Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C12

Advanced Subsidiary

Tuesday 9 January 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

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1. Given that

$$y = \frac{2x^{\frac{2}{3}} + 3}{6}, \quad x > 0$$

find, in the simplest form,

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(2)**

(b) $\int y \, \mathrm{d}x$ **(3)**

Question Number	Scheme	Notes	Marks
1	$y=\dfrac{2x^{\frac{2}{3}}+3}{6}$		
(a)	$x^{\frac{2}{3}} \to x^{-\frac{1}{3}}$	For reducing the power of $x^{\frac{2}{3}}$ by 1 which may be implied by e.g. $x^{\frac{2}{3}} \to x^{\frac{2}{3}-1}$ and no other powers of x	M1
	Note that some candidates think $\dfrac{2x^{\frac{2}{3}}+3}{6}=2x^{\frac{2}{3}}+3+6$ but the M mark can still score for $x^{\frac{2}{3}} \to x^{-\frac{1}{3}}$		
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{2}{9}x^{-\frac{1}{3}}$	Correct expression. Allow equivalent exact, simplified forms e.g. $\dfrac{2x^{-\frac{1}{3}}}{9}, \dfrac{2}{9x^{\frac{1}{3}}}, \dfrac{2}{9\sqrt[3]{x}}$. Allow 0.222… or 0.2 with a dot over the 2 for $\dfrac{2}{9}$.	A1
	Ignore what they use to indicate differentiation and ignore subsequent working following a fully correct answer.		
			(2)
(b)	**Must be integrating the given function in (b), not their answer to part (a)**		
	$x^{\frac{2}{3}} \to x^{\frac{5}{3}}$ or $k \to kx$	Increases the power by 1 for one term from $x^{\frac{2}{3}} \to x^{\frac{5}{3}}$ or $k \to kx$. May be implied by e.g. $x^{\frac{2}{3}} \to x^{\frac{2}{3}+1}$. This must come from correct work, so integrating numerator and denominator e.g. $\dfrac{2x^{\frac{2}{3}}+3}{6} \to \dfrac{...x^{\frac{5}{3}}+...x}{6x}$ is M0	M1
	Note that some candidates think $\dfrac{2x^{\frac{2}{3}}+3}{6}=2x^{\frac{2}{3}}+3+6$ but the M mark can still score for $x^{\frac{2}{3}} \to x^{\frac{5}{3}}$ or $k \to kx$		
	$\dfrac{3}{5}\times\dfrac{2}{6}x^{\frac{5}{3}}$ or $\dfrac{3}{6}x$	One correct term which may be un-simplified, including the power. So, $\dfrac{2}{6}\times\dfrac{x^{1+\frac{2}{3}}}{1+\frac{2}{3}}$ would be acceptable for this mark.	A1
	$\dfrac{1}{5}x^{\frac{5}{3}}+\dfrac{1}{2}x+c$	All correct and simplified including + c all appearing on one line. (c/6 is acceptable for c) Allow $\sqrt[3]{x^5}$ for $x^{\frac{5}{3}}$ but not x^1 for x. Allow 0.2 for $\dfrac{1}{5}$ and 0.5 for $\dfrac{1}{2}$	A1
	Ignore any spurious integral signs and/or dx's and ignore subsequent working following a fully correct answer.		
			(3)
			Total 5

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2. A sequence is defined by

$$u_1 = 1$$
$$u_{n+1} = 2 - 3u_n \quad n \geqslant 1$$

(a) Find the value of u_2 and the value of u_3

(2)

(b) Calculate the value of $\sum_{r=1}^{4}(r - u_r)$

(3)

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Question Number	Scheme	Notes	Marks
2	**Mark (a) and (b) together**		
(a)	$u_2 = -1,\ u_3 = 5$	As (a) and (b) are marked together, these can score as part of their calculation in (b) if – 1 and 5 are clearly the second and third terms.	B1, B1
			(2)
(b)	$u_4 = 2 - 3\times"5"(= -13)$	Correct attempt at the 4th term (can score anywhere) and may be implied by their calculation below)	M1
	$\sum_{r=1}^{4}(r-u_r) = \pm\{(1-1)+(2-"-1")+(3-"5")+(4-"-13")\}$ or $\sum_{r=1}^{4}(r-u_r) = \sum_{r=1}^{4}r - \sum_{r=1}^{4}u_r = \pm\{(1+2+3+4)-(1+"-1"+"5"+"-13")\}$		dM1
	A correct method for the sum or (– sum). Allow minor slips or mis-reads of their values but the intention must be clear. **Dependent on the first method mark.**		
	$=18$	cso	A1
			(3)
			Total 5

Question Number	Scheme	Notes	Marks
3(a)	$\left(3x^{\frac{1}{2}}\right)^4 = 81x^2$	B1: Obtains $ax^n,\ (a, n \neq 0)$ where a = 81 or n = 2 B1: $81x^2$	B1B1
	Do not isw so for example $\left(3x^{\frac{1}{2}}\right)^4 = 81x^2 = 9x$ scores B0B0		
			(2)
(b)	$\frac{2y^7 \times (4y)^{-2}}{3y} = \frac{y^4}{24}$	B1: Obtains $ay^n,\ (a, n \neq 0)$ where $a = \frac{1}{24}$ or n = 4 (Allow 0.41666… or 0.416 with a dot over the 6 for $\frac{1}{24}$) B1: $\frac{y^4}{24}$ (Allow $\frac{1y^4}{24}$)	B1B1
	Do not isw – mark their final answer		
			(2)
			Total 4

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3. Simplify fully

(a) $\left(3x^{\frac{1}{2}}\right)^4$

(2)

(b) $\dfrac{2y^7 \times (4y)^{-2}}{3y}$

(2)

Question Number	Scheme	Notes	Marks
2	**Mark (a) and (b) together**		
(a)	$u_2 = -1,\ u_3 = 5$	As (a) and (b) are marked together, these can score as part of their calculation in (b) if – 1 and 5 are clearly the second and third terms.	B1, B1
			(2)
(b)	$u_4 = 2 - 3\times"5"(=-13)$	Correct attempt at the 4th term (can score anywhere) and may be implied by their calculation below)	M1
	$\sum_{r=1}^{4}(r-u_r) = \pm\{(1-1)+(2-"-1")+(3-"5")+(4-"-13")\}$ or $\sum_{r=1}^{4}(r-u_r) = \sum_{r=1}^{4}r - \sum_{r=1}^{4}u_r = \pm\{(1+2+3+4)-(1+"-1"+"5"+"-13")\}$		dM1
	A correct method for the sum or (– sum). Allow minor slips or mis-reads of their values but the intention must be clear. **Dependent on the first method mark.**		
	$=18$	cso	A1
			(3)
			Total 5

Question Number	Scheme	Notes	Marks
3(a)	$\left(3x^{\frac{1}{2}}\right)^4 = 81x^2$	B1: Obtains $ax^n, (a, n \neq 0)$ where a = 81 or n = 2 B1: $81x^2$	B1B1
	Do not isw so for example $\left(3x^{\frac{1}{2}}\right)^4 = 81x^2 = 9x$ scores B0B0		
			(2)
(b)	$\frac{2y^7\times(4y)^{-2}}{3y} = \frac{y^4}{24}$	B1: Obtains $ay^n, (a, n \neq 0)$ where $a = \frac{1}{24}$ or n = 4 (Allow 0.41666… or 0.416 with a dot over the 6 for $\frac{1}{24}$) B1: $\frac{y^4}{24}$ (Allow $\frac{1y^4}{24}$)	B1B1
	Do not isw – mark their final answer		
			(2)
			Total 4

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4. The equation

$$(p - 2)x^2 + 8x + (p + 4) = 0, \quad \text{where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 + 2p - 24 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

Question Number	Scheme	Notes	Marks
4(a)	$b^2-4ac=8^2-4(p-2)(p+4)$	Attempts to use b^2-4ac with at least two of *a*, *b* or *c* correct. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $b^2=4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac. There must be no *x*'s.	M1
	$8^2-4(p-2)(p+4)<0$	For a correct un-simplified inequality in any form that is not the final printed answer or a positive constant multiple of the final printed answer with no incorrect previous statements.	A1
	$64<4p^2+8p-32$		
	$p^2+2p-24>0$*	Correct solution with intermediate working and no errors with the inequality sign appearing correctly before the final printed answer.	A1*
			(3)
(b)	$p^2+2p-24=0\Rightarrow p=.......$ $(p+1)^2-1-24=0\Rightarrow p=.......$ $(p=)\dfrac{-2\pm\sqrt{2^2-4\times1\times(-24)}}{2\times1}$	For an attempt to solve $p^2+2p-24=0$ **(not their quadratic)** leading to **two** critical values. See general guidance for solving a 3TQ when awarding this method mark. May be implied by their critical values.	M1
	$p=4,-6$	Correct critical values	A1
	$p<"-6",\ p>"4"$	Chooses the outside region for their two critical values. Look for $p<$ their -6, $p>$ their 4. This could be scored from $4<p<-6$ or $-6>p>4$. Evidence is to be taken from their answers not from a diagram. Allow e.g. $p\le"-6"$, $p\ge"4"$	M1
	$p<-6$ or $p>4$ $p<-6 \quad p>4$ $p<-6,\ p>4$ $p<-6;\ p>4$ $p<-6\ \cup\ p>4$ $(-\infty,-6),\ (4,\infty)$ $]-\infty,-6[,\ ]4,\infty[$	Correct inequalities e.g. answers as shown. Note that $p<-6$ **<u>and</u>** $p>4$ would score M1A0 as would $4<p<-6$ or $-6>p>4$ or $p<-6\ \cap\ p>4$. Apply isw where possible.	A1
	Allow letter other than *p* to be used in (b) but the final A mark requires answers in terms of *p* only. Correct answer only scores full marks in (b)		
			(4)
			Total 7

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5. (*In this question, solutions based entirely on graphical or numerical methods are not acceptable*.)

(i) Solve, for $0 < \theta < \frac{\pi}{2}$

$$5\sin 3\theta - 7\cos 3\theta = 0$$

Give each solution, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 360°$

$$9\cos^2 x + 5\cos x = 3\sin^2 x$$

Give each solution, in degrees, to one decimal place.

(6)

Question Number	Scheme	Notes	Marks
5(i)	$5\sin 3\theta - 7\cos 3\theta = 0 \Rightarrow \tan 3\theta = \frac{7}{5}$	M1: Reaches $\tan ... = k$ where $k \neq 0$ A1: $\tan ... = \frac{7}{5}$	M1A1
	$3\theta = 0.95054...$		dM1
	$3\theta = \tan^{-1}\left(\text{their } \frac{7}{5}\right)$ leading to a value of 3θ. Must be 3θ here but this may be implied if they divide their values by 3 (you may need to check). **Dependent on the first method mark.**		
	$\theta = 0.317$ **or** $\theta = 1.36$	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	$\theta = 0.317$ **and** $\theta = 1.36$ **only**	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	Alternative 1 for (i):		
	$5\sin 3\theta - 7\cos 3\theta = \sqrt{74}\sin(3\theta - 0.9505...)$	M1: Correct method using addition formula A1: $\sqrt{74}\sin(3\theta - 0.9505...)$	M1A1
	$3\theta - 0.9505... = 0,\ \pi$	$3\theta - \text{their } \alpha = \sin^{-1}(0)$. **Dependent on the first method mark.**	dM1
	$\theta = 0.317$ **or** $\theta = 1.36$	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	$\theta = 0.317$ **and** $\theta = 1.36$ **only**	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	Special case: If **both** answers are given in degrees allow A1A0 but needs to be awrt 18.2 **and** awrt 78.2)		
	Alternative 2 for (i):		
	$5\sin 3\theta = 7\cos 3\theta \Rightarrow 25\sin^2 ... = 49\cos^2 ...$ or $5\sin 3\theta - 7\cos 3\theta = 0 \Rightarrow 25\sin^2 ... - 49\cos^2 ... = 0$ M1: Obtains $p\sin^2 ... = q\cos^2 ...$ or $p\sin^2 ... - q\cos^2 ... = 0 \quad p, q > 0$		M1
	$\sin ... = (\pm)\frac{7}{\sqrt{74}}$ or $\cos ... = (\pm)\frac{5}{\sqrt{74}}$ $\pm(awrt\ 0.8)$ $\pm(awrt\ 0.6)$	Correct value for $\sin ...$ or $\cos ...$	A1
	$3\theta = 0.95054...$		dM1
	$3\theta = \sin^{-1}\left(\text{their } \frac{7}{\sqrt{74}}\right)$ or $3\theta = \cos^{-1}\left(\text{their } \frac{5}{\sqrt{74}}\right)$ leading to a value of 3θ. **Dependent on the first M.**		
	$\theta = 0.317$ **or** $\theta = 1.36$	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	$\theta = 0.317$ **and** $\theta = 1.36$ **only**	Awrt 0.317 (Allow awrt 0.101π) **or** Awrt 1.36 (Allow awrt 0.434π)	A1
	Special case: If **both** answers are given in degrees allow A1A0 but needs to be awrt 18.2 and awrt 78.2). If they give answers in degrees and radians, the radians answers take precedence. **For an otherwise fully correct solution, the final mark can be withheld for extra answers in range. Ignore extra answers outside the range. Answers <u>only</u> scores no marks.**		
			(5)

5(ii)	$9\cos^2 x + 5\cos x = 3\sin^2 x$		
	$9\cos^2 x + 5\cos x = 3\left(1-\cos^2 x\right)$	Uses $\sin^2 x = \pm 1 \pm \cos^2 x$	M1
	$12\cos^2 x + 5\cos x - 3 = 0$	Correct 3 term quadratic equation. Allow equivalent equations with terms collected e.g. $12\cos^2 x + 5\cos x = 3$	A1
	$(3\cos x - 1)(4\cos x + 3) = 0$ $\Rightarrow (\cos x) = ...$	Solves their 3TQ in cos x to obtain at least one value. See general guidance for solving a 3TQ when awarding this method mark. **Dependent on the first method mark.**	dM1
	$\cos x = \frac{1}{3}, -\frac{3}{4}$	Correct values for cos x	A1
	$x = 70.5, \quad 289.5, \quad 138.6, \quad 221.4$	A1: Any 2 correct solutions (awrt) A1: All 4 answers (awrt)	A1A1
	Special case: If **all** answers are given in radians allow A1A0 but needs to be awrt 1.2, 5.1, 2.4, 3.9 **For an otherwise fully correct solution, the final mark can be withheld for extra answers in range. Ignore extra answers outside the range.** **Answers <u>only</u> scores no marks.**		
			(6)
			Total 11

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6.

$$f(x) = ax^3 - 8x^2 + bx + 6$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$ there is no remainder.

When $f(x)$ is divided by $(x - 2)$ the remainder is -12

(a) Find the value of a and the value of b.

(5)

(b) Factorise $f(x)$ completely.

(4)

Question Number	Scheme	Notes	Marks
6(a)	$\mathrm{f}(\pm 1) = \ldots$ or $\mathrm{f}(\pm 2) = \ldots$	Attempts f(±1) or f(±2)	M1
	$a(-1)^3 - 8(-1)^2 + b(-1) + 6 = 0$	Allow un-simplified but do not condone missing brackets unless later work implies a correct expression.	A1
	$a(2)^3 - 8(2)^2 + b(2) + 6 = -12$	Allow un-simplified	A1
	$a + b = -2,\ 4a + b = 7$ $\Rightarrow a = 3,\ b = -5$	M1: Solves two linear equations in *a* and *b* simultaneously to obtain values for *a* and *b*. A1: Correct values	M1A1
	Alternative by long division:		
	$(ax^3 - 8x^2 + bx + 6) \div (x+1) \rightarrow$ remainder $f(a,b)$ or $(ax^3 - 8x^2 + bx + 6) \div (x-2) \rightarrow$ remainder $g(a,b)$ Attempts long division by either expression to obtain a remainder in terms of *a* and *b*		M1
	$-a - b - 2 = 0$	Allow un-simplified but do not condone missing brackets unless later work implies a correct expression.	A1
	$8a + 2b - 26 = -12$	Allow un-simplified	A1
	$a + b = -2,\ 4a + b = 7$ $\Rightarrow a = 3,\ b = -5$	M1: Solves simultaneously A1: Correct values	M1A1
			(5)
(b)	$(x+1)(ax^2 + kx + \ldots)$	Uses (*x* + 1) as a factor and obtains at least the first 2 terms of a quadratic with an ax^2 term and an *x* term. This might be by inspection or by long division.	M1
	$(x+1)(3x^2 - 11x + 6)$	Correct quadratic factor	A1
	$3x^2 - 11x + 6 = (3x-2)(x-3)$	Attempt to factorise their 3 term quadratic according to the general guidance, even if there was a remainder and (*x* + 1) must have been used as a factor.	M1
	Note that $3x^2 - 11x + 6 = (x - \frac{2}{3})(x-3)$ scores M0 here but $3x^2 - 11x + 6 = 3(x - \frac{2}{3})(x-3)$ is fine for M1		
	$(\mathrm{f}(x) =)(x+1)(3x-2)(x-3)$ or $(\mathrm{f}(x) =)3(x+1)(x - \frac{2}{3})(x-3)$	Fully correct factorisation. The factors need to appear together all on one line and no commas in between.	A1
	<u>Answers with no working in (b):</u> $\mathrm{f}(x) = 3x^3 - 8x^2 - 5x + 6 = (x+1)(3x-2)(x-3)$ scores full marks $\mathrm{f}(x) = 3x^3 - 8x^2 - 5x + 6 = (x+1)(x - \frac{2}{3})(x-3)$ scores a special case M1A1M0A0		
	<u>Just</u> writing down roots of the cubic scores no marks.		
	Ignore any "= 0" and also ignore any subsequent attempts to solve f(*x*) = 0 once the factorised form is seen.		
			(4)
			Total 9

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7.

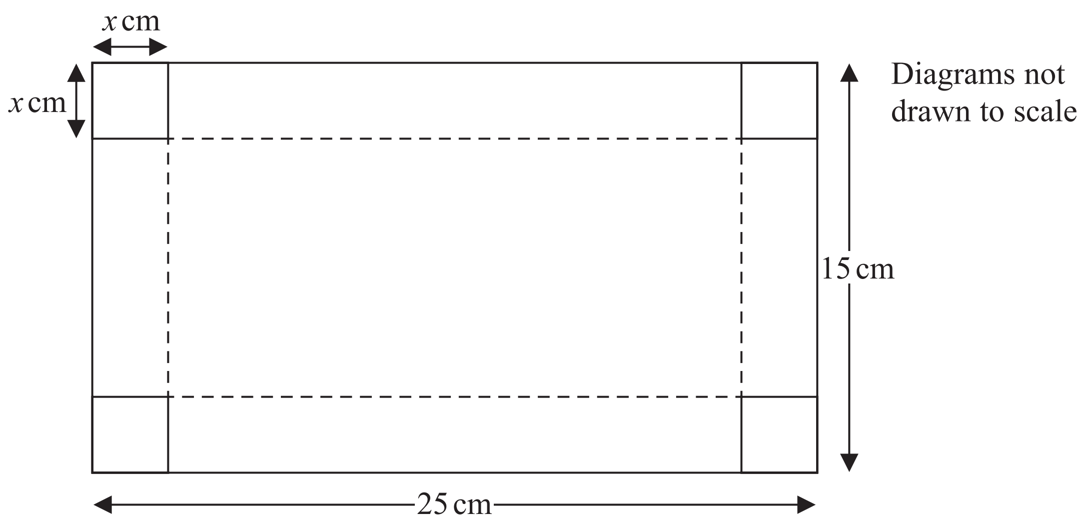

Figure 1

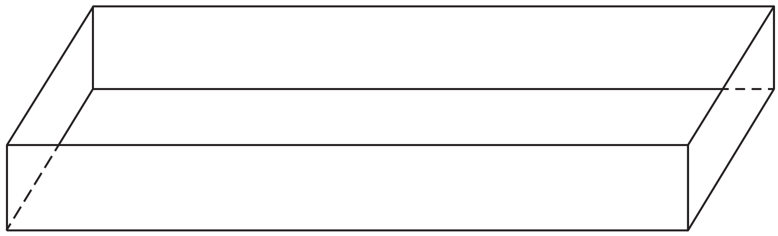

Figure 2

Figure 1 shows a rectangular sheet of metal of negligible thickness, which measures 25 cm by 15 cm. Squares of side x cm are cut from each corner of the sheet and the remainder is folded along the dotted lines to make an open cuboid box, as shown in Figure 2.

(a) Show that the volume, V cm^3, of the box is given by

$$V = 4x^3 - 80x^2 + 375x$$

(2)

(b) Use calculus to find the value of x, to 3 significant figures, for which the volume of the box is a maximum.

(4)

(c) Justify that this value of x gives a maximum value for V.

(2)

(d) Find, to 3 significant figures, the maximum volume of the box.

(2)

Question Number	Scheme	Notes	Marks
7(a)	$(V=)x(25-2x)(15-2x)$	Correct method for the volume. It must be a correct statement for the volume.	M1
	$(V)=x(375-80x+4x^2)=4x^3-80x^2+375x$* Allow the terms of $4x^3-80x^2+375x$ to be in any order.		A1*
	Completes correctly to printed answer with no errors including bracketing errors E.g. $V=25x-2x^2(15-2x)=4x^3-80x^2+375x$ scores M1A0 "V =" or e.g. "Volume = " must appear at some point.		
	$V=x(25-2x)(15-2x)=4x^3-80x^2+375x$ scores M1A0 (lack of working) $V=x(25-2x)(15-2x)=(25x-2x^2)(15-2x)=4x^3-80x^2+375x$ scores M1A1		
			(2)
	<u>Mark (b), (c) and (d) together</u> so that continued work with x = 3.03.. in (c) and (d) can be taken as evidence that the candidate has chosen this value in (b). **Allow e.g. $\frac{dy}{dx}$ for $\frac{dV}{dx}$ and/or $\frac{d^2y}{dx^2}$ for $\frac{d^2V}{dx^2}$**		
(b)	$\left(\frac{dV}{dx}=\right)12x^2-160x+375$	M1: $x^n \to x^{n-1}$ seen at least once A1: Correct derivative	M1A1
	$\frac{dV}{dx}=0 \Rightarrow x=\frac{160\pm\sqrt{7600}}{24}$	Puts $\frac{dV}{dx}=0$ (may be implied) and attempts to solve a 3 term quadratic to find x. May be implied by correct values.	M1
	x = 3.03, 10.3 but $0<x<7.5$ so x = 3.03	Identifies awrt 3.03 only as the required value.	A1
			(4)
(c)	$\left(\frac{d^2V}{dx^2}=\right)24x-160=24(3.03)-160$	Attempts the second derivative $(x^n \to x^{n-1})$ and substitutes at least one **positive** value of x from their $\frac{dV}{dx}=0$	M1
	$\frac{d^2V}{dx^2}=24(3.03)-160 \Rightarrow \frac{d^2V}{dx^2}<0$ ∴ maximum Fully correct proof for the maximum using a correct second derivative and using x = awrt 3 only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"		A1
	Allow alternatives e.g. considers **values** of V at, and either side of "3.03" or **values** of dV/dx either side of "3.03"		
			(2)
(d)	$V=4(3.03)^3-80(3.03)^2+375(3.03)$	Substitutes a (positive) x from their $\frac{dV}{dx}=0$ into the given V or a "version" of V.	M1
	$V=513$	Awrt 513	A1
	Note that V = awrt 513 only scores M1A1		
			(2)
			Total 10

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8.

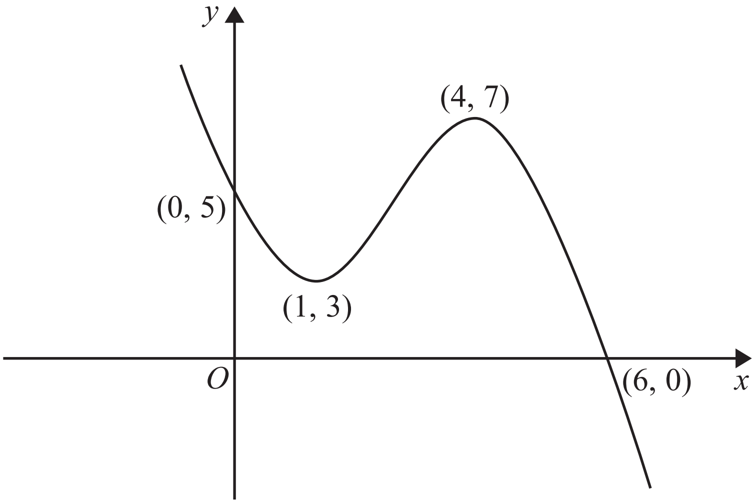


Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve crosses the y-axis at the point (0, 5) and crosses the x-axis at the point (6, 0).

The curve has a minimum point at (1, 3) and a maximum point at (4, 7).

On separate diagrams, sketch the curve with equation

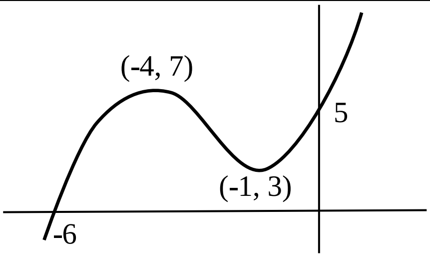
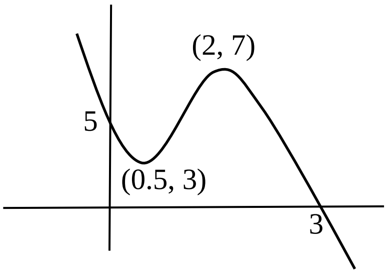
(a) $y = f(-x)$

(3)

(b) $y = f(2x)$

(3)

On each diagram, show clearly the coordinates of any points of intersection of the curve with the two coordinate axes and the coordinates of the stationary points.

Question Number	Scheme	Notes	Marks
8(a)			
	Reflection in the *y*-axis. Needs to be a positive cubic with one maximum and one minimum in the second quadrant. The curve must at least reach both axes. It should be a curve and not a set of straight lines.		B1
	Passes through $(-6, 0)$ and $(0, 5)$. Allow – 6 and 5 to be marked in the correct places and allow $(0, -6)$ and $(5, 0)$ as long as they are in the correct places. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other intercepts. **If there is any ambiguity, the sketch takes precedence but if the correct coordinates are seen in the script, allow sign errors when transferring them to the sketch.**		B1
	Maximum at $(-4, 7)$ and minimum at $(-1, 3)$ in the second quadrant. Must be seen as correct coordinate pairs or as numbers marked on the axes that clearly indicate the position of the maximum or minimum. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other turning points. **If there is any ambiguity, the sketch takes precedence but if the correct coordinates are seen in the script, allow sign errors when transferring them to the sketch.**		B1
			(3)
(b)			
	A stretch in the *x* direction. Need to see $(x, y) \rightarrow (kx, y)$ where $k \neq 1$ for all points seen. There must be no evidence of a change in ant y coordinates. The curve must at least reach both axes. It should be a curve and not a set of straight lines.		B1
	Passes through (3, 0) and (0, 5). Allow 3 and 5 to be marked in the correct places and allow (0, 3) and (5, 0) as long as they are in the correct places. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other intercepts. **If there is any ambiguity, the sketch takes precedence.**		B1
	Minimum at $\left(\frac{1}{2},\ 3\right)$ and maximum at (2, 7). in the first quadrant. Must be seen as correct coordinate pairs or as numbers marked on the axes that clearly indicate the position of the maximum or minimum. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other turning points. **If there is any ambiguity, the sketch takes precedence.**		B1
			(3)
			Total 6

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9. The first term of a geometric series is 20 and the common ratio is 0.9

(a) Find the value of the fifth term.
(2)

(b) Find the sum of the first 8 terms, giving your answer to one decimal place.
(2)

Given that $S_\infty - S_N < 0.04$, where S_N is the sum of the first N terms of this series,

(c) show that $0.9^N < 0.0002$
(4)

(d) Hence find the smallest possible value of N.
(2)

Question Number	Scheme	Notes	Marks
9(a)	$t_5 = ar^{n-1} = 20\times 0.9^{5-1} = 13.122$	M1: Use of a correct formula with $a = 20$, $r = 0.9$ and $n = 5$. Can be implied by a correct answer. A1: 13.122 or $\frac{6561}{500}$. Apply isw but just 13.1 is A0.	M1A1
	MR: Some are misreading fifth as fifteenth or fiftieth and find $t_{15} = ar^{n-1} = 20\times 0.9^{15-1} = 4.57...$ or $t_{15} = ar^{n-1} = 20\times 0.9^{50-1} = 0.114...$ Allow M1A0 in these cases. Listing: Need to see a fully correct attempt to find the fifth term e.g. 20, 18, 16.2, 14.58, 13.122 Must reach awrt 13 and intermediate decimals may not bee seen)		
	Just 13.122 with no working scores both marks		
			(2)
(b)	$S_8 = \frac{a(1-r^n)}{1-r} = \frac{20(1-0.9^8)}{1-0.9} = 113.9$	M1: Use of a correct formula with $a = 20$, $r = 0.9$ and $n = 8$ A1: 113.9 **only**	M1A1
	Listing: Need to see a fully correct method e.g. 20 + 18 + 16.2 + 14.58 + …+ 9.565938 = 113.9 (May be implied by awrt 114)		
			(2)
(c)	$S_\infty = \frac{20}{1-0.9}(=200)$	Correct S_∞ which can be simplified or un-simplified.	B1
	$200 - \frac{20(1-0.9^N)}{1-0.9} < 0.04$	M1: Attempts $S_\infty - S_N < 0.04$ (allow n for N) using $a = 20$ and $r = 0.9$ A1: Correct inequality in any form in terms of N or n only.	M1A1
	Note that $\frac{20}{1-0.9} - \frac{20(1-0.9^N)}{1-0.9} < 0.04$ scores B1M1A1		
	$0.9^N < 0.0002*$	Reaches the printed answer with intermediate working and with **<u>no errors or incorrect statements</u>**	A1*
			(4)
(d)	$(N >)\frac{\log 0.0002}{\log 0.9} \Rightarrow N = 81$	M1: Correct attempt to find N ignoring what they use for ">" i.e. they could be using < or =. Look for $(N =)\frac{\log 0.0002}{\log 0.9}$ or $(N =)\log_{0.9} 0.0002$ May be implied by awrt 81 A1: 81only. Accept 81 only or $N/n = 81$ but not $N/n > 81$.	M1A1
	81 <u>only</u> with no working scores both marks		
			(2)
			Total 10

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10. (i) Use the laws of logarithms to solve the equation

$$3\log_8 2 + \log_8(7 - x) = 2 + \log_8 x$$

(5)

(ii) Using algebra, find, in terms of logarithms, the exact value of y for which

$$3^{2y} + 3^{y+1} = 10$$

(5)

Question Number	Scheme	Notes	Marks
10(i)	**Examples:** $3\log_8 2=\log_8 2^3$, $3\log_8 2=\log_8 8$ $3\log_8 2=1$, $\log_8 2=\frac{1}{3}$, $2=\log_8 64$	Demonstrates a law or property of logs on either of the constant terms.	B1
	Examples: $\log_8(7-x)-\log_8 x=\log_8\frac{(7-x)}{x}$ $\log_8 64+\log_8 x=\log_8 64x$ $\log_8 8+\log_8(7-x)=\log_8 8(7-x)$	Demonstrates the addition or subtraction law of logs on two terms, at least one of which is in terms of *x*.	B1
	For the B marks above, look for work as described and award the marks where possible. If there is some correct and some incorrect work, do not look to penalise for the incorrect statements.		
	$\log_8 8(7-x)=\log_8 64x$, $\log_8\frac{(7-x)}{x}=1$, $\log_8\frac{(7-x)}{8x}=0$, $\log_8\frac{8(7-x)}{x}=2$ Correct processing leading to one of these equations or the equivalent. **NB needs to be a correct equation.**		M1
	$8(7-x)=64x$, $\frac{(7-x)}{x}=8$, $\frac{7-x}{8x}=1$, $\frac{8(7-x)}{x}=64$ **Correct equation with logs removed**		A1
	$x=\frac{7}{9}$	Accept equivalents but must be exact e.g. $\frac{56}{72}$ or 0.777… or 0.7 with a dot over the 7	A1
			(5)
(ii)	$3^{2y}+3^{y+1}=10$		
	$3^y\times3^y+3\times3^y=10$ or $3^y(3^y+3)=10$ or $(3^y)^2+3\times3^y=10$ or $x=3^y\Rightarrow x^2+3x=10$ A correct quadratic in *x* (or 3^y)		B1
	$x^2+3x-10=0\Rightarrow x=...$	Correct attempt to solve a quadratic equation of the form $ax^2+bx\pm10=0$ (may be a letter other than *x* or may be 3^y etc.)	M1
	$x=2$ or $x=2$ and -5	Correct values.	A1
	$3^y=2\Rightarrow y=\log_3 2$ or $\frac{\log 2}{\log 3}$	Correct use of logs. Need to see $3^y=k\Rightarrow y=\log_3 k$ or $\frac{\log k}{\log 3}, k>0$ which may be implied by awrt 0.63. Allow lg and ln for log.	dM1
	$y=\log_3 2$ or $y=\frac{\log 2}{\log 3}$	Cao (And no incorrect work using "-5"). Give BOD but penalise very sloppy notation e.g. log3(2) for $\log_3 2$ if necessary.	A1
			(5)
			Total 10

(ii) Way 2	$3^{2y}+3^{y+1}=10$		
	$3^{2y}+3^{y+1}=\left(3^2\right)^y+3(9)^{0.5y}$ $\Rightarrow 9^y+3(9)^{0.5y}=10$	Correct quadratic in $9^{0.5y}$	B1
	$x^2+3x-10=0\Rightarrow x=2\left(\text{or } -5\right)$	M1: Correct attempt to solve a quadratic equation of the form $ax^2+bx-10=0$ (may be a letter other than x or may be $9^{0.5y}$ etc.) A1: Correct solution(s)	M1A1
	$9^{0.5y}=2\Rightarrow 0.5y=\log_9 2$ or $\dfrac{\log 2}{\log 9}$	Correct use of logs. Need to see $9^{0.5y}=k\Rightarrow 0.5y=\log_9 k$ or $\dfrac{\log k}{\log 9}, k>0$	dM1
	$y=2\log_9 2$ or $y=\dfrac{2\log 2}{\log 9}$	Cao (And no incorrect work using "-5")	A1
			(5)

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11. The circle C has equation

$$x^2 + y^2 - 8x - 10y + 16 = 0$$

The centre of C is at the point T.

(a) Find

(i) the coordinates of the point T,

(ii) the radius of the circle C.

(4)

The point M has coordinates (20, 12).

(b) Find the exact length of the line MT.

(2)

Point P lies on the circle C such that the tangent at P passes through the point M.

(c) Find the exact area of triangle MTP, giving your answer as a simplified surd.

(3)

Question Number	Scheme	Notes	Marks
	Mark (a)(i) and (ii) together		
11(a)(i)	$(x \pm 4)^2$ **and** $(y \pm 5)^2$		M1
	Attempts to complete the square on x and y or sight of $(x\pm 4)^2$ **and** $(y \pm 5)^2$. May be implied by a centre of $(\pm 4, \pm 5)$. Or if considering $x^2 + y^2 + 2gx + 2fy + c = 0$, centre is $(\pm g, \pm f)$.		
	Centre is (4, 5)	Correct centre	A1
	Correct answer scores both marks		
(ii)	$r^2 = (\pm"4")^2 + (\pm"5")^2 - 16$ **(Must be -16)**		M1
	Must reach: $r^2 = \text{their } (\pm 4)^2 + \text{their } (\pm 5)^2 - 16$ or $r = \sqrt{\text{their } (\pm 4)^2 + \text{their } (\pm 5)^2 - 16}$ or if using $x^2 + y^2 + 2gx + 2fx + c = 0$, $r^2 = g^2 + f^2 - c$ or $r = \sqrt{g^2 + f^2 - c}$ Must clearly be identifying the radius or radius^2 May be implied by a correct radius.		
	$r = 5$		A1
	Correct answer scores both marks		
			(4)
(b)	$MT^2 = (20 - "4")^2 + (12 - "5")^2 \; (= 305)$	Fully correct method using Pythagoras for MT or MT^2	M1
	Other methods may be seen for finding MT. E.g. $\tan\theta = \frac{7}{16} \Rightarrow \theta = 23.6...,\ MT = \frac{7}{\sin\theta} = 17.46...$ Needs a fully correct method for MT		
	$MT = \sqrt{305}$	Must be exact	A1
	Beware incorrect work leading to a correct answer e.g. $MT^2 = \sqrt{(20-4)^2} + \sqrt{(12-5)^2} = \sqrt{256} + \sqrt{49} = \sqrt{305}$ **scores M0**		
			(2)
(c)	$(MP^2) = MT^2 - "5"^2$	Correct method for MP or MP^2 where MT > "5"	M1
	Area $MTP = \frac{1}{2} \times "5" \times "\sqrt{280}"$	Correct triangle area method	M1
	$5\sqrt{70}$	cao	A1
			(3)
	Alternative for (c):		
	$\cos PTM = \frac{"5"}{\sqrt{"305"}}$ $\sin PMT = \frac{"5"}{\sqrt{"305"}}$	Correct method for angle PTM or PMT (NB PTM = 73.36…, PMT = 16.63…)	M1
	Area $MTP = \frac{1}{2} \times "5" \times "\sqrt{305}" \times \sqrt{\frac{56}{61}}$	Correct triangle area method. May not work with exact values but needs to be a fully correct method using their values.	M1
	$5\sqrt{70}$	Cao. Note that $5\sqrt{70} = 41.83..$ which might imply a correct method.	A1
			Total 9

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12. The line l_1 has equation $x + 3y - 11 = 0$

The point A and the point B lie on l_1

Given that A has coordinates $(-1, p)$ and B has coordinates $(q, 2)$, where p and q are integers,

(a) find the value of p and the value of q,

(2)

(b) find the length of AB, giving your answer as a simplified surd.

(2)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB.

(c) Find an equation for l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

Question Number	Scheme	Notes	Marks
12(a)	$p = 4$ **or** $q = 5$	One correct value. May be implied by e.g. when $x = -1$, $y = 4$ **or** when $y = 2$, $x = 5$	B1
	$p = 4$ **and** $q = 5$	Both correct values. May be implied by e.g. when $x = -1$, $y = 4$ **and** when $y = 2$, $x = 5$	B1
			(2)
(b)	$AB^2 = (\text{"4"}-2)^2 + (-1-\text{"5"})^2$ or $AB = \sqrt{(\text{"4"}-2)^2 + (-1-\text{"5"})^2}$	Correct Pythagoras method using (– 1, "4") and ("5", 2) to find AB or AB^2	M1
	$(AB) = 2\sqrt{10}$	$2\sqrt{10}$ only	A1
			(2)
(c)	$M = \left(\frac{-1+\text{"5"}}{2}, \frac{\text{"4"}+2}{2}\right) = (2,\ 3)$	Correct midpoint method. May be implied by at least one correct coordinate if no working is shown.	M1
	Gradient of $l_1 = -\frac{1}{3}$	Correct gradient of l_1. Allow equivalent exact expressions. May be implied by a correct perpendicular gradient.	B1
	Perpendicular gradient = 3	Correct perpendicular gradient rule. This can be awarded for a correct value or a correct method e.g. $m = \frac{-1}{-\frac{1}{3}}$ or $\frac{-1}{3} \times m = -1 \Rightarrow m = ...$	M1
	$y - \text{"3"} = \text{"3"}(x - \text{"2"})$ or $y = mx + x \Rightarrow \text{"3"} = \text{"3"} \times \text{"2"} + c \Rightarrow c = ...$	Correct straight line method **using their midpoint** and a "changed" gradient. If using $y = mx + c$, they must reach as far as a value for c.	M1
	$y = 3x - 3$	cao	A1
			(5)
	Alternative for last 4 marks of (c):		
	$3x - y + c = 0$	B1: "$3x - y$" M1: $3x - y + c = 0$	B1M1
	$3(2) - 3 + c = 0 \Rightarrow c = -3$	Correct method to find c using their values	M1
	$y = 3x - 3$	cao	A1
			Total 9

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13.

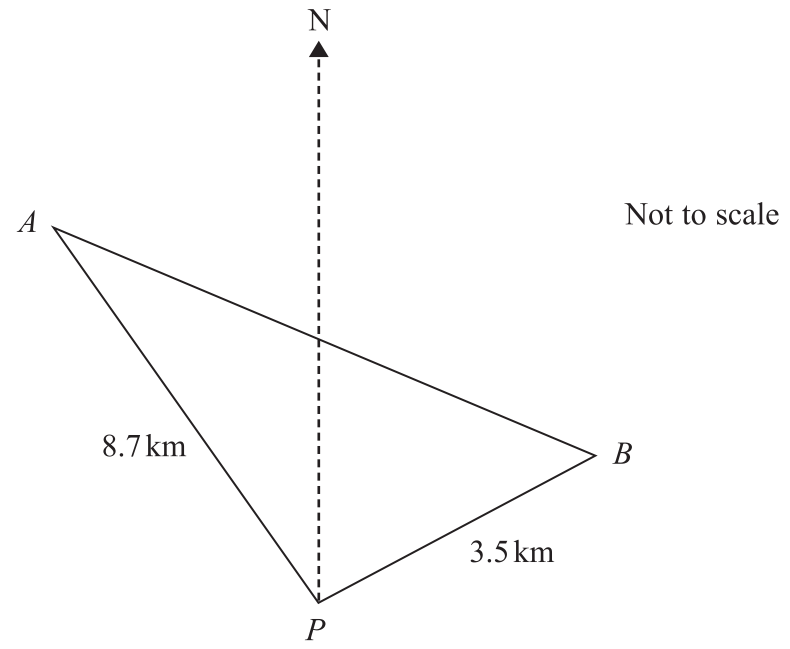


Figure 4

Figure 4 shows the position of two stationary boats, A and B, and a port P which are assumed to be in the same horizontal plane.

Boat A is 8.7 km on a bearing of 314° from port P.

Boat B is 3.5 km on a bearing of 052° from port P.

(a) Show that angle APB is 98°

(1)

(b) Find the distance of boat B from boat A, giving your answer to one decimal place.

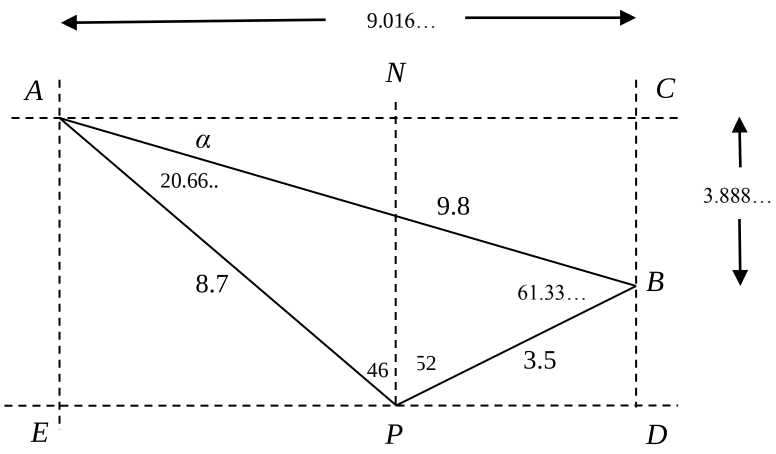
(2)

(c) Find the bearing of boat B from boat A, giving your answer to the nearest degree.

(4)

Question Number	Scheme	Notes	Marks
13(a)	(APN =) 360° – 314° = 46° (APB =) 46° + 52° = 98° or (Reflex APB) = 314° – 52° = 262° (APB =) 360° – 262° = 98° or Shows on a sketch the 314 and 46 And states 46° + 52° = 98°	Correct explanation that **explains why** APN is 46° (e.g. 360° – 314°) and adds that to 52° **or** shows/states that reflex APB = 262° and so APB = 360° – 262° = 98°. Do not be overly concerned how they use the letters to reference angles as long as the correct calculations are seen. **Do not allow the use of AB = 9.8 from (b).**	B1
			(1)
(b)	$\left(AB^2=\right)8.7^2+3.5^2-2\times8.7\times3.5\cos98^\circ$	Correct use of cosine rule. You can ignore the lhs for this mark so just look for $8.7^2+3.5^2-2\times8.7\times3.5\cos98^\circ$	M1
	AB = 9.8 (km)	Awrt 9.8 km (you can ignore their intermediate value for AB^2 provided awrt 9.8 is obtained for AB)	A1
			(2)
(c) Way 1	$\frac{"9.8"}{\sin98^\circ}=\frac{3.5}{\sin PAB}$ or $3.5^2=8.7^2+"9.8"^2-2\times8.7\times"9.8"\cos PAB$ $\Rightarrow PAB=...$	Correct sine or cosine rule method to obtain angle PAB. May be implied by awrt 21°	M1
	$PAB=20.66...^\circ$	Allow awrt 21°. May be implied by a correct bearing.	A1
	Bearing is 180° – "20.66°" – 46°	Fully correct method	M1
	= 113° or 114°	Awrt 113° or awrt 114°	A1
(c) Way 2	$\frac{"9.8"}{\sin98^\circ}=\frac{8.7}{\sin PBA}$ or $8.7^2=3.5^2+"9.8"^2-2\times3.5\times"9.8"\cos PBA$ $\Rightarrow PBA=...$	Correct sine or cosine rule method to obtain angle PBA. May be implied by awrt 61° or 62°	M1
	$PBA=61.33...^\circ$	Allow awrt 61° or awrt 62°. May be implied by a correct bearing.	A1
	Bearing is 52° + "61.33...°"	Fully correct method	M1
	= 113° or 114°	Awrt 113° or awrt 114°	A1
			(4)
(c) Way 3	Let α = Bearing – 90°		
	$\tan\alpha=\frac{BC}{AC}=\frac{8.7\cos46^\circ-3.5\cos52^\circ}{8.7\sin46^\circ+3.5\sin52^\circ}$	Correct method for α	M1
	$\alpha=23.33^\circ$	Allow awrt 23°. May be implied by a correct bearing.	A1
	Bearing is 90° + "23.33°"	Fully correct method	M1
	= 113° or 114°	Awrt 113° or awrt 114°	A1
			(4)
			Total 7

Diagram for Q13



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14.

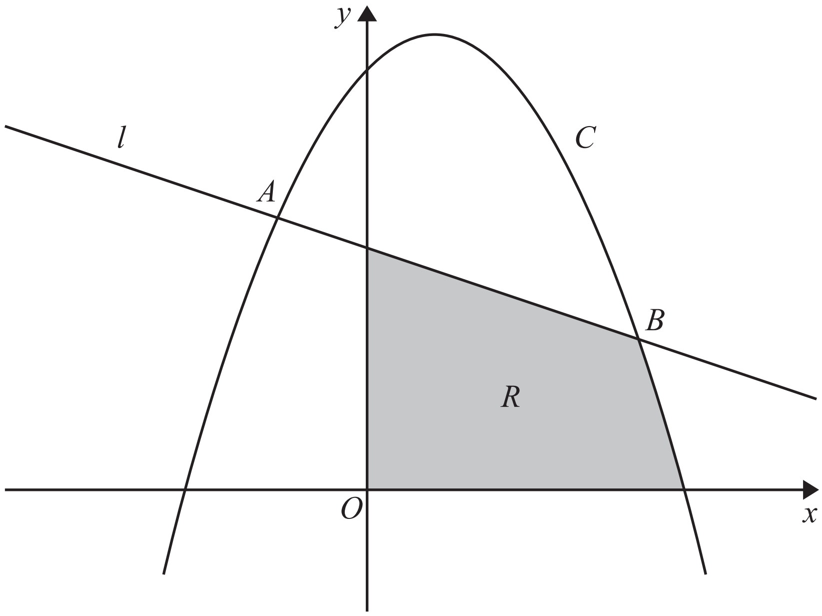


Figure 5

Figure 5 shows a sketch of part of the line l with equation $y = 8 - x$ and part of the curve C with equation $y = 14 + 3x - 2x^2$

The line l and the curve C intersect at the point A and the point B as shown.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

The region R, shown shaded in Figure 5, is bounded by the coordinate axes, the line l, and the curve C.

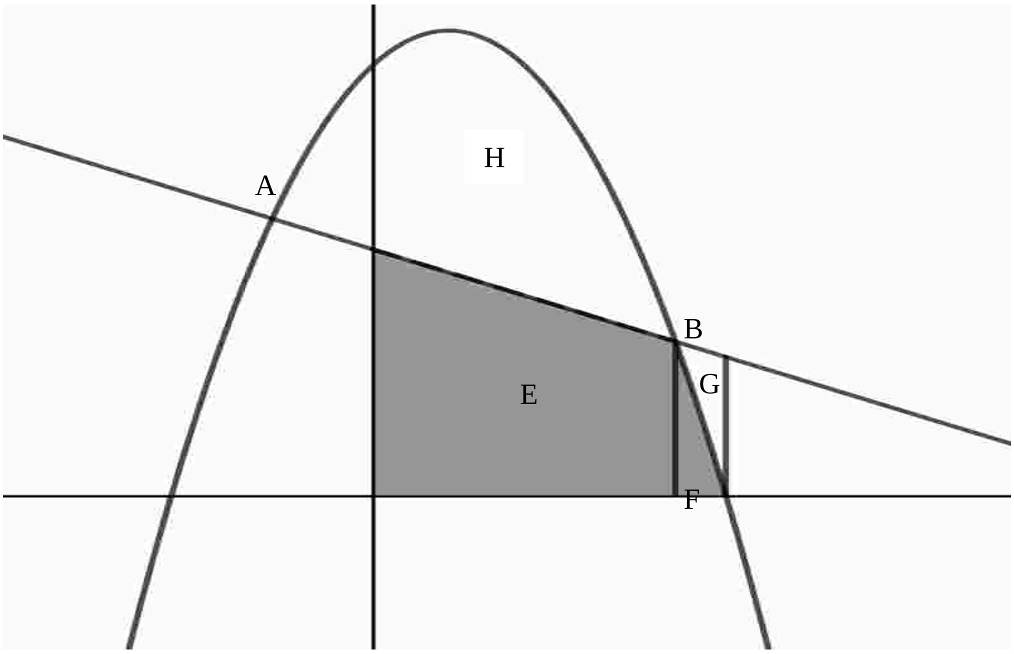
(b) Use algebraic integration to calculate the exact area of R.

(8)

P 5 0 7 1 3 A 0 4 6 5 2

Question Number	Scheme	Notes	Marks
14	$y = 8 - x,\ \ y = 14 + 3x - 2x^2$		
(a)	$8 - x = 14 + 3x - 2x^2$ or $y = 14 + 3(8 - y) - 2(8 - y)^2$	Uses the given line and curve to obtain an equation in one variable.	M1
	$2x^2 - 4x - 6 = 0 \Rightarrow x = ...$ or $2y^2 - 28y + 90 = 0 \Rightarrow y = ...$	Solves their 3TQ as far as x = ... or y = … **Dependent on the first method mark**.	dM1
	$x = -1,\ \ x = 3$ or $y = 5,\ \ y = 9$	Correct x values or correct y values	A1
	$(-1,\ 9) \quad (3,\ 5)$	ddM1: Solves for y or x using at least one value of x or y. **Dependent on both previous method marks**. A1: Correct coordinates which do not need to be paired so just look for correct values.	ddM1A1
	Special case: <u>Fully correct</u> answers only with <u>no working</u> scores M0M0A0M1A1		
			(5)

(b)



WAY 1	**Adds areas E and F**		
	$x=0 \Rightarrow y=8$ or $\int(8-x)\,\mathrm{d}x = 8x - \frac{x^2}{2}$	Correct y intercept which may be seen on the diagram or correct integration of $8 - x$	B1
	$14+3x-2x^2=0 \Rightarrow x=3.5$	Correct value - may be seen on the diagram.	B1
	$\int(14+3x-2x^2)\,\mathrm{d}x = 14x+\frac{3x^2}{2}-\frac{2x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on at least two terms for the curve C A1: Correct integration	M1A1
	$[...]_{"3"}^{"3.5"} = \left(49+\frac{147}{8}-\frac{343}{12}\right)-\left(42+\frac{27}{2}-18\right)$ $\left(=\frac{31}{24}\right)$	Correct use of their limits "3" and "3.5" either way round on their integrated curve C. Must be a "changed" function.	M1
	Trapezium: $\frac{1}{2}\times"3"("8"+"5")\left(=\frac{39}{2}\right)$ or $\left[8x-\frac{x^2}{2}\right]_0^{"3"} = 8(3)-\frac{(3)^2}{2}(-0)$	Correct method for the area of the trapezium between $x = 0$ and $x = 3$ using their values. If using the integration, the integration must be correct and used correctly.	M1
	Area $R = \frac{39}{2}+\frac{31}{24}=\frac{499}{24}$	dM1: Adds their trapezium area and integrated area (dependent on <u>**all**</u> previous method marks) A1: Allow exact equivalents e.g. $20\frac{19}{24}$	**d**M1A1

WAY 2	**Adds areas E, F and H and subtracts area H**		
	$\pm(\text{curve}-\text{line})=\pm\left(14+3x-2x^2-(8-x)\right)$		B1
	$14+3x-2x^2=0\Rightarrow x=3.5$	Correct value - may be seen on the diagram.	B1
	$\int\left(14+3x-2x^2\right)\mathrm{d}x=14x+\frac{3x^2}{2}-\frac{2x^3}{3}(+c)$ or $\int\pm(\text{curve}-\text{line})\mathrm{d}x=\pm\left(\text{"}6x+2x^2-\frac{2x^3}{3}\text{"}\right)(+c)$	M1: $x^n\to x^{n+1}$ on at least two terms for the curve C or their $\pm(\text{curve}-\text{line})$ A1: Correct integration but allow correct ft integration for slips on their $\pm(\text{curve}-\text{line})$ (ignore + c)	M1A1
	$[...]_0^{\text{"3.5"}}=\left(49+\frac{147}{8}-\frac{343}{12}\right)-(0)\left(=\frac{931}{24}\right)$	Correct use of their upper limit "3.5" and 0 (which may be implied) either way round on their integrated curve C. Must be a "changed" function.	M1
	$\left[6x+2x^2-\frac{2x^3}{3}\right]_0^{\text{"3"}}=6(3)+2(3)^2-\frac{2(3)^3}{3}(-0)$		M1
	Correct use of their "3" and 0 (which may be implied) either way round on their integrated ±(curve – line). Must be a "changed" function.		
	Area $R=\frac{931}{24}-18=\frac{499}{24}$	dM1: Subtracts (curve – line) area from curve area (dependent on **all** previous method marks) A1: Allow exact equivalents e.g. $20\frac{19}{24}$	**d**M1A1

WAY 3	**Adds areas E, F and G and subtracts area G**		
	$x=0\Rightarrow y=8$ or $\pm(\text{line} - \text{curve})=\pm\left(8-x-\left(14+3x-2x^2\right)\right)$ or or $\int(8-x)\,dx=8x-\frac{x^2}{2}$	Correct y intercept - may be seen on the diagram. Or correct $\pm(\text{curve}-\text{line})$ or correct integration of $8-x$	B1
	$14+3x-2x^2=0\Rightarrow x=3.5$	Correct value - may be seen on the diagram.	B1
	$\int\pm(\text{line} - \text{curve})\,dx=\pm\left(\frac{2x^3}{3}-6x-2x^2\right)(+c)$	M1: $x^n\to x^{n+1}$ on at least two terms for their $\pm(\text{curve}-\text{line})$ A1: Correct integration but allow correct ft integration for slips on their $\pm(\text{curve}-\text{line})$ (ignore + c)	M1A1
	$\left["\frac{2x^3}{3}-6x-2x^2"\right]_{"3"}^{"3.5"}=\frac{2("3.5")^3}{3}-6("3.5")-2("3.5")^2-\left(\frac{2("3")^3}{3}-6("3")-2("3")^2\right)$		M1
	Correct use of their "3" and "3.5" either way round on their integrated ±(curve – line). Must be a "changed" function.		
	Trapezium: $\frac{1}{2}\times"3.5"("8"+"4.5")\left(=\frac{175}{8}\right)$ or $\left[8x-\frac{x^2}{2}\right]_0^{"3.5"}=8(3.5)-\frac{(3.5)^2}{2}(-0)$	Correct method for the area of the trapezium between x = 0 and x = "3.5" using their values. If using the integration, the integration must be correct and used correctly.	M1
	Area $R=\frac{175}{8}-\frac{13}{12}=\frac{499}{24}$	dM1: Subtracts (line – curve) area from trapezium area (dependent on **all** previous method marks) A1: Allow exact equivalents e.g. $20\frac{19}{24}$	**d**M1A1
			(8)
			Total 13

Q14(b) COMBINED SCHEME

B1 $x = 0 \rightarrow y = 8$ (May be seen on the diagram)

OR: Correct integration of $8 - x$, giving $8x - \frac{x^2}{2}$

OR: $\pm(curve - line) = \pm(14 + 3x - 2x^2 - (8 - x))$

B1 $14 + 3x - 2x^2 = 0 \rightarrow x = 3.5$ (May be seen on the diagram).

M1 Integration of the curve quadratic or their $\pm(curve - line)$ quadratic expression with $x^n \rightarrow x^{n+1}$ for at least two terms.

A1 Completely correct integration of the quadratic expression, even if mistakes have been made in 'simplifying' their quadratic expression. Ignore "+ c". (So the M1A1 is essentially given for correct integration).

N.B. "integrated curve" = $"\left(14x + \frac{3x^2}{2} - \frac{2x^3}{3}\right)"$

"integrated (curve – line)" = $"\left(6x + 2x^2 - \frac{2x^3}{3}\right)"$

Next two M marks for any one of the following three variations, with correct use of their limits on their integrated function (must be a "changed" function) or correct method for the appropriate trapezium using their values:

M1	1(i)	$["integrated\ curve"]_{"3"}^{"3.5"} = \cdots$	$\left(\frac{31}{24}\right)$
M1	1(ii)	$\left[8x - \frac{x^2}{2}\right]_0^{"3"} = \cdots$ or $\frac{1}{2} \times "3" \times ("8 + "5")$	$\left(\frac{39}{2}\right)$
M1	2(i)	$["integrated\ curve"]_0^{"3.5"} = \cdots$	$\left(\frac{931}{24}\right)$
M1	2(ii)	$["integrated\ \pm(curve - line)"]_0^{"3"} = \cdots$	(18)
M1	3(i)	$["integrated\ \pm(line - curve)"]_{"3"}^{"3.5"} = \cdots$	$\left(\frac{13}{12}\right)$
M1	3(ii)	$\left[8x - \frac{x^2}{2}\right]_0^{"3.5"} = \cdots$ or $\frac{1}{2} \times "3.5" \times ("8 + "4.5")$	$\left(\frac{175}{8}\right)$

dM1 (Dependent on all previous method marks). Attempts the correct combination, which must be either 1(i) + 1(ii), or 2(i) – 2(ii), or 3(ii) – 3(i).

A1 $\frac{499}{24}$ or exact equivalent, e.g. $20\frac{19}{24}$

Leave blank

15. The binomial expansion, in ascending powers of x, of $(1 + kx)^n$ is

$$1 + 36x + 126kx^2 + \ldots$$

where k is a non-zero constant and n is a positive integer.

(a) Show that $nk(n - 1) = 252$

(2)

(b) Find the value of k and the value of n.

(5)

(c) Using the values of k and n, find the coefficient of x^3 in the binomial expansion of $(1 + kx)^n$

(3)

Question Number	Scheme	Notes	Marks
15	$(1+kx)^n = 1 + nkx + \frac{n(n-1)}{2}k^2x^2$		
(a)	$\frac{n(n-1)}{2}k^2 = 126k$ or $\frac{n(n-1)}{2}k = 126k$ or ${}^nC_2k^2 = 126k$ or ${}^nC_2k = 126k$ Compares x^2 terms using one of these forms, with or without the x^2.		M1
	$kn(n-1) = 252*$ Obtains the printed equation from $\frac{n(n-1)}{2}k^2 = 126k$ or $\frac{n(n-1)}{2}k^2x^2 = 126kx^2$		A1*
	<u>Note that these are acceptable proofs:</u> $\frac{n(n-1)}{2}k^2x^2$ followed by $\frac{n(n-1)}{2}k = 126 \Rightarrow nk(n-1) = 252$ $\frac{n(n-1)}{2}k^2x^2$ followed by $n(n-1)k^2 = 252k \Rightarrow nk(n-1) = 252$		
			(2)
(b)	$nk = 36$	Correct equation (oe). **Can score anywhere.**	B1
	$36(n-1) = 252$ or $36\left(\frac{36}{k} - 1\right) = 252$	Uses a valid method with their nk = 36 and the given equation to obtain an equation in n or k only. It must be a correct algebraic method allowing for sign and/or arithmetic slips only.	M1
	$36n - 36 = 252 \Rightarrow n = 8$ or $\frac{36}{k} - 1 = 7 \Rightarrow k = 4.5$	dM1: Solves, using a correct method, to obtain a value for n or k A1: Correct value for n or k	dM1A1
	$n = 8 \Rightarrow k = 4.5$ or $k = 4.5 \Rightarrow n = 8$	Correct values for n and k	A1
	Special Case: Some candidates have a second term of *nx* which gives *n* = 36 and then solve $kn(n-1) = 252$ to give *k* = 0.2. <u>This scores a special case of B1.</u> **Generally, to score the method marks, candidates must be solving 2 equations in *n* and *k*.**		
			(5)
(c)	$\frac{n(n-1)(n-2)}{3!}k^3\left(x^3\right)$	Correct coefficient. May be implied by $56k^3$ or ${}^{"8"}C_3"k"^3$ with or without x^3. If no working is shown, you may need to check their values.	B1ft
	$= \frac{8(8-1)(8-2)}{3!}4.5^3 = ...$	Substitutes their values correctly including integer n, $n > 3$, to obtain a value for the coefficient of x^3. Must be a correct calculation for the x^3 coefficient for their values.	M1
	$= 5103$	Allow $5103x^3$	A1
	Answer only of 5103 scores B1M1A1		
			(3)
			Total 10