Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C12
Advanced Subsidiary

Wednesday 25 May 2016 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/1/1/1/1/
P 4 6 7 1 3 R A 0 1 5 2

PEARSON

Leave blank

1. The first three terms in ascending powers of x in the binomial expansion of $(1 + px)^8$ are given by

$$1 + 12x + qx^2$$

where p and q are constants.

Find the value of p and the value of q.

(5)

Question Number	Scheme	Marks
1	$(1+px)^8 = 1+8(px)+\frac{8\times 7}{2!}(px)^2$	M1
	Compares coefficients in $x \Rightarrow 8p = 12 \Rightarrow p = 1.5$	M1A1
	Compares coefficients in $x^2 \Rightarrow q = 28p^2 \Rightarrow q = 63$	M1A1
		(5) **(5 marks)**

(a)

M1 Uses the power series expansion/ binomial expansion with the correct form for terms 2 and 3. You may ignore the first term in this question.
Accept the correct coefficient with the correct power of x for terms 2 and 3.

$(1+px)^8 = 1+8(..x)+\frac{8\times 7}{2!}(..x)^2$

Allow missing bracket on x^2 term.

Allow for $(1+px)^8 = 1+\binom{8}{1}(..x)+\binom{8}{2}(..x)^2$ or equivalent.

Allow sight of $\binom{8}{1}(..x)$ and $\binom{8}{2}(..x)^2$ separated by commas

M1 Sets their coefficient in x equal to 12 $\Rightarrow 8p = 12 \Rightarrow p = ...$
It is not dependent on the previous M but it must be of the form $kp = 12 \Rightarrow p = ...$

A1 $p = 1.5$ or equivalent such as $\frac{12}{8}$

M1 Sets q equal to their coefficient of x^2 (which must include a p or a p^2) then substitutes in their value of p leading to $q =$

A1 $q = 63$

Leave blank

2. Find the range of values of x for which

(a) $4(x - 2) \leqslant 2x + 1$

(2)

(b) $(2x - 3)(x + 5) > 0$

(3)

(c) **both** $4(x - 2) \leqslant 2x + 1$ **and** $(2x - 3)(x + 5) > 0$

(1)

Question Number	Scheme	Marks
2(a)	$4(x-2)\leqslant 2x+1 \Rightarrow 4x-8\leqslant 2x+1$	
	$\Rightarrow x\leqslant 4.5$	M1A1
		(2)
(b)	$(2x-3)(x+5)>0$	
	Roots are $1.5, -5$	B1
	Chooses outsides $x<-5, x>1.5$	M1A1
		(3)
(c)	$x<-5, 1.5<x\leqslant 4.5$	B1
		(1)
		(6 marks)

(a)

M1 Proceeds as far as $x\leqslant ...$ after firstly multiplying out brackets oe. Condone $x<...$ for this mark
Minimum expectation is that you see $4x-8...2x+1 \Rightarrow x...c$ where ... is $\leqslant$ or <

A1 $x\leqslant 4.5$ or equivalent in set notation such as $\{x : x\leqslant 4.5\}$ $x\in(-\infty, 4.5]$. Accept just $(-\infty, 4.5]$

(b)

B1 Critical values are 1.5, -5

M1 Chooses the outside values of their critical values. You may well see candidates multiply out the brackets and factorise incorrectly. They can score this mark for choosing the 'outsides'
Do not allow this mark from just a diagram. You must see the inequalities.
Accept for the method mark $x\leqslant -5, x\geqslant 1.5$

A1 Accept any of '$x<-5, x>1.5$' '$x<-5$ or $x>1.5$' '$\{x : x<-5 \cup x>1.5\}$'
'$\{x : -\infty<x<-5 \cup 1.5<x<\infty\}$' or their exact equivalents

Do not accept on its own (without seeing any of the above)
'$x<-5$ and $x>1.5$" '$-5>x>1.5$' '$\{x : x<-5 \cap x>1.5\}$'

(c)

B1 cao Accept any of
'$x<-5, 1.5<x\leqslant 4.5$' '$x<-5$ or $1.5<x\leqslant 4.5$' '$\{x : x<-5 \cup 1.5<x\leqslant 4.5\}$'
'$x\in -\infty<x<-5 \cup 1.5<x\leqslant 4.5$' or their exact equivalents.

There must be just two distinct regions represented by just two inequalities.
If a candidate writes $x\leqslant 4.5$ $x<-5, x>1.5$ it is B0

Leave blank

3. Answer this question without a calculator, showing all your working and giving your answers in their simplest form.

(i) Solve the equation

$$4^{2x+1} = 8^{4x}$$

(3)

(ii) (a) Express

$$3\sqrt{18} - \sqrt{32}$$

in the form $k\sqrt{2}$, where k is an integer.

(2)

(b) Hence, or otherwise, solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n}$$

(2)

Question Number	Scheme	Marks
3 (i)	Either $4^{2x+1}=2^{2(2x+1)}$ and $8^{4x}=2^{3\times 4x}$ or $8^{4x}=4^{\frac{3}{2}\times 4x}$	M1
	$2(2x+1)=12x \Rightarrow x=\dfrac{1}{4}$	dM1A1
		(3)
(ii)(a)	$3\sqrt{18}-\sqrt{32}=9\sqrt{2}-4\sqrt{2}=5\sqrt{2}$	M1A1
		(2)
(b)	$\sqrt{n}=5\sqrt{2}\Rightarrow n=\left(5\sqrt{2}\right)^2=25\times 2=50$	M1A1
		(2)
		(7 marks)
Alt 3 (i)	Taking logs of both sides and proceeding to $(2x+1)\log 4=4x\log 8$ $\Rightarrow x=\dfrac{\log 4}{4\log 8-2\log 4}$	M1
	$\Rightarrow x=\dfrac{\log 4}{\log 256}=\dfrac{1}{4}$	dM1A1
		(3)

(i)

M1 Writes both sides as powers of 2 or equivalent Eg $2^{2(2x+1)}=2^{3\times 4x}$

Alternatively writes both sides as powers of 4 or 8 or 64. Eg $8^{4x}=4^{\frac{3}{2}\times 4x}$

Note that expressions such as $2^{2+(2x+1)}=2^{3+4x}$ would be M0

Condone poor (or missing) brackets $2^{2\times 2x+1}=2^{3}$ but not incorrect index work eg $4^{2x+1}=8^{\frac{1}{2}(2x+1)}$

It is possible to use logs. most commonly with base 2 or 4. Using logs it is for reaching a linear form of the equation, again condoning poor bracketing .

$4^{2x+1}=8^{4x}\Rightarrow \log 4^{2x+1}=\log 8^{4x}\Rightarrow (2x+1)\log 4=4x\log 8$

dM1 Dependent upon the previous M. It is for equating the indices and proceeding to $x=..$

Condone sign/bracketing errors when manipulating the equation but not processing errors

If logs are used they must be evaluated without a calculator. Lengthy decimals would be evidence of this and would be dM0

$(2x+1)\log_2 4=4x\log_2 8\Rightarrow (2x+1)\times 2=4x\times 3\Rightarrow x=..$

$4^{2x+1}=8^{4x}\Rightarrow 2x+1=4x\log_4 8\Rightarrow 2x+1=\dfrac{3}{2}\times 4x\Rightarrow x=..$ is fine

A1 $x=\dfrac{1}{4}$ or equivalent

(ii)(a) Mark part (ii) as one complete question. Marks in (a) can be gained from (b)

M1 Writes either $\sqrt{18}=3\sqrt{2}$ or $3\sqrt{18}=9\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$

If the candidate writes $3\sqrt{18}-\sqrt{32}=k\sqrt{2}$ it can be scored for $\dfrac{3\sqrt{18}}{\sqrt{2}}=9$ or $\dfrac{\sqrt{32}}{\sqrt{2}}=4$

A1 $5\sqrt{2}$ or states $k=5$

The answer without working (the M1) would be 0 marks

(ii)(b)

M1 Moves from $\sqrt{n}=k\sqrt{2}$ to $n=2k^2$

Also accept for this mark $\sqrt{n}=\sqrt{50}$ or indeed $\sqrt{50}$ on its own

A1 $(n=)50$

Leave blank

4.

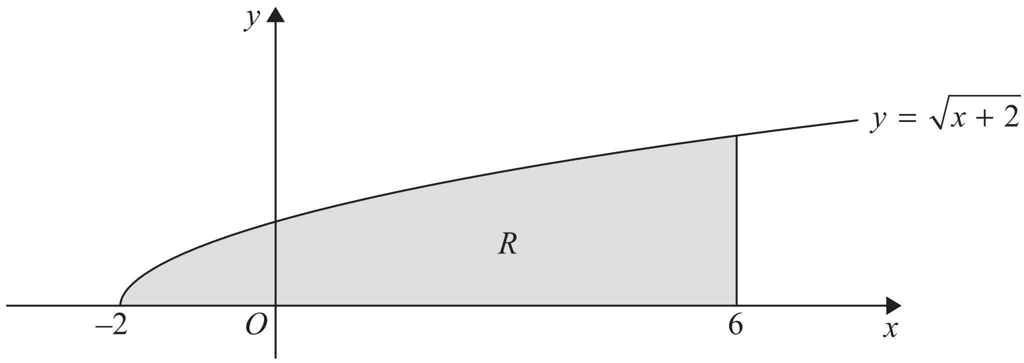


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{x+2}$, $x \geqslant -2$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 6$

The table below shows corresponding values of x and y for $y = \sqrt{x+2}$

x	–2	0	2	4	6
y	0	1.4142	2		2.8284

(a) Complete the table above, giving the missing value of y to 4 decimal places.
(1)

(b) Use the trapezium rule, with all of the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 3 decimal places.
(3)

Use your answer to part (b) to find approximate values of

(c) (i) $\displaystyle\int_{-2}^{6} \frac{\sqrt{x+2}}{2}\, dx$

(ii) $\displaystyle\int_{-2}^{6} \left(2 + \sqrt{x+2}\right) dx$
(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question Number	Scheme	Marks
4(a)	Awrt 2.4495	B1 **(1)**
(b)	Strip width =2	B1
	Area $\approx \dfrac{\text{'Strip width'}}{2}\{0+2.8284+2\times(1.4142+2+\text{'}2.4495\text{'})\}=$ awrt 14.556	M1A1 **(3)**
(c)(i)	$\frac{1}{2}(\text{b})=7.278$ Allow awrt 7.28	B1ft **(1)**
(ii)	$\int_{-2}^{6} 2\,\mathrm{d}x=[2x]_{-2}^{6}=12-(-4)=16$	M1
	$\int_{-2}^{6}\left(2+\sqrt{x+2}\right)\mathrm{d}x=\int_{-2}^{6}2\,\mathrm{d}x+(\text{b})=16+(\text{b})=30.556$ Allow awrt 30.56	dM1 A1ft **(3)**
		(8 marks)

(a)

B1 Awrt 2.4495

(b)

B1 Uses a strip width of 2 units. This may be embedded in the trapezium formula.
It may be implied by sight of $\frac{2}{2}\{\text{...........}\}$ or $1\times\{\text{...........}\}$

M1 Uses the correct form of the bracket within the trapezium rule.
Look for $(0)+2.8284+2\times(1.4142+2+\text{'}2.4495\text{'})$

A1 awrt 14.556

If separate trapezia are used then you should see the sum of 4 trapezia. B1 Width or h =2 and M1 for the correct values of 'a' and 'b' for each one.

(c)(i)

B1ft Look for $\frac{1}{2}(\text{b})$. Follow through on their answer to (b) but you may allow accuracy to 2dp
Also allow from adapting their trapezium rule in part (b) with all terms halved

(c)(ii)

M1 Look for an attempt to either integrate '2' by writing $[2x]_{-2}^{6}$ oe. It may be embedded within a longer integral. Alternatively finds the area of a rectangle with height 2 and length of $(6-(-2))$ which is implied by the sight of 16
Do not allow $\int 2\,\mathrm{d}x=2$
Allow an attempt to use the trapezium rule in part (b) with 2 being added to each value inside the bracket.

dM1 Adds together their answer for the integral of 2 to their answer for (b).
It is dependent upon the previous M
If their trapezium rule is adapted then this mark is scored at the point when the value is calculated

A1ft $16+(\text{b})$. Accept this for all 3 marks as long as no incorrect working seen. Allow accuracy to 2dp.

Leave blank

5. (i)
$$U_{n+1} = \frac{U_n}{U_n - 3}, \quad n \geqslant 1$$

Given $U_1 = 4$, find

(a) U_2

(1)

(b) $\sum_{n=1}^{100} U_n$

(2)

(ii) Given

$$\sum_{r=1}^{n} (100 - 3r) < 0$$

find the least value of the positive integer n.

(3)

Question Number	Scheme	Marks
5(i)		
(a)	$(U_2) = \dfrac{4}{4-3} = 4$	B1
		(1)
(b)	$\sum_{n=1}^{100} U_n = 100 \times 4 = 400$	M1A1
		(2)
5(ii)	$\sum_{r=1}^{n}(100-3r) < 0 \Rightarrow 97+94+91+...(100-3r) < 0$	
	$\sum$AP with $a=97, d=-3, n=n, S<0 \Rightarrow 0 = \frac{n}{2}(2\times 97+(n-1)\times -3) < 0$	M1
	$\Rightarrow \frac{n}{2}(197-3n) < 0 \Rightarrow n > 65.\dot{6}$	dM1
	$\Rightarrow n = 66$	A1
		(3) **(6 marks)**
(ii) ALT I	$\sum_{r=1}^{n}(100-3r) < 0 \Rightarrow \sum_{r=1}^{n} 3r > \sum_{r=1}^{n} 100$	
	$\Rightarrow 3\dfrac{n(n+1)}{2} > 100n$	M1 M1A1
	$\Rightarrow n > 65.\dot{6} \Rightarrow n = 66$	

(i)(a)

B1 States that U_2 is 4. Accept $\frac{4}{1}$ but not $\frac{4}{4-3}$ and remember to isw.

Note that $U_1 = 4$ so be sure that you don't award this B1

(i)(b)

M1 Uses the method that $\sum_{n=1}^{100} U_n = k \times 4$ where $k = 100$ or 99

You may see the AP formula being used which is fine as long as $a = 4,\ d = 0$ and $n = 99/100$

Look for expression of the form $\frac{100}{2}\{2\times 4 + 99\times 0\}$ OR $\frac{100}{2}\{4+4\}$

A1 400

(ii)

M1 Uses $S...\frac{n}{2}(2a+(n-1)d)$ with ... as $=$ or $>$ or $<$ and $S=0$ $a=97$ or 100, $d=-3$

Alternatively uses $S...\frac{n}{2}(a+l)$ with ... as $=$ or $>$ or $<$ and $S=0$ $a=97$ or 100, $l=100-3n$

A solution should not appear to come from the value of the nth term or in fact any linear equation. This is M0

dM1 Dependent upon previous M. Scored for a solution of their quadratic equation in *n*.
Accept n = , n > , n<

A1 $n=66$ cso

....

Part (ii) Alt 1 Using the formula $\sum_{1}^{n} r=\frac{n(n+1)}{2}$

M1 For attempting to solve $\sum_{r=1}^{n} 3r ... \sum_{r=1}^{n} 100$ where .. is = or $>$ or $<$ by writing $\sum_{1}^{n} r=\frac{n(n+1)}{2}$ and $\sum_{r=1}^{n} 100=100n$. Condone the 3 in the $\sum_{r=1}^{n} 3r$ "disappearing"

dM1 Dependent upon previous M. Scored for a solution to their quadratic equation in *n*.
Accept n = , n > , n<

A1 $n=66$ cso

....

Part (ii) Alt 2 Using the fact that the sum is (less than) zero when the negative terms add up to more than the positive terms in the sequence

M1 Sum of positive terms = $97+94+..............+1=\frac{33}{2}(97+1)=[1617]$

Sum of negative terms = $-2+-5........$

$$\frac{n}{2}\{2\times 2+3(n-1)\}>'1617'$$

dM1 Dependent upon last mark. It is for solving the quadratic equation with usual methods and adding on 33 terms (the number of positive terms)

$\frac{n}{2}\{2\times 2+3(n-1)\}>'1617' \Rightarrow n>32.$ So total number of terms is 33+33=66

A1 cso 66

Leave blank

6. (a) Show that $\dfrac{x^2 - 4}{2\sqrt{x}}$ can be written in the form $Ax^p + Bx^q$, where A, B, p and q are constants to be determined.

(3)

(b) Hence find

$$\int \frac{x^2 - 4}{2\sqrt{x}}\,\mathrm{d}x, \quad x > 0$$

giving your answer in its simplest form.

(4)

Question Number	Scheme	Marks
6(a)	$\dfrac{x^2-4}{2\sqrt{x}}=\dfrac{x^2}{2\sqrt{x}}-\dfrac{4}{2\sqrt{x}}=\dfrac{1}{2}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}$	M1A1A1
		(3)
(b)	$\displaystyle\int\frac{x^{\frac{3}{2}}}{2}-2x^{-\frac{1}{2}}\,\mathrm{d}x=\frac{x^{\frac{5}{2}}}{2\times 2.5}-2\frac{x^{\frac{1}{2}}}{0.5}(+c)$	M1 A1ft A1
	$=\dfrac{x^{\frac{5}{2}}}{5}-4x^{\frac{1}{2}}+c$	B1
		(4) **(7 marks)**

(a)

M1 Attempt to divide by $2\sqrt{x}$ to get exactly two terms (not three)
This can be implied by any of A, B, p or q being correct.

A1 Two of A, B, p or q correct. Look for two of the four numbers $\dfrac{x^{\frac{3}{2}}}{2}-2x^{-\frac{1}{2}}$ oe

Allow the power as $\dfrac{3}{2}$ appearing as $2-\dfrac{1}{2}$

A1 Completely correct expression $\dfrac{x^{\frac{3}{2}}}{2}-2x^{-\frac{1}{2}}$ or equivalent. Eg accept $0.5x^{1.5}-2x^{-0.5}$
The powers must now be simplified.

(b)

M1 Increases a fractional index by one. Do not allow if the candidate integrates the numerator and denominator of the original function

A1ft One of the fractional terms correct unsimplified.
You may follow through on any term with a fractional index.

A1 Both terms correct unsimplified
Allow the powers and coefficients such as $\dfrac{5}{2}$ appearing as $\left(\dfrac{3}{2}+1\right)$ for this mark

B1 $=\dfrac{x^{\frac{5}{2}}}{5}-4x^{\frac{1}{2}}+c$ or exact equivalent such as $0.2x^{2.5}-4x^{0.5}+c$

Leave blank

7.

$$f(x) = 3x^3 + ax^2 + bx - 10, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that $(x - 2)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $2a + b = -7$

(2)

Given also that when $f(x)$ is divided by $(x + 1)$ the remainder is -36

(b) find the value of a and the value of b.

(4)

$f(x)$ can be written in the form

$$f(x) = (x - 2)Q(x), \text{ where } Q(x) \text{ is a quadratic function.}$$

(c) (i) Find $Q(x)$.

(ii) Prove that the equation $f(x) = 0$ has only one real root.

You must justify your answer and show all your working.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question Number	Scheme	Marks
7(a)	Sets $\mathrm{f}(\pm 2)=0$	M1
	$\mathrm{f}(2)=0 \Rightarrow 24+4a+2b-10=0 \Rightarrow 4a+2b=-14 \Rightarrow 2a+b=-7$	A1*
		(2)
(b)	Sets $\mathrm{f}(\pm 1)=-36$	M1
	$\mathrm{f}(-1)=-36 \Rightarrow -3+a-b-10=-36 \Rightarrow a-b=-23$ oe	A1
	Solves simultaneously to get both a and b	dM1
	$a=-10, \quad b=13$	A1
		(4)
(c)(i)	Divides their f(x) by $(x-2)$ to get a quadratic	M1
	$\mathrm{f}(x)=(x-2)(3x^2-4x+5)$ or $\mathrm{Q}(x)=3x^2-4x+5$	A1
(ii)	Calculates b^2-4ac on their $\mathrm{Q}(x)$ or solves their Q $(x)=0$	M1
	$(3x^2-4x+5)$ has no roots as $b^2-4ac=16-60<0$. Hence f(x) has 1 root	A1*
		(4)
		(10 marks)

(a)

M1 Sets $f(\pm 2)=0$

The = 0 may be implied by later working.

A1* $f(2)=0\Rightarrow 2a+b=-7$. **There must be at least one intermediate line between these two statements**. Accept $f(2)=0\Rightarrow 3\times 2^3+a\times 2^2+b\times 2-10=0\Rightarrow 2a+b=-7$

Note that this is a given result.

(b)

M1 Sets $f(\pm 1)=-36$

If a candidate attempts division look for a minimum of

$$x+1\overline{)3x^3+ax^2+bx-10}\quad\text{quotient: }3x^2+(a-3)x......$$

$$\overline{\qquad\qquad}$$
$$f(a,b)$$

before the candidate sets their remainder, which must be a function of a and b equal to -36

A1 $f(-1)=-36\Rightarrow a-b=-23$ or equivalent. The equation does not need to be simplified but the indices must have been dealt with correctly

In division the remainder may appear in the form $-10-(b-a+3)=-36$

dM1 Solves simultaneously to get values for both *a* and *b*. Don't be too concerned with the process as long as you see values for *a* and *b* coming from two equations in both *a* and *b*. It is dependent upon the previous M

A1 $a=-10,\quad b=13$

(c)(i)

M1 For dividing their f(*x*) by $(x-2)$ to get a $f(x)=(x-2)\times$ quadratic

If it is attempted by division look for the first two terms following through on their *a*

$$x-2\overline{)3x^3+'a'x^2+'b'x-10}\quad\text{quotient: }3x^2+(a+6)x......$$

If it attempted by inspection only check first and last terms. Look for $f(x)=(x-2)\left(3x^2...x+5\right)$

A1 $f(x)=(x-2)\left(3x^2-4x+5\right)$ or $Q(x)=3x^2-4x+5$

(c)(ii)

M1 Scored for an attempt at finding the number of roots of **their** 3TQ $Q(x)$.

This can be attempted by calculating the value of their b^2-4ac, Eg $b^2-4ac=16-60=-44$

or using the formula/ completing the square to find the roots. Eg. $ax^2+bx+c=0\Rightarrow x=..,..$

A1* cso. All aspects must be correct including $\mathbf{f(x)=(x-2)\left(3x^2-4x+5\right)}$ **with proof** that $\left(3x^2-4x+5\right)$ has no roots **and hence** f(*x*) has only 1 root (at *x*=2)

Leave blank

8. In this question the angle θ is measured in degrees throughout.

(a) Show that the equation

$$\frac{5 + \sin\theta}{3\cos\theta} = 2\cos\theta, \qquad \theta \neq (2n+1)90°, \quad n \in \mathbb{Z}$$

may be rewritten as

$$6\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(b) Hence solve, for $-90° < \theta < 90°$, the equation

$$\frac{5 + \sin\theta}{3\cos\theta} = 2\cos\theta$$

Give your answers to one decimal place, where appropriate.

(4)

Question Number	Scheme	Marks
8(a)	$\left(\dfrac{5+\sin\theta}{3\cos\theta}=2\cos\theta\right)\Rightarrow 5+\sin\theta=6\cos^2\theta$	M1
	$\Rightarrow 5+\sin\theta=6(1-\sin^2\theta)\Rightarrow 6\sin^2\theta+\sin\theta-1=0$	dM1A1*
		(3)
(b)	$6\sin^2\theta+\sin\theta-1=0\Rightarrow(3\sin\theta-1)(2\sin\theta+1)=0$	M1
	$(\sin\theta)=+\dfrac{1}{3},-\dfrac{1}{2}$	A1
	$\theta=19.5°,-30°$	dM1,A1
		(4)
		(7 marks)

(a)

M1 Attempts to cross multiply to form an equation in the form $5+\sin\theta=A\cos^2\theta$

dM1 Dependent upon previous M. For using $\cos^2\theta=\pm1\pm\sin^2\theta$ to get an equation in just $\sin\theta$

A1* This is a given answer. All aspects must be correct.
Mixed variables, say x's and θ's would lose this mark.
An otherwise correct solution with $\cos^2\theta$ or $(\cos\theta)^2$ written as $\cos\theta^2$ would also be M1 dM1 A0

(b)

M1 Attempts to factorise, usual rules. Accept answers by formula or just written down from a calculator. Accept answers/factors given in terms of x for this mark

A1 For the values $+\frac{1}{3},-\frac{1}{2}$
These do not have to be simplified and can be implied by correct answers for θ

dM1 Calculates at least one value of θ from their '$\sin\theta$'.
It is dependent upon the previous M. You may have to check with a calculator.
This may be implied by either $\theta=19.5°,-30°$ from a correct quadratic

A1 Both $\theta=19.5°$ and $-30°$ with this accuracy and no additional solutions inside the range.
Condone the answer $\theta=19.5°$ and $-30.0°$
Ignore any solutions outside the range.

Leave blank

9. The first term of a geometric series is 6 and the common ratio is 0.92

For this series, find

(a) (i) the 25$^{\text{th}}$ term, giving your answer to 2 significant figures,

(ii) the sum to infinity.

(4)

The sum to n terms of this series is greater than 72

(b) Calculate the smallest possible value of n.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question Number	Scheme	Marks
9 (a)(i)	Attempts to use $u_n = ar^{n-1} \Rightarrow u_{25} = 6\times 0.92^{24} =$ awrt 0.81	M1A1
(ii)	Attempts to use $S_\infty = \dfrac{a}{1-r} \Rightarrow S_\infty = \dfrac{6}{1-0.92} = 75$	M1A1
		(4)
(b)	Sets $S_n > 72 \Rightarrow \dfrac{6(1-0.92^n)}{1-0.92} > 72$ Accept $\dfrac{6(1-0.92^n)}{1-0.92} = 72$	M1
	$0.92^n < 0.04$ Accept $0.92^n = 0.04$	A1
	Takes log's $n > \dfrac{\log 0.04}{\log 0.92}$ Accept $n = \dfrac{\log 0.04}{\log 0.92}$	dM1
	n=39	A1
		(4)

(a) (i)

M1 Attempts 6×0.92^{24} or $6\times 0.92^{25-1}$

A1 awrt 0.81

(a)(ii)

M1 Attempts to use $S_\infty = \dfrac{a}{1-r}$ with a =6 and r=0.92 $S_\infty = \dfrac{6}{1-0.92}$

A1 75

(b)

M1 Sets $S_n > 72 \Rightarrow \dfrac{6(1-0.92^n)}{1-0.92} > 72$ Accept $\dfrac{6(1-0.92^n)}{1-0.92} = 72$ or $\dfrac{6(1-0.92^n)}{1-0.92} < 72$

A1 Proceeds to $0.92^n \,....\, 0.04$ where .... is either = or > or <

dM1 Proceeds from $a^n \,....\, b$ to $n \,....\, \log_a b$ or $n \,....\, \dfrac{\log b}{\log a}$ where ….is =, > or <

The values of a and b must be positive

Allow this mark in cases where the candidate has incorrectly multiplied 6 by 0.92^n to form an equation in the form $5.52^n \,....\, k$ (k>0)and solves by correctly taking logs

A1 cso n =39. All aspects must be correct including the inequalities on each line if they have been used. It is acceptable to use '=' however

Do not accept n = 38.6 for this mark

....

Note: Trial and Improvement can gain all 4 marks as long as n =39 is stated

M1A1 For sight of a trial at 38 or 39 using the sum formula

$\dfrac{6(1-0.92^{38})}{1-0.92} =$ awrt 71.8 or $\dfrac{6(1-0.92^{39})}{1-0.92} =$ awrt 72.1

M1A1 For trial at 38 and 39 using the sum formula, and conclusion that n=39

Leave blank

10. The curve C has equation $y = \sin\left(x + \frac{\pi}{4}\right), \quad 0 \leqslant x \leqslant 2\pi$

(a) On the axes below, sketch the curve C.

(2)

(b) Write down the exact coordinates of all the points at which the curve C meets or intersects the x-axis and the y-axis.

(3)

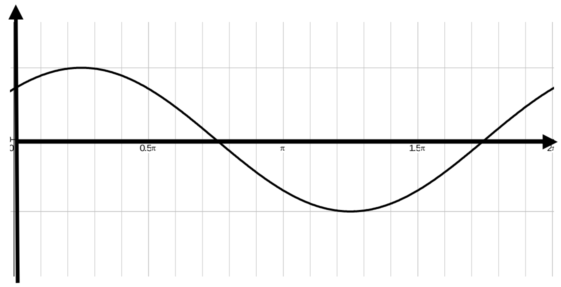
(c) Solve, for $0 \leqslant x \leqslant 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answers in the form $k\pi$, where k is a rational number.

(4)

P 4 6 7 1 3 R A 0 2 6 5 2

Question Number	Scheme	Marks
10(a)	 2π Harmonic shape	M1
	Position	A1
		(2)
(b)	$\left(0,\dfrac{\sqrt{2}}{2}\right),\left(\dfrac{3\pi}{4},0\right),\left(\dfrac{7\pi}{4},0\right)$	B1 B1 B1
		(3)
(c)	$\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{\sqrt{3}}{2} \Rightarrow x=\arcsin\left(\dfrac{\sqrt{3}}{2}\right)-\dfrac{\pi}{4}=\dfrac{\pi}{3}-\dfrac{\pi}{4}=\dfrac{\pi}{12}$	M1A1
	$x=\pi-\dfrac{\pi}{3}-\dfrac{\pi}{4}=\dfrac{5\pi}{12}$	M1A1
		(4)
		(9 marks)

(a) Look only at the part of the graph between 0 and 2π .
If there is no x scale shown, then assume that 2π is at the point where their graph stops.
If the sketch continues past the x label on the x axis, assume 2π is at the point of the x of the x label.

M1 Graph is harmonic. Could be part of a cycle, or several cycles but needs at least one max and one min in the range $0 \leqslant \theta \leqslant 2\pi$. Do not penalise graphs which are not in the correct position in relation to either axis

A1 Requires only one cycle between 0 and 2π .
The sketch needs: a positive y intercept and a positive y value at 2π , with a positive gradient at both 0 and 2π .The maximum must be positive and the minimum must be negative.

(b) B1B1B1 Score 1,1,0 for **ANY** 2 out of 3 and 1,0,0 for any 1 out of 3
Each answer is cao. No decimals are accepted unless the exact value is also given.
Accept points marked on the graph.
Accept coordinates with zeroes (as given) or for example $\dfrac{\sqrt{2}}{2}$ marked on the y axis or an obvious statement such as y intercept is ... or $y =$
For both correct answers in degrees withhold one mark.
If there are extra answers in the range, in addition to the correct answers, withhold one mark.
If answers are given in text and on diagram, the text takes precedence.

(c)

M1 Takes invsin and subtracts $\dfrac{\pi}{4}$ in an attempt to find one value of x. Look for $x=\dfrac{\pi}{3}-\dfrac{\pi}{4}\Rightarrow x=..$
Accept answers in degrees ($x=60-45\Rightarrow x=..$) and decimals for the method mark.
Do not accept mixed degrees/radians.

A1 One of $\dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$

M1 Correct attempt to get a second value of x from their first value. Accept $x+\dfrac{\pi}{4}=\pi-their\dfrac{\pi}{3}\Rightarrow x=..$
Accept degrees or decimals for this method mark

A1 Both $\dfrac{\pi}{12}$ and $\dfrac{5\pi}{12}$ and no other values within the range. Ignore additional answers outside the range.

Leave blank

11.

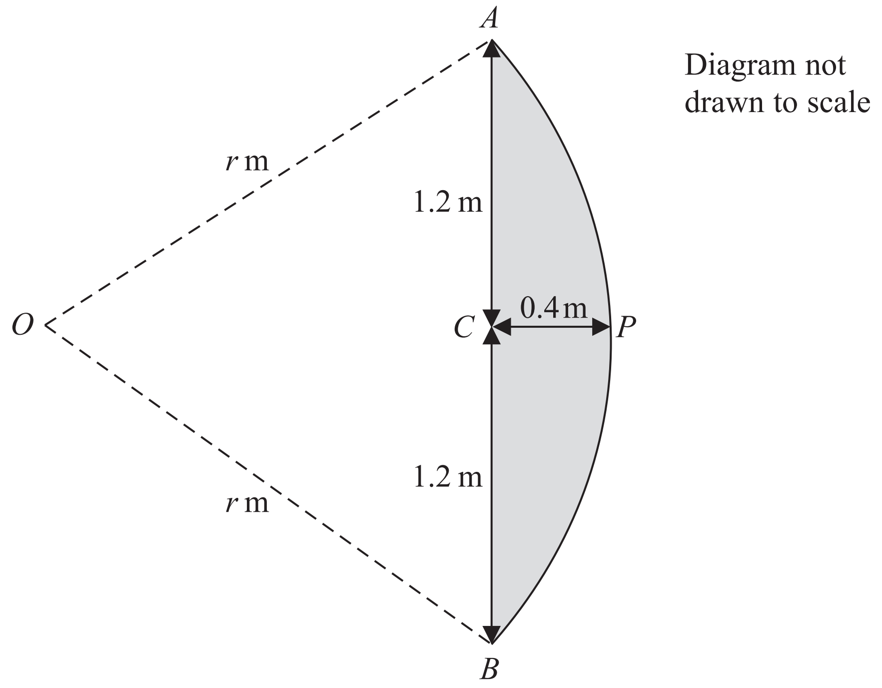


Figure 2

Figure 2 shows the design for a sail *APBCA*.

The curved edge *APB* of the sail is an arc of a circle centre *O* and radius *r* m.

The straight edge *ACB* is a chord of the circle.

The height *AB* of the sail is 2.4 m.

The maximum width *CP* of the sail is 0.4 m.

(a) Show that $r = 2$

(2)

(b) Show, to 4 decimal places, that angle $AOB = 1.2870$ radians.

(2)

(c) Hence calculate the area of the sail, giving your answer, in m^2, to 3 decimal places.

(4)

Question Number	Scheme	Marks
11 (a)	States $r^2 = 1.2^2 + (r-0.4)^2$	M1
	$0.8r = 1.60 \Rightarrow r = 2.$	A1*
		(2)
(b)	Attempt to find the angle or 1/2 angle	
	$'\frac{1}{2}\theta' = \arcsin\left(\frac{1.2}{2}\right) \Rightarrow '\frac{1}{2}\theta' = awrt\,37°\quad awrt\;0.64\;\text{rads}$	M1
	cso $AOB = 1.2870$	A1*
		(2)
(c)	Attempts to find area of sector using degree or radian formula	
	Area sector $= \frac{'73.74'}{360} \times \pi \times 2^2$ or $\frac{1}{2} \times 2^2 \times '1.2870'$	M1
	Attempts area of triangle using the correct formula	
	$Area = \frac{1}{2} \times 2 \times 2 \times \sin '73.74°'$ or $Area = \frac{1}{2} \times 2 \times 2 \times \sin '1.2870'$	M1
	Area of sail = Sector -Triangle using the correct combination	
	$= \frac{1}{2} \times 2^2 \times '1.29' - \frac{1}{2} \times 2^2 \times \sin '1.29' = 0.654\left(m^2\right)$	dM1, A1
		(4)
		(8 marks)

(a)

M1 Attempts Pythagoras with lengths r, $(r-0.4)$ and 1.2 in the correct positions within the formula

Alternatively uses the given answer and attempts Pythagoras' theorem with $1.6, 1.2$ *and* 2

A1* Proceeds to $r = 2$ with no errors

If the alternative method is used, then there must be a statement such as hence true, $r = 2$

(b)

M1 Attempts to find either the angle or half angle in the sector in either degrees or radians

For the half angle accept $\arctan\left(\frac{1.2}{1.6}\right)$, $\arcsin\left(\frac{1.2}{2}\right)$, $\arccos\left(\frac{1.6}{2}\right)$

For the whole angle accept $2.4^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos\theta \Rightarrow \theta = ..$ or similar.

A1* cso. This is a given answer $AOB = 1.2870$

Allow from a value where ½ the angle lacks 4 dp of accuracy

(c)

M1 Correct method to find the area of the sector with radius 2 and angle 1.2870 radians

If angle was found in degrees use $= \frac{'73.74'}{360} \times \pi \times 2^2 = (2.574)$

If angle was found in radians use $= \frac{1}{2} \times 2^2 \times '1.2870'$

M1 Correct method to find the area of the isosceles triangle with lengths 2 and angle 1.2870

For the triangle *AOB* accept $\frac{1}{2} \times 2 \times 2 \times \sin '73.74'$ or $\frac{1}{2} \times 2 \times 2 \times \sin '1.2870' = (1.92)$

Also for triangle *AOB* accept use of $\frac{1}{2}bh = \frac{1}{2} \times 2.4 \times 1.6$. Note $\frac{1}{2}bh = \frac{1}{2} \times 2.4 \times 2$ is M0

dM1 Attempts the area of the sail using the correct combinations of sector - triangle.

It is dependent upon both previous M's

If the candidate uses the segment formula $\frac{1}{2} \times 2^2 \times '1.29' - \frac{1}{2} \times 2^2 \times \sin '1.29'$ all three M's can be awarded

A1 awrt $0.654\left(m^2\right)$

Leave blank

12.

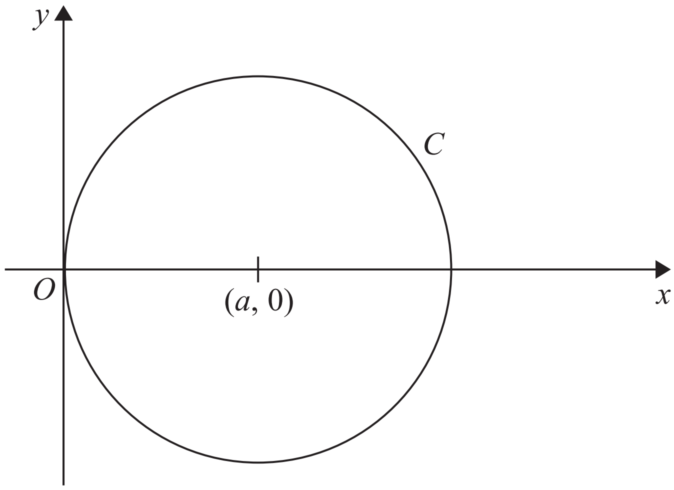


Figure 3

Figure 3 shows a circle C

C touches the y-axis and has centre at the point $(a, 0)$ where a is a positive constant.

(a) Write down an equation for C in terms of a

(2)

Given that the point $P(4, –3)$ lies on C,

(b) find the value of a

(3)

Question Number	Scheme	Marks
12(a)	Writes C as $(x-a)^2+(y-0)^2=a^2$	M1A1 **(2)**
(b)	Subs $(4,-3)$ $\Rightarrow (4-a)^2+(-3-0)^2=a^2$	M1
	$\Rightarrow 16-8a+a^2+9=a^2$	
	$\Rightarrow 25=8a$	
	$\Rightarrow a=\dfrac{25}{8}$	dM1A1 **(3)**
		(5 marks)

Mark parts (a) and (b) together. Award marks in (a) from (b) and vice versa, but see note

(a)

M1 Attempts to find the equation of C centre (a,0) radius a. Accept $(x\pm a)^2+y^2=a^2$ oe

If the alternative form of the circle is used accept $x^2+y^2\pm 2ax=a^2-a^2$

Allow for the M1 $(x\pm a)^2+(y\pm 0)^2=r^2$

A1 Writes C as $(x-a)^2+(y-0)^2=a^2$ or equivalent $x^2+y^2-2ax=0$.

(b)

M1 Subs $x=4$ and $y=-3$ into their circle equation for C which must be of the form $(x\pm a)^2+(y\pm 0)^2=a^2$

dM1 Proceeds to a linear equation in 'a' and reaches a=.... Condone numerical slips

A1 $a=\dfrac{25}{8}$ Accept exact alternatives

Note: There are some candidates who write the equation of the circle as $(x-a)^2+(y-0)^2=r^2$ in part (a)

This is M1 A0

However in part (b) they substitute $(4,-3)$ and write down $(4-a)^2+(-3)^2=a^2$

We will allow them to score all 3 marks in part (b).

Had they written $(x-a)^2+y^2=a^2$ in (b) we would allow them to score all 5 marks

Leave blank

13. (a) Show that the equation

$$2\log_2 y = 5 - \log_2 x \qquad x > 0, y > 0$$

may be written in the form $y^2 = \dfrac{k}{x}$ where k is a constant to be found.

(3)

(b) Hence, or otherwise, solve the simultaneous equations

$$2\log_2 y = 5 - \log_2 x$$

$$\log_x y = -3$$

for $x > 0, y > 0$

(5)

Question Number	Scheme	Marks
13 (a)	$2\log_2 y = 5 - \log_2 x \Rightarrow \log_2 y^2 = 5 - \log_2 x$	M1
	$\Rightarrow \log_2 y^2 = \log_2 32 - \log_2 x \Rightarrow \log_2 y^2 = \log_2\left(\dfrac{32}{x}\right)$	
	$\Rightarrow y^2 = \dfrac{32}{x}$	M1A1
		(3)
(b)	$\log_x y = -3 \Rightarrow y = x^{-3}$	M1
	Sub $y = x^{-3}$ into $y^2 = \dfrac{32}{x} \Rightarrow x^{-6} = \dfrac{32}{x} \Rightarrow x^5 = \dfrac{1}{32} \Rightarrow x = \dfrac{1}{2}$	M1A1
	Sub $x = \dfrac{1}{2}$ into either eqn $\Rightarrow y = 8$	M1A1
		(5) **(8 marks)**
Alt (b)	Sub $y^2 = \dfrac{32}{x}$ into $\log_x y = -3 \Rightarrow \log_x \sqrt{\dfrac{32}{x}} = -3$	2nd M1
	$\Rightarrow \sqrt{\dfrac{32}{x}} = x^{-3}$	1st M1
	$\Rightarrow x^5 = \dfrac{1}{32} \Rightarrow x = \dfrac{1}{2}$	A1

(a)

M1 Uses one correct log law.

Eg Uses the index law and writes $2\log_2 y = \log_2 y^2$.

Alternatively writes 5 as $\log_2 32$. This may well come from $\log_2 ... = 5 \Rightarrow ... = 32$

Note that $2\log_2 y + \log_2 x = 2\log_2 xy$ is M0

M1 Uses two correct log laws

Award for $\log_2 y^2 = \log_2(32) - \log_2 x$

or $\log_2 x + \log_2 y^2 = 5 \Rightarrow \log_2 xy^2 = 5$

A1 Proceeds correctly to $y^2 = \dfrac{32}{x}$

(b)

M1 Undoes the log in the second equation $\log_x y = -3 \Rightarrow y = x^{-3}$

This may well appear later in the solution

M1 Combines both equations to form a single equation in one variable.

A1 $x = \dfrac{1}{2}$ or $y = 8$. Condone a solution $y = \pm 8$ for this mark

M1 Substitutes their $x = \dfrac{1}{2}$ into an equation to find y.

Alternatively substitutes their $y = 8$ into an equation to find x

A1 $x = \dfrac{1}{2}$ and $y = 8$ only. Note $x = \dfrac{1}{2}$ and $y = \pm 8$ is A0

SC. If a candidate uses $y = \dfrac{k}{x}$ with $\log_x y = -3$ this can potentially score M1: Undoing logs, M0: as combining the equations has been made easier, A0: M1: If they substitute their x to find y and vice versa followed by A0: scoring 10010

Leave blank

14.

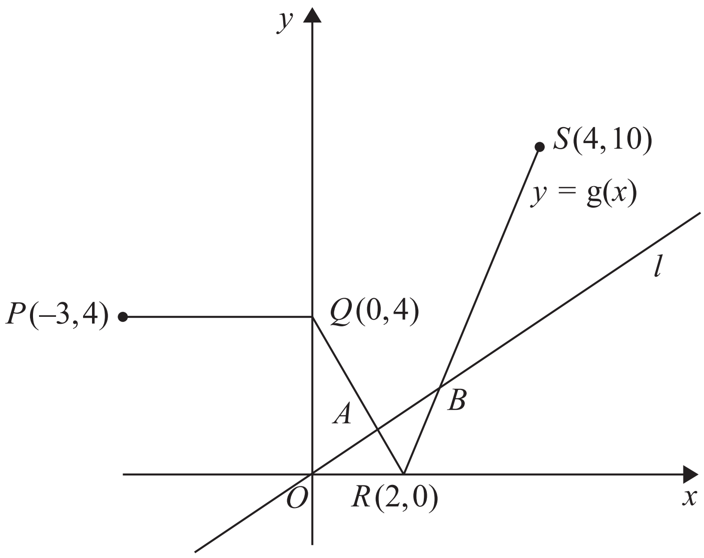


Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, $-3 \leqslant x \leqslant 4$ and part of the line l with equation $y = \frac{1}{2}x$

The graph of $y = \mathrm{g}(x)$ consists of three line segments, from $P(-3,4)$ to $Q(0,4)$, from $Q(0,4)$ to $R(2,0)$ and from $R(2,0)$ to $S(4,10)$.

The line l intersects $y = \mathrm{g}(x)$ at the points A and B as shown in Figure 4.

(a) Use algebra to find the x coordinate of the point A and the x coordinate of the point B.

Show each step of your working and give your answers as exact fractions.

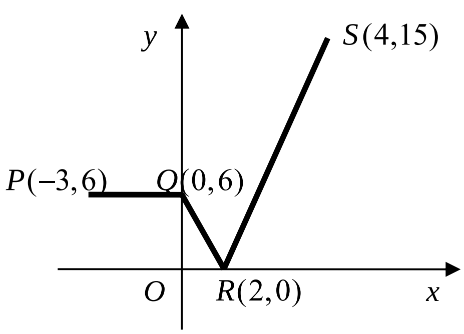
(6)

(b) Sketch the graph with equation

$$y = \frac{3}{2}\mathrm{g}(x), \quad -3 \leqslant x \leqslant 4$$

On your sketch show the coordinates of the points to which P, Q, R and S are transformed.

(2)

Question Number	Scheme	Marks
14(a)	Sets $\frac{1}{2}x = ax+4$ where $a<0$	M1
	Solves $\frac{1}{2}x = -2x+4 \Rightarrow \frac{5}{2}x = 4 \Rightarrow x = \frac{8}{5}$ oe	dM1A1
	Sets $\frac{1}{2}x = 5x+b$ where $b<0$	M1
	Solves $\frac{1}{2}x = 5x-10 \Rightarrow \frac{9}{2}x = 10 \Rightarrow x = \frac{20}{9}$ oe	dM1A1
		(6)
(b)	 Any two points correct	B1
	Same 'shape' with 4 points correct	B1
		(2)
		(8 marks)

(a)

M1 Attempts the smaller solution. Accept setting $\frac{1}{2}x = ax+4$ where $a<0$

dM1 Sets $\frac{1}{2}x = -2x+4$ and proceeds to $x = ..$ by collecting terms. Condone errors

A1 $x = \frac{8}{5}$ oe. Accept 1.6

M1 Attempts to find the larger solution. Accept setting $\frac{1}{2}x = 5x+b$ where $b<0$

dM1 Sets $\frac{1}{2}x = 5x-10$ and proceeds to $x = ..$ by collecting terms. Condone errors

A1 $x = \frac{20}{9}$ Accept exact equivalents such as $2\frac{2}{9}$ but not 2.2 or $2.\dot{2}$

(b)

B1 Any two points correct either in the text or on a sketch. Accept 6 and 2 written on the correct axes

B1 Shape + all four points correct.
Watch for candidates who adapt the given diagram. This is acceptable
A diagram can be labelled with *P*, *Q*, *R* and *S* and coordinates given for *P*,*Q*,*R* and *S* in the body of the script. If they are given on the diagram and in the body of the script the diagram takes precedence.

Leave blank

15.

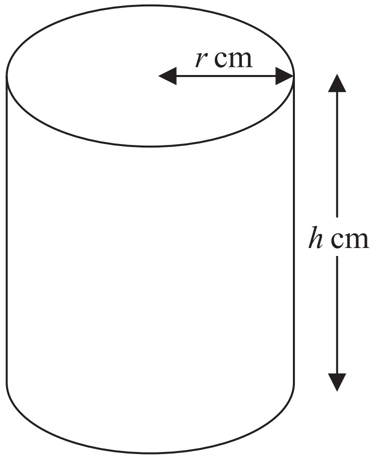


Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height *h* cm and radius *r* cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm^3 of water when full.

(a) Show that the total external surface area, S cm^2, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(c) Justify that the value of S you found in part (b) is a minimum.

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $\quad 60000=\pi r^2 h \Rightarrow h=\dfrac{60000}{\pi r^2}$	M1
	Subs in $S=\pi r^2+2\pi rh \Rightarrow S=\pi r^2+2\pi r\times\dfrac{60000}{\pi r^2}$	M1
	$\Rightarrow S=\pi r^2+\dfrac{120000}{r}$	A1*
		(3)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}r}=2\pi r-\dfrac{120000}{r^2}$	M1A1
	$\Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}r}=0 \Rightarrow r^3=\dfrac{120000}{2\pi} \Rightarrow r=awrt\ 27(\text{cm})$	dM1A1
	$\Rightarrow S=\pi\times\text{"}26.7\text{"}^2+\dfrac{120000}{\text{"}26.7\text{"}}=awrt\ 6730(\text{cm}^2)$	dM1 A1
		(6)
(c)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}=2\pi+\left.\dfrac{240000}{r^3}\right\|_{r=26.7}=awrt\,19>0\Rightarrow \text{Minimum}$	M1A1
		(2)
		(11 marks)

(a)

M1 Uses $60000=\pi r^2 h \Rightarrow h=..$ Alternatively uses $60000=\pi r^2 h \Rightarrow \pi rh=..$
Condone errors on the number of zeros but the formula must be correct

M1 Score for the attempt to substitute any $h=..$ or $\pi rh=..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000=\dfrac{1}{3}\pi r^2 h\Rightarrow h=..\right)$ into $S=k\pi r^2+2\pi rh$ where $k=1$ or 2 to get S in terms of r
Allow if S is called something else such as A.

A1* Completes proof with no errors (or omissions) $S=\pi r^2+\dfrac{120000}{r}$.
Allow from $S=\pi r^2+\dfrac{2V}{r}$ if quoted. $S=$ must be somewhere in the proof

(b) Condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout (b)

M1 Differentiate the **given** function and gets one power correct.

A1 $\left(\frac{\mathrm{d}S}{\mathrm{d}r}\right) = 2\pi r - \frac{120000}{r^2}$

dM1 Sets their $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$, multiplies by r^2 and proceeds with a correct method to $r^3 = ..$
The = 0 may be implied by their working

A1 $r = \text{awrt } 27$ or $r = \sqrt[3]{\frac{120000}{2\pi}}$ but may also be implied by a correct final answer

dM1 Substitutes their positive value of $r = ..$ into $S = \pi r^2 + \frac{120000}{r}$. It is dependent upon the first M1

A1 Awrt 3*sf* $6730\left(\text{cm}^2\right)$

(c) Condone second derivative appearing as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for both marks in (c)

M1 Subs positive $r = ..$ achieved in part (b) into $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = A \pm \frac{B}{r^3}$, $A, B \neq 0$ and calculates its value

Alternatively achieves $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = A \pm \frac{B}{r^3}$ and consider sign

A1 $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ with a correct substitution of their r in $\frac{\mathrm{d}^2S}{\mathrm{d}r^2}$ and a statement $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum. (This is allowed from incorrect positive r values)

Or $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ with a correct value of $\frac{\mathrm{d}^2S}{\mathrm{d}r^2}$ for their r and a statement $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum.

Or $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ and refers to $r > 0$ to state $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum.

....

Alt (c) by using the first derivative either side of 26.7

M1:Attempts to find the **numerical** value of $\frac{\mathrm{d}S}{\mathrm{d}r} = Ar \pm \frac{B}{r^2}$ either side of a positive r achieved in part (b)

A1: $\frac{\mathrm{d}S}{\mathrm{d}r} = 2\pi r - \frac{120000}{r^2}$ and at $r = a \Rightarrow \frac{dS}{dR} < 0$ at $r = b \Rightarrow \frac{dS}{dR} > 0$ hence Minimum (where $a < 26.7 < b$)

....

Alt (c) using the value of S either side of 26.7

M1:Attempts to find the **numerical** value of $S = \pi r^2 + \frac{120000}{r}$ either side of a positive r achieved in (b)

A1:At $r = a \Rightarrow S_{26.7} < S_a$ at $r = a \Rightarrow S_{26.7} < S_b$ hence Minimum (where $a < 26.7 < b$)

....

Leave blank

16.

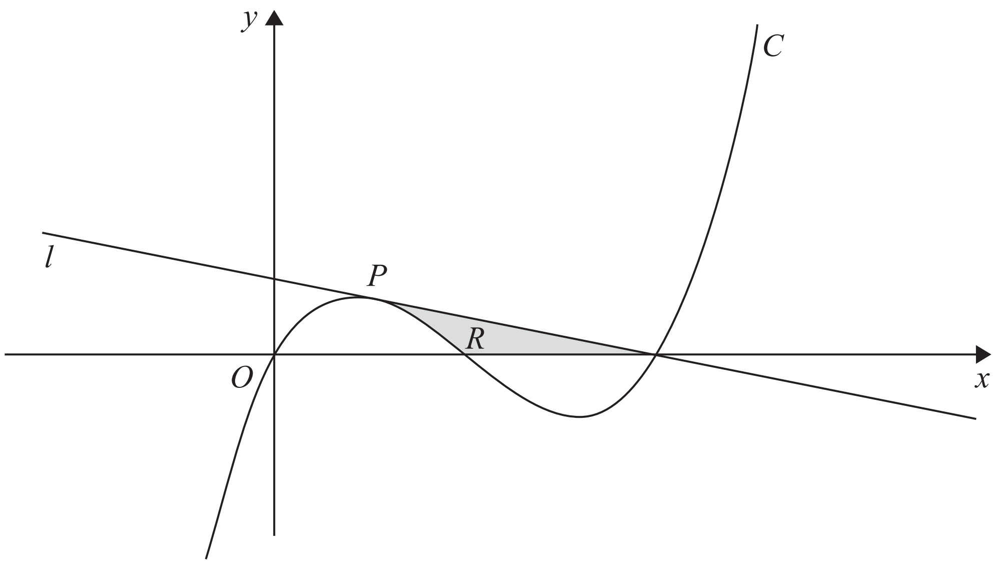


Figure 6

Figure 6 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 2)$$

The point P lies on C and has x coordinate $\frac{1}{2}$

The line l, as shown on Figure 6, is the tangent to C at P.

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Use part (a) to find an equation for l in the form $ax + by = c$, where a, b and c are integers. **(4)**

The finite region R, shown shaded in Figure 6, is bounded by the line l, the curve C and the x-axis.

The line l meets the curve again at the point (2, 0)

(c) Use integration to find the exact area of the shaded region R. **(6)**

Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $\quad 60000=\pi r^2 h\Rightarrow h=\dfrac{60000}{\pi r^2}$	M1
	Subs in $S=\pi r^2+2\pi rh\Rightarrow S=\pi r^2+2\pi r\times\dfrac{60000}{\pi r^2}$	M1
	$\Rightarrow S=\pi r^2+\dfrac{120000}{r}$	A1*
		(3)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}r}=2\pi r-\dfrac{120000}{r^2}$	M1A1
	$\Rightarrow\dfrac{\mathrm{d}S}{\mathrm{d}r}=0\Rightarrow r^3=\dfrac{120000}{2\pi}\Rightarrow r=awrt\ 27(\text{cm})$	dM1A1
	$\Rightarrow S=\pi\times"26.7"^2+\dfrac{120000}{"26.7"}=awrt\ 6730(\text{cm}^2)$	dM1 A1
		(6)
(c)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}=2\pi+\left.\dfrac{240000}{r^3}\right\vert_{r=26.7}=awrt\,19>0\Rightarrow\text{Minimum}$	M1A1
		(2) **(11 marks)**

(a)

M1 Uses $60000=\pi r^2h\Rightarrow h=..$ Alternatively uses $60000=\pi r^2h\Rightarrow\pi rh=..$
Condone errors on the number of zeros but the formula must be correct

M1 Score for the attempt to substitute any $h=..$ or $\pi rh=..$ from a dimensionally correct formula for V $\left(\text{Eg. }60000=\dfrac{1}{3}\pi r^2h\Rightarrow h=..\right)$ into $S=k\pi r^2+2\pi rh$ where $k=1$ or 2 to get S in terms of r
Allow if S is called something else such as A.

A1* Completes proof with no errors (or omissions) $S=\pi r^2+\dfrac{120000}{r}$.
Allow from $S=\pi r^2+\dfrac{2V}{r}$ if quoted. $S=$ must be somewhere in the proof

(b) Condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout (b)

M1 Differentiate the **given** function and gets one power correct.

A1 $\left(\frac{\mathrm{d}S}{\mathrm{d}r}\right) = 2\pi r - \frac{120000}{r^2}$

dM1 Sets their $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$, multiplies by r^2 and proceeds with a correct method to $r^3 = ..$

The = 0 may be implied by their working

A1 $r = \text{awrt } 27$ or $r = \sqrt[3]{\frac{120000}{2\pi}}$ but may also be implied by a correct final answer

dM1 Substitutes their positive value of $r = ..$ into $S = \pi r^2 + \frac{120000}{r}$. It is dependent upon the first M1

A1 Awrt 3*sf* $6730\left(\text{cm}^2\right)$

(c) Condone second derivative appearing as $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ for both marks in (c)

M1 Subs positive $r = ..$ achieved in part (b) into $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = A \pm \frac{B}{r^3}$, $A, B \neq 0$ and calculates its value

Alternatively achieves $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = A \pm \frac{B}{r^3}$ and consider sign

A1 $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ with a correct substitution of their r in $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2}$ and a statement $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum. (This is allowed from incorrect positive r values)

Or $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ with a correct value of $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2}$ for their r and a statement $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum.

Or $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} = 2\pi + \frac{240000}{r^3}$ and refers to $r > 0$ to state $\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} > 0 \Rightarrow$ Minimum.

....

Alt (c) by using the first derivative either side of 26.7

M1:Attempts to find the **numerical** value of $\frac{\mathrm{d}S}{\mathrm{d}r} = Ar \pm \frac{B}{r^2}$ either side of a positive r achieved in part (b)

A1: $\frac{\mathrm{d}S}{\mathrm{d}r} = 2\pi r - \frac{120000}{r^2}$ and at $r = a \Rightarrow \frac{dS}{dR} < 0$ at $r = b \Rightarrow \frac{dS}{dR} > 0$ hence Minimum (where $a < 26.7 < b$)

....

Alt (c) using the value of S either side of 26.7

M1:Attempts to find the **numerical** value of $S = \pi r^2 + \frac{120000}{r}$ either side of a positive r achieved in (b)

A1:At $r = a \Rightarrow S_{26.7} < S_a$ at $r = a \Rightarrow S_{26.7} < S_b$ hence Minimum (where $a < 26.7 < b$)

....

Leave blank

16.

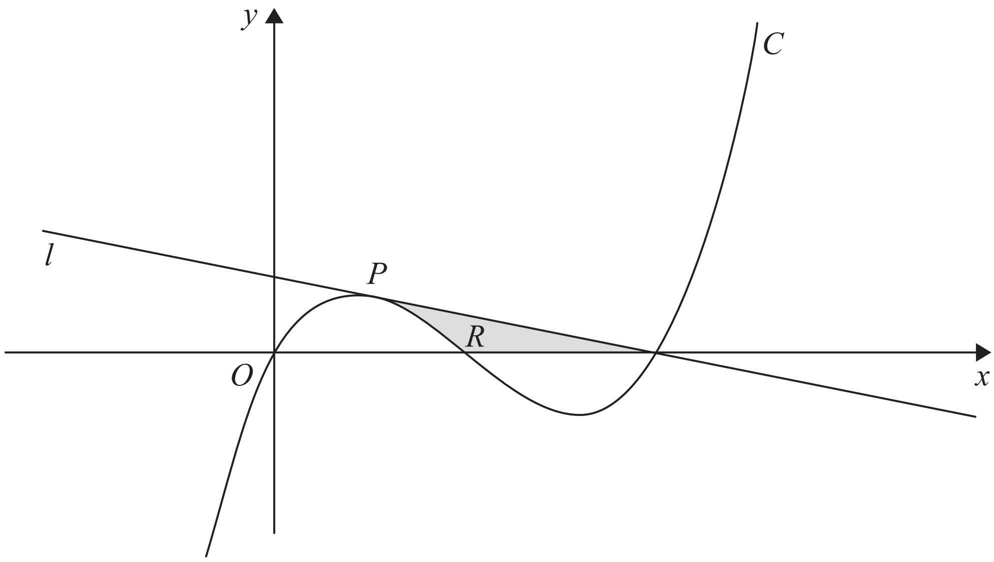


Figure 6

Figure 6 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 2)$$

The point P lies on C and has x coordinate $\frac{1}{2}$

The line l, as shown on Figure 6, is the tangent to C at P.

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Use part (a) to find an equation for l in the form $ax + by = c$, where a, b and c are integers. **(4)**

The finite region R, shown shaded in Figure 6, is bounded by the line l, the curve C and the x-axis.

The line l meets the curve again at the point $(2, 0)$

(c) Use integration to find the exact area of the shaded region R. **(6)**

Question Number	Scheme	Marks
16(a)	$y = x(x-1)(x-2) = x^3 - 3x^2 + 2x \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 3x^2 - 6x + 2$	M1 A1
		(2)
(b)	When $x = \frac{1}{2}$, $y = \frac{3}{8}$	B1
	Sub $x = \frac{1}{2}$ into $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{x=\frac{1}{2}} = \frac{3}{4} - 3 + 2 = -\frac{1}{4}$	M1
	Uses gradient and $\left(\frac{1}{2}, \frac{3}{8}\right) \Rightarrow y - "\frac{3}{8}" = "-\frac{1}{4}"\left(x - \frac{1}{2}\right) \Rightarrow 4y + x = 2$	M1A1
		(4)
(c)	Later….	

(a)

M1 Attempts to multiply out $x(x-1)(x-2)$ and differentiate each term.
Look for a cubic expression being differentiated into a quadratic expression

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 3x^2 - 6x + 2$. Accept exact equivalents such as $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 3x^2 - 4x - 2x + 2$.

(b)

B1 When $x = \frac{1}{2}$, $y = \frac{3}{8}$ oe

M1 Substitute $x = \frac{1}{2}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and either states /uses this as a gradient $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$, not the y - coordinate

M1 Uses their gradient, $x = \frac{1}{2}$ and their $y = \frac{3}{8}$ to find the equation of the tangent.
Look for $y - "\frac{3}{8}" = "-\frac{1}{4}"(x-2)$
If the form $y = mx + c$ is used the candidate must proceed as far as $c = ..$
Finding the equation of the normal is M0

A1 $k(4y + x = 2)$ where k is an integer
Alternatively accept a statement that $a = 1, b = 4$ *and* $c = 2$ or multiples thereof.

If the gradient is found by using $\left(\frac{1}{2}, \frac{3}{8}\right)$ and (2,0), treat as a special case where candidates can potentially score B1 M0 M1 A0. They have not satisfied the demand of the question.

If the candidate uses $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{x=\frac{1}{2}}$ and the point (2,0) they can score all marks. Score as follows

1st M1: Substitute $x = \frac{1}{2}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and either states /uses as a gradient $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ and not the y - coordinate

2nd M1: Uses their gradient with $(2,0)$ to form the equation of the tangent

B1: Any correct equation followed by A1: $k(4y + x = 2)$ where k is an integer.

Question Number	Scheme	Marks
(c)	**Candidate considers area under curve from $x=\frac{1}{2}$ and $x=1$**	
Main	$\int y\mathrm{d}x=\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}(+c)$	M1 A1
	Area under curve between $x=\frac{1}{2}$ and $x=1$ $\left[\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right]_{\frac{1}{2}}^{1}=\left(\frac{7}{64}\right)$	M1
	Area under line between $x=\frac{1}{2}$ and $x=2$ $=\frac{1}{2}\times\left(2-\frac{1}{2}\right)\times\frac{3}{8}=\left(\frac{9}{32}\right)$	M1
	Area of $R=\frac{1}{2}\times\left(2-\frac{1}{2}\right)\times\frac{3}{8}-\left[\left(\frac{1}{4}-1+1\right)-\left(\frac{1}{64}-\frac{1}{8}+\frac{1}{4}\right)\right]=\frac{9}{32}-\frac{7}{64}=\frac{11}{64}$	dM1A1
		(6) **(12 marks)**

(c) **Main scheme: Use if the candidate attempts to find the area under the curve**

M1 Attempts $\int x(x-1)(x-2)\mathrm{d}x$ by first multiplying and then integrating to a quartic form

A1 **Correct** and unsimplified$=\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}(+c)$

M1 Uses limits of $x=\frac{1}{2}$ and $x=1$ with their answer to $\int x(x-1)(x-2)\mathrm{d}x$.

M1 Uses the correct method to find the area under the line between $x=\frac{1}{2}$ and $x=2$

If the triangle is used then look for $\frac{1}{2}\times\left(2-\frac{1}{2}\right)\times"\frac{3}{8}"=\left(\frac{9}{32}\right)$ or $\frac{1}{2}\times\frac{3}{2}\times"\frac{3}{8}"=\left(\frac{9}{32}\right)$

If the equation of the line is used it must be correct. Look for an evaluation of

$\int_{\frac{1}{2}}^{2}\left(\frac{1}{2}-\frac{1}{4}x\right)\mathrm{d}x=\left[\frac{1}{2}x-\frac{1}{8}x^2\right]_{\frac{1}{2}}^{2}$ with correct limits and correct integration

dM1 Score for the correct combination of areas including the limits....... in this case the above areas must be subtracted. It is dependent upon all 3 M's

A1 $\frac{11}{64}$

Question Number	Scheme	Marks
(c)	**Candidate considers area between line and curve $x = \frac{1}{2}$ to $x = 1$**	
ALT 1	$\int \left("\frac{1}{2} - \frac{1}{4}x" \right) - \left(x(x-1)(x-2) \right) \mathrm{d}x = \left(\frac{1}{2}x - \frac{1}{8}x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right)$	M1A1
	Area between line and curve between $x = \frac{1}{2}$ and $x = 1$ is $\left[\left(\frac{1}{2}x - \frac{1}{8}x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right) \right]_{\frac{1}{2}}^{1} = \left(\frac{3}{64} \right)$	M1
	Area under line between $x = 1$ and $x = 2$ $= \frac{1}{2} \times (2-1) \times \frac{1}{4} = \frac{1}{8}$	M1
	Area of R = $\frac{1}{8} + \left[\left(\frac{1}{8} \right) - \left(\frac{5}{64} \right) \right] = \frac{1}{8} + \frac{3}{64} = \frac{11}{64}$	dM1A1
		(6)

M1 Attempts $\int \left("\frac{1}{2} - \frac{1}{4}x" \right) - \left(x(x-1)(x-2) \right) \mathrm{d}x$ either way around, multiplies out and integrates to a quartic form

A1 Answer **correct** $\left(\frac{1}{2}x - \frac{1}{8}x^2 \right) - \left(\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} \right)$ or its simplified form $\left(\frac{1}{2}x - \frac{9}{8}x^2 + x^3 - \frac{1}{4}x^4 \right)$

Do not follow through on their tangent line in this case

M1 Uses the limits $x = \frac{1}{2}$ and $x = 1$ with their answer to $\int \left("\frac{1}{2} - \frac{1}{4}x" \right) - \left(x(x-1)(x-2) \right) \mathrm{d}x$

M1 Uses the correct method to find the area under the line between $x = 1$ and $x = 2$

If the triangle is used look for $\frac{1}{2} \times (2-1) \times \frac{1}{4} = \left(\frac{1}{8} \right)$

If the equation of the line is used it must be correct. Look for an evaluation of

$$\int_1^2 \left(\frac{1}{2} - \frac{1}{4}x \right) \mathrm{d}x = \left[\frac{1}{2}x - \frac{1}{8}x^2 \right]_1^2$$ with correct limits and integration.

dM1 Score for the correct combination of areas including limits.....in this case the above areas must be added. It is dependent upon all 3 M's

A1 $\frac{11}{64}$

ALT 2	**Candidate considers area between line and curve from $x = \frac{1}{2}$ to $x = 2$.** $\int\left(\text{"}\frac{1}{2}-\frac{1}{4}x\text{"}\right)-(x(x-1)(x-2))\,\mathrm{d}x=\left(\frac{1}{2}x-\frac{1}{8}x^2\right)-\left(\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right)$	M1A1
	Total area between line and curve $=\left[\left(\frac{1}{2}x-\frac{1}{8}x^2\right)-\left(\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right)\right]_{\frac{1}{2}}^{2}=\left(\frac{27}{64}\right)$	M1
	Area under curve $=\left[\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right]_1^2=\left(-\frac{1}{4}\right)$	M1
	Area of R $= \frac{27}{64}-\frac{1}{4}=\frac{11}{64}$	dM1A1
		(6)

M1 Attempts $\int\left(\text{"}\frac{1}{2}-\frac{1}{4}x\text{"}\right)-(x(x-1)(x-2))\,\mathrm{d}x$ either way around, multiplies out and integrates to a quartic form

A1 Answer correct $\left(\frac{1}{2}x-\frac{1}{8}x^2\right)-\left(\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right)$ or its simplified form $\left(\frac{1}{2}x-\frac{9}{8}x^2+x^3-\frac{1}{4}x^4\right)$

Do not follow through on their tangent line in this case

M1 Uses the limits $x=\frac{1}{2}$ and $x=2$ with their answer to $\int\left(\text{"}\frac{1}{2}-\frac{1}{4}x\text{"}\right)-(x(x-1)(x-2))\,\mathrm{d}x$

M1 Evaluates $\pm\int_1^2 x(x-1)(x-2)\,\mathrm{d}x=\pm\left[\frac{x^4}{4}-x^3+x^2\right]_1^2$ Both the limits and integration must be correct

dM1 Score for the correct combination of areas including limits.....in this case you need to see the equivalent to $\left(\frac{27}{64}\right)+-\frac{1}{4}$ or $\left(\frac{27}{64}\right)-\left|-\frac{1}{4}\right|$ It is dependent upon all 3 M's

A1 $\frac{11}{64}$

……………………………………………………………………………………………………..

Useful guide: (but please use the scheme and notes)

1) Scan through candidates answer to ascertain whether they are integrating just the curve or the line – the curve to decide the method. Looking at their limits of integration can help you decide between alt 1 and alt 2
2) Marks are awarded like this

M1: Attempt to integrate appropriate function to reach a quartic
A1: Correct answer to the integration. There is no follow through
M1: Correct limits used in the integrated function
M1: Finds area under line (main or alt 1) either by using the correct triangle or using the correct f function and limits.
Finds area under curve (alt 2) using correct function and limits
dM1: Correct combination of areas. **It is dependent upon all 3 M's**
A1: Correct answer

If an incomplete method is shown always score by the method that gives the candidate more marks. If you cannot determine the method that a candidate has used then please use the review system and your team leader will advise.