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Surname	Ot	her names
Pearson Edexcel nternational Advanced Level	Centre Number	Candidate Number
Core Matr	nemati	
Advanced Subsidia	ry	
Advanced Subsidia Wednesday 25 May 2016 –	ry • Morning	Paper Reference
Advanced Subsidia Wednesday 25 May 2016 – Time: 2 hours 30 minutes	ry - Morning	Paper Reference WMA01/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Summe Past Pape	r 2016 www.mystudybro.com This resource was created and owned by Pearson Edexcel	Mathematics C12
1.	The first three terms in ascending powers of x in the binomial expansion of $(1 + 1)^8$	Leave
	$(1 + px)^{\circ}$ are given by	
	$1 + 12x + qx^2$	
	where p and q are constants.	
	Find the value of p and the value of q .	(5)
2		

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P 4 6 7 1 3 R A 0 2 5 2

Question Number	Scheme	Marks
1	$(1+px)^8 = 1+8(px) + \frac{8\times7}{2!}(px)^2$	M1
	Compares coefficients in $x \Rightarrow 8 p = 12 \Rightarrow p = 1.5$	M1A1
	Compares coefficients in $x^2 \implies q = 28 p^2 \implies q = 63$	M1A1
		(5)
		(5 marks)

(a)

M1 Uses the power series expansion/ binomial expansion with the correct form for terms 2 and 3. You may ignore the first term in this question.

Accept the correct coefficient with the correct power of *x* for terms 2 and 3.

$$(1+px)^8 = 1+8(..x) + \frac{8\times7}{2!}(..x)^2$$

Allow missing bracket on x^2 term.

Allow for
$$(1 + px)^8 = 1 + \binom{8}{1}(..x) + \binom{8}{2}(..x)^2$$
 or equivalent.

Allow sight of $\binom{8}{1}$ (...x) and $\binom{8}{2}$ (...x)² separated by commas

M1 Sets their coefficient in x equal to $12 \Rightarrow 8p = 12 \Rightarrow p = ...$ It is not dependent on the previous M but it must be of the form $kp = 12 \Rightarrow p = ...$

A1
$$p = 1.5$$
 or equivalent such as $\frac{12}{8}$

M1 Sets q equal to their coefficient of x^2 (which must include a p or a p^2) then substitutes in their value of p leading to q =

A1
$$q = 63$$

Summer Past Paper	2016 www.mystudybro.com This resource was created and owned by Pearson Edexcel	Mathematics C12 WMA01
2. I	ind the range of values of x for which	Leave blank
(a) $4(x-2) \le 2x+1$	(2)
(b) $(2x-3)(x+5) > 0$	(3)
(c) both $4(x-2) \le 2x+1$ and $(2x-3)(x+5) > 0$	(1)
4		
	I IAAIIAAI III AIAIA AIIII IAAAI IIAAA IIIAIIA	

Past Paper (Mark Scheme) This resource was created and owned by Pearson Edexcel

Question Number	Scheme	Marks	5
2(a)	$4(x-2), 2x+1 \Longrightarrow 4x-8, 2x+1 \\ \Longrightarrow x, 4.5$	M1A1	
(b)	(2x-3)(x+5) > 0		(2)
	Roots are 1.5, -5	B1	
	Chooses outsides $x < -5$, $x > 1.5$	M1A1	
			(3)
(c)	$x < -5, 1.5 < x_{,,}$ 4.5	B1	
			(1)
		(6 ma	rks)

(a)

- M1 Proceeds as far as $x_{,,...}$ after firstly multiplying out brackets oe. Condone x < ... for this mark Minimum expectation is that you see $4x 8...2x + 1 \Rightarrow x...c$ where ... is ,, or <
- A1 x, 4.5 or equivalent in set notation such as $\{x : x, 4.5\}$ $x \in (-\infty, 4.5]$. Accept just $(-\infty, 4.5]$
- (b)
- B1 Critical values are 1.5, -5
- M1 Chooses the outside values of their critical values. You may well see candidates multiply out the brackets and factorise incorrectly. They can score this mark for choosing the 'outsides' Do not allow this mark from just a diagram. You must see the inequalities. Accept for the method mark $x_{,,} -5, x_{..} 1.5$
- A1 Accept any of x < -5, x > 1.5' x < -5 or x > 1.5' $\{x : x < -5 \cup x > 1.5\}'$ $\{x : -\infty < x < -5 \cup 1.5 < x < \infty\}$ or their exact equivalents

Do not accept on its own (without seeing any of the above) 'x < -5 and x > 1.5'' ' $x < -5 \cap x > 1.5$ '' ' $x : x < -5 \cap x > 1.5$ ''

(c)

B1 cao Accept any of

x < -5, 1.5 < x, 4.5' x < -5 or 1.5 < x, 4.5' $(x : x < -5 \cup 1.5 < x, 4.5')$ $x \in -\infty < x < -5 \cup 1.5 < x, 4.5'$ or their exact equivalents.

There must be just two distinct regions represented by just two inequalities. If a candidate writes $x_{,,}$ 4.5 x < -5, x > 1.5 it is B0

Summ	ner 2016 www.mystudybro.com	n Mathematics C12
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3.	Answer this question without a calculator, showing all you your answers in their simplest form.	r working and giving
	(i) Solve the equation	
	$4^{2x+1} = 8^{4x}$	(3)
	(ii) (a) Express	
	$3\sqrt{18} - \sqrt{32}$	
	in the form $k\sqrt{2}$, where k is an integer.	(2)
	(b) Hence, or otherwise, solve	
	$3\sqrt{18} - \sqrt{32} = \sqrt{n}$	(2)
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P 4 6 7 1 3 R A 0 6 5 2

6

Question Number	Scheme	Marks	
3 (i)	Either $4^{2x+1} = 2^{2(2x+1)}$ and $8^{4x} = 2^{3 \times 4x}$ or $8^{4x} = 4^{\frac{3}{2} \times 4x}$	M1	
	$2(2x+1) = 12x \Longrightarrow x = \frac{1}{4}$	dM1A1	
(ii)(a)	$3\sqrt{18} - \sqrt{32} = 9\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$	M1A1	(3)
(b)	$\sqrt{n} = 5\sqrt{2} \Longrightarrow n = (5\sqrt{2})^2 = 25 \times 2 = 50$	M1A1	(2)
		(7 marks)	(2)
Alt 3 (i)	Taking logs of both sides and proceeding to $(2x+1)\log 4 = 4x\log 8$	M1	
	$\Rightarrow x = \frac{\log 4}{4\log 8 - 2\log 4}$		
	$\Rightarrow x = \frac{\log 4}{\log 256} = \frac{1}{4}$	dM1A1	
		((3)

(i)

M1 Writes both sides as powers of 2 or equivalent Eg $2^{2(2x+1)} = 2^{3\times 4x}$

Alternatively writes both sides as powers of 4 or 8 or 64. Eg $8^{4x} = 4^{\frac{3}{2} \times 4x}$ Note that expressions such as $2^{2+(2x+1)} = 2^{3+4x}$ would be M0

Condone poor (or missing) brackets $2^{2\times 2x+1} = 2^3$ but not incorrect index work eg $4^{2x+1} = 8^{\frac{1}{2}(2x+1)}$ It is possible to use logs. most commonly with base 2 or 4. Using logs it is for reaching a linear form of the equation, again condoning poor bracketing.

 $4^{2x+1} = 8^{4x} \Longrightarrow \log 4^{2x+1} = \log 8^{4x} \Longrightarrow (2x+1)\log 4 = 4x\log 8$

dM1 Dependent upon the previous M. It is for equating the indices and proceeding to x = ..Condone sign/bracketing errors when manipulating the equation but not processing errors If logs are used they must be evaluated without a calculator. Lengthy decimals would be evidence of this and would be dM0

$$(2x+1)\log_2 4 = 4x\log_2 8 \Rightarrow (2x+1) \times 2 = 4x \times 3 \Rightarrow x = ..$$
$$4^{2x+1} = 8^{4x} \Rightarrow 2x+1 = 4x\log_4 8 \Rightarrow 2x+1 = \frac{3}{2} \times 4x \Rightarrow x = ..$$
 is fine

A1 $x = \frac{1}{4}$ or equivalent

(ii)(a) Mark part (ii) as one complete question. Marks in (a) can be gained from (b) M1 Writes either $\sqrt{18} = 3\sqrt{2}$ or $3\sqrt{18} = 9\sqrt{2}$ or $\sqrt{32} = 4\sqrt{2}$

M1 Writes either $\sqrt{18} = 3\sqrt{2}$ or $3\sqrt{18} = 9\sqrt{2}$ or $\sqrt{32} = 4\sqrt{2}$

If the candidate writes $3\sqrt{18} - \sqrt{32} = k\sqrt{2}$ it can be scored for $\frac{3\sqrt{18}}{\sqrt{2}} = 9$ or $\frac{\sqrt{32}}{\sqrt{2}} = 4$

- A1 $5\sqrt{2}$ or states k = 5The answer without working (the M1) would be 0 marks (ii)(b)
- M1 Moves from $\sqrt{n} = k\sqrt{2}$ to $n = 2k^2$ Also accept for this mark $\sqrt{n} = \sqrt{50}$ or indeed $\sqrt{50}$ on its own
- A1 (n=)50

Past Paper

4.

Mathematics C12



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Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{x+2}, x \ge -2$

The finite region *R*, shown shaded in Figure 1, is bounded by the curve, the *x*-axis and the line x = 6

The table below shows corresponding values of x and y for $y = \sqrt{x+2}$

x	-2	0	2	4	6
У	0	1.4142	2		2.8284

(a) Complete the table above, giving the missing value of y to 4 decimal places.

- (1)
- (b) Use the trapezium rule, with all of the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 3 decimal places.

(3)

Use your answer to part (b) to find approximate values of



(4)



Question Number	Scheme	Marks
4 (a)	Awrt 2.4495	B1
(b)	Strip width' $(0 + 2.8284 + 2)(1.4142 + 2 + 12.4405')) = avert 14.556$	(1) B1
	Area $\approx \frac{1}{2} \{0 + 2.8284 + 2 \times (1.4142 + 2 + 2.4495)\} = awit 14.556$	(3)
(c)(i)	$\frac{1}{2}(b) = 7.278$ Allow awrt 7.28	B1ft
	6	(1)
(ii)	$\int_{-2}^{2} 2 dx = [2x]_{-2}^{-2} = 12 - (-4) = 16$	MI
	$\int_{-2}^{0} (2 + \sqrt{x+2}) dx = \int_{-2}^{0} 2 dx + (b) = 16 + (b) = 30.556$ Allow awrt 30.56	dM1 A1ft
		(3) (8 marks)
(a) B1 Awr	t 2.4495	

(b)

B1 Uses a strip width of 2 units. This may be embedded in the trapezium formula.

It may be implied by sight of $\frac{2}{2}$ {.....} or 1×{.....}

M1 Uses the correct form of the bracket within the trapezium rule. Look for $(0) + 2.8284 + 2 \times (1.4142 + 2 + '2.4495')$

A1 awrt 14.556

If separate trapezia are used then you should see the sum of 4 trapezia. B1 Width or h = 2 and M1 for the correct values of 'a' and 'b' for each one.

(c)(i)

B1ft Look for $\frac{1}{2}(b)$. Follow through on their answer to (b) but you may allow accuracy to 2dp

Also allow from adapting their trapezium rule in part (b) with all terms halved

(c)(ii)

M1 Look for an attempt to either integrate '2' by writing $[2x]_{-2}^{6}$ oe. It may be embedded within a longer integral. Alternatively finds the area of a rectangle with height 2 and length of (6-(-2)) which is implied by the sight of 16

Do not allow $\int 2 \, \mathrm{d}x = 2$

Allow an attempt to use the trapezium rule in part (b) with 2 being added to each value inside the bracket.

- dM1 Adds together their answer for the integral of 2 to their answer for (b).It is dependent upon the previous MIf their trapezium rule is adapted then this mark is scored at the point when the value is calculated
- A1ft 16+(b). Accept this for all 3 marks as long as no incorrect working seen. Allow accuracy to 2dp.

Mathematics C12

(1)

(2)

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blank

5. (i)

 $U_{n+1} = \frac{U_n}{U_n - 3}, \quad n \ge 1$

Given $U_1 = 4$, find

(a)
$$U_2$$

(b)
$$\sum_{n=1}^{100} U_n$$

 $\sum_{r=1}^{n} (100 - 3r) < 0$

find the least value of the positive integer n.

(3)

12	
	P 4 6 7 1 3 R A 0 1 2 5 2

Question Number	Scheme	Marks
5(i)		
(a)	$(U_2) = \frac{4}{4} = 4$	B1
	4-5	(1)
	$\sum_{i=1}^{100} V_{i} = 100 \times 4 - 400$	
(b)	$\sum_{n=1}^{N} U_n = 100 \times 4 = 400$	MIAI
		(2)
5(ii)	$\sum_{n=1}^{n} (100-3r) < 0 \Rightarrow 97+94+91+(100-3r) < 0$	
C(II)	r=1	
	\sum AP with $a = 97, d = -3, n = n, S < 0 \Rightarrow 0 = \frac{n}{2}(2 \times 97 + (n-1) \times -3) < 0$	M1
	$\Rightarrow \frac{n}{2}(197 - 3n) < 0 \Rightarrow n > 65.6$	dM1
	\Rightarrow $n = 66$	A1
		(3)
(ii)	<u> </u>	(o marks)
ALT I	$\sum (100 - 3r) < 0 \Longrightarrow \sum 3r > \sum 100$	
	r=1 $r=1$ $r=1$	
	$\rightarrow 2^{n(n+1)} > 100 r$	M1
	$\Rightarrow 3 \frac{100n}{2} > 100n$	M1A1
	\Rightarrow $n > 65.6 \Rightarrow n = 66$	

(i)(a)

B1 States that U_2 is 4. Accept $\frac{4}{1}$ but not $\frac{4}{4-3}$ and remember to isw. Note that $U_1 = 4$ so be sure that you don't award this B1

(i)(b)

M1 Uses the method that $\sum_{n=1}^{100} U_n = k \times 4$ where k = 100 or 99 You may see the AP formula being used which is fine as long as a = 4, d = 0 and n = 99/100Look for expression of the form $\frac{100}{2} \{2 \times 4 + 99 \times 0\}$ OR $\frac{100}{2} \{4 + 4\}$ A1 400 (ii)

M1 Uses
$$S \dots \frac{n}{2}(2a + (n-1)d)$$
 with ... as $= \text{ or } > \text{ or } <$ and $S = 0$ $a = 97$ or 100, $d = -3$
Alternatively uses $S \dots \frac{n}{2}(a+l)$ with ... as $= \text{ or } > \text{ or } <$ and $S = 0$ $a = 97$ or 100, $l = 100 - 3n$
A solution should not appear to come from the value of the nth term or in fact any linear equation.
This is M0
dM1 Dependent upon previous M. Scored for a solution of their quadratic equation in n .
Accept $n = , n > , n <$
A1 $n = 66$ cso

Part (ii) Alt 1 Using the formula
$$\sum_{1}^{n} r = \frac{n(n+1)}{2}$$

M1 For attempting to solve
$$\sum_{r=1}^{n} 3r \dots \sum_{r=1}^{n} 100$$
 where \dots is $=$ or $<$ by writing $\sum_{1}^{n} r = \frac{n(n+1)}{2}$ and

$$\sum_{r=1}^{n} 100 = 100n$$
. Condone the 3 in the $\sum_{r=1}^{n} 3r$ "disappearing"

dM1 Dependent upon previous M. Scored for a solution to their quadratic equation in *n*. Accept n = , n > , n <

A1 $n = 66 \operatorname{cso}$

.....

Part (ii) Alt 2 Using the fact that the sum is (less than) zero when the negative terms add up to more than the positive terms in the sequence

M1 Sum of positive terms = $97 + 94 + \dots + 1 = \frac{33}{2}(97 + 1) = [1617]$ Sum of negative terms = -2 + -5.....

$$\frac{n}{2} \{2 \times 2 + 3(n-1)\} > '1617'$$

dM1 Dependent upon last mark. It is for solving the quadratic equation with usual methods and adding on 33 terms (the number of positive terms)

$$\frac{n}{2} \{2 \times 2 + 3(n-1)\} > '1617' \Rightarrow n > 32.$$
 So total number of terms is 33+33=66

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Leave blank 6. (a) Show that $\frac{x^2 - 4}{2\sqrt{x}}$ can be written in the form $Ax^p + Bx^q$, where A, B, p and q are constants to be determined. (3) (b) Hence find $\int \frac{x^2 - 4}{2\sqrt{x}} \mathrm{d}x, \quad x > 0$ giving your answer in its simplest form. (4) 16 P 4 6 7 1 3 R A 0 1 6 5 2

Question Number	Scheme	Marks
6(a)	$\frac{x^2 - 4}{2\sqrt{x}} = \frac{x^2}{2\sqrt{x}} - \frac{4}{2\sqrt{x}} = \frac{1}{2}x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$	M1A1A1 (3)
(b)	$\int \frac{x^{\frac{3}{2}}}{2} - 2x^{-\frac{1}{2}} dx = \frac{x^{\frac{5}{2}}}{2 \times 2.5} - 2\frac{x^{\frac{1}{2}}}{0.5}(+c)$	M1 A1ft A1
	$=\frac{x^{\frac{5}{2}}}{5}-4x^{\frac{1}{2}}+c$	B1
		(4) (7 marks)

(a)

- M1 Attempt to divide by $2\sqrt{x}$ to get exactly two terms (not three) This can be implied by any of *A*, *B*, *p* or *q* being correct.
- A1 Two of *A*, *B*, *p* or *q* correct. Look for two of the four numbers $\frac{x^{\frac{3}{2}}}{2} 2x^{-\frac{1}{2}}$ oe Allow the power as $\frac{3}{2}$ appearing as $2 - \frac{1}{2}$ A1 Completely correct expression $\frac{x^{\frac{3}{2}}}{2} - 2x^{-\frac{1}{2}}$ or equivalent. Eg accept $0.5x^{1.5} - 2x^{-0.5}$ The powers must now be simplified.

(b)

- M1 Increases a fractional index by one. Do not allow if the candidate integrates the numerator and denominator of the original function
- A1ft One of the fractional terms correct unsimplified. You may follow through on any term with a fractional index.
- A1 Both terms correct unsimplified Allow the powers and coefficients such as $\frac{5}{2}$ appearing as $\left(\frac{3}{2}+1\right)$ for this mark

B1
$$=\frac{x^{\frac{5}{2}}}{5} - 4x^{\frac{1}{2}} + c$$
 or exact equivalent such as $0.2x^{2.5} - 4x^{0.5} + c$

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7		Leave blank
7.	$f(x) = 3x^3 + ax^2 + bx - 10$, where a and b are constants.	
	Given that $(x - 2)$ is a factor of $f(x)$,	
	(a) use the factor theorem to show that $2a + b = -7$	(2)
	Given also that when $f(x)$ is divided by $(x + 1)$ the remainder is -36	
	(b) find the value of <i>a</i> and the value of <i>b</i> .	(4)
	f(x) can be written in the form	
	f(x) = (x - 2)Q(x), where $Q(x)$ is a quadratic function.	
	(c) (i) Find $Q(x)$.	
	(ii) Prove that the equation $f(x) = 0$ has only one real root.	
	You must justify your answer and show all your working.	(4)
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18		

Question Number	Scheme	Marks
7(a)	Sets $f(\pm 2) = 0$	M1
	$f(2) = 0 \Longrightarrow 24 + 4a + 2b - 10 = 0 \Longrightarrow 4a + 2b = -14 \Longrightarrow 2a + b = -7$	A1*
		(2)
(b)	Sets $f(\pm 1) = -36$	M1
	$f(-1) = -36 \Longrightarrow -3 + a - b - 10 = -36 \Longrightarrow a - b = -23 \qquad \text{oe}$	A1
	Solves simultaneously to get both a and b	dM1
	a = -10, b = 13	A1
		(4)
(c)(i)	Divides their $f(x)$ by $(x-2)$ to get a quadratic	M1
	$f(x) = (x-2)(3x^2-4x+5)$ or $Q(x) = 3x^2-4x+5$	A1
(ii)	Calculates $b^2 - 4ac$ on their Q(x) or solves their Q(x) = 0	M1
	$(3x^2 - 4x + 5)$ has no roots as $b^2 - 4ac = 16 - 60 < 0$. Hence $f(x)$ has 1 root	A1*
		(4) (10 marks)

(a)

M1 Sets $f(\pm 2) = 0$

The = 0 may be implied by later working.

A1* $f(2) = 0 \Rightarrow 2a + b = -7$. There must be at least one intermediate line between these two statements. Accept $f(2) = 0 \Rightarrow 3 \times 2^3 + a \times 2^2 + b \times 2 - 10 = 0 \Rightarrow 2a + b = -7$ Note that this is a given result.

(b)

M1 Sets $f(\pm 1) = -36$

If a candidate attempts division look for a minimum of

$$\frac{3x^2 + (a-3)x....}{x+1)3x^3 + ax^2 + bx - 10}$$

f(a,b)

before the candidate sets their remainder, which must be a function of a and b equal to -36

- A1 $f(-1) = -36 \Rightarrow a b = -23$ or equivalent. The equation does not need to be simplified but the indices must have been dealt with correctly In division the remainder may appear in the form -10 (b a + 3) = -36
- dM1 Solves simultaneously to get values for both *a* and *b*. Don't be too concerned with the process as long as you see values for *a* and *b* coming from two equations in both *a* and *b*. It is dependent upon the previous M

A1
$$a = -10, b = 13$$

(c)(i)

M1 For dividing their f(x) by (x-2) to get a $f(x) = (x-2) \times$ quadratic If it is attempted by division look for the first two terms following through on their a $\frac{3x^2 + (a+6)x....}{x-2)3x^3 + a'x^2 + b'x - 10}$

If it attempted by inspection only check first and last terms. Look for $f(x) = (x-2)(3x^2...x+5)$ $f(x) = (x-2)(3x^2-4x+5)$ or $Q(x) = 3x^2-4x+5$

(c)(ii)

A1

M1 Scored for an attempt at finding the number of roots of **their** 3TQ Q(x). This can be attempted by calculating the value of their $b^2 - 4ac$, Eg $b^2 - 4ac = 16 - 60 = -44$ or using the formula/ completing the square to find the roots. Eg. $ax^2 + bx + c = 0 \Rightarrow x = ..., ..$ A1* cso. All aspects must be correct including $\mathbf{f}(x) = (x-2)(3x^2 - 4x + 5)$ with proof that

 $(3x^2 - 4x + 5)$ has no roots **and hence** f(x) has only 1 root (at x=2)

Summer 2016 www.mystudybro.com **Mathematics C12** This resource was created and owned by Pearson Edexcel Past Paper WMA01 Leave blank 8. In this question the angle θ is measured in degrees throughout. (a) Show that the equation $\frac{5+\sin\theta}{3\cos\theta} = 2\cos\theta, \qquad \theta \neq (2n+1)90^\circ, \quad n \in \mathbb{Z}$ may be rewritten as $6\sin^2\theta + \sin\theta - 1 = 0$ (3) (b) Hence solve, for $-90^{\circ} < \theta < 90^{\circ}$, the equation $\frac{5+\sin\theta}{3\cos\theta} = 2\cos\theta$ Give your answers to one decimal place, where appropriate. (4)



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WMA01

Question Number	Scheme	Marks
8(a)	$\left(\frac{5+\sin\theta}{3\cos\theta}=2\cos\theta\right) \Longrightarrow 5+\sin\theta=6\cos^2\theta$	M1
	$\Rightarrow 5 + \sin \theta = 6(1 - \sin^2 \theta) \Rightarrow 6\sin^2 \theta + \sin \theta - 1 = 0$	dM1A1* (3)
(b)	$6\sin^2\theta + \sin\theta - 1 = 0 \Longrightarrow (3\sin\theta - 1)(2\sin\theta + 1) = 0$	M1
	$(\sin\theta) = +\frac{1}{3}, -\frac{1}{2}$	A1
	$\theta = 19.5^{\circ}, -30^{\circ}$	dM1,A1
		(4) (7 marks)

(a)

- M1 Attempts to cross multiply to form an equation in the form $5 + \sin \theta = A \cos^2 \theta$
- dM1 Dependent upon previous M. For using $\cos^2 \theta = \pm 1 \pm \sin^2 \theta$ to get an equation in just $\sin \theta$
- A1* This is a given answer. All aspects must be correct. Mixed variables, say x's and θ 's would lose this mark. An otherwise correct solution with $\cos^2 \theta$ or $(\cos \theta)^2$ written as $\cos \theta^2$ would also be M1 dM1 A0

(b)

- M1 Attempts to factorise, usual rules. Accept answers by formula or just written down from a calculator. Accept answers/factors given in terms of *x* for this mark
- A1 For the values $+\frac{1}{3}, -\frac{1}{2}$

These do not have to be simplified and can be implied by correct answers for θ Calculates at least one value of θ from their 'sin θ '.

- dM1 Calculates at least one value of θ from their'sin θ' . It is dependent upon the previous M. You may have to check with a calculator. This may be implied by either $\theta = 19.5^{\circ}, -30^{\circ}$ from a correct quadratic
- A1 Both $\theta = 19.5^{\circ}$ and -30° with this accuracy and no additional solutions inside the range. Condone the answer $\theta = 19.5^{\circ}$ and -30.0° Ignore any solutions outside the range.

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The first term of a geometric series is 6 and the common ratio is 0.92	
For this series, find	
(a) (i) the 25 th term, giving your answer to 2 significant figures,	
(ii) the sum to infinity.	(4)
The sum to <i>n</i> terms of this series is greater than 72	
The sum to n terms of this series is greater than 72	
(b) Calculate the smallest possible value of <i>n</i> .	(4)

Ques Num	stion nber	Scheme	Marks	
9 (a)(i)	Attempts to use $u_n = ar^{n-1} \implies u_{25} = 6 \times 0.92^{24} = \text{awrt } 0.81$	M1A1	
	(ii)	Attempts to use $S_{\infty} = \frac{a}{1-r} \Longrightarrow S_{\infty} = \frac{6}{1-0.92} = 75$	M1A1	
((b)	Sets $S_n > 72 \Rightarrow \frac{6(1-0.92^n)}{1-0.92} > 72$ Accept $\frac{6(1-0.92^n)}{1-0.92} = 72$	(4) M1	
		$0.92^n < 0.04$ Accept $0.92^n = 0.04$	A1	
		Takes log's $n > \frac{\log 0.04}{\log 0.92}$ Accept $n = \frac{\log 0.04}{\log 0.92}$	dM1	
		n=39	A1 (4)	
(a) (i)	• • •	-6.002^{24} -6.002^{25-1}		
MI	Atte	mpts 6×0.92^{-1} or 6×0.92^{-2}		
A1	awrt	0.81		
(a)(ii)				
M1	Atte	mpts to use $S_{\infty} = \frac{a}{1-1}$ with $a = 6$ and $r = 0.92$ $S_{\infty} = \frac{6}{1-0.02}$		
A1 (b)	1 75 $1-r$ $1-0.92$			
M1	1 Sets $S_n > 72 \Rightarrow \frac{6(1-0.92^n)}{1-0.92} > 72$ Accept $\frac{6(1-0.92^n)}{1-0.92} = 72$ or $\frac{6(1-0.92^n)}{1-0.92} < 72$			
A1	1 Proceeds to $0.92^n \dots 0.04$ where \dots is either = or > or <			
dM1	A1 Proceeds from $a^n \dots b$ to $n \dots \log_a b$ or $n \dots \frac{\log b}{\log a}$ where \dots is =, > or <			
	The	values of a and b must be positive		
A1	 Allow this mark in cases where the candidate has incorrectly multiplied 6 by 0.92ⁿ to form an equation in the form 5.52ⁿ k (k>0)and solves by correctly taking logs 1 cso n =39. All aspects must be correct including the inequalities on each line if they have been 			
	used Do n	. It is acceptable to use '=' however not accept $n = 38.6$ for this mark		
Note [.] 7	Trial :	and Improvement can gain all 4 marks as long as $n = 39$ is stated		
N 1 1 1				
MIAI	F(6)	or signt of a trial at 58 or 59 using the sum formula $(1-0.92^{38})$ $6(1-0.92^{39})$		
		$\frac{1}{1-0.92}$ = awrt 71.8 or $\frac{0(1-0.92)}{1-0.92}$ = awrt 72.1		

M1A1 For trial at 38 and 39 using the sum formula, and conclusion that *n*=39

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10. The curve *C* has equation
$$y = \sin\left(x + \frac{\pi}{4}\right), \quad 0 \le x \le 2\pi$$

- (a) On the axes below, sketch the curve C.
- (b) Write down the exact coordinates of all the points at which the curve Cmeets or intersects the x-axis and the y-axis.
- (3)

(2)

(c) Solve, for $0 \le x \le 2\pi$, the equation

$$\sin\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answers in the form $k\pi$, where k is a rational number.

(4)





(a) Look only at the part of the graph between 0 and 2π.
 If there is no x scale shown, then assume that 2π is at the point where their graph stops.
 If the sketch continues past the x label on the x axis, assume 2π is at the point of the x of the x label.

- M1 Graph is harmonic. Could be part of a cycle, or several cycles but needs at least one max and one min in the range $0, \theta, 2\pi$. Do not penalise graphs which are not in the correct position in relation to either axis
- A1 Requires only one cycle between 0 and 2π . The sketch needs: a positive y intercept and a positive y value at 2π , with a positive gradient at both 0 and 2π . The maximum must be positive and the minimum must be negative.
- (b) B1B1B1 Score 1,1,0 for ANY 2 out of 3 and 1,0,0 for any 1 out of 3 Each answer is cao. No decimals are accepted unless the exact value is also given. Accept points marked on the graph.

Accept coordinates with zeroes (as given) or for example $\frac{\sqrt{2}}{2}$ marked on the y axis or an obvious

statement such as y intercept is ... or y =

For both correct answers in degrees withhold one mark.

If there are extra answers in the range, in addition to the correct answers, withhold one mark. If answers are given in text and on diagram, the text takes precedence.

- (c)
- M1 Takes invsin and subtracts $\frac{\pi}{4}$ in an attempt to find one value of x. Look for $x = \frac{\pi}{3} \frac{\pi}{4} \Rightarrow x = ..$ Accept answers in degrees ($x = 60 - 45 \Rightarrow x = ..$) and decimals for the method mark. Do not accept mixed degrees/radians.
- A1 One of $\frac{\pi}{12}$ or $\frac{5\pi}{12}$
- M1 Correct attempt to get a second value of x from their first value. Accept $x + \frac{\pi}{4} = \pi their \frac{\pi}{3} \Rightarrow x = ..$ Accept degrees or decimals for this method mark
- A1 Both $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ and no other values within the range. Ignore additional answers outside the range.

Mathematics C12

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Questi Numb	on er Scheme	Marks
11 (a) States $r^2 = 1.2^2 + (r - 0.4)^2$ $0.8r = 1.60 \Rightarrow r = 2.$ Attempt to find the angle or 1/2 angle	M1 A1* (2)
	$\frac{1}{2}\theta' = \arcsin\left(\frac{1.2}{2}\right) \Rightarrow \frac{1}{2}\theta' = awrt 37^{\circ} awrt \ 0.64 \text{ rads}$	M1
(6)	$\cos AOB = 1.2870$	A1* (2)
	Area sector $=\frac{'73.74'}{360} \times \pi \times 2^2$ or $\frac{1}{2} \times 2^2 \times '1.2870'$	M1
	Attempts area of triangle using the correct formula $Area = \frac{1}{2} \times 2 \times 2 \times \sin' 73.74^{\circ}' \text{ or } Area = \frac{1}{2} \times 2 \times 2 \times \sin' 1.2870'$	M1
	Area of sail = Sector -Triangle using the correct combination = $\frac{1}{2} \times 2^2 \times 1.29' - \frac{1}{2} \times 2^2 \times \sin 1.29' = 0.654 (m^2)$	dM1, A1
		(4) (8 marks)
(a) M1 4 A1* 1 (b) M1 4 A1* (c) M1 (c) M1 (c) I	Attempts Pythagoras with lengths r , $(r-0.4)$ and 1.2 in the correct positions within the Alternatively uses the given answer and attempts Pythagoras' theorem with 1.6,1.2 Proceeds to $r = 2$ with no errors if the alternative method is used, then there must be a statement such as hence true, a Attempts to find either the angle or half angle in the sector in either degrees or radian. For the half angle accept $arctan\left(\frac{1.2}{1.6}\right)$, $arcsin\left(\frac{1.2}{2}\right)$, $arccos\left(\frac{1.6}{2}\right)$. For the whole angle accept $2.4^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos \theta \Rightarrow \theta =or similar. eso. This is a given answer AOB = 1.2870. Allow from a value where \frac{1}{2} the angle lacks 4 dp of accuracy. Correct method to find the area of the sector with radius 2 and angle 1.2870 radians if angle was found in degrees use =\frac{173.74'}{360} \times \pi \times 2^2 = (2.574) if angle was found in radians use =\frac{1}{2} \times 2^2 \times 1.2870'.$	the formula and 2 r =2 ns
M1 (1)	Correct method to find the area of the isosceles triangle with lengths 2 and angle 1.2 For the triangle <i>AOB</i> accept $\frac{1}{2} \times 2 \times 2 \times \sin'73.74'$ or $\frac{1}{2} \times 2 \times 2 \times \sin'1.2870' = (1.92)$ Also for triangle <i>AOB</i> accept use of $\frac{1}{2}bh = \frac{1}{2} \times 2.4 \times 1.6$. Note $\frac{1}{2}bh = \frac{1}{2} \times 2.4 \times 2$ is N Attempts the area of the sail using the correct combinations of sector - triangle.	870 M0
A1 a	f the candidate uses the segment formula $\frac{1}{2} \times 2^2 \times 1.29' - \frac{1}{2} \times 2^2 \times \sin 1.29'$ all three warded wrt $0.654(m^2)$	M's can be

Mathematics C12



P 4 6 7 1 3 R A 0 3 4 5 2

Question Number	Scheme	Marks	
12(a)	Writes C as $(x-a)^2 + (y-0)^2 = a^2$	M1A1	
(b)	Subs $(4, -3) \implies (4-a)^2 + (-3-0)^2 = a^2$	M1	(2)
	$\Rightarrow 16 - 8a + a^2 + 9 = a^2$ $\Rightarrow 25 = 8a$		
	$\Rightarrow 25 - 54$ $\Rightarrow a = \frac{25}{2}$	dM1A1	(3)
	δ	(5 marks)	(3)

Mark parts (a) and (b) together. Award marks in (a) from (b) and vice versa, but see note (a)

M1 Attempts to find the equation of *C* centre (*a*,0) radius *a*. Accept $(x \pm a)^2 + y^2 = a^2$ oe If the alternative form of the circle is used accept $x^2 + y^2 \pm 2ax = a^2 - a^2$ Allow for the M1 $(x \pm a)^2 + (y \pm 0)^2 = r^2$

A1 Writes C as
$$(x-a)^2 + (y-0)^2 = a^2$$
 or equivalent $x^2 + y^2 - 2ax = 0$.

- (b)
- M1 Subs x = 4 and y = -3 into their circle equation for C which must be of the form $(x \pm a)^2 + (y \pm 0)^2 = a^2$
- dM1 Proceeds to a linear equation in 'a' and reaches a=... Condone numerical slips A1 $a = \frac{25}{8}$ Accept exact alternatives

Note: There are some candidates who write the equation of the circle as $(x-a)^2 + (y-0)^2 = r^2$ in part (a) This is M1 A0 However in part (b) they substitute (4,-3) and write down $(4-a)^2 + (-3)^2 = a^2$ We will allow them to score all 3 marks in part (b).

Had they written $(x-a)^2 + y^2 = a^2$ in (b) we would allow them to score all 5 marks

WMA01

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This resource was created and owned by Pearson Edexcel Past Paper Leave blank 13. (a) Show that the equation $2\log_2 y = 5 - \log_2 x$ x > 0, y > 0may be written in the form $y^2 = \frac{k}{x}$ where k is a constant to be found. (3) (b) Hence, or otherwise, solve the simultaneous equations $2\log_2 y = 5 - \log_2 x$ $\log_x y = -3$ for x > 0, y > 0(5) 36 P 4 6 7 1 3 R A 0 3 6 5 2

Ques Num	tion ber	Scheme	Marks
13 ((a)	$2\log_2 y = 5 - \log_2 x \Longrightarrow \log_2 y^2 = 5 - \log_2 x$	M1
		$\Rightarrow \log_2 y^2 = \log_2 32 - \log_2 x \Rightarrow \log_2 y^2 = \log_2 \left(\frac{32}{x}\right)$ $\Rightarrow y^2 = \frac{32}{x}$	M1A1
(b)	$\log_x y = -3 \Longrightarrow y = x^{-3}$	(3) M1
		Sub $y = x^{-3}$ into $y^2 = \frac{32}{x} \Rightarrow x^{-6} = \frac{32}{x} \Rightarrow x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$	M1A1
		Sub $x = \frac{1}{2}$ into either eqn $\Rightarrow y = 8$	M1A1
			(5) (8 marks)
Alt	(b)	Sub $y^2 = \frac{32}{x}$ into $\log_x y = -3 \Longrightarrow \log_x \sqrt{\frac{32}{x}} = -3$	2nd M1
		$\Rightarrow \sqrt{\frac{32}{x}} = x^{-3}$	1st M1
		$\Rightarrow x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$	A1
M1 M1	Eg U Alter Note Uses Awa or lo	one correct log law. Uses the index law and writes $2 \log_2 y = \log_2 y^2$. matively writes 5 as $\log_2 32$. This may well come from $\log_2 = 5 \Rightarrow = 32$ that $2 \log_2 y + \log_2 x = 2 \log_2 xy$ is M0 two correct log laws rd for $\log_2 y^2 = \log_2 (32) - \log_2 x$ $\log_2 x + \log_2 y^2 = 5 \Rightarrow \log_2 xy^2 = 5$	
A1	Proc	eeds correctly to $y^2 = \frac{32}{x}$	
(b) M1 M1	Undo This Com	bes the log in the second equation $\log_x y = -3 \Rightarrow y = x^{-3}$ may well appear later in the solution bines both equations to form a single equation in one variable.	
A1	$x = -\frac{1}{2}$	$\frac{1}{2}$ or $y = 8$. Condone a solution $y = \pm 8$ for this mark	
M1	Subs	titutes their $x = \frac{1}{2}$ into an equation to find y.	
	Alter	matively substitutes their $y = 8$ into an equation to find x	
A1	$x = \frac{1}{2}$	$\frac{1}{2}$ and $y = 8$ only. Note $x = \frac{1}{2}$ and $y = \pm 8$ is A0	
SC. If a	a cano	didate uses $y = \frac{k}{x}$ with $\log_x y = -3$ this can potentially score M1: Undoing logs	s, M0: as
combir followe	ning the design of the design	he equations has been made easier, A0: M1: If they substitute their x to find y a A0: scoring 10010	and vice versa

Mathematics C12

WMA01



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Q(0,4) to R(2,0) and from R(2,0) to S(4,10).

The line *l* intersects y = g(x) at the points *A* and *B* as shown in Figure 4.

(a) Use algebra to find the x coordinate of the point A and the x coordinate of the point B.

Show each step of your working and give your answers as exact fractions.

(6)

(b) Sketch the graph with equation

$$y = \frac{3}{2}g(x), \quad -3 \leqslant x \leqslant 4$$

On your sketch show the coordinates of the points to which P, Q, R and S are transformed.

(2)



Question Number	Scheme	Marks
14(a)	Sets $\frac{1}{2}x = ax + 4$ where $a < 0$	M1
	Solves $\frac{1}{2}x = -2x + 4 \Longrightarrow \frac{5}{2}x = 4 \Longrightarrow x = \frac{8}{5}$ oe	dM1A1
	Sets $\frac{1}{2}x = 5x + b$ where $b < 0$	M1
	Solves $\frac{1}{2}x = 5x - 10 \Rightarrow \frac{9}{2}x = 10 \Rightarrow x = \frac{20}{9}$ oe	dM1A1
		(6)
	<i>y S</i> (4,15)	
(b)	P(-3, 6) = O(0, 6) Any two points correct	B1
	$\begin{array}{c c} & & \\ \hline & \\ O & R(2,0) & x \end{array}$ Same 'shape' with 4 points correct	B1
		(2)
		(8 marks)

(a)

- M1 Attempts the smaller solution. Accept setting $\frac{1}{2}x = ax + 4$ where a < 0
- dM1 Sets $\frac{1}{2}x = -2x + 4$ and proceeds to x = ... by collecting terms. Condone errors
- A1 $x = \frac{8}{5}$ oe. Accept 1.6
- M1 Attempts to find the larger solution. Accept setting $\frac{1}{2}x = 5x + b$ where b < 0
- dM1 Sets $\frac{1}{2}x = 5x 10$ and proceeds to x = ... by collecting terms. Condone errors

A1
$$x = \frac{20}{9}$$
 Accept exact equivalents such as $2\frac{2}{9}$ but not 2.2 or 2.2

- (b)
- B1 Any two points correct either in the text or on a sketch. Accept 6 and 2 written on the correct axes
- B1 Shape + all four points correct. Watch for candidates who adapt the given diagram. This is acceptable A diagram can be labelled with P, Q, R and S and coordinates given for P,Q,R and S in the body of the script. If they are given on the diagram and in the body of the script the diagram takes precedence.

Mathematics C12



Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height h cm and radius r cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60000 cm³ of water when full.

(a) Show that the total external surface area, $S \text{ cm}^2$, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(2)

(c) Justify that the value of S you found in part (b) is a minimum.

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Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $60000 = \pi r^2 h \Rightarrow h = \frac{60000}{\pi r^2}$	M1
	Subs in $S = \pi r^2 + 2\pi rh \Longrightarrow S = \pi r^2 + 2\pi r \times \frac{60000}{\pi r^2}$	M1
	$\implies S = \pi r^2 + \frac{120000}{r}$	A1*
		(3)
(b)	$\frac{\mathrm{d}S}{\mathrm{d}r} = 2\pi r - \frac{120000}{r^2}$	M1A1
	$\Rightarrow \frac{\mathrm{d}S}{\mathrm{d}r} = 0 \Rightarrow r^3 = \frac{120000}{2\pi} \Rightarrow r = awrt\ 27(\mathrm{cm})$	dM1A1
	$\Rightarrow S = \pi \times "26.7"^{2} + \frac{120000}{"26.7"} = awrt\ 6730(\text{cm}^{2})$	dM1 A1
		(6)
(c)	$\left. \frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3} \right _{r=26.7} = awrt 19 > 0 \Longrightarrow \mathrm{Minimum}$	M1A1
		(2)
		(11 marks)

(a)

- M1 Uses $60000 = \pi r^2 h \Rightarrow h = ..$ Alternatively uses $60000 = \pi r^2 h \Rightarrow \pi r h = ..$ Condone errors on the number of zeros but the formula must be correct
- M1 Score for the attempt to substitute any h = .. or $\pi rh = ..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000 = \frac{1}{3}\pi r^2 h \Longrightarrow h = ..\right)$ into $S = k\pi r^2 + 2\pi rh$ where k = 1 or 2 to get S in terms of r

Allow if *S* is called something else such as *A*.

A1* Completes proof with no errors (or omissions) $S = \pi r^2 + \frac{120000}{r}$.

Allow from $S = \pi r^2 + \frac{2V}{r}$ if quoted. S = must be somewhere in the proof

Alt (c) using the value of S either side of 26.7

M1:Attempts to find the **numerical** value of $S = \pi r^2 + \frac{120000}{r}$ either side of a positive *r* achieved in (b) A1:At $r = a \Rightarrow S_{26.7} < S_a$ at $r = a \Rightarrow S_{26.7} < S_b$ hence Minimum (where a < 26.7 < b) Past Paper

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Figure 6

Figure 6 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-2)$$

The point *P* lies on *C* and has *x* coordinate $\frac{1}{2}$

The line *l*, as shown on Figure 6, is the tangent to *C* at *P*.

(a) Find
$$\frac{dy}{dx}$$
 (2)

(b) Use part (a) to find an equation for *l* in the form ax + by = c, where *a*, *b* and *c* are integers.

The finite region R, shown shaded in Figure 6, is bounded by the line l, the curve C and the *x*-axis.

The line l meets the curve again at the point (2, 0)

(c) Use integration to find the exact area of the shaded region R.

(6)

(4)





Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $60000 = \pi r^2 h \Rightarrow h = \frac{60000}{\pi r^2}$	M1
	Subs in $S = \pi r^2 + 2\pi rh \Longrightarrow S = \pi r^2 + 2\pi r \times \frac{60000}{\pi r^2}$	M1
	$\implies S = \pi r^2 + \frac{120000}{r}$	A1*
		(3)
(b)	$\frac{\mathrm{d}S}{\mathrm{d}r} = 2\pi r - \frac{120000}{r^2}$	M1A1
	$\Rightarrow \frac{\mathrm{d}S}{\mathrm{d}r} = 0 \Rightarrow r^3 = \frac{120000}{2\pi} \Rightarrow r = awrt\ 27(\mathrm{cm})$	dM1A1
	$\Rightarrow S = \pi \times "26.7"^{2} + \frac{120000}{"26.7"} = awrt\ 6730(\text{cm}^{2})$	dM1 A1
		(6)
(c)	$\left. \frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3} \right _{r=26.7} = awrt 19 > 0 \Longrightarrow \mathrm{Minimum}$	M1A1
		(2)
		(11 marks)

(a)

- M1 Uses $60000 = \pi r^2 h \Rightarrow h = ..$ Alternatively uses $60000 = \pi r^2 h \Rightarrow \pi r h = ..$ Condone errors on the number of zeros but the formula must be correct
- M1 Score for the attempt to substitute any h = .. or $\pi rh = ..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000 = \frac{1}{3}\pi r^2 h \Longrightarrow h = ..\right)$ into $S = k\pi r^2 + 2\pi rh$ where k = 1 or 2 to get S in terms of r

Allow if *S* is called something else such as *A*.

A1* Completes proof with no errors (or omissions) $S = \pi r^2 + \frac{120000}{r}$.

Allow from $S = \pi r^2 + \frac{2V}{r}$ if quoted. S = must be somewhere in the proof

Alt (c) using the value of S either side of 26.7

M1:Attempts to find the **numerical** value of $S = \pi r^2 + \frac{120000}{r}$ either side of a positive *r* achieved in (b) A1:At $r = a \Rightarrow S_{26.7} < S_a$ at $r = a \Rightarrow S_{26.7} < S_b$ hence Minimum (where a < 26.7 < b) Past Paper

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Figure 6

Figure 6 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-2)$$

The point *P* lies on *C* and has *x* coordinate $\frac{1}{2}$

The line l, as shown on Figure 6, is the tangent to C at P.

(a) Find
$$\frac{dy}{dx}$$
 (2)

(b) Use part (a) to find an equation for *l* in the form ax + by = c, where *a*, *b* and *c* are integers.

The finite region R, shown shaded in Figure 6, is bounded by the line l, the curve C and the *x*-axis.

The line l meets the curve again at the point (2, 0)

(c) Use integration to find the exact area of the shaded region R.

(6)

(4)



Question Number	Scheme	Marks
16(a)	$y = x(x-1)(x-2) = x^3 - 3x^2 + 2x \Rightarrow \left(\frac{dy}{dx}\right) = 3x^2 - 6x + 2$	M1 A1
(b)	When $x = \frac{1}{2}, y = \frac{3}{8}$	(2) B1
	Sub $x = \frac{1}{2}$ into $\left. \frac{dy}{dx} \right _{x=\frac{1}{2}} = \frac{3}{4} - 3 + 2 = -\frac{1}{4}$	M1
	Uses gradient and $\left(\frac{1}{2}, \frac{3}{8}\right) \Rightarrow y - "\frac{3}{8}" = "-\frac{1}{4}"\left(x - \frac{1}{2}\right) \Rightarrow 4y + x = 2$	M1A1
(c)	Later	(4)
$(a) M1 AtteLooA1 \left(\frac{dy}{dy}\right)$	mpts to multiply out $x(x-1)(x-2)$ and differentiate each term. k for a cubic expression being differentiated into a quadratic expression $x^2 - 6x + 2$. Accept exact equivalents such as $\left(\frac{dy}{dx}\right) - 3x^2 - 4x - 2x + 2$	
(h) \sqrt{dx}	$\int -3x - 6x + 2$. Accept exact equivalents such as $\left(\frac{1}{dx}\right)^{-3x} - 4x - 2x + 2$.	
B1 Whe	$x = \frac{1}{2}, y = \frac{3}{8}$ oe	
M1 Subs	stitute $x = \frac{1}{2}$ into their $\frac{dy}{dx}$ and either states /uses this as a gradient $\left(\frac{dy}{dx}\right)$, not the	e y - coordinate
M1 Uses	s their gradient, $x = \frac{1}{2}$ and their $y = \frac{3}{8}$ to find the equation of the tangent.	
Loo	k for $y - \frac{3}{8} = \frac{-1}{4}(x-2)$	
If th	e form $y = mx + c$ is used the candidate must proceed as far as $c =$	
A1 $k(4$	y + x = 2) where k is an integer	
Alte	rnatively accept a statement that $a = 1, b = 4$ and $c = 2$ or multiples thereof.	
If the gradie	ent is found by using $\left(\frac{1}{2}, \frac{3}{8}\right)$ and (2,0), treat as a special case where candidates of	can potentially
score B1 M	0 M1 A0. They have not satisfied the demand of the question.	
If the candio	date uses $\frac{dy}{dx}\Big _{x=\frac{1}{2}}$ and the point (2,0) they can score all marks. Score as follows	
1st M1: Sub	estitute $x = \frac{1}{2}$ into their $\frac{dy}{dx}$ and either states /uses as a gradient $\left(\frac{dy}{dx}\right)$ and not the	e y - coordinate
2nd M1: U B1: Any	ses their gradient with $(2,0)$ to form the equation of the tangent correct equation followed by A1: $k(4y+x=2)$ where k is an integer.	

Question Number	Scheme	Marks
(c)	Candidate considers area under curve from $x = \frac{1}{2}$ and $x = 1$	
Main	$\int y dx = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}(+c)$	M1 A1
	Area under curve between $x = \frac{1}{2}$ and $x = 1 \left[\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right]_{\frac{1}{2}}^{1} = \left(\frac{7}{64} \right)$	M1
	Area under line between $x = \frac{1}{2}$ and $x = 2 = \frac{1}{2} \times \left(2 - \frac{1}{2}\right) \times \frac{3}{8} = \left(\frac{9}{32}\right)$	M1
	Area of $R = \frac{1}{2} \times \left(2 - \frac{1}{2}\right) \times \frac{3}{8} - \left[\left(\frac{1}{4} - 1 + 1\right) - \left(\frac{1}{64} - \frac{1}{8} + \frac{1}{4}\right)\right] = \frac{9}{32} - \frac{7}{64} = \frac{11}{64}$	dM1A1
		(6) (12 marks)

(c) Main scheme: Use if the candidate attempts to find the area under the curve

M1 Attempts $\int x(x-1)(x-2) dx$ by first multiplying and then integrating to a quartic form

A1 **Correct** and unsimplified =
$$\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}(+c)$$

M1 Uses limits of
$$x = \frac{1}{2}$$
 and $x = 1$ with their answer to $\int x(x-1)(x-2)dx$.

M1 Uses the correct method to find the area under the line between $x = \frac{1}{2}$ and x = 2

If the triangle is used then look for $\frac{1}{2} \times \left(2 - \frac{1}{2}\right) \times \left(\frac{3}{8}\right) = \left(\frac{9}{32}\right)$ or $\frac{1}{2} \times \frac{3}{2} \times \left(\frac{3}{8}\right) = \left(\frac{9}{32}\right)$

If the equation of the line is used it must be correct. Look for an evaluation of

 $\int_{\underline{1}}^{\underline{1}} \left(\frac{1}{2} - \frac{1}{4}x\right) dx = \left[\frac{1}{2}x - \frac{1}{8}x^2\right]_{\underline{1}}^2$ with correct limits and correct integration

- dM1 Score for the correct combination of areas including the limits...... in this case the above areas must be subtracted. It is dependent upon all 3 M's
- A1 $\frac{11}{64}$

Question Number	Scheme	Marks
(c)	Candidate considers area between line and curve $x = \frac{1}{2}$ to $x = 1$	
ALT 1	$\int \left(\left(\frac{1}{2} - \frac{1}{4}x^{*}\right) - \left(x(x-1)(x-2)\right) dx = \left(\frac{1}{2}x - \frac{1}{8}x^{2}\right) - \left(\frac{x^{4}}{4} - 3\frac{x^{3}}{3} + 2\frac{x^{2}}{2}\right)$	M1A1
	Area between line and curve between $x = \frac{1}{2}$ and $x = 1$ is	
	$\left[\left(\frac{1}{2}x - \frac{1}{8}x^2\right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}\right)\right]_{\frac{1}{2}}^{1} = \left(\frac{3}{64}\right)$	M1
	Area under line between $x = 1$ and $x = 2 = \frac{1}{2} \times (2-1) \times \frac{1}{4} = \frac{1}{8}$	M1
	Area of $\mathbf{R} = \frac{1}{8} + \left[\left(\frac{1}{8} \right) - \left(\frac{5}{64} \right) \right] = \frac{1}{8} + \frac{3}{64} = \frac{11}{64}$	dM1A1
		(6)

- M1 Attempts $\int \left(\left\| \frac{1}{2} \frac{1}{4} x \right\| \right) (x(x-1)(x-2)) dx$ either way around, multiplies out and integrates to a quartic form
- A1 Answer correct $\left(\frac{1}{2}x \frac{1}{8}x^2\right) \left(\frac{x^4}{4} 3\frac{x^3}{3} + 2\frac{x^2}{2}\right)$ or its simplified form $\left(\frac{1}{2}x \frac{9}{8}x^2 + x^3 \frac{1}{4}x^4\right)$ Do not follow through on their tangent line in this case

M1 Uses the limits
$$x = \frac{1}{2}$$
 and $x = 1$ with their answer to $\int \left(\frac{1}{2} - \frac{1}{4}x^{*} \right) - (x(x-1)(x-2)) dx$

M1 Uses the correct method to find the area under the line between x = 1 and x = 2If the triangle is used look for $\frac{1}{2} \times (2-1) \times \frac{1}{4} = \left(\frac{1}{8}\right)$

If the equation of the line is used it must be correct. Look for an evaluation of

$$\int_{1}^{2} \left(\frac{1}{2} - \frac{1}{4}x\right) dx = \left[\frac{1}{2}x - \frac{1}{8}x^{2}\right]_{1}^{2}$$
 with correct limits and integration.

dM1 Score for the correct combination of areas including limits....in this case the above areas must be added. It is dependent upon all 3 M's

A1
$$\frac{11}{64}$$

ALT 2Candidate considers area between line and curve from
$$x = \frac{1}{2}$$
 to $x = 2$.

$$\int \left(\frac{1}{2} - \frac{1}{4} x^n \right) - (x(x-1)(x-2)) dx = \left(\frac{1}{2} x - \frac{1}{8} x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right)$$

Total area between line and curve $= \left[\left(\frac{1}{2} x - \frac{1}{8} x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right) \right]_{\frac{1}{2}}^2 = \left(\frac{27}{64} \right)$

Area under curve $= \left[\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right]_1^2 = \left(-\frac{1}{4} \right)$

Area of $\mathbf{R} = \frac{27}{64} - \frac{1}{4} = \frac{11}{64}$

M1

Attempts $\int \left(\frac{n}{2} - \frac{1}{4} x^n \right) - (x(x-1)(x-2)) dx$ either way around, multiplies out and integrates to a quartic form

A1

Answer correct $\left(\frac{1}{2} x - \frac{1}{8} x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right)$ or its simplified form $\left(\frac{1}{2} x - \frac{9}{8} x^2 + x^3 - \frac{1}{4} x^4 \right)$

Do not follow through on their tangent line in this case

M1

Uses the limits $x = \frac{1}{2}$ and $x = 2$ with their answer to $\int \left(\frac{n}{2} - \frac{1}{4} x^n \right) - (x(x-1)(x-2)) dx$

M1

Evaluates $\pm \int_{1}^{2} x(x-1)(x-2) dx = \pm \left[\frac{x^4}{4} - x^3 + x^2 \right]_{1}^{2}$

Both the limits and integration must be correct dM1

Score for the correct combination of areas including limits....in this case you need to see the equivalent to $\left(\frac{27}{64} \right) + -\frac{1}{4}$ or $\left(\frac{27}{64} \right) - \left| -\frac{1}{4} \right|$

It is dependent upon all 3 M's

A1

A1

A1

A1

B1

B2

B3

B3

B4

B5

B5

B4

Useful guide: (but please use the scheme and notes)

- 1) Scan through candidates answer to ascertain whether they are integrating just the curve or the line the curve to decide the method. Looking at their limits of integration can help you decide between alt 1 and alt 2
- 2) Marks are awarded like this
- M1: Attempt to integrate appropriate function to reach a quartic
- A1: Correct answer to the integration. There is no follow through
- M1: Correct limits used in the integrated function
- M1: Finds area under line (main or alt 1) either by using the correct triangle or using the correct f function and limits.
 - Finds area under curve (alt 2) using correct function and limits
- dM1: Correct combination of areas. It is dependent upon all 3 M's
- A1: Correct answer

If an incomplete method is shown always score by the method that gives the candidate more marks. If you cannot determine the method that a candidate has used then please use the review system and your team leader will advise.