Write your name here

Surname | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

# Core Mathematics C12

## Advanced Subsidiary

Monday 10 October 2016 – Morning
**Time: 2 hours 30 minutes**

Paper Reference
**WMA01/01**

**You must have:**
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

P48253A

1/1/1/1/

P 4 8 2 5 3 A 0 1 4 8

PEARSON

Leave blank

**1.**

$$f(x) = 3x^2 + x - \frac{4}{\sqrt{x}} + 6x^{-3}, \quad x > 0$$

Find $\int f(x)\,dx$, simplifying each term.

**(5)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

| Question | Scheme | Marks |
|---|---|---|
| **1.** | $\mathrm{f}(x) = 3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}$ | |
| | $\int\left(3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}\right)\mathrm{d}x = \frac{3x^3}{3} + \frac{x^2}{2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-2}}{-2}(+c)$ | M1 A1A1A1 |
| | $= x^3 + \frac{x^2}{2} - 8x^{\frac{1}{2}} - 3x^{-2} + c$ | A1 |
| | | **[5]** |
| | | **5 marks** |

**Notes**

**M1**: Attempt to integrate original f($x$)– one power increased $x^n \to x^{n+1}$

**A1**: **Two** of the four terms in $x$ correct un simplified or simplified– (ignore no constant here). They may be listed.

$3x^2 \to 3\frac{x^3}{3}$ is acceptable for an un simplified term BUT $3x^2 \to 3\frac{x^{2+1}}{2+1}$ isn't

**A1**: **Three terms** correct **(may be) unsimplified.** They may be listed separately

**A1: All four terms correct (may be) unsimplified on a single line.**

**A1** **cao: All four** terms **correct simplified** with **constant** of integration on a single line. You may isw after sight of correct answer.

Leave blank

**2.** Find, giving your answer to 3 significant figures where appropriate, the value of $x$ for which

(a) $7^{2x} = 14$

**(3)**

(b) $\log_5(3x + 1) = -2$

**(2)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

| Question | Scheme | Marks |
|---|---|---|
| **2.** | (a) $2x\log 7=\log 14$ or $x\log 49=\log 14$ or $2x=\log_7 14$ | M1 |
| | $x=\dfrac{\log 14}{2\log 7}$ = awrt 0.678 | M1A1 |
| | | **(3)** |
| | (b) $3x+1=5^{-2}$ | M1 |
| | So $x=-\dfrac{8}{25}$ or $-0.32$ | A1 |
| | | **(2)** |
| | | **5 marks** |
| | **Notes** | |

**(a)**
**M1:** Uses logs and brings down *x* correctly
**M1:** Makes *x* the subject correctly. This must follow a method that did involve taking logs
**A1:** Accept awrt 0.678 (N.B. Correct answer with no working implies two previous marks)

**(b)**
**M1:** Uses powers correctly to undo log. Accept $3x+1=5^{-2}$ or equivalent such as $3x+1=0.04$
**A1**: Correct answer (Correct answer implies method mark). Accept $-0.320$

Leave blank

**3. Answer this question without the use of a calculator and show your method clearly.**

(i) Show that

$$\sqrt{45} - \frac{20}{\sqrt{5}} + \sqrt{6}\sqrt{30} = 5\sqrt{5}$$

**(2)**

(ii) Show that

$$\frac{17\sqrt{2}}{\sqrt{2} + 6} = 3\sqrt{2} - 1$$

**(3)**

| Question | Scheme | Marks |
|---|---|---|
| **3 (i)** | $\sqrt{45}-\dfrac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}$ | |
| | $=\sqrt{9}\sqrt{5}-\dfrac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}+\sqrt{6}\sqrt{6}\sqrt{5}$ $=3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$ | M1 |
| | $=5\sqrt{5}$ | A1* |
| | | **[2]** |
| **(ii)** | $\text{LHS}=\dfrac{17\sqrt{2}(\sqrt{2}-6)}{(\sqrt{2}+6)(\sqrt{2}-6)}$ | M1 |
| | $=\dfrac{17\times 2-17\times 6\sqrt{2}}{2-36}$ oe | A1 |
| | $=\dfrac{34-102\sqrt{2}}{-34}$ $=3\sqrt{2}-1$* | A1* |
| | | **[3]** |
| | | **5 marks** |
| | **Notes** | |

(i)

**M1:** Shows at least **one term** on LHS as multiple of $\sqrt{5}$ with a correct intermediate step

Look for $\sqrt{45}=\sqrt{9}\times\sqrt{5}$ *or* $\sqrt{3\times3\times5}=3\sqrt{5}$, or even $45=3\times3\times5$ or $9\times5$ followed by $\sqrt{45}=3\sqrt{5}$

$\dfrac{20}{\sqrt{5}}=\dfrac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}$ or $\dfrac{20\sqrt{5}}{5}=4\sqrt{5}$ or $\dfrac{4\times5}{\sqrt{5}}=4\sqrt{5}$

$\sqrt{6}\sqrt{30}=\sqrt{6}\sqrt{6}\sqrt{5}$ or $\sqrt{6}\sqrt{30}=\sqrt{180}=\sqrt{36\times5}=6\sqrt{5}$

or even $180=2\times2\times3\times3\times5$ followed by $\sqrt{180}=6\sqrt{5}$

**A1*:** All three terms must have the intermediate step with $3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$ followed by $5\sqrt{5}$

**Special Case: Score M1 A0 for** $\sqrt{45}-\dfrac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}=3\sqrt{5}-4\sqrt{5}+6\sqrt{5}=5\sqrt{5}$ without the intermediate steps

**Alternative method:**

**M1:** Multiplies all terms by $\sqrt{5}$ to achieve $\sqrt{45}\times\sqrt{5}-20+\sqrt{5}\sqrt{6}\sqrt{30}=5\sqrt{5}\sqrt{5}$ and simplifies any one of the above terms to 15, -20, 30 or 25 showing the intermediate step

**A1:** All terms simplified showing the intermediate step (see main scheme on how to apply) followed by 15 – 20 + 30 = 25, and minimal conclusion eg. hence true

(ii)

**M1:** Multiply numerator and denominator by $\sqrt{2}-6$ or $6-\sqrt{2}$

**A1:** Multiplies out to a correct (unsimplified) answer. For example allow $=\dfrac{17\times2-17\times6\sqrt{2}}{2-36}$

**A1**: The denominator must be simplified so $\dfrac{34-17\times6\sqrt{2}}{-34}$ or similar such as $\dfrac{17\times2-102\sqrt{2}}{-34}$ is seen before you see the given answer $3\sqrt{2}-1$. There is no need to 'split' into two separate fractions.

**Alternative method:**

**M1:** Alternatively multiplies the rhs by $\left(\sqrt{2}+6\right)\left(3\sqrt{2}-1\right)$

**A1:** Correct unsimplified rhs Accept $3\times2-6+18\sqrt{2}-\sqrt{2}$

**A1*:** Simpifies rhs to $17\sqrt{2}$ **and** gives a minimal conclusion e.g. hence true or hence $\dfrac{17\sqrt{2}}{(\sqrt{2}+6)}=3\sqrt{2}-1$

Leave blank

**4.** $$f(x) = 6x^3 - 7x^2 - 43x + 30$$

(a) Find the remainder when f($x$) is divided by

(i) $2x + 1$

(ii) $x - 3$

**(4)**

(b) Hence factorise f($x$) completely.

**(4)**

| Question | Scheme | Marks |
|---|---|---|
| **4.** | $f(x) = 6x^3 - 7x^2 - 43x + 30$ | |
| (a)(i) | Attempts $f(\pm\frac{1}{2})$ Or Use long division as far as remainder | M1 |
| | Remainder = 49 | A1 |
| (a)(ii) | Attempts $f(\pm 3)$ Or Use long division as far as remainder | M1 |
| | Remainder = 0 | A1 |
| | | **[4]** |
| (b) | $6x^3 - 7x^2 - 43x + 30 = (x-3)(6x^2 + 11x - 10)$ | M1 A1 |
| | $(6x^2 + 11x - 10) = (ax+b)(cx+d)$ where $ac = "6"$ and $bd = "-10"$ | M1 |
| | $= (x-3)(2x+5)(3x-2)$ | A1 |
| | | **[4]** |
| | | **8 marks** |
| | **Notes** | |

(a)(i)

**M1**: Attempts $f(\pm\frac{1}{2})$ or attempts long division $2x+1\overline{)6x^3 - 7x^2 - 43x + 30}$ with quotient $3x^2 + ................$ and remainder $R$, and achieves a numerical $R$

**A1**: cao Accept $f\left(-\frac{1}{2}\right) = 49$ or even just 49 for both marks

If the candidate has attempted long division they must be stating **the remainder** = 49 or $R$ = 49

(a)(ii)

**M1:** Attempts $f(\pm 3)$

Or attempts long division. See above for application of this mark. This time quotient must start $6x^2$

**A1**: cao Accept $f(3) = 0$ or even just 0 for both marks

If the candidate has attempted long division they must be stating **the remainder** = 0 or $R$ = 0

(b)

**M1**: Recognises ($x$ - 3) is factor and obtains quadratic factor with two correct terms by any correct method.

If division is used look for a minimum of the first two terms $x-3\overline{)6x^3 - 7x^2 - 43x + 30}$ with quotient $6x^2 \pm 11x..............$ and first line $6x^2 - 18x$

If factorisation is used look for correct first and last terms $6x^3 - 7x^2 - 43x + 30 = (x-3)(6x^2.....x \pm 10)$

**A1**: Correct quadratic

**M1**: Attempt to factorise their quadratic

**A1**: **cao** – need all three factors together. Do not penalise candidates who go on to state the roots.

Allow $6(x-3)\left(x+\frac{5}{2}\right)\left(x-\frac{2}{3}\right)$ following $(x-3)(6x^2 + 11x - 10)$

Note: There may be candidates who just write down the factors from their GC. The question did state hence so we need to be careful here and see some correct work.

$6x^3 - 7x^2 - 43x + 30 = (x-3)\left(x-\frac{2}{3}\right)\left(x+\frac{5}{2}\right)$ presumably from the roots is M0A0M0A0

$6x^3 - 7x^2 - 43x + 30 = 6(x-3)\left(x-\frac{2}{3}\right)\left(x+\frac{5}{2}\right)$ with no working can score M1A0M1A0

Leave blank

**5.** (a) Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of

$$\left(3 - \frac{ax}{2}\right)^5$$

where $a$ is a positive constant. Give each term in its simplest form.

**(4)**

Given that, in the expansion, the coefficient of $x$ is equal to the coefficient of $x^3$,

(b) find the exact value of $a$ in its simplest form.

**(3)**

| Question | Scheme | Marks |
| --- | --- | --- |
| **5.** | (a) $\left(3-\frac{ax}{2}\right)^5=3^5+\binom{5}{1}3^4.\left(-\frac{ax}{2}\right)+\binom{5}{2}3^3.\left(-\frac{ax}{2}\right)^2+\binom{5}{3}3^2.\left(-\frac{ax}{2}\right)^3...$ | M1 |
| | $=243,-\frac{405}{2}ax+\frac{135}{2}a^2x^2-\frac{45}{4}a^3x^3...$ | B1, A1, A1 |
| | | **[4]** |
| | (b) $\frac{405}{2}a=\frac{45}{4}a^3$ | M1 |
| | $a^2=\frac{810}{45}=18$ or equivalent | A1 |
| | $a=3\sqrt{2}$ | A1 |
| | | **[3]** |
| | | **7 marks** |
| | **Notes** | |

**(a)**

**M1**: The **method** mark is awarded for an attempt at Binomial to get the second and/or third and/or fourth term.

You need to see the **correct** binomial coefficient combined with correct power of *x*. e.g. $\binom{5}{2}..x^2$

Condone bracket errors. Accept any notation for ${}^5C_1$, ${}^5C_2$ and ${}^5C_3$, e.g. $\binom{5}{1}$, $\binom{5}{2}$ and $\binom{5}{3}$

or 5, 10 and 10 from Pascal's triangle.

The mark can be applied in the same way if $3^5$ is taken out as a factor.

**B1:** For the first term of 243. (writing just $3^5$ is **B0 ).**

**A1:** is cao and is for **two correct and simplified terms** from $-\frac{405}{2}ax$, $+\frac{135}{2}a^2x^2$ and $-\frac{45}{4}a^3x^3...$

Allow two correct from $-\frac{405}{2}(ax)$, $+\frac{135}{2}(ax)^2$ and $-\frac{45}{4}(ax)^3...$ with the brackets.

Allow decimals. Allow lists

**A1:** is c.a.o and is for **all** of the terms correct and simplified.

Allow $+\frac{135}{2}(ax)^2$ and $-\frac{45}{4}(ax)^3...$ (ignore $x^4$ terms)

Allow decimal equivalents $-202.5ax+67.5a^2x^2-11.25a^3x^3...$ Allow listing.

**(b)**

**M1:** Puts their coefficient of *x* equal to their coefficient of $x^3$ (There should be no *x* terms)

**A1:** This is cao for obtaining $a^2$ or *a* correctly (may be unsimplified)

**A1:** This is cao for $a=3\sqrt{2}$ Condone $a=\pm3\sqrt{2}$

We will condone all 3 marks to be scored in (b) from a solution in (a) where all signs are +ve

$=243+\frac{405}{2}ax+\frac{135}{2}a^2x^2+\frac{45}{4}a^3x^3...$

Leave blank

**6.** A sequence is defined by

$$u_1 = 36$$

$$u_{n+1} = \frac{2}{3}u_n, \qquad n \geqslant 1$$

(a) Find the exact simplified values of $u_2$, $u_3$ and $u_4$

**(2)**

(b) Write down the common ratio of the sequence.

**(1)**

(c) Find, giving your answer to 4 significant figures, the value of $u_{11}$

**(2)**

(d) Find the exact value of $\sum_{i=1}^{6} u_i$

**(2)**

(e) Find the value of $\sum_{i=1}^{\infty} u_i$

**(2)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

| Question | Scheme | Marks |
|---|---|---|
| **6.** (a) | $u_2 = 24,\ u_3 = 16$ and $u_4 = \frac{32}{3}$ | M1, A1 **[2]** |
| (b) | $r = \frac{2}{3}$ | B1 **[1]** |
| (c) | $u_{11} = ar^{10} = 36 \times (r)^{10}$ . | M1 |
| | $u_{11} = ar^{10} = 36 \times \left(\frac{2}{3}\right)^{10} = \left(\frac{4096}{6561}\right)$ | |
| | $= 0.6243$ | A1 **[2]** |
| (d) | $\sum_{i=1}^{6} u_i = \frac{36(1-(\frac{2}{3})^6)}{1-\frac{2}{3}}$ or $\sum_{i=1}^{6} u_i = 36+24+16+\frac{32}{3}+u_5+u_6$ | M1 |
| | $= 98\frac{14}{27}$ | A1cao **[2]** |
| (e) | $\sum_{i=1}^{\infty} u_i = \frac{36}{1-\frac{2}{3}} = 108$ | M1 A1 **[2]** |
| | | **9 marks** |

**Notes**

**(a)**

**M1:** Attempt to use formula correctly at least twice. It may be seen for example in $u_3$ and $u_4$

**A1**: All three correct exact simplified answers. Allow $10.\dot{6}$

**(b)**

**B1:** Accept $\frac{2}{3}$ or equivalent such as $\frac{24}{36}$ Allow awrt 0.667

**(c)**

**M1**: Uses $u_{11} = ar^{10} = 36 \times (r)^{10}$ with their $r$

**A1**: Accept awrt 0.6243 or $\frac{4096}{6561}$

**(d)**

**M1**: Uses correct sum formula with $a$ = 36 and their $r$ **or** alternatively for adding their first six terms.
FYI Sight of 36, 24, 16, 10.7, 7.1, 4.7 followed by 98.5 implies this mark. (You may only see the first 4 terms in part a)

**A1:** Obtains $= 98\frac{14}{27}$ (must be exact). For information $\frac{2660}{27} = 98\frac{14}{27}$ Allow $98.\dot{5}1\dot{8}$

**(e)**

**M1**: Uses correct sum to infinity formula with $a$ = 36 and either $r = \frac{2}{3}$ or their $r$ as long as $|r| < 1$

**A1**: Obtains 108 (must be exact)

Leave blank

**7.** (a) Sketch the graph of $y = 3^{x-2}$, $x \in \mathbb{R}$
Give the exact values for the coordinates of the point where your graph crosses the $y$-axis.

**(2)**

The table below gives corresponding values of $x$ and $y$, for $y = 3^{x-2}$
The values of $y$ are rounded to 3 decimal places where necessary.

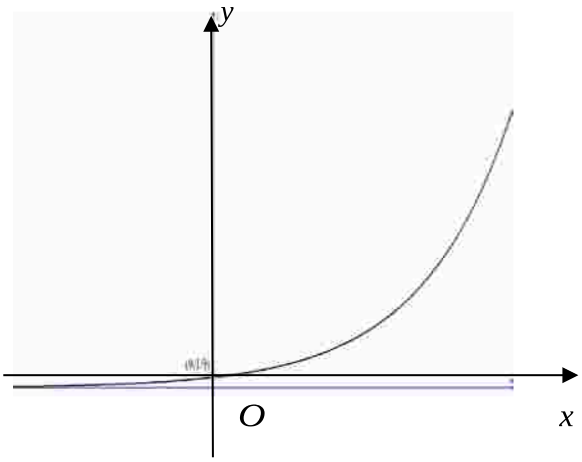
| $x$ | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 0.192 | 0.333 | 0.577 | 1 | 1.732 | 3 |

(b) Use the trapezium rule with all the values of $y$ from the table to find an approximate value for

$$\int_{0.5}^{3} 3^{x-2}\, dx$$

Give your answer to 2 decimal places.

**(4)**

| Question | Scheme | Marks |
|---|---|---|
| **7.** (a) |  Shape and position correct; (0, 1/9) correct | B1 B1 **[2]** |
| (b) | State $h$ = 0.5, or use of $\frac{1}{2}\times 0.5$ ; | B1 aef |
| | $\underline{\{0.192+3+2(\ 0.333+0.577+1+1.732\ )\}}$ For structure of $\{..............\}$ ; | M1A1 |
| | $\frac{1}{2}\times 0.5\times\underline{\{10.476\}}$ = awrt 2.62 | A1 **[4]** |
| | | **6 marks** |

**Notes**

(a)

**B1:** Curve just in quadrant one and two with a gradient that is approaching zero at the lhs and increases as $x$ increases. Curves that **just** cross the $y$ axis into quadrant 2 may be penalised. As a rule of thumb expect it reach at least as far as $x$ = -1.

**B1:** The point (0, 1/9) lies on the curve.
Accept 1/9 marked on the y axis. Accept a statement when $x$ = 0, $y$ = 1/9
Do not accept $3^{-2}$ or 0.11. Condone $\left(0, 0.\dot{1}\right)$

(b)

**B1:** For using $\frac{1}{2}\times 0.5$ or $h$ = 0.5 or equivalent such as (1-0.5)

**M1:** Scored for the sight of the correct structure for the outer bracket.
You need to see the first $y$ value plus the last $y$ value plus 2 times a bracket containing the sum of the remaining $y$ values with no additional values.
If the only mistake is a copying error or is to omit one of the remaining y values then this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however).
$\frac{1}{2}\times 0.5\times 0.192+3+2(\ 0.333+0.577+1+1.732\ )$ or awrt 8.08 implies this mark

**A1:** For $\{0.192+3+2(\ 0.333+0.577+1+1.732\ )\}$ **or** $\{(0.192+3)+2(\ 0.333+0.577+1+1.732\ )\}$ **oe**

**A1:** For answer which rounds to 2.62. Correct answer implies all 4 marks

NB: Separate trapezia may be used: B1 for 0.5, M1 for 1/2 $h(a+b)$ used 4 or 5 times followed by A1 (if it is all correct ) and A1 as before.

Leave blank

**8.**

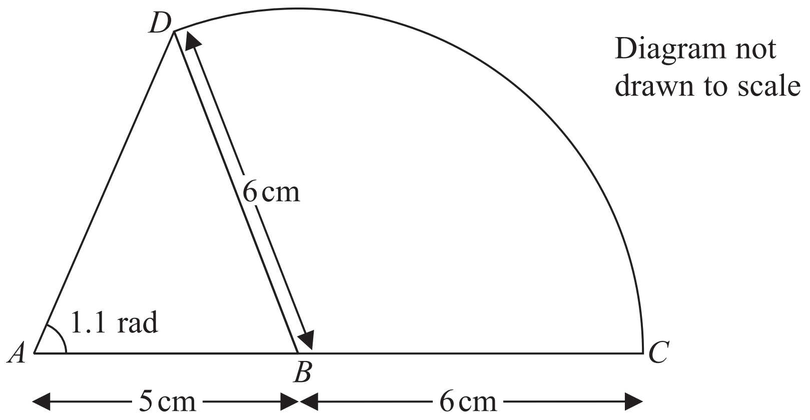



**Figure 1**

The compound shape *ABCDA*, shown in Figure 1, consists of a triangle *ABD* joined along its edge *BD* to a sector *DBC* of a circle with centre *B* and radius 6 cm.
The points *A*, *B* and *C* lie on a straight line with *AB* = 5 cm and *BC* = 6 cm.
Angle *DAB* = 1.1 radians.

(a) Show that angle *ABD* = 1.20 radians to 3 significant figures.

**(4)**

(b) Find the area of the compound shape, giving your answer to 3 significant figures.

**(4)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

| Question | Scheme | Marks |
|---|---|---|
| **8.** (a) | $\frac{\sin D}{5} = \frac{\sin 1.1}{6}$ | M1 |
| | $\sin D = 0.74267$ so $D = 0.84$ | M1, A1 |
| | $B = \pi - (1.1 + 0.84) = 1.20$ * | A1* |
| | | **[4]** |
| (b) | Uses angle $DBC$ = $\pi - 1.2 = \text{awrt } 1.94$ | B1 |
| | Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times '1.94'$ $(=34.9)$ or Area of triangle ABD $= \frac{1}{2}\times 5\times 6\times \sin 1.2$ $(=14.0)$ | M1 |
| | Total area is $\frac{1}{2}\times 6^2 \times '1.94' + \frac{1}{2}\times 5\times 6\times \sin 1.2$ | dM1 |
| | $= 48.9\text{cm}^2$ | A1 |
| | | **[4]** |
| | | **8 marks** |
| | **Notes** | |

(a)

**M1:** Uses sine rule – the sides and angles must be in the correct positions

**M1:** Makes sin $D$ the subject and uses inverse sine (in degrees or radians)

**A1:** Accept awrt 0.84 or in degrees accept answers truncating $47.9..°$ or rounding to $48.0°$

**A1*:** Answer is printed so should see either $\pi - (1.1 + \text{awrt } 0.84)$ **or** $\pi - 1.1 - \text{awrt } 0.84$ before you see 1.20

If the question was changed to degrees look for accuracy to one decimal places throughout the question for the final A1 mark. So $1.1 \text{ rads} = \text{awrt } 63.0°$ and $(180 - \text{awrt } 63.0 - \text{awrt } 48.0) = \text{awrt } 69.0..\times\frac{\pi}{180} = 1.20$

**There are many ways to attempt this question: For example**

**M1:** Uses cosine rule $6^2 = 5^2 + x^2 - 2\times 5\times x\cos 1.1$ (where $x$ = $AD$) and attempts to solve to find $x$. For information $x \approx 6.29$

**M1:** Uses cosine rule $\cos B = \frac{6^2 + 5^2 - \text{their } '6.29'^2}{2\times 6\times 5}$

**A1:** Achieves $\cos B = \frac{6^2 + 5^2 - (\text{awrt } 6.29)^2}{2\times 6\times 5}$

**A1:** 1.20*

(b)

**B1:** Uses angles on a straight line formula. Score for $\pi - 1.2$ or allow awrt 1.94 as evidence.
If converted to degrees accept awrt $111.2°$ as evidence

**M1:** Uses a correct area formula for the sector **or** a correct area formula for the triangle.
You may follow through on an incorrectly found angle $DBC$
For example $2\pi - 1.2$ is acceptable but $180° - 1.2$ is not as it is using mixed units.
If the angle was found in degrees, the correct formula must be used.
For the triangle the correct combinations of sides and angle should be attempted.
e.g. You may see the area of triangle $ABD = \frac{1}{2}5\times(\text{their } 6.29)\times \sin 1.1$ or $\frac{1}{2}6\times(\text{their } 6.29)\times \sin(\text{their } ADB)$

**dM1:** Adds together a correct area formula for the sector **and** a correct area formula for the triangle.
You may follow through on an incorrectly found angle $DBC$ or $ADB$

**A1:** Accept awrt 48.9 (do not need units)

Leave blank

**9.** In a large theatre there are 20 rows of seats.

The number of seats in the first row is $a$, where $a$ is a constant.

In the second row the number of seats is $(a + d)$, where $d$ is a constant. In the third row the number of seats is $(a + 2d)$, and on each subsequent row there are $d$ more seats than on the previous row. The number of seats in each row forms an arithmetic sequence.

The **total** number of seats in the first 10 rows is 395

(a) Use this information to show that $10a + 45d = 395$

**(1)**

The **total** number of seats in the first 18 rows is 927

(b) Use this information to write down a second simplified equation relating $a$ and $d$.

**(2)**

(c) Solve these equations to find the value of $a$ and the value of $d$.

**(3)**

(d) Find the number of seats in the 20th row of the theatre.

**(2)**

| Question | Scheme | Marks |
|---|---|---|
| **9.** (a) | Uses $\frac{n}{2}(2\times a+(n-1)d)$ with $n$ = 10 to give 10$a$ + 45$d$ = 395 * | B1* **[1]** |
| (b) | Uses $\frac{n}{2}(2\times a+(n-1)d)$ with $n$ = 18 and $S$=927 | M1 |
| | Obtain $18a+153d=927$ or $2a+17d=103$ | A1 **[2]** |
| (c) | Solves simultaneous equations to find either $a$ or $d$ | M1 |
| | $a$ = 26 and $d$ = 3 | A1, A1 **[3]** |
| (d) | Uses $a+(n-1)d$ with $n$ = 20 | M1 |
| | = 83 | A1 **[2]** |
| | | **8 marks** |

**Notes**

Mark the whole question as one.

(a)

**B1:** Use the correct formula for the sum of an AP with $n$ = 10, $S$ =395 AND proceeds to the given answer.
It is acceptable for the 395 to appear just at the answer stage.
Could use formula with $n$ = 10, $S$ =395 and $l$ = $a$+9$d$
It is OK to list but minimum would be $a+a+d+a+2d.....+a+9d=395$

(b)

**M1:** Obtain a correct second equation e.g. $927=\frac{18}{2}(2\times a+(18-1)d)$ or equivalent. Condone a slip on the 927.
Note that if the candidate reads 927 as 972 they will only have access to M marks in this question. This is due to the fact that with this number, the values of $a$ and $d$ would be fractional and this could not occur as they must be integers

**A1:** A simplified equation so accept either $18a+153d=927$ or $2a+17d=103$
Sight of one of these scores both marks.

(c)

**M1:** Solves simultaneous equations to find either $a$ or $d$.
Do not concern yourself with the process as calculators are allowed on this paper so score if they proceed to either $a$ and/or $d$

**A1:** Obtains correct $a$ or $d$ (just one)

**A1:** Obtains correct $a$ and $d$ (both)

(d)

**M1:** Uses correct formula for $n$ th term using their $a$ and $d$ but with $n$ = 20. Look for $'a'+19'd'$

**A1:** Correct answer

Leave blank

**10.** (a) Given that

$$8\tan x = -3\cos x$$

show that

$$3\sin^2 x - 8\sin x - 3 = 0$$

**(3)**

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$8\tan 2\theta = -3\cos 2\theta$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

| Question | Scheme | Marks |
|---|---|---|
| **10. (a)** | Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$ | M1 |
| | Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$ | M1 |
| | So $8\sin x = -3 + 3\sin^2 x$ and $3\sin^2 x - 8\sin x - 3 = 0$* | A1 * **[3]** |
| **(b)** | Solves the three term quadratic "$3\sin^2 x - 8\sin x - 3 = 0$" | M1 |
| | So $(\sin x) = -\frac{1}{3}$ (or 3) | A1 |
| | $(2\theta) = -19.47$ or $199.47$ or $340.53$ | dM1 |
| | $\theta = 99.7, 170.3, 279.7$ or $350.3$ | A1, A1 **[5]** |
| | | **8 marks** |
| | **Notes** | |

**(a)**

**M1:** Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$ or equivalent

**M1:** Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$

May also be seen $8\tan x = -3\cos x \Rightarrow 8\tan x = -3\sqrt{1-\sin^2 x}$

**A1:** Proceeds to given answer with no errors.
(This is a given answer so do not tolerate bracketing or notation errors such as $\cos^2 x$ written as $\cos x^2$ or sin $x$ appearing as sin )

**(b)**

**M1:** Solving quadratic by usual methods (see notes).
If the formula is quoted it must be correct but allow solutions from calculators.

**A1:** You only need to see $-\frac{1}{3}$ .
This is an intermediate answer so condone $-\frac{1}{3}$ appearing as awrt $-0.333$
Condone errors on the lhs so accept for this mark $x/a/\theta = -\frac{1}{3}, \sin x = -\frac{1}{3}, \sin 2x = -\frac{1}{3}$

**dM1:** Uses inverse sine to obtain an answer for $2\theta$ .
This may appear as answers for $x$. The only stipulation is that $\text{invsin } k, |k| < 1$
It is dependent upon seeing a correct method of solving their quadratic
Accept answers rounding to 1 dp for $2\theta$ e.g. awrt $-19.5$ or 199.5 or 340.5.
It may also be implied by a correct answer for $\theta$ e.g. awrt $-9.7$ or 99.7 or 170.2

**A1:** Two correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or $350.3$

**A1:** All four correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or $350.3$

Leave blank

**11.** The equation $5x^2 + 6 = k(13x^2 - 12x)$, where $k$ is a constant, has two distinct real roots.

(a) Show that $k$ satisfies the inequality

$$6k^2 + 13k - 5 > 0$$

**(4)**

(b) Find the set of possible values for $k$.

**(4)**

| Question | Scheme | Marks |
|---|---|---|
| **11.** | | |
| **(a)** | $(13k-5)x^2-12kx-6=0$ or $(5-13k)x^2+12kx+6=0$ | B1 |
| | Uses $b^2-4ac$ with $a=\pm13k\pm5$, $b=\pm12k$ and $c=\pm6$ | M1 |
| | And states $b^2-4ac>0$ with $a=\pm(13k-5)$, $b=\pm12k$ and $c=\pm6$ | A1ft |
| | Proceeds correctly with no errors to $6k^2+13k-5>0$ * | A1* |
| | | **[4]** |
| **(b)** | Attempts to solve $6k^2+13k-5=0$ to give $k=$ | M1 |
| | $\Rightarrow$ Critical values, $k=\frac{1}{3},\ \frac{-5}{2}$ | A1 |
| | $6k^2+13k-5>0$ gives $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$ | M1 A1 |
| | | **[4]** |
| | | **8 marks** |
| | **Notes** | |

**(a)**

**B1:** Expresses equation as three term quadratic in $x$. $(13k-5)x^2-12kx-6=0$ **oe.**
The equals 0 may be implied by subsequent work. Allow $(5-13k)x^2+12kx+6=0$
Allow an equation of the form $13kx^2-5x^2-12kx-6(=0)$ as long as it is followed by $a=13k-5$........

**M1:** Attempts $b^2-4ac$ with $a=\pm13k\pm5$, $b=\pm12k$ and $c=\pm6$
or uses quadratic formula to solve equation
or uses the discriminant on two sides of an equation or inequation e.g. $b^2=4ac$ or $b^2<4ac$

**A1:** Uses the discriminant condition, eg $b^2-4ac>0$ or $b^2>4ac$ with $a=\pm13k\pm5$, $b=\pm12k$ and $c=\pm6$

**A1***: Proceeds to given answer with no errors. AG. Condone missing = 0 on the equation
Condone a solution where $(13k-5)x^2-12kx-6=0$ is followed by $144k^2+24(13k-5)>0$
Watch for $a=13k-5$, $b=+12k$ and $c=-6$ which does give the correct inequality but loses the final A1*

**(b)**

**M1:** Uses factorisation, formula, or completion of square method to find two values for $k$,
or finds two **correct** answers with no obvious method for **their** three term quadratic

**A1:** Obtains $k=\frac{1}{3},\ \frac{-5}{2}$ accept -2.5, 0.333 (awrt) here but need exact answer for final A1.
Also condone $x=\frac{1}{3},\ \frac{-5}{2}$ for this mark .

**M1:** Chooses outside region **(** $k<$ Their Lower Limit $k>$ Their Upper Limit ) for appropriate 3 term quadratic inequality . Do not award simply for diagram or table.
Award if final answer is $k\geqslant\frac{1}{3}$ (or) $k\leqslant\frac{-5}{2}$ or $\frac{1}{3}<k<\frac{-5}{2}$
Condone $x$ appearing instead of $k$

**A1:** $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$ $\left(k\neq\frac{5}{13}\right)$ must be exact and must be $k$.
Must be two separate inequalities and not be $k>\frac{1}{3}$ **and** $k<\frac{-5}{2}$

Leave blank

**12.**

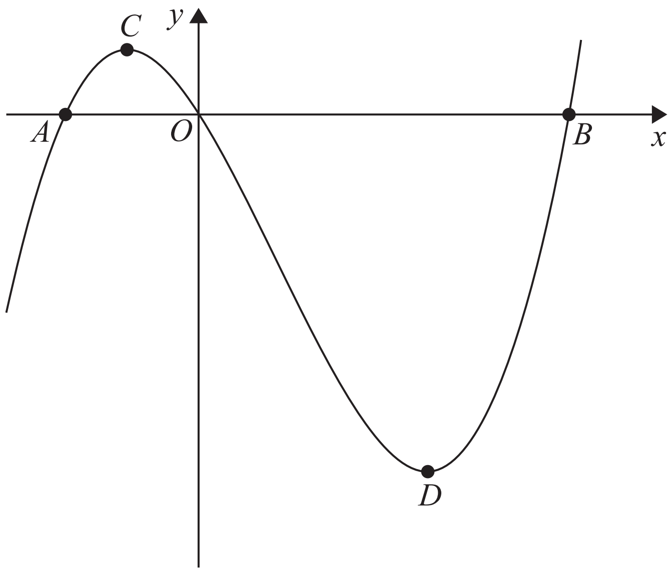



Diagram not drawn to scale

**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{x^3 - 9x^2 - 81x}{27}$$

The curve crosses the $x$-axis at the point $A$, the point $B$ and the origin $O$.
The curve has a maximum turning point at $C$ and a minimum turning point at $D$.

(a) Use algebra to find exact values for the $x$ coordinates of the points $A$ and $B$.
**(4)**

(b) Use calculus to find the coordinates of the points $C$ and $D$.
**(6)**

The graph of $y = f(x + a)$, where $a$ is a constant, has its minimum turning point on the $y$-axis.

(c) Write down the value of $a$.
**(1)**

| Question | Scheme | Marks |
|---|---|---|
| **12** (a) | $\mathrm{f}(x)=\dfrac{x^3-9x^2-81x}{27}=0 \Rightarrow x(x^2-9x-81)=0$ | M1 |
| | $x=\dfrac{9\pm\sqrt{81+324}}{2}$ | dM1 |
| | $x=\dfrac{9\pm\sqrt{405}}{2}$ or $x=\dfrac{9\pm 9\sqrt{5}}{2}$ | A1 A1 **[4]** |
| (b) | Differentiates (usual rules), correctly and sets = 0 $\mathrm{f}'(x)=3x^2-18x-81=0$ | M1, A1 |
| | Solves $\mathrm{f}'(x)=0$ (or multiple) $\Rightarrow$ $x$ = 9 and -3 | dM1 A1 |
| | Substitutes one of their values for $x$ into f($x$) | ddM1 |
| | $x$ = 9 $y$ = -27 and $x$ =-3 $y$ = 5 | A1 **[6]** |
| (c) | $a$ = 9 | B1 **[1]** |
| | | **11 marks** |

**Notes**

**(a)**

**M1:** Attempts to solve f($x$) = 0, by taking out a factor of (/cancelling by ) $x$ and obtaining a quadratic factor.
Allow on $x\left(\dfrac{x^2}{27}-\dfrac{9x}{27}-\dfrac{81}{27}\right)=0$ or just the numerator $x(x^2-9x-81)=0$
This is implied by sight of $x^2-9x-81=0$

**dM1:** Uses formula or completion of square method to find at least one value for $x$ , for **their** three term quadratic. Factorisation is M0. Note that their 3 term quadratic equation may be $\dfrac{1}{27}x^2-\dfrac{1}{3}x-3=0$

**A1:** One correct solution – need not be fully simplified. So allow $x=\dfrac{9+\sqrt{405}}{2}$ but not $x=\dfrac{9+\sqrt{81+324}}{2}$

**A1**: Two correct solutions – need not be simplified or attributed correctly to $A$ or $B$.

**Special case:** If a candidate takes out a common factor of $x$ and uses a calculator to write down the exact surd answers to the quadratic they have used (a limited) amount of algebra. Decimals would not be awarded for this SC. We will therefore score this SC M1 M1 A0 A0 for 2 out of 4. $x(x^2-9x-81)=0\Rightarrow x=\dfrac{9\pm 9\sqrt{5}}{2}$ Just writing down the answers with no working scores 0 marks

**(b)**

**M1:** Differentiates f($x$) to a 3 term quadratic
You may see confusion over the 27 but score for $\mathrm{f}'(x)$ being a 3 term quadratic

**A1**: Differentiates correctly and sets correct derivative = 0
$3x^2-18x-81=0$ or any multiple thereof. For example it may be common to see $\dfrac{3x^2}{27}-\dfrac{18x}{27}-\dfrac{81}{27}=0$

**dM1:** Solves quadratic to give two solutions. It is dependent upon the previous M.
Allow any appropriate method including the use of a calculator.
Condone $\dfrac{x^2}{9}-\dfrac{2x}{3}-3=0\Rightarrow(x-9)(x+3)=0$

**A1 :** Gives **both** 9 and $-3$

**ddM1:** Substitute at least one of their values of $x$ (obtained from a solution of $\mathrm{f}'(x)=0$) into f($x$) to give $y$ = .

**A1:** Gives both $-27$ and 5 (arising from $x$ values of 9 and $-3$) (Do not require coordinates).
Again they do not need to be attributed correctly to $C$ or $D$

**(c)**

**B1:** For $a$ = 9 only (no ft)

Leave blank

**13.** The circle $C$ has centre $A(1, -3)$ and passes through the point $P(8, -2)$.

(a) Find an equation for the circle $C$.

**(4)**

The line $l_1$ is the tangent to $C$ at the point $P$.

(b) Find an equation for $l_1$, giving your answer in the form $y = mx + c$

**(4)**

The line $l_2$, with equation $y = x + 6$, is the tangent to $C$ at the point $Q$.

(c) Find the coordinates of the point $Q$.

**(5)**

| Question | Scheme | | Marks |
|---|---|---|---|
| **13** (a) | See $(x\pm1)^2+(y\pm3)^2=r^2$ | Or see $x^2+y^2\pm2x\pm6y+c=0$ | M1 |
| | **Attempt** $\sqrt{(8-1)^2+(-2-(-3))^2}$ **or** $(8-1)^2+(-2-(-3))^2$ | Substitute $(8,-2)$ into equation | M1 |
| | $(x-1)^2+(y+3)^2,=50$ | $x^2+y^2-2x+6y-40=0$ | A1, A1 **[4]** |
| (b) | Gradient of $AP$ = $\frac{1}{7}$ | | B1 |
| | So gradient of tangent is $-7$ | | M1 |
| | Equation of tangent is $(y+2)=-7(x-8)$ | | dM1 |
| | $y=-7x+54$ or $m$ = -7, $c$ =54 | | A1 **[4]** |
| | Way 1 | Way 2 | |
| (c) | $y=x+6$ meets circle when $(x-1)^2+(x+9)^2=50$ or when $(y-7)^2+(y+3)^2=50$ | As tangent has gradient 1 $AQ$ has gradient -1 and $\frac{y-(-3)}{x-1}=-1$ | M1 |
| | i.e. $2x^2+16x+32=0$ **or** when $2y^2-8y+8=0$ | $y+x=-2$ | A1 |
| | Solve to give $x$ or $y$ = | Solve $y+x=-2$ with $y=x+6$ or alternatively solve $y+x=-2$ with the equation of the circle to give $x$ or $y$ = | M1 |
| | Substitute to give $y$ = (**or** $x$ = ) | | dM1 |
| | $(-4, 2)$ only | | A1 **[5]** |
| | | | **13 marks** |
| | Notes | | |

(a)

**M1** : Scored for centre at $(1,-3)\Rightarrow(x\pm1)^2+(y\pm3)^2=...$ or $x^2+y^2\pm2x\pm6y+...=0$

**M1:** Scored for an attempt at finding the radius or the radius $^2$ (see scheme).
It need not be in the equation It can be <u>implied</u> by $\sqrt{50}$ *or* $5\sqrt{2}$ or 50
If the form $x^2+y^2\pm2x\pm6y+c=0$ is used it is for substituting $(8,-2)$ into the equation

**A1:** LHS or RHS correct $(x-1)^2+(y+3)^2,=...$ or $(x\pm a)^2+(y\pm b)^2,=50$ $x^2+y^2-2x+6y...=0$

A**1:** Correct equation. Accept $(x-1)^2+(y+3)^2=50$ or $x^2+y^2-2x+6y-40=0$ or $x^2+y^2-2x+6y=40$

(b)

**B1 :** Obtain 1/7 . Implied by use of – 7 in their tangent

**M1:** Uses negative reciprocal

**dM1**: Linear equation through point (8, -2) with their negative reciprocal gradient

**A1**: cao

(c)

**M1**: Eliminates $x$ or $y$ from two relevant equations, that is whose intersection is $Q$.

**A1:** Correct quadratic in $x$ or in $y$

**M1:** Solves (with usual rules) to give first variable. The first M must have been scored

**dM1:** Substitute in either (relevant) equation to give second coordinate, dependent upon both previous M's

**A1**: Correct answer accept $x=-4$ , $y=2$ . Withhold this if two answers given

Leave blank

**14.**

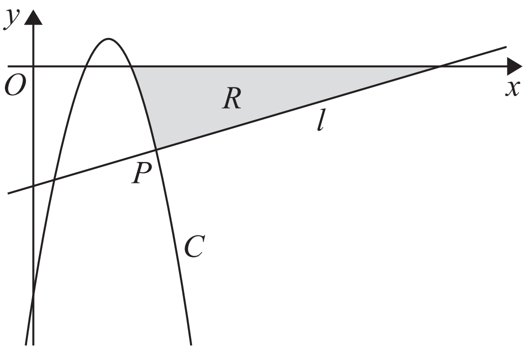



**Figure 3**

Figure 3 shows a sketch of the curve $C$ with equation $y = -x^2 + 6x - 8$. The normal to $C$ at the point $P(5, -3)$ is the line $l$, which is also shown in Figure 3.

(a) Find an equation for $l$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(5)**

The finite region $R$, shown shaded in Figure 3, is bounded below by the line $l$ and the curve $C$, and is bounded above by the $x$-axis.

(b) Find the exact value of the area of $R$.

**(6)**

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

| Question | Scheme | Marks |
|---|---|---|
| **14.** | $y = -x^2 + 6x - 8$ | |
| (a) | $\frac{dy}{dx} = -2x + 6$ and substitutes $x = 5$ to give gradient = $m$ = $-4$ | M1 A1 |
| | Normal has gradient $\frac{-1}{m} = \left(\frac{1}{4}\right)$ | M1 |
| | Equation of normal is $(y+3) = "\frac{1}{4}"(x-5)$ so $x - 4y - 17 = 0$ | dM1 A1 **[5]** |
| (b) | $\int -x^2 + 6x - 8\,dx = -\frac{x^3}{3} + 6\frac{x^2}{2} - 8x$ | M1 |
| | The Line meets the $x$-axis at 17 | B1 |
| | The Curve meets the $x$-axis at 4 | B1 |
| | Uses correct limits correctly for their integral<br>i.e. $\left[-\frac{x^3}{3} + 6\frac{x^2}{2} - 8x\right]_4^5 = -\frac{5^3}{3} + 6\frac{5^2}{2} - 8\times 5 - \left(-\frac{4^3}{3} + 6\frac{4^2}{2} - 8\times 4\right)$ | M1 |
| | Finds area above line, using area of triangle or integration $= \frac{1}{2}\times 3\times("17" - 5)$ | M1 |
| | Area of $R$ = $18 + 1\frac{1}{3} = 19\frac{1}{3}$ | A1 |
| | | **[6]** |
| | | **11 marks** |

| | **Notes** | |
|---|---|---|

(a)

**M1:** Differentiates to give $\frac{dy}{dx} = \pm 2x \pm 6$ and substitutes $x = 5$

**A1:** Obtains answer $-4$.

**M1**: Uses negative reciprocal of their numerical $\frac{dy}{dx}$ (follow through). M1 must have been awarded

**dM1:** Linear equation through point $(5,-3)$ with their **changed** gradient.

Dependent upon the first M, so you would allow for $(y+3) = 4(x-5)$ following an answer of $-4$

**A1:** cao accept $k(x-4y-17)=0$ where $k$ is a positive or negative integer

Candidates who work with a gradient of $\pm 2$ from their $\frac{dy}{dx} = \pm 2x \pm 6$ will score 0 marks in this part of the question.

(b)

**M1**: Integrates a quadratic expression correctly.
If they integrate (line -curve) follow through on their new quadratic
The terms including the coefficients must be correct for their quadratic

**B1:** Obtains 17 for the point where the line meets the $x$ - axis

**B1:** Finds that the curve meets the $x$ axis at 4.
You may score this for $y = 0 \Rightarrow x = 2, 4$ ignoring even an incorrect 2
Also allow for a limit in the integral.
You may even score this if 4 appears (in the correct place) on the diagram

**M1:** Uses the limits 4 and 5 in their integrated function

If a candidate writes down $\int_4^5 \pm(-x^2+6x-8)\,dx = \pm\frac{4}{3}$ (from a GC) we will allow them to score this mark.

**M1:** Finds appropriate area above the line for their attempted integral, so

if they integrate just curve look for area of triangle $= \frac{1}{2} \times 3 \times$ "*their* $17-5$" or $\int_5^{"17"} "\left(\frac{1}{4}x - \frac{17}{4}\right)" dx = \left[\frac{1}{8}x^2 - \frac{17}{4}x\right]_5^{"17"}$

if they integrate (line - curve) from 4 to 5, then the triangle would be $= \frac{1}{2} \times$ *their* "$\frac{13}{4}$" $\times$ "*their* $17-4$"

**A1**: correct work leading to $19\frac{1}{3}$

**A candidate who does the integration on a GC can potentially score M0 B1 B1 M1 M1 A0**

Leave blank

**15.**

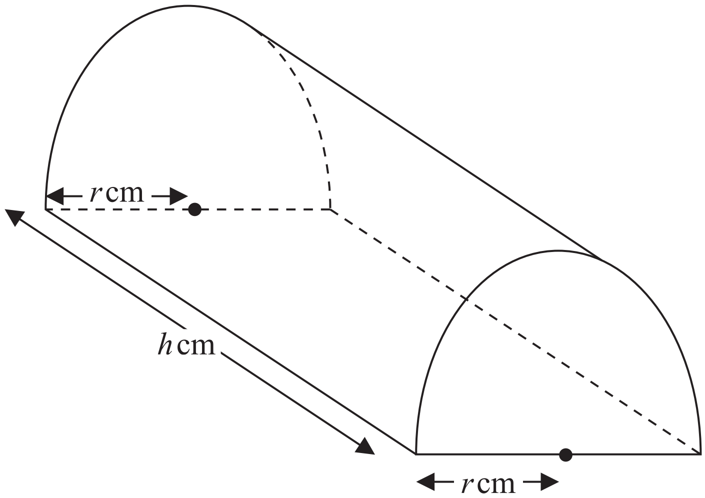



**Figure 4**

Figure 4 shows a solid wooden block. The block is a right prism with length $h$ cm. The cross-section of the block is a semi-circle with radius $r$ cm.

The total surface area of the block, including the curved surface, the two semi-circular ends and the rectangular base, is $200\,\text{cm}^2$

(a) Show that the volume $V\,\text{cm}^3$ of the block is given by

$$V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$$

**(5)**

(b) Use calculus to find the maximum value of $V$. Give your answer to the nearest $\text{cm}^3$.

**(6)**

(c) Justify, by further differentiation, that the value of $V$ that you have found is a maximum.

**(2)**

| Question | Scheme | Marks |
|---|---|---|
| **15** (a) | $200 = \pi r^2 + \pi rh + 2rh$ | M1 A1 |
| | $(h =)\dfrac{200 - \pi r^2}{\pi r + 2r}$ or $(rh =)\dfrac{200 - \pi r^2}{\pi + 2}$ | dM1 |
| | $V = \dfrac{1}{2}\pi r^2 h =$ | M1 |
| | $\Rightarrow V = \dfrac{\pi r^2(200 - \pi r^2)}{2(2r + \pi r)} = \dfrac{\pi r(200 - \pi r^2)}{4 + 2\pi}$ * | A1 cso * |
| | | **[5]** |
| (b) | $\dfrac{dV}{dr} = \dfrac{200\pi - 3\pi^2 r^2}{4 + 2\pi}$ Accept awrt $\dfrac{dV}{dr} = 61.1 - 2.9r^2$ | M1 A1 |
| | $\dfrac{200\pi - 3\pi^2 r^2}{4 + 2\pi} = 0$ or $200\pi - 3\pi^2 r^2 = 0$ leading to $r^2 =$ | dM1 |
| | $r = \sqrt{\dfrac{200}{3\pi}}$ or answers which round to 4.6 | dM1 A1 |
| | $V = 188$ | B1 |
| | | **[6]** |
| (c) | $\dfrac{d^2V}{dr^2} = \dfrac{-6\pi^2 r}{4 + 2\pi}$, and sign considered Accept $\dfrac{d^2V}{dr^2} =$ awrt $-5.8r$ | M1 |
| | $\left.\dfrac{d^2V}{dr^2}\right\|_{r=..} = -27 < 0$ and therefore maximum | A1 |
| | | **[2]** |
| | | **13 marks** |

| | **Notes** | |
|---|---|---|

**(**a)

**M1**: Sets total surface area equal to 200 with at least two correct terms.
Note that $200 = 2\pi r^2 + \pi rh$ or even $200 = \pi r^2 + \pi rh + \pi r^2$ does not mean that two terms are correct.

**A1**: Completely correct $200 = \pi r^2 + \pi rh + 2rh$

**dM1**: Makes $h$ or $rh$ the subject of their formula which must have had two terms in $h$
This is dependent upon the previous M1

**M1**: Gives formula for volume. This may be implied by sight of $V = \frac{1}{2}\pi r^2 \times$ their $h$

**A1***: cso – substitutes for $r$ or for $rh$ correctly and proceeds correctly to $V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$

(b) **Parts b and c can be scored together**

**M1:** Attempts to differentiate $V$ or numerator of $V$ Accept $\frac{dV}{dr} = A \pm Br^2$

You may see $(4 + 2\pi)\frac{dV}{dr} = A \pm Br^2$ if candidates multiply by $(4 + 2\pi)$ first

**A1:** Accept any equivalent correct answer or correct numerator if only this was considered.
Also accept decimals.

**dM1:** Setting $\frac{dV}{dr}$=0 and finding a value for $r^2$ using correct mathematics (May be implied by answer).

Note that you may not see $r^2$. It is acceptable to go straight to $r$. Allow $\frac{dy}{dx} = 0$

**dM1:** Using square root to find $r$. Dependent upon all previous M's.
An answer of 5 for $r$ following a correct derivative may imply this mark as some candidates find r to the nearest cm rather than V to the nearest cm$^3$.
If you don't see incorrect work you may award this mark.

**A1 :** For any equivalent correct answer. Accept $r = \sqrt{\frac{200}{3\pi}}$ or awrt 4.6

Correct answer implies previous two M marks

**B1 :** Obtain $V$= 188 Exact answer only. Do not accept, for example, 187.8

(c)

**M1**: Score for either a second derivative of $\frac{d^2V}{dr^2} = \pm Cr$ and considers the sign.

It can be implied by $\frac{\pi r(200 - \pi r^2)}{4 + 2\pi} \to A \pm Br^2 \to \pm Cr$ and a consideration of the sign

Or a second derivative of $\frac{d^2V}{dr^2} = \pm Cr$ and substitutes in their value of '$r$' from (b)

Or a completely correct second derivative $\frac{d^2V}{dr^2} = \frac{-6\pi^2 r}{4 + 2\pi}$ accept $\frac{d^2V}{dr^2} = \text{awrt} -5.76r$

**A1**: Clear statements and conclusion. For both marks

(1) $\frac{d^2V}{dr^2}$ must be correct (see above), not just the numerator.

(2) A statement (which could be implied) that when their $r$ (which does not need to be correct) is substituted into $\frac{d^2V}{dr^2}$ then $\frac{d^2V}{dr^2}$ is either negative or < 0

(3) and a minimal conclusion such as hence maximum

For example, accept for both marks $\frac{d^2V}{dr^2} = -5.76r$ When $r = 4.5 \Rightarrow \frac{d^2V}{dr^2} < 0,$ hence max