Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C12

Advanced Subsidiary

Tuesday 10 January 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 4 8 3 2 4 A 0 1 5 2

Pearson

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1. Given $y = \frac{x^3}{3} - 2x^2 + 3x + 5$

(a) find $\frac{dy}{dx}$, simplifying each term.

(3)

(b) Hence find the set of values of x for which $\frac{dy}{dx} > 0$

(4)

Question Number	Scheme		Marks
1(a)	$\left(\frac{dy}{dx}=\right)\frac{3x^2}{3}-2\times 2x+3$	M1: $x^n \to x^{n-1}$ or $5\to 0$ A1: **Any 3** of the 4 terms differentiated correctly - this could be 2 terms correct and $5\to 0$ (allow simplified or un-simplified for this mark, including $3x^0$ for 3)	M1A1
	$\left(\frac{dy}{dx}=\right)x^2-4x+3$	Cao. All 3 terms correct and simplified and on the same line and no + 0. (Do not allow $1x^2$ for x^2 or x^1 for x or $3x^0$ for 3). Condone poor notation e.g. omission of dy/dx = … or if they use y = …	A1
	Candidates who multiply by 3 before differentiating: e.g. $\left(\frac{x^3}{3}-2x^2+3x+5\right)\times 3=x^3-6x^2+9x+15\Rightarrow\frac{dy}{dx}=3x^2-12x+9$ Scores M1A0A0 but could recover in (a) if they then divide by 3 **If they persist with** $\frac{dy}{dx}=3x^2-12x+9$ **in (b), allow full recovery in (b)**		
			(3)
(b)	$x^2-4x+3=0\Rightarrow x=1,\ 3$	M1: Attempt to solve their 3TQ from part (a) as far as x = … (see general guidance for solving a 3TQ). If no working is shown and the roots are incorrect for their 3TQ, score M0 here but the second method mark below is still available. A1: Correct values (may be implied by their inequalities e.g. a correct quadratic followed by just $x > 1$ and $x > 3$ could score M1A1 here)	M1A1
	$x<"1",\quad x>"3"$	**Chooses outside region** (*x* < their lower limit *x* > their upper limit). Do not award simply for diagram or table.	M1
	$x<1,\quad x>3$ Correct answer. Allow the correct regions separated by a comma or written separately and allow other notation e.g. $(-\infty,1)\cup(3,\infty)$. Do not allow $1>x>3$ or $x<1$ **and** $x>3$ (These score M1A0). ISW if possible e.g. $x>3,\quad x<1$ followed by $1>x>3$ can score M1A1. $x>3,\quad x>1$ followed by $x>3$ (or) $x<1$ can score M1A1. **Fully correct answer with no working scores both marks. Answers that are otherwise correct but use** $\le,\ge$ **lose final mark as would** $[-\infty,1]\cup[3,\infty]$.		A1
			(4)
			(7 marks)

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2. A circle, with centre C and radius r, has equation

$$x^2 + y^2 - 8x + 4y - 12 = 0$$

Find

(a) the coordinates of C,

(2)

(b) the exact value of r.

(2)

The circle cuts the y-axis at the points A and B.

(c) Find the coordinates of the points A and B.

(3)

Question Number	Scheme		Marks
2 (a)	**Mark (a) and (b) together**		
	$(x\pm4)...(y\pm2)$	Attempts to complete the square on x and y or sight of $(x\pm4)$ **and** $(y\pm2)$. May be implied by a centre of $(\pm4,\pm2)$. Or if considering $x^2+y^2+2gx+2fy+c=0$, centre is $(\pm g,\pm f)$.	M1
	Centre $C=(4,-2)$	Correct centre (allow x = 4, y = -2) But not g = .., f = … or p = …, q = … etc.	A1
	Correct answer scores both marks		
			(2)
(b)	$r^2=12+(\pm4)^2+(\pm2)^2$	**Must reach:** $r^2=12+\text{their }(\pm4)^2+\text{their }(\pm2)^2$ or $r=\sqrt{12+\text{their }(\pm4)^2+\text{their }(\pm2)^2}$ or if considering $x^2+y^2+2gx+2fx+c=0$, $r^2=g^2+f^2-c$ or $r=\sqrt{g^2+f^2-c}$ Must clearly be identifying the radius or radius^2 May be implied by a correct exact radius or awrt 5.66	M1
	$r=\sqrt{32}$	$r=\sqrt{32}$. Accept exact equivalents such as $4\sqrt{2}$. r = … not needed but must clearly be the radius. Do not allow $\pm\sqrt{32}$ unless minus is rejected	A1
	Correct answer scores both marks		
			(2)
(c)	$x=0\Rightarrow y^2+4y-12=0$	Correct quadratic. Allow $16+(y+2)^2=32$	B1
	$(y+6)(y-2)=0\Rightarrow y=...$	Attempts to solve a 3TQ that has come from substituting x = 0 or y = 0 into the given equation or their 'changed' equation. May be implied by correct answers for their quadratic.	M1
	$y=2,-6$ or $(0,2)$ and $(0,-6)$	Correct y values or correct coordinates. Accept sight of these for all 3 marks if no incorrect working seen but must clearly be y values or correct coordinates. This may be implied by the correct roots of a quadratic in y.	A1
			(3)
			(7 marks)

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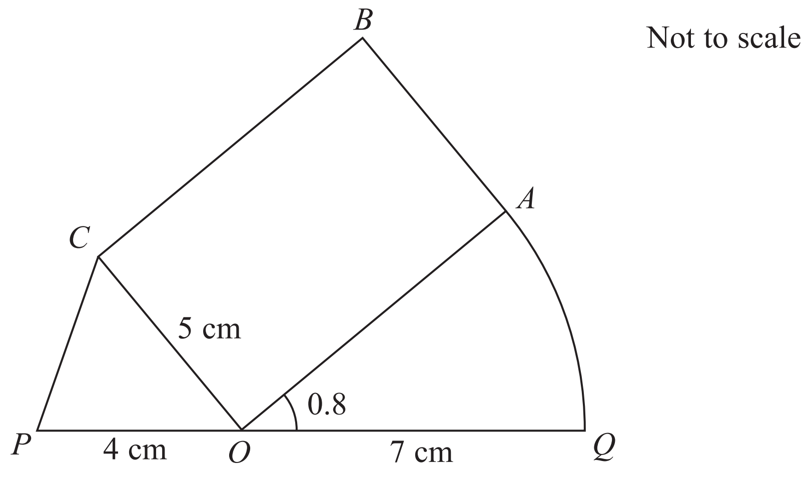

Figure 1

The shape *POQABCP*, as shown in Figure 1, consists of a triangle *POC*, a sector *OQA* of a circle with radius 7 cm and centre *O*, joined to a rectangle *OABC*.

The points *P*, *O* and *Q* lie on a straight line.

$PO = 4$ cm, $CO = 5$ cm and angle $AOQ = 0.8$ radians.

(a) Find the length of arc *AQ*.

(2)

(b) Find the size of angle *POC* in radians, giving your answer to 3 decimal places.

(2)

(c) Find the perimeter of the shape *POQABCP*, in cm, giving your answer to 2 decimal places.

(4)

Question Number	Scheme		Marks
3(a)	$S = r\theta = 7\times 0.8 = 5.6\text{(cm)}$	M1: Uses $S = r\theta$ A1: 5.6 oe e.g. 28/5	M1A1
	Note that if the 0.8 is converted to degrees e.g. $0.8\times\frac{180}{\pi} = 45.8366...$, this angle may be rounded or truncated when attempting $\frac{45.8366...}{360}\times 2\times\pi\times 7$ for the M1 so allow A1 for awrt 5.6		
			(2)
(b)	$\angle POC = \frac{\pi}{2} - 0.8 = \text{awrt } 0.771$	M1: Attempts to find $\frac{\pi}{2} - 0.8$ or $\pi - \frac{\pi}{2} - 0.8.$ Allow an attempt to find θ from $\theta + \frac{\pi}{2} + 0.8 = \pi.$ Accept as evidence awrt 0.77 A1: awrt 0.771	M1A1
	Answers in degrees **only** can score M1A0 e.g. $180 - 90 - 0.8\times\frac{180}{\pi}(= 44.163...)$		
			(2)
(c)	$4^2 + 5^2 - 2\times 4\times 5\cos'0.771'$ or $\sqrt{4^2 + 5^2 - 2\times 4\times 5\cos'0.771'}$	Correct use of the cosine rule to find *CP* or CP^2. NB 0.771 radians is awrt 44 degrees. **Ignore lhs for this mark** and look for e.g. $4^2 + 5^2 - 2\times 4\times 5\cos'0.771$ or $44'$	M1
	$CP^2 = 4^2 + 5^2 - 2\times 4\times 5\cos 0.771$ or $CP = \sqrt{4^2 + 5^2 - 2\times 4\times 5\cos 0.771}$	A correct expression for *CP* or CP^2 **with lhs consistent with rhs**. Allow awrt 0.77 radians or awrt 44 degrees. (May be implied if a correct numerical value is used in subsequent work)	A1
	Perimeter = $4 + 5 + 2\times 7 + '5.6' + '3.5'$	$4 + 5 + 2\times 7 + \text{their } AQ + \text{their } CP$. Need to see all 6 lengths but may be implied by e.g. $23 + '5.6' + '3.5'$	M1
	= 32.11 (cm)	Awrt 32.11 (ignore units)	A1
			(4)
			(8 marks)

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4. An arithmetic series has first term a and common difference d.

Given that the sum of the first 9 terms is 54

(a) show that

$$a + 4d = 6$$

(2)

Given also that the 8th term is half the 7th term,

(b) find the values of a and d.

(4)

Question Number	Scheme		Marks
4 (a)	$S_9 = 54$ $\Rightarrow 54 = \frac{9}{2}(2a+8d)$ or $\Rightarrow 54 = \frac{9}{2}(a+a+8d)$	Uses a correct sum formula with $n = 9$ and $S_9 = 54$	M1
	$\Rightarrow a+4d = 6$*	cso	A1*
	Listing:		
	$a+a+d+a+2d+....+a+8d=54$ $\Rightarrow 9a+36d=54$ Scores M1 for attempting to sum 9 terms (both lines needed) or $a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d+a+7d+a+8d=54$ Scores M1 on its own and then A1 if they complete correctly.		
			(2)
(b)	$a+7d = \frac{1}{2}(a+6d)$ or $\frac{1}{2}(a+7d) = a+6d$	Uses $t_8 = \frac{1}{2}t_7$ or $\frac{1}{2}t_8 = t_7$ to produce one of these equations.	M1
	$\Rightarrow 6-4d+7d = \frac{1}{2}(6-4d+6d)$ $\Rightarrow d = ...$	Uses the **given equation from (a)** and their second linear **<u>equation</u>** in a and d and proceeds to find a value for either a or d.	M1
	$\Rightarrow d = -1.5, a = 12$	A1: Either $d = -1.5$ (*oe*) or $a = 12$ A1: Both $d = -1.5$ (*oe*) and $a = 12$	A1A1
	Note that use of $\frac{1}{2}t_8 = t_7$ in (b) gives a = 30 and d = -6		
			(4)
			(6 marks)

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5. (a) Given that

$$y = \log_3 x$$

find expressions in terms of y for

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3 \sqrt{x}$

Write each answer in its simplest form.

(3)

(b) Hence or otherwise solve

$$2\log_3\left(\frac{x}{9}\right) - \log_3 \sqrt{x} = 2$$

(4)

Question Number	Scheme		Marks
5 (a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-2$	M1: $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9$ or $\log_3\left(\frac{x}{9}\right)=\log_3 x+\log_3\frac{1}{9}$ Correct use of the subtraction rule or addition rule. Ignore the presence or absence of a base and any spurious "= 0"	M1A1
		A1: $y-2$	
	An answer left as $\log_3 3^{y-2}$ scores M1A0		
	Note that $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-\log_3 9$ scores M1A0		
(ii)	$\log_3\sqrt{x}=\log_3 x^{\frac{1}{2}}=\frac{1}{2}\log_3 x=\frac{1}{2}y$	$\frac{1}{2}y$ or equivalent	B1
			(3)
(b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\Rightarrow 2(y-2)-\frac{1}{2}y=2$ Uses their answers from part (a) to create a **linear** equation in *y* (condone poor use of brackets e.g. $2(y-2)=2y-2$ and also the slip $(y-2)-\frac{1}{2}y=2$ for this mark)		M1
	$\Rightarrow y=4$	Correct value for *y*.	A1
	Note that arriving at $(y-2)^2-\frac{1}{2}y=2$ above scores M0 (not linear) but does have a solution *y* = 4 so look out for *y* = 4 not being derived correctly.		
	$\log_3 x=4\Rightarrow x=3^4$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1
			(4)
			(7 marks)
Alt 1 (b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)$ or $2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\log_3 x-2\log_3 9-\log_3\sqrt{x}=\log_3\frac{x^2}{\sqrt{x}}+...$ Combines **two log terms in *x* correctly** to obtain a single log term		M1
	$\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)=2$ or $\log_3\left(\frac{x^2}{\sqrt{x}}\right)=6$	Correct equation	A1
	$\left(\frac{(x/9)^2}{\sqrt{x}}\right)=3^2$ or $\left(\frac{x^2}{\sqrt{x}}\right)=3^6$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1

Alt 2 (b) Uses $x=3^y$	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\log_3\left(\frac{3^y}{9}\right)-\log_3 3^{\frac{y}{2}}=\log_3\left(\frac{3^{\frac{3y}{2}}}{81}\right)$ Combines logs correctly		M1
	$\log_3\left(\frac{3^{\frac{3y}{2}}}{81}\right)=2\Rightarrow y=4$	Correct value for y	A1
	$\log_3 x=4\Rightarrow x=3^4$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1

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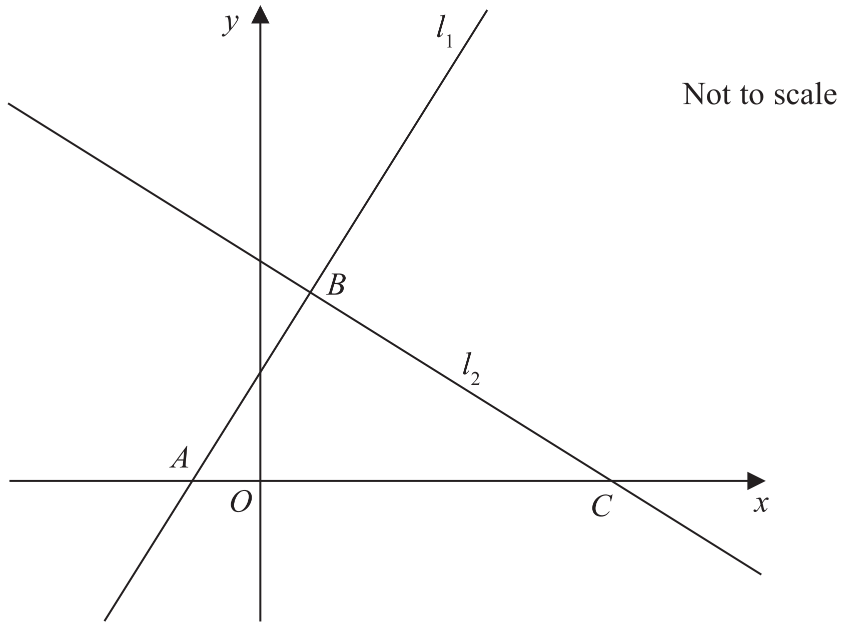

Figure 2

The straight line l_1 has equation $2y = 3x + 5$

The line l_1 cuts the x-axis at the point A, as shown in Figure 2.

(a) (i) State the gradient of l_1

(ii) Write down the x coordinate of point A.

(3)

Another straight line l_2 intersects l_1 at the point B with x coordinate 1 and crosses the x-axis at the point C, as shown in Figure 2.

Given that l_2 is perpendicular to l_1

(b) find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers,

(5)

(c) find the exact area of triangle ABC.

(3)

Question Number	Scheme		Marks
6(a)(i)	$\frac{3}{2}$	Accept exact equivalents	B1
(ii)	$y=0,\quad 3x+5=0\Rightarrow x=-\frac{5}{3}$	M1: Sets $y=0$ and attempts to find x. Accept as evidence $3x+5=0\Rightarrow x=..$ or awrt -1.7 A1: $x=-\frac{5}{3}$ or exact equivalent including 1.6 **recurring (i.e. a clear dot over the 6)**	M1A1
			(3)
(b)	Gradient $l_2=-\frac{1}{"\frac{3}{2}"}=-\frac{2}{3}$	Uses $m_2=-\frac{1}{m_1}$ to find the gradient of l_2 (may be implied by their line equation). Allow an attempt to find m_2 from $m_1\times m_2=-1$.	M1
	Point B has y coordinate of 4	This may be embedded within the equation of the line but must be seen in part (b).	B1
	e.g. $y-'4'='-\frac{2}{3}'(x-1)$ or $\frac{y-'4'}{x-1}='-\frac{2}{3}'$	A correct straight line method with a changed gradient and their point $(1,'4')$. There must have been attempt to find the y coordinate of B. If using $y = mx + c$, must reach as far as finding a value for c.	M1
	e.g. $y-4=-\frac{2}{3}(x-1)$ or $\frac{y-4}{x-1}=-\frac{2}{3}$	A correct un-simplified equation	A1
	$2x+3y-14=0$	Accept $A(2x+3y-14)=0$ where A is an integer. Terms can be in any order but must have '= 0'.	A1
			(5)
Alt (b)	Gradient $l_2=-\frac{1}{"\frac{3}{2}"}=-\frac{2}{3}$	Uses $m_2=-\frac{1}{m_1}$ to find the gradient of l_2 as before	M1
	$\frac{3}{2}x+\frac{5}{2}=-\frac{2}{3}x+c$	A correct statement for $l_1=l_2$	B1
	$x=1\Rightarrow c=\frac{14}{3}$	Substitutes x = 1 to find a value for c	M1
	$y=-\frac{2}{3}x+\frac{14}{3}$	Correct equation	A1
	$2x+3y-14=0$	Accept $A(2x+3y-14)=0$ where A is an integer.	A1

(c)	$y=0 \Rightarrow 2x-14=0 \Rightarrow x=7$	Attempts to find C using y = 0 in the equation obtained in part (b)	M1
	Attempts Area of triangle using $\frac{1}{2}\times AC\times\left(y \text{ coord of } B\right)$ $=\frac{1}{2}\times\left('7'+'\frac{5}{3}'\right)\times'4'$ or Attempts Area of triangle using 2 triangles $\frac{1}{2}\times\left(1+'\left(\frac{5}{3}\right)'\right)\times\left(y \text{ coord of } B\right)+\frac{1}{2}\times\left('7'-1\right)\times\left(y \text{ coord of } B\right)$ If they make a second/different attempt to find the y coordinate of B then still allow this mark.		M1
	$=\frac{52}{3}$	Area $=\frac{52}{3}$ or exact equivalent e.g $17\frac{1}{3}$ or 17.3 **recurring (i.e. a clear dot over the 3)**	A1
			(3)
			(11 marks)
Way 2 6(c)	$y=0 \Rightarrow 2x-14=0 \Rightarrow x=7$	Attempts to find C using y = 0 in the equation obtained in part (b)	M1
	Attempts area of triangle using $\frac{1}{2}AB\times BC=\frac{1}{2}\times\sqrt{\frac{208}{9}}\times\sqrt{52}$ A complete method for the area including correct attempts at finding AB and BC using their values.		M1
	$=\frac{52}{3}$	Area $=\frac{52}{3}$ or exact equivalent e.g. $17\frac{1}{3}$ or 17.3 **recurring (i.e. a clear dot over the 3)**	A1
			(3)
Way 3 6(c)	$y=0 \Rightarrow 2x-14=0 \Rightarrow x=7$	Attempts to find C using y = 0 in the equation obtained in part (b)	M1
	$\frac{1}{2}\begin{vmatrix}1 & 7 & -\frac{5}{3} & 1\\ 4 & 0 & 0 & 4\end{vmatrix}=\frac{1}{2}\left\|-\frac{20}{3}-28\right\|$	Uses shoelace method. Must see a correct method including ½.	M1
	$=\frac{52}{3}$	Area $=\frac{52}{3}$ or exact equivalent e.g $17\frac{1}{3}$ or 17.3 **recurring (i.e. a clear dot over the 3)**	A1
			(3)

Way 4 6(c)	$y=0 \Rightarrow 2x-14=0 \Rightarrow x=7$	Attempts to find C using y = 0 in the equation obtained in part (b)	M1
	$\int_{-\frac{5}{3}}^{1}\left(\frac{3x}{2}+\frac{5}{2}\right)\mathrm{d}x+\int_{1}^{7}\left(-\frac{2x}{3}+\frac{14}{3}\right)\mathrm{d}x$ $=\left[\frac{3x^2}{4}+\frac{5}{2}x\right]_{-\frac{5}{3}}^{1}+\left[-\frac{2x^2}{6}+\frac{14}{3}x\right]_{1}^{7}$ $=\left(\frac{3}{4}+\frac{5}{2}\right)-\left(\frac{75}{36}-\frac{25}{6}\right)+\left(-\frac{49}{3}+\frac{98}{3}\right)-\left(-\frac{1}{3}+\frac{14}{3}\right)$ A complete method using their values with correct integration on l_1 and their l_2: Finds the area under the given line between their -5/3 and 1 and adds the area under their l_2 between 1 and their 7.		M1
	$=\frac{52}{3}$	Area $=\frac{52}{3}$ or exact equivalent e.g. $17\frac{1}{3}$ or 17.3 **recurring (i.e. a clear dot over the 3)**	A1
			(3)

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7. (i) Find

$$\int \frac{2 + 4x^3}{x^2}\,dx$$

giving each term in its simplest form.

(4)

(ii) Given that k is a constant and

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k\right)dx = 30$$

find the exact value of k.

(5)

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Question Number	Scheme		Marks
7 (i)	$\frac{2+4x^3}{x^2}=\frac{2}{x^2}+4x=2x^{-2}+4x$	Attempts to split the fraction. This can be awarded for $\frac{2}{x^2}$ or $\frac{4x^3}{x^2}$ or **may** be implied by the sight of one correct index e.g px^{-2} or qx **providing one of these terms is obtained correctly. So for example** $\frac{2+4x^3}{x^2}=2+4x^3+x^{-2}$ would be M0 as the x^{-2} **has been obtained incorrectly.**	M1
	$\int 2x^{-2}+4x\,\mathrm{d}x=2\times\frac{x^{-1}}{-1}+4\times\frac{x^2}{2}(+c)$	dM1: $x^n\to x^{n+1}$ on any term. **Dependent** on the first M. A1: At least one term correct, simplified or un-simplified. Allow powers **and** coefficients to be un-simplified e.g. $2\times\frac{x^{-2+1}}{-1}$, $+4\times\frac{x^{1+1}}{2}$	dM1A1
	$=-\frac{2}{x}+2x^2+c$	All correct and simplified including the + c. Accept equivalents such as $-2x^{-1}+2x^2+c$	A1
			(4)
	There are no marks in (ii) for use of the trapezium rule – must use integration		
(ii)	$\int\left(\frac{4}{\sqrt{x}}+k\right)\mathrm{d}x$ $=\int\left(4x^{-0.5}+k\right)\mathrm{d}x=4\frac{x^{0.5}}{0.5}+kx(+c)$	M1: Integrates to obtain either $\alpha x^{0.5}$ or kx A1: Correct integration (simplified or un-simplified). Allow powers **and** coefficients to be un-simplified e.g. $4\frac{x^{-0.5+1}}{0.5}$. There is no need for + c	M1A1
	$\left[4\frac{x^{0.5}}{0.5}+kx\right]_2^4=30\Rightarrow\left(8\sqrt{4}+4k\right)-\left(8\sqrt{2}+2k\right)=30$ Substitutes both $x=4$ and $x=2$ into <ins>**changed**</ins> expression involving k, subtracts either way round and sets equal to 30 Condone poor use or omission of brackets when subtracting.		M1
	$2k+16-8\sqrt{2}=30\ \Rightarrow k=7+4\sqrt{2}$	**dd**M1: Attempts to solve for k from a **linear** equation in k. **Dependent upon both M's and need to have seen** $\int k\,\mathrm{d}x\to kx$. A1: $7+4\sqrt{2}$ or exact equivalent e.g. $7+2^{2.5}$, $7+4\times2^{0.5}$	**dd**M1A1
			(5)
			(9 marks)

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8. $$f(x) = 2x^3 - 5x^2 - 23x - 10$$

(a) Find the remainder when f(x) is divided by (x – 3).
(2)

(b) Show that (x + 2) is a factor of f(x).
(2)

(c) Hence fully factorise f(x).
(4)

(d) Hence solve

$$2(3^{3t}) - 5(3^{2t}) - 23(3^t) = 10$$

giving your answer to 3 decimal places.
(2)

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Question Number	Scheme		Marks
8(a)	$f(3)=2(3)^3-5(3)^2-23(3)-10$ or $x-3\overline{)2x^3-5x^2-23x-10}$ with quotient $2x^2+\ldots\ldots\ldots$, ... , ...	Attempts to calculate $f(\pm 3)$ or divides by $(x-3)$. For long division need to see minimum as shown with a constant remainder.	M1
	(Remainder =) -70	-70	A1
			(2)
	Mark (b) and (c) together		
(b)	$f(-2)=2(-2)^3-5(-2)^2-23(-2)-10$ Or $x+2\overline{)2x^3-5x^2-23x-10}$ with quotient $2x^2+\ldots\ldots\ldots$, ... , ...	Attempts $f(\pm 2)$ or divides by $(x+2)$. For long division need to see minimum as shown with a constant remainder.	M1
	Remainder = 0, hence $x+2$ is a factor	Obtains a remainder zero and makes a conclusion (not just a tick or e.g. QED). Do not need to refer to the remainder in the conclusion but a zero remainder must have been obtained. (May be seen in a preamble)	A1*
	Note that **just** f(-2) = 0 therefore (x + 2) is a factor scores M0A0 as there must be some evidence of a calculation		
			(2)
(c)	$\dfrac{2x^3-5x^2-23x-10}{(x+2)}=ax^2+bx+c$	Divides f(x) by $(x+2)$ or compares coefficients or uses inspection to obtain a quadratic expression with $2x^2$ as the first term.	M1
	$2x^2-9x-5$	Correct quadratic seen	A1
	$f(x)=(x+2)(2x+1)(x-5)$ **d**M1: Attempt to factorise their 3TQ $\left(2x^2\ldots\right)$. The usual rules apply here so if $2x^2-9x-5$ is factorised as $(x-5)\left(x+\frac{1}{2}\right)$, this scores M0 unless the factor of 2 appears later. A1: $(x+2)(2x+1)(x-5)$ oe e.g. $2(x+2)\left(x+\frac{1}{2}\right)(x-5)$. All factors together on one line. Must appear here and not in (d). Ignore subsequent attempts to solve.		**d**M1A1
	SC: This is a hence question but we will allow a special case of 1100 for candidates in this part who use their graphical calculators to get roots of -2, -0.5 and 5 and write down the <u>correct</u> factorised form.		

Question Number	Scheme		Marks
	But note that if all that is seen is $(x+2)\left(x+\frac{1}{2}\right)(x-5)$ this scores 1000		
			(4)

(d)	$3^t = '5' \Rightarrow t\log 3 = \log '5'$	Solves $3^t = k$ where $k > 0$ and follows from their (c) to obtain $t\log 3 = \log k$. Accept sight of $t = \log_3 k$ where $k > 0$ and follows from their (c)	M1
	$\Rightarrow t = \text{awrt } 1.465$ only	$t = \text{awrt } 1.465$ and no other solutions	A1
			(2)
			(10 marks)

Question Number	Scheme		Marks
9(a)	$f(x) = 8x^{-1} + \frac{1}{2}x - 5$ $\Rightarrow f'(x) = -8x^{-2} + \frac{1}{2}$	M1: $-8x^{-2}$ or $\frac{1}{2}$ A1: Fully correct $f'(x) = -8x^{-2} + \frac{1}{2}$ **(may be un-simplified)**	M1A1
	Sets $-8x^{-2} + \frac{1}{2} = 0 \Rightarrow x = 4$	M1: Sets their $f'(x) = 0$ i.e. a "changed" function (may be implied by their work) and proceeds to find x. A1: $x = 4$ (Allow $x = \pm 4$)	M1A1
	$(4, -1)$	Correct coordinates (allow $x = 4$, $y = -1$). Ignore their (-4, …)	A1
			(5)
(b)(i)	$(x=)\,2,\ 8$	$x = 2$ and $x = 8$ **only**. Do **not** accept as coordinates here.	B1
(b)(ii)	$(4,\ 1)$	$(4,\ 1)$ or follow through on their solution in (a). Accept $(x, y+2)$ from their (x, y). **With no other points.**	B1ft
(b)(iii)	$(x=)\,2,\ \frac{1}{2}$	Both answers are needed and **accept** $(2,\ 0)$, $\left(\frac{1}{2},\ 0\right)$ here. Ignore any reference to the image of the turning point.	B1
			(3)
			(8 marks)

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9.

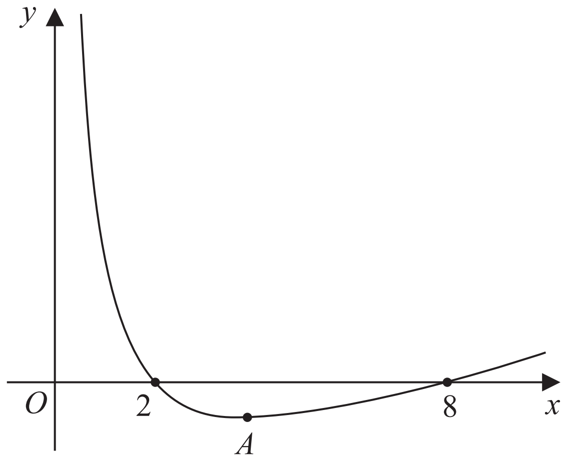

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{8}{x} + \frac{1}{2}x - 5, \qquad 0 < x \leqslant 12$$

The curve crosses the x-axis at (2, 0) and (8, 0) and has a minimum point at A.

(a) Use calculus to find the coordinates of point A.

(5)

(b) State

(i) the roots of the equation $2\mathrm{f}(x) = 0$

(ii) the coordinates of the turning point on the curve $y = \mathrm{f}(x) + 2$

(iii) the roots of the equation $\mathrm{f}(4x) = 0$

(3)

Question Number	Scheme		Marks
	But note that if all that is seen is $(x+2)(x+\frac{1}{2})(x-5)$ **this scores 1000**		
			(4)
(d)	$3^t='5'\Rightarrow t\log 3=\log'5'$	Solves $3^t=k$ where $k>0$ and follows from their (c) to obtain $t\log 3=\log k$. Accept sight of $t=\log_3 k$ where $k>0$ and follows from their (c)	M1
	$\Rightarrow t=\text{awrt } 1.465$ only	$t=\text{awrt } 1.465$ and no other solutions	A1
			(2)
			(10 marks)

Question Number	Scheme		Marks
9(a)	$f(x)=8x^{-1}+\frac{1}{2}x-5$ $\Rightarrow f'(x)=-8x^{-2}+\frac{1}{2}$	M1: $-8x^{-2}$ or $\frac{1}{2}$ A1: Fully correct $f'(x)=-8x^{-2}+\frac{1}{2}$ **(may be un-simplified)**	M1A1
	Sets $-8x^{-2}+\frac{1}{2}=0\Rightarrow x=4$	M1: Sets their $f'(x)=0$ i.e. a "changed" function (may be implied by their work) and proceeds to find *x*. A1: $x=4$ (Allow $x=\pm 4$)	M1A1
	$(4,-1)$	Correct coordinates (allow *x* = 4, *y* = −1). Ignore their (-4, …)	A1
			(5)
(b)(i)	$(x=)\,2,\ 8$	$x=2$ and $x=8$ **only**. Do **not** accept as coordinates here.	B1
(b)(ii)	$(4,\ 1)$	$(4,\ 1)$ or follow through on their solution in (a). Accept $(x, y+2)$ from their (x, y). **With no other points.**	B1ft
(b)(iii)	$(x=)\,2,\ \frac{1}{2}$	Both answers are needed and **accept** $(2,\ 0)$, $\left(\frac{1}{2},\ 0\right)$ here. Ignore any reference to the image of the turning point.	B1
			(3)
			(8 marks)

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10. The first 3 terms, in ascending powers of x, in the binomial expansion of $(1 + ax)^{20}$ are given by

$$1 + 4x + px^2$$

where a and p are constants.

(a) Find the value of a.
(2)

(b) Find the value of p.
(2)

One of the terms in the binomial expansion of $(1 + ax)^{20}$ is qx^4, where q is a constant.

(c) Find the value of q.
(2)

Question Number	Scheme		Marks
	Mark (a) and (b) together		
10(a)	$(1+ax)^{20} = 1^{20} + {}^{20}C_1 1^{19}(ax)^1 + {}^{20}C_2 1^{18}(ax)^2$. Note that the notation $\binom{20}{1}$ may be seen for ${}^{20}C_1$ etc.		
	${}^{20}C_1 1^{19}(ax)^1 = 4x \Rightarrow 20a = 4 \Rightarrow a = 0.2$	M1: Uses either ${}^{20}C_1(1^{19})(ax)^1 = 4x^1$ or $20a = 4$ to obtain a value for a.	M1A1
		A1: $a = 0.2$ or equivalent	
			(2)
(b)	${}^{20}C_2 1^{18}(ax)^2 = px^2$ $\Rightarrow \frac{20\times19}{2}\times('0.2')^2 = p$ $\Rightarrow p = ...$	Uses ${}^{20}C_2(1^{18})(ax)^2 = px^2$ and their value of a to find a value for p. Condone the use of a rather than a^2 in finding p. Maybe implied by an attempt to find a value for $190a^2$ or $190a$. Note: ${}^{20}C_{18}$ can be used for ${}^{20}C_2$	M1
	$p = 7.6$	Accept equivalents such as $\frac{38}{5}, \frac{190}{25}$	A1
			(2)
(c)	Term is ${}^{20}C_4 1^{16}(ax)^4 \Rightarrow q = ...$	Identifies the correct term and uses their value of a to find a value for q. Condone the use of a rather than a^4. Must be an attempt to calculate ${}^{20}C_4 a^4$ or ${}^{20}C_4 a$ or ${}^{20}C_{16} a^4$ or ${}^{20}C_{16} a$	M1
	$q = {}^{20}C_4 \times 0.2^4 = \frac{969}{125} = (7.752)$	$q = \frac{969}{125}$ or exact equivalent e.g. $7.752,\ 7\frac{94}{125}$. $q = \frac{969}{125}x^4$ scores A0 but $qx^4 = \frac{969}{125}x^4$ scores A1.	A1
			(2)
			(6 marks)

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11. In this question solutions based entirely on graphical or numerical methods are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$,

$$3\cos^2 x + 1 = 4\sin^2 x$$

giving your answers in radians to 2 decimal places.

(5)

(ii) Solve, for $0 \leqslant \theta < 360°$,

$$5\sin(\theta + 10°) = \cos(\theta + 10°)$$

giving your answers in degrees to one decimal place.

(5)

Question Number	Scheme		Marks
11(i)	$3\cos^2 x + 1 = 4(1-\cos^2 x)$ or $3(1-\sin^2 x)+1 = 4\sin^2 x$ or $3+\tan^2 x + 1 = 4\tan^2 x$ or $3\frac{\cos 2x+1}{2}+1 = 4\frac{1-\cos 2x}{2}$	Uses $\sin^2 x = 1-\cos^2 x$ to produce an equation in $\cos^2 x$ or uses $\cos^2 x = 1-\sin^2 x$ to produce an equation in $\sin^2 x$ or uses $\cos^2 x+\sin^2 x = 1$ and divides by $\cos^2 x$ to produce an equation in $\tan^2 x$ or uses $\sin^2 x$ and $\cos^2 x$ in terms of cos2*x*. Condone missing brackets.	M1
	$\Rightarrow \cos^2 x = \frac{3}{7}$ or $\sin^2 x = \frac{4}{7}$ or $\tan^2 x = \frac{4}{3}$ or $\cos 2x = -\frac{1}{7}$	Correct value for $\cos^2 x$ or $\sin^2 x$ or $\tan^2 x$ or cos2*x*. This may be implied by $\cos x = \sqrt{\frac{3}{7}}$ or $\sin x = \sqrt{\frac{4}{7}}$ or $\tan x = \sqrt{\frac{4}{3}}$	A1
	$\Rightarrow \cos x = \pm\sqrt{\frac{3}{7}} \Rightarrow x = \cos^{-1}\left(\sqrt{\frac{3}{7}}\right)$ A correct order of operations to obtain a correct expression for *x*. E.g. $\cos^2 x = p \Rightarrow \cos x = \sqrt{p} \Rightarrow x = \cos^{-1}\sqrt{p}$ or $\sin^2 x = p \Rightarrow \sin x = \sqrt{p} \Rightarrow x = \sin^{-1}\sqrt{p}$ or $\tan^2 x = p \Rightarrow \tan x = \sqrt{p} \Rightarrow x = \tan^{-1}\sqrt{p}$ or $\cos 2x = p \Rightarrow 2x = \cos^{-1} p \Rightarrow x = \frac{1}{2}\cos^{-1} p$ This may be implied by one correct answer for their values.		M1
	$\Rightarrow x = \text{awrt } 0.86, 2.28, 4.00, 5.43$	A1: Any two of awrt 0.86, 2.28, 4.00, 5.43 A1: All four of awrt $0.86, 2.28, 4.00, 5.43$ with no additional solutions in the range and ignore solutions outside the range.	A2,1,0
	Note that answers in degrees are: 49.11, 130.89, 229.11, 310.89 Allow A1 for awrt two of these but deduct the final A mark. For answers given as awrt $0.27\pi, 0.73\pi, 1.27\pi, 1.73\pi$, allow A1 only for any 2 of these but deduct the final A mark.		
			(5)

(ii)	$5\sin(\theta+10°)=\cos(\theta+10°)$ $\Rightarrow \tan(\theta+10°)=0.2$	M1: Reaches $\tan(...)=\alpha$ where α is a constant including zero. A1: $\tan(...)=0.2$	M1A1
	$\Rightarrow \theta=\tan^{-1}(0.2)-10°$	For the correct order of operations to produce one value for θ. Accept $\theta=\tan^{-1}(\alpha)-10,\ \alpha\neq 0$ or one correct answer as evidence. **Dependent on the first M.**	**d**M1
	$\Rightarrow \theta=$ awrt $1.3°, 181.3°$	A1: One of awrt $\theta=1.3, 181.3$ A1: Both awrt $\theta=1.3, 181.3$ and no other solutions in range and ignore solutions outside the range.	A1A1
	Note that final answers in **radians** in (ii) cannot score the final 2 A marks but the earlier marks are available (maximum 11100)		
			(5)
			(10 marks)
	Alternative 1 for (ii) by squaring:		
	$5\sin(...)=\cos(...)$ $\Rightarrow 25\sin^2(...)=\cos^2(...)$ $\Rightarrow 25(1-\cos^2(...))=\cos^2(...)$ or $25\sin^2(...)=1-\sin^2(...)$ Leading to $\sin^2(...)=...$ or $\cos^2(...)=...$	Squares both sides, replaces $\sin^2(...)$ by $1-\cos^2(...)$ or replaces $\cos^2(...)$ by $1-\sin^2(...)$ and reaches $\sin^2(...)=...$ or $\cos^2(...)=...$	M1
	$\sin^2(...)=\frac{1}{26}$ or $\cos^2(...)=\frac{25}{26}$	Correct value for $\sin^2(...)$ or $\cos^2(...)$. This may be implied by $\sin(...)=\frac{1}{\sqrt{26}}$ or $\cos(...)=\sqrt{\frac{25}{26}}$	A1
	$\theta=\sin^{-1}\frac{1}{\sqrt{26}}-10°$ or $\theta=\cos^{-1}\frac{5}{\sqrt{26}}-10°$	For the correct order of operations to produce one value for θ as shown or accept one correct answer as evidence. **Dependent on the first M.**	**d**M1
	$\Rightarrow \theta=1.3°, 181.3°$	A1: One of awrt $\theta=1.3, 181.3$ A1: Both awrt $\theta=1.3, 181.3$ and no other solutions in range and ignore solutions outside the range.	A1A1
	Note that final answers in **radians** in (ii) cannot score the final 2 A marks but the earlier marks are available (maximum 11100)		

	Alternative 2 for (ii) Using the addition formulae		
Alt (ii)	$5\sin\theta\cos10+5\cos\theta\sin10=\cos\theta\cos10-\sin\theta\sin10$ Uses the correct addition formulae on both sides and rearranges to tan(…) =		M1
	$\tan\theta=\dfrac{\cos10-5\sin10}{5\cos10+\sin10}=(0.0229)$	Correct value for $\tan\theta$	A1
	$\tan\theta=0.0229\Rightarrow\theta=...$	Uses arctan to produce one value for θ. **Dependent on the first M.**	**d**M1
	$\Rightarrow\theta=1.3°, 181.3°$	A1: One of awrt $\theta=1.3, 181.3$ A1: Both awrt $\theta=1.3, 181.3$ and no other solutions in range and ignore solutions outside the range.	A1A1
	Note that final answers in **radians** in (ii) cannot score the final 2 A marks but the earlier marks are available (maximum 11100)		
			(5)

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12.

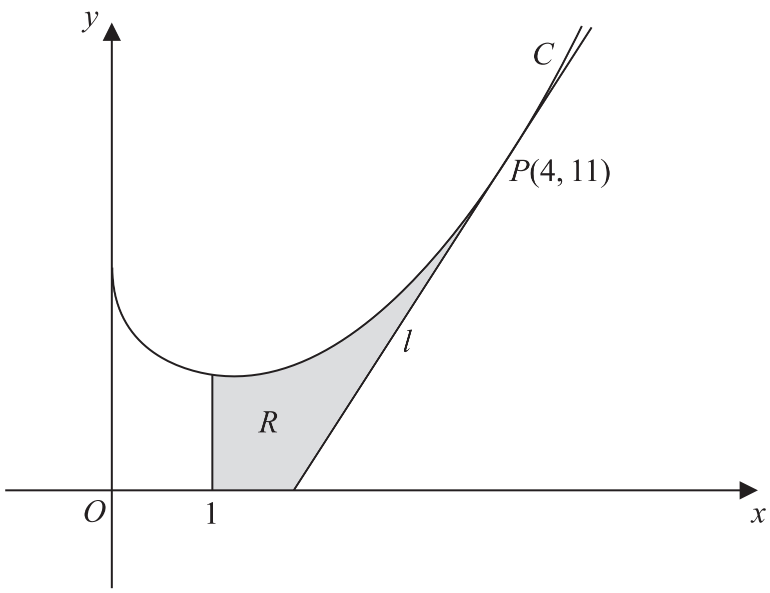


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7, \quad x > 0$$

The point P lies on C and has coordinates (4, 11).

Line l is the tangent to C at the point P.

(a) Use calculus to show that l has equation $y = 5x - 9$

(5)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line $x = 1$, the x-axis and the line l.

(b) Find, by using calculus, the area of R, giving your answer to 2 decimal places.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

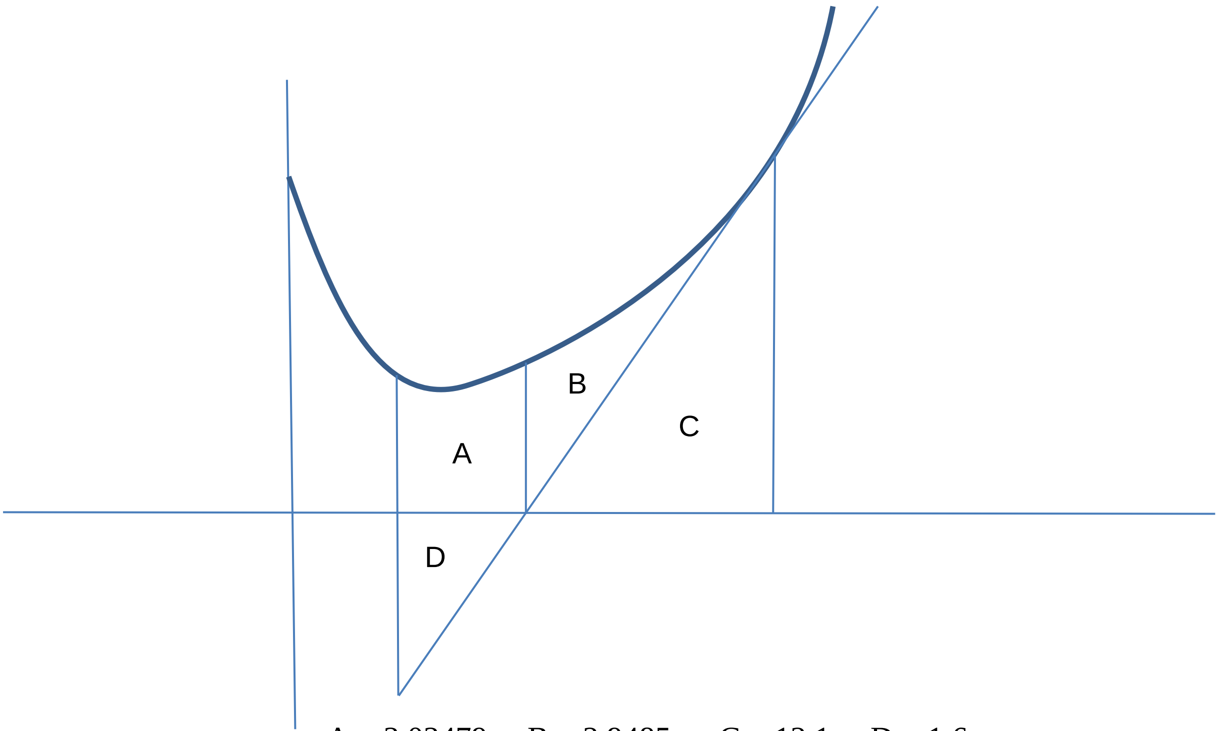
Question Number	Scheme		Marks
12(a)	$y=\frac{3}{4}x^2-4\sqrt{x}+7 \Rightarrow \frac{dy}{dx}=\frac{3}{2}x-2x^{-0.5}$	M1: Differentiates to obtain at least one correct power for one of the terms in *x*. (may be un-simplified) e.g. $x^2 \to x^{2-1}$ or $\sqrt{x} \to x^{\frac{1}{2}-1}$ A1: Correct derivative. Allow un-simplified e.g. $2\times\frac{3}{4}x^{2-1}$ or $-4\times\frac{1}{2}x^{\frac{1}{2}-1}$	M1A1
	At $x = 4$ $\frac{dy}{dx}=\frac{3}{2}(4)-2(4)^{-0.5}=...$	Substitutes $x = 4$ into a changed function in an attempt to find the gradient.	M1
	$y-11="5"(x-4)$ or $y=mx+c \Rightarrow 11="5"\times 4+c \Rightarrow c=...$	Correct straight line method using (4, 11) correctly placed and their d*y*/d*x* at $x = 4$ for the tangent **not the normal**. If using $y = mx + c$, must reach as far as finding a value for *c*. **Dependent on the previous M.**	**d**M1
	$y=5x-9$	Correct printed equation with no errors seen. **Beware of the "5" appearing from wrong working.**	A1*
	<u>Important Note:</u> **Following a correct derivative, if candidate states $x = 4$ so d*y*/d*x* = 5, this is fine if they then complete correctly – allow full marks. However, following a correct derivative, if the candidate <u>just</u> states d*y*/d*x* = 5 and then proceeds to obtain the correct straight line equation, the final mark can be withheld. Some evidence is needed that the candidate is considering the gradient at $x = 4$.**		
			(5)

For part (b), in all cases, look to apply the appropriate scheme that gives the candidate the best mark

	Finds area under curve between 1 and 4 and subtracts triangle C (see diagram at end)		
(b) Way 1	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7\,dx = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3}\times\frac{3}{4}x^{2+1}$, $-\frac{2}{3}\times 4x^{0.5+1}$, $7x^1$ and + c is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle $= \frac{1}{2}\times(4 - '1.8')\times 11 = (12.1)$ Correct method for the area of a triangle - look for $\frac{1}{2}\times(4-'1.8')\times 11$ This may be implied by the evaluation of $\int_{'1.8'}^{4} 5x - 9\,dx = \left[5\frac{x^2}{2} - 9x\right]_{'1.8'}^{4}$		M1
	Correct method for area = Area A + Area B + Area C – Area C $\left(\frac{1}{4}4^3 - \frac{8}{3}\times 4^{1.5} + 7\times 4\right) - \left(\frac{1}{4}1^3 - \frac{8}{3}\times 1^{1.5} + 7\times 1\right) - '12.1'$ Correct combination of areas. **Dependent on both previous method marks.**		**dd**M1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)
			(11 marks)

	Finds area under curve between 1 and "1.8" and adds "line – curve" or "curve – line" between "1.8" and 4		
(b) **Way 2**	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7 \, \mathrm{d}x = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3}\times\frac{3}{4}x^{2+1}$, $-\frac{2}{3}\times 4x^{0.5+1}$, $7x^1$ and + c is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be seen on a diagram.	B1
	Area between "1.8" and 4 = $\pm\int_{'1.8'}^{4}\left(\frac{3}{4}x^2 - 4\sqrt{x} + 7\right) - (5x - 9)\, \mathrm{d}x = \pm\left[\frac{1}{4}x^3 - \frac{8}{3}x^{1.5} - \frac{5x^2}{2} + 16x\right]_{'1.8'}^{4}$ $= \frac{56}{3} - 15.7182...(= 2.9485...)$ Attempts to integrate "curve – line" or "line – curve", substitute the limits "1.8" and 4 and subtracts.		M1
	Correct method for area = Area A + Area B $\left(\left(\frac{1}{4}"1.8"^3 - \frac{8}{3}"1.8"^{1.5} + 7\times"1.8"\right) - \left(\frac{1}{4}1^3 - \frac{8}{3}1^{1.5} + 7\times 1\right) + '2.9485...'\right)$ Correct combination of areas. **Dependent on both previous method marks.**		**dd**M1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)

	Uses "line – curve" or "curve – line" between 1 and 4 and subtracts triangle below x axis	
(b) Way 3	$\pm\left(\frac{3}{4}x^2-4\sqrt{x}+7-5x+9\right)=\pm\left(\frac{3}{4}x^2-4\sqrt{x}-5x+16\right)$ $\pm\int\frac{3}{4}x^2-4\sqrt{x}-5x+16\,\mathrm{d}x=\pm\left(\frac{1}{4}x^3-\frac{8}{3}x^{1.5}-\frac{5x^2}{2}\right)+kx(+c)$ M1: $x^n\to x^{n+1}$ on any term. May be un-simplified e.g. $x^2\to x^{2+1}$, $x^{0.5}\to x^{0.5+1}$, $x\to x^{1+1}$, $16\to16x^1$. If terms are not collected when subtracting then the same condition applies. A1: Correct integration **as shown**. May be un-simplified for coefficients and powers and + c is not required.	M1A1
	Tangent meets x axis at $x=1.8$ — This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle $=\frac{1}{2}\times('1.8'-1)\times\lvert5\times1-9\rvert=(1.6)$ Correct method for the area of a triangle - look for $\frac{1}{2}\times('1.8'-1)\times\lvert5\times1-9\rvert$	M1
	Correct method for area = Area A + Area B + Area D – Area D $\left(\left(\frac{1}{4}4^3-\frac{8}{3}4^{1.5}-\frac{5\times4^2}{2}+16\times4\right)-\left(\frac{1}{4}1^3-\frac{8}{3}1^{1.5}-\frac{5\times1^2}{2}+16\times1\right)-'1.6'\right)$ Correct combination of areas. **Dependent on both previous method marks.**	**dd**M1
	= awrt 5.98 — Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
		(6)



A = 3.03479, B = 2.9485, C = 12.1, D = 1.6

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13. (a) On separate axes sketch the graphs of

(i) $y = c^2 - x^2$

(ii) $y = x^2(x - 3c)$

where c is a positive constant.

Show clearly the coordinates of the points where each graph crosses or meets the x-axis and the y-axis.

(5)

(b) Prove that the x coordinate of any point of intersection of

$$y = c^2 - x^2 \quad \text{and} \quad y = x^2(x - 3c)$$

where c is a positive constant, is given by a solution of the equation

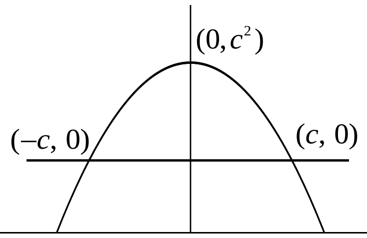
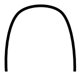
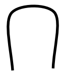
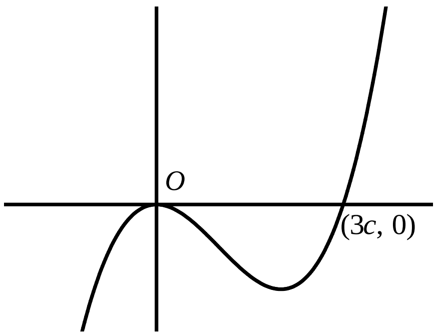
$$x^3 + (1 - 3c)x^2 - c^2 = 0$$

(2)

Given that the graphs meet when $x = 2$

(c) find the exact value of c, writing your answer as a fully simplified surd.

(4)

Question Number	Scheme	Marks
13(a)(i)		
	 Or  shape anywhere but not  The maximum must be smooth and not form a point and the branches must not clearly turn back in on themselves. **or** A continuous graph passing through or touching at the points (-c, 0), (c, 0) and (0, c^2). They can appear on their sketch or within the body of the script but there must be a sketch. Allow these marked as –c, c and c^2 in the correct places. Allow (0, -c), (0, c) and (c^2, 0) as long as they are marked in the correct places. If there is any ambiguity, the sketch takes precedence.	B1
	A fully correct diagram with the curve in the correct position and the intercepts and shape as described above. The maximum must be on the y-axis and the branches must extend below the x-axis.	B1
(a)(ii)	**There must be a sketch to score any marks in (a)**	
	  Shape. A positive cubic with only one maximum and one minimum. The curve must be smooth at the maximum and at the minimum (not pointed).	B1
	A smooth curve that touches or meets the x-axis at the origin and $(3c, 0)$ in the correct place and no other intersections. The origin does not need to be marked but the $(3c, 0)$ does. Allow 3c or $(0,\ 3c)$ to be marked in the correct place. May appear on their sketch or within the body of the script. If there is any ambiguity, the sketch takes precedence.	B1
	Maximum at the origin (allow the maximum to form a point or cusp)	B1
	There must be a sketch to score any marks in (a)	**(5)**
(b)	Intersect when $x^2(x-3c) = c^2 - x^2 \Rightarrow x^3 - 3cx^2 = c^2 - x^2$ Sets equations equal to each other and attempts to multiply out the bracket or vice versa	M1
	$x^3 + x^2 - 3cx^2 - c^2 = 0$ $\Rightarrow x^3 + (1-3c)x^2 - c^2 = 0*$ Collects to one side (may be implied), factorises the x^2 terms and obtains printed answer with no errors. **There must be an intermediate line of working.** **Allow** $x^3 + x^2(1-3c) - c^2 = 0$ or $0 = x^3 + (1-3c)x^2 - c^2$ or $0 = x^3 + x^2(1-3c) - c^2$	A1*
		(2)

(c)	$8+4(1-3c)-c^2=0$	Substitutes $x=2$ to give a **correct** un-simplified form of the equation.	M1
	$c^2+12c-12=0$	Correct 3 term quadratic. Allow any equivalent form with the terms collected (may be implied)	A1
	$(c+6)^2-36-12=0 \Rightarrow c=...$ or $c=\dfrac{-12\pm\sqrt{12^2-4\times1\times(-12)}}{2}$	Solves their 3TQ by using the formula or completing the square **only**. This may be implied by a correct **exact** answer for their 3TQ. (May need to check)	M1
	$4\sqrt{3}-6$	$c=4\sqrt{3}-6$ or $c=-6+4\sqrt{3}$ **only**	A1
			(4)
			(11 marks)

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14. A geometric series has a first term a and a common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in a barrel at the end of a year is 7% less than the volume at the start of the year.

At the start of the first year, a barrel is filled with 180 litres of the liquid.

(b) Show that the amount of the liquid in this barrel at the end of 5 years is approximately 125.2 litres.

(2)

At the start of each year a new identical barrel is filled with 180 litres of the liquid so that, at the end of 20 years, there are 20 barrels containing varying amounts of the liquid.

(c) Calculate the total amount of the liquid, to the nearest litre, in the 20 barrels at the end of 20 years.

(3)

Question Number	Scheme		Marks
14 (a)	Allow the use of S or S_n throughout without penalty. $S = a + ar + ar^2 + ...ar^{n-1}$ **and** $rS = ar + ar^2 + ar^3 + ...ar^n$ There must be a minimum of '3' terms and must include the first and the nth term. Condone for this mark only $S = a + ar + ar^2 + ...ar^n$ **and** $rS = ar + ar^2 + ar^3 + ...ar^{n+1}$ and allow commas instead of +'s but see note below.		M1
	$S - rS = a - ar^n$	Subtracts either way around. **As a special case allow** $S - rS = a + ar^n$. For this mark, their S and their rS must be different but it must be S and rS they are considering with possible missing terms or slips.	M1
	$\Rightarrow S(1-r) = a(1-r^n) \Rightarrow S = \dfrac{a(1-r^n)}{(1-r)}$	**d**M1: **Dependent upon both previous M's**. It is for taking out a common factor of S and achieving $S = \ldots$ A1*: Fully correct proof with **no errors or omissions. The use of commas instead of +'s is an error.** $S = \dfrac{a(r^n - 1)}{(r-1)}$ without reaching the printed answer is A0	**d**M1A1*
			(4)
(a) Way 2	$S = \dfrac{(a + ar + ar^2 + ...ar^{n-1})(1-r)}{1-r}$	Gives a minimum of '3' terms and must include the first and the nth and multiplies top and bottom by $1 - r$	M1
	$S = \dfrac{a + ar + ar^2 + ...ar^{n-1} - ar - ar^2 - ... - ar^n}{1-r}$	Expands the top with a minimum of '3' terms in each and must include the first and the nth term	M1
	$S = \dfrac{a(1-r^n)}{(1-r)}$	dM1: Dependent upon both previous M's. It is for taking out a common factor of a on top and achieving $S = \ldots$ A1*: Fully correct proof with **no errors or omissions. The use of commas instead of +'s is an error.** $S = \dfrac{a(r^n - 1)}{(r-1)}$ without reaching the printed answer is A0	dM1A1

(b)	$U = 180 \times 0.93^n$ with $n = 4$ or 5	Attempts $U = 180 \times 0.93^n$ with $n = 4$ or 5. Accept $U = 167.4 \times 0.93^n$ with $n = 3$ or 4 Allow 93% for 0.93	M1
	$U_5 = 180 \times (0.93)^5 = 125.2$ (litres)	Cso. Awrt 125.2	A1*
	Allow 93% or 1 – 7% for 0.93		
			(2)
(c)	Attempts $S_n = \frac{a(1-r^n)}{(1-r)}$ with any combination of: $n = 20/21$ $a = 180/167.4$ and $r = 0.93$ Allow 93% for 0.93		M1
	$S = \frac{167.4(1-0.93^{20})}{(1-0.93)}$ or $S = 180 \times \frac{0.93(1-0.93^{20})}{(1-0.93)}$ or $S = \frac{180(1-0.93^{21})}{(1-0.93)} - 180$ A correct numerical expression for the sum (may be implied by awrt 1831) Allow 93% or 1 – 7% for 0.93		A1
	1831 (litres)	1831 **only** (Ignore units). Do not isw here, so 1831 followed by $1831 \times 20 = ...$ scores A0.	A1
			(3)
			(9 marks)

Listing:

(b)	Sight of awrt 180, 167, 156, 145, 135, 125	Starts with 180 and multiplies by 0.93 either 4 or 5 times showing each result at least to the nearest litre and chooses the 5th or 6th term	M1
	$U_5 = 125.2$ (litres)	Must see all values accurate to 1dp: e.g. awrt 180, 167.4, 155.7, 144.8, (134.6 or 134.7), 125.2	A1*
			(2)
(c)	$\text{Total} = 180 \times 0.93 + 180 \times 0.93^2 + ... + 180 \times 0.93^{19} + 180 \times 0.93^{20} = ...$ Finds an expression for the sum of 20 or 21 terms		M1
	All sums accurate to awrt 1dp 167.4+155.7+144.8+134.6+125.2+.....42.2 A correct numerical expression for the sum (may be implied by awrt 1831)		A1
	1831 (litres)	1831 **only** (Ignore units). Do not isw here, so 1831 followed by $1831 \times 20 = ...$ scores A0.	A1
			(3)

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15.

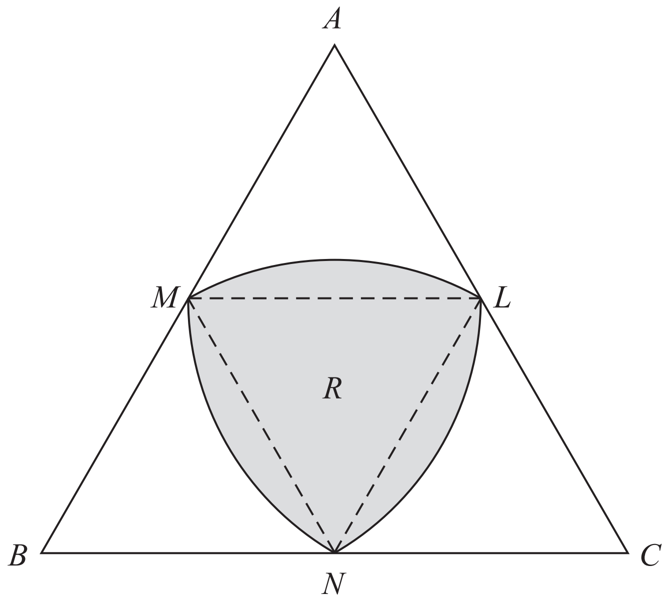

Figure 5

Figure 5 shows the design for a logo.

The logo is in the shape of an equilateral triangle ABC of side length $2r$ cm, where r is a constant.

The points L, M and N are the midpoints of sides AC, AB and BC respectively.

The shaded section R, of the logo, is bounded by three curves MN, NL and LM.

The curve MN is the arc of a circle centre L, radius r cm.

The curve NL is the arc of a circle centre M, radius r cm.

The curve LM is the arc of a circle centre N, radius r cm.

Find, in cm^2, the area of R. Give your answer in the form kr^2, where k is an exact constant to be determined.

(5)

Question Number	Scheme		Marks
15	Area of triangle = $\frac{1}{2}\times(2r)^2\sin\left(\frac{\pi}{3}\text{ or }60\right)$ or $\frac{1}{2}\times(r)^2\sin\left(\frac{\pi}{3}\text{ or }60\right)$ Correct method for the area of either triangle. Ignore any reference to which triangle they are finding the area of.		M1
	Area of sector = $\frac{1}{2}\times r^2\times\frac{\pi}{3}$	Use of the sector formula $\frac{1}{2}r^2\theta$ with $\theta=\frac{\pi}{3}$ which may be embedded within a segment	M1
	Area R = Sector + 2 Segments $=\frac{1}{2}r^2\times\frac{\pi}{3}+2\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$ Area R = Triangle + 3 Segments $=\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}+3\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$ Area R = 3 Sectors − 2 Triangles $=3\times\frac{1}{2}r^2\times\frac{\pi}{3}-2\times\left(\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)$ Area R = Big triangle − 3 White bits $=\frac{1}{2}\times(2r)^2\frac{\sqrt{3}}{2}-3\times\left(\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}-\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\times\frac{\sqrt{3}}{2}\right)\right)$ M1: A fully correct method (may be implied by a final answer of awrt $0.705r^2$) A1: Correct exact expression - for this to be scored $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ must be seen		M1A1
	$=\frac{1}{2}\pi r^2-\frac{\sqrt{3}}{2}r^2=r^2\left(\frac{1}{2}\pi-\frac{\sqrt{3}}{2}\right)$	Cso (Allow $\frac{r^2}{2}\left(\pi-\sqrt{3}\right)$ or any exact equivalent with r^2 taken out as a common factor)	A1
			(5 marks)