-2}^{4} = \left(106\frac{2}{3}\right) - \left(-37\frac{1}{3}\right)$	dM1 A1
	Full method (awarded on line 1) So area R_1 = 144	M1 A1
		[6]
(b)	Attempts to find the area of R_2 using correct limits $\pm\left[-4x^2 - 32x + \frac{4x^3}{3}\right]_{4}^{5} = \left(106\frac{2}{3}\right) - \left(93\frac{1}{3}\right)$	M1
	So area of R_2 = $13\tfrac{1}{3}$	A1
	Total Area shaded = $144 + 13\tfrac{1}{3} = 157\tfrac{1}{3}$	A1 **[3]**
(c)	(k) = 10.8 oe	B1 **[1]**
	Notes	

Way 1: Integrates separately. Note that this is now scored M1 A1 M1 A1 M1 A1
(a)

M1: Correct integration method for $\int (4x^2+6)\mathrm{d}x$ – increase power by one

A1: $\dfrac{4x^3}{3}+6x$

dM1: Uses limits 4 and -2 within an integrated function– see some embedded values unless implied by correct answer
A1: For achieving 132
M1: For a full attempt at the area of R_1
Look for the area of the trapezium or the area under line (by integration) and subtract their 132

Eg. $\frac{1}{2}\times 6\times(22+70)-\text{"132"}$ or $\left[4x^2+38x\right]_{-2}^{4}-\text{"132"}$ either way around.

A1: 144
(b)

M1: Uses limits 5 and 4 either way round in their $\dfrac{4x^3}{3}+6x$ and subtracts (or subtracts from) the area of a trapezium $\frac{1}{2}\times1\times(\text{"78"}+70)$ The 78 must have been attempted using a correct method. (Not using the quadratic function)

Alternatively Uses limits 5 and 4 either way round in their $\dfrac{4x^3}{3}+6x$ and their $4x^2+38x$ and subtracts either way

A1: For $\pm\ 13\frac{1}{3}$ **(may be implied by final answer)**

Allow alternative/international forms $13.\dot{3}$ *and* $13.\overline{3}$ for recurring

A1: For $157\frac{1}{3}$
(c)
B1: For 10.8 oe

....

Way 2 : Integrates a combined function (Eg. line -curve) Note that this is now scored M1 A1 M1 A1 M1 A1
M1: Attempts to combine (hopefully subtract) and integrate . Correct integration method – increase power by one seen at least once. Condone bracketing error. It can be scored if they add. (Penultimate M mark is not scored)

A1: For $\pm\left(4x^2+32x-\dfrac{4}{3}x^3\right)$ This may be left un-simplified

dM1: Uses the limits 4 and -2 within an integrated function– see some working either embedded value or (...) - (...)

A1: Correct values seen, either embedded or as in scheme (...) - (...) for $\pm\left(4x^2+32x-\dfrac{4}{3}x^3\right)$

M1: For a full method. This is implied by line 1 with the functions subtracted. Condone bracketing issues
A1: For 144 following correct work.
(b)

M1: Uses limits 5 and 4 either way round in their $\pm\left(4x^2+32x-\dfrac{4}{3}x^3\right)$ or the result of their subtracted functions

A1: For $\pm\ 13\frac{1}{3}$ **(may be implied by final answer)** You may see the alternative forms for recurring.

A1: For $157\frac{1}{3}$
(c)
B1: For 10.8 oe

....

Note the demand of the question is "use integration"

If candidate writes Area = $\int_{-2}^{4}\left\{(8x+38)-(4x^2+6)\right\}\mathrm{d}x=144$ they can score SC M0 A0 M1 A0 M1 A0

If candidate writes Area = $\int_{-2}^{4}\left\{(8x+38)-(4x^2+6)\right\}\mathrm{d}x=\left[4x^2+38x-\dfrac{4}{3}x^3-6x\right]_{-2}^{4}=144$ they can score M1 A1 M1 A1 M1 A1

....