Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number ☐☐☐☐☐ Candidate Number ☐☐☐☐

Core Mathematics C12

Advanced Subsidiary

Tuesday 12 January 2016 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. A sequence of numbers $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 6, \qquad n \geqslant 1$$

Given that $u_1 = 2$

(a) find the value of u_3

(2)

(b) evaluate $\sum_{i=1}^{4} u_i$

(3)

January 2016
Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1(a)	$u_2 = 2\times 2 - 6 = -2, \qquad u_3 = 2\times(-2) - 6 = -10$ or $u_3 = 2\times(2\times 2-6)-6 = -10$	M1 A1
		[2]
(b)	$\sum_{i=1}^{4} u_i = 2 + (-2) + (-10)$	M1
	$+(-26)$	A1ft
	$= -36$	A1
		[3]
		5 marks
	Notes	
(a)	**M1:** Attempt to use the given formula correctly at least once. This may be implied by a correct value for u_2 or a value for u_3 which follows through from their u_2 or implied by correct answer for u_3 **A1**: u_3 correct **and no incorrect work seen**	
(b)	**M1:** Uses sum of the 3 numerical terms from part (a) (may be implied by correct answer for their terms). Attempting to sum an AP here is M0. **A1ft**: obtains u_4 correctly (may be attempted in part (a)) and adds to sum of the first three terms from part (a) **A1**: -36 cao (-36 implies both A marks)	
	Special Cases: Some candidates attempt $u_2 + u_3 + u_4 + u_5$ in part (b) – **allow M1 only** Some candidates mis-copy one of their terms from part (a) into part (b) – **allow M1 only**	

2. (i) Given that $\frac{49}{\sqrt{7}} = 7^a$, find the value of a.

(2)

(ii) Show that $\frac{10}{\sqrt{18} - 4} = 15\sqrt{2} + 20$

You must show all stages of your working.

(3)

Question Number	Scheme			Marks
2(i)	**Way 1:**	**Way 2:**	**Way 3:**	
	$\frac{49}{\sqrt{7}}=\frac{7^2}{7^{\frac{1}{2}}}=7^{2-\frac{1}{2}}$	$7\sqrt{7}=7^{1+\frac{1}{2}}$	$7^a=\frac{49}{\sqrt{7}}\Rightarrow a=\frac{\log\frac{49}{\sqrt{7}}}{\log 7}$ or $7^a=\frac{49}{\sqrt{7}}\Rightarrow a=\log_7\frac{49}{\sqrt{7}}$	M1
	$(a=)1\frac{1}{2}$ (oe) or see answer $=7^{1\frac{1}{2}}$			A1
				[2]
(ii)	**Way 1:**	**Way 2:**		
	$\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$	$(15\sqrt{2}+20)(\sqrt{18}-4)$		M1
	$=\frac{\cdots}{2}$	$=15\sqrt{36}-60\sqrt{2}+20\sqrt{18}-80$		B1
	$\frac{10}{\sqrt{18}-4}=5\left(3\sqrt{2}+4\right)=15\sqrt{2}+20*$	$=90-60\sqrt{2}+60\sqrt{2}-80$ = 10 so $\frac{10}{\sqrt{18}-4}=15\sqrt{2}+20*$		A1cso
				[3]
				5 marks
	Notes			

(i)	**Way 1:**	**Way 2:**	**Way 3:**
	M1: Subtracts **their** powers of 7	**M1:** Cancels fraction to $7\sqrt{7}$ and adds **their** powers of 7	**M1:** Correct use of logs to obtain a correct expression for *a*
	A1: cao (answer only is 2 marks) Do not allow work with inexact decimals for this mark e.g. $49\times 7^{-\frac{1}{2}}=18.52\Rightarrow\log 18.52=1.4999...\Rightarrow a=1.5$ scores M1A0		
(ii)	**Way 1:**	**Way 2:**	
	M1: Multiply numerator and denominator by $\sqrt{18}+4$ or equivalent. The statement $\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$ is sufficient but do not allow $\frac{10(\sqrt{18}+4)}{\sqrt{18}-4(\sqrt{18}+4)}$ unless missing brackets are implied by subsequent work. **B1:** Correctly obtains ±2 in the denominator (**Must follow M1 – i.e. treat as A1**). May be implied by e.g. $\frac{10(\sqrt{18}+4)}{18-16}=5\left(\sqrt{18}+4\right)$ **A1**: Correct result with no errors seen and $\sqrt{18}=3\sqrt{2}$ **used before their final answer.** Note that for **Way 1**, correct work leading to $5\sqrt{18}+20$ followed by $15\sqrt{2}+20$ with no intermediate step would lose the final mark	**M1:** Attempts to expand $(15\sqrt{2}+20)(\sqrt{18}-4)$ to obtain at least 3 (not necessarily correct) terms **B1:** All 4 terms correct (**Must follow M1 – i.e. treat as A1**) **A1**: Obtains 10 with no errors and $\sqrt{18}=3\sqrt{2}$ seen or implied by e.g. $20\sqrt{18}=60\sqrt{2}$ **and** conclusion that states the given answer i.e. not just 10 = 10	

Leave blank

3. Find, using calculus and showing each step of your working,

$$\int_1^4 \left(6x - 3 - \frac{2}{\sqrt{x}}\right) \mathrm{d}x$$

(5)

Question Number	Scheme	Marks
3.	$\int\left(6x-3-\frac{2}{\sqrt{x}}\right)\mathrm{d}x=\frac{6x^2}{2}-3x-\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}+(c)$	M1 A1 A1
	$\left[\frac{6x^2}{2}-3x-\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}+(c)\right]_1^4=(28)-(-4)=32$	M1 A1
		[5]
		5 marks
	Notes	
	M1: Attempt to integrate original f(x) – at least one power increased $x^n\to x^{n+1}$ **A1: Two** of the three terms **correct un-simplified or simplified** (**Constant not required**) **A1:** All three terms **correct un-simplified or simplified** (**Constant not required**) **M1:** Substitutes limits 4 and 1 into their ‘changed’ function and subtracts the right way round **A1:** 32 cao **(32 + c is A0)** **The question requires the use of calculus so a correct answer only scores no marks)**	

Question Number	Scheme	Marks
4.	$a + 3d = 3$ **OR** $\frac{6}{2}(2a+5d)=27$	M1 A1
	$a + 3d = 3$ **AND** $\frac{6}{2}(2a+5d)=27$	A1
	Eliminates one variable to find a or d from 2 equations in a and d	dM1
	Obtains $a = 12$ **or** $d = -3$	A1
	Obtains $a = 12$ **and** $d = -3$	A1
		[6]
		6 marks
	Notes	
	M1A1: Writes down a correct (possibly un-simplified) equation for 4th term **or** for sum of the first 6 terms. Allow the individual terms to be added for the sum e.g. $a+a+d+a+2d+a+3d+a+4d+a+5d=27$ **A1cao:** A correct equation for 4th term **and** a correct equation for the sum (allow either to be un-simplified) **dM1:** Eliminates one variable from two equations in a and d to find either a or d (see note below) **A1:** One variable correct (This implies previous M mark) **A1:** Both variables correct Note that if both equations are correct and there is **no working** and the values of a and d are both incorrect, this scores dM0. Also if either or both equations is/are incorrect and values of a and d are obtained with **no working** this also scores dM0.	

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4. The 4th term of an arithmetic sequence is 3 and the sum of the first 6 terms is 27

Find the first term and the common difference of this sequence.

(6)

Question Number	Scheme	Marks
3.	$\int\left(6x-3-\frac{2}{\sqrt{x}}\right)\mathrm{d}x=\frac{6x^2}{2}-3x-\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}+(c)$	M1 A1 A1
	$\left[\frac{6x^2}{2}-3x-\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}+(c)\right]_1^4=(28)-(-4)=32$	M1 A1
		[5]
		5 marks
	Notes	
	M1: Attempt to integrate original f(x) – at least one power increased $x^n\rightarrow x^{n+1}$ **A1: Two** of the three terms **correct un-simplified or simplified** (**Constant not required**) **A1:** All three terms **correct un-simplified or simplified** (**Constant not required**) **M1:** Substitutes limits 4 and 1 into their ‘changed’ function and subtracts the right way round **A1:** 32 cao **(32 + c is A0)** **The question requires the use of calculus so a correct answer only scores no marks)**	

Question Number	Scheme	Marks
4.	$a+3d=3$ **OR** $\frac{6}{2}(2a+5d)=27$	M1 A1
	$a+3d=3$ **AND** $\frac{6}{2}(2a+5d)=27$	A1
	Eliminates one variable to find a or d from 2 equations in a and d	dM1
	Obtains $a=12$ **or** $d=-3$	A1
	Obtains $a=12$ **and** $d=-3$	A1
		[6]
		6 marks
	Notes	
	M1A1: Writes down a correct (possibly un-simplified) equation for 4th term **or** for sum of the first 6 terms. Allow the individual terms to be added for the sum e.g. $a+a+d+a+2d+a+3d+a+4d+a+5d=27$ **A1cao:** A correct equation for 4th term **and** a correct equation for the sum (allow either to be un-simplified) **dM1:** Eliminates one variable from two equations in a and d to find either a or d (see note below) **A1:** One variable correct (This implies previous M mark) **A1:** Both variables correct Note that if both equations are correct and there is **no working** and the values of a and d are both incorrect, this scores dM0. Also if either or both equations is/are incorrect and values of a and d are obtained with **no working** this also scores dM0.	

Leave blank

5. (a) Sketch the graph of $y = \sin 2x, \quad 0 \leqslant x \leqslant \frac{3\pi}{2}$

Show the coordinates of the points where your graph crosses the x-axis.

(2)

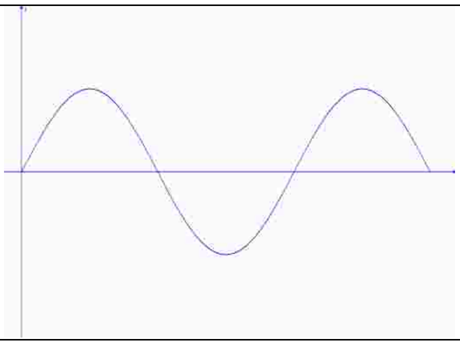
The table below gives corresponding values of x and y, for $y = \sin 2x$.
The values of y are rounded to 3 decimal places where necessary.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$
y	0	0.5	0.866	1

(b) Use the trapezium rule with all the values of y from the table to find an approximate value for

$$\int_0^{\frac{\pi}{4}} \sin 2x \, \mathrm{d}x$$

(3)

<table>
<tr><th>Question number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">5(a)</td><td rowspan="2"></td><td>Sketch of a positive sine curve- passing through O with at least one complete cycle from O. Condone different amplitudes above and below the x-axis.</td><td>B1</td></tr>
<tr><td>Correct shape with one and a half cycles as shown (from O to $\frac{3\pi}{2}$) and crossing the x–axis at $\frac{\pi}{2}$ and π</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>[2]</td></tr>
<tr><td rowspan="5">(b)</td><td colspan="2"><table><tr><td>x</td><td>0</td><td>$\frac{\pi}{12}$</td><td>$\frac{\pi}{6}$</td><td>$\frac{\pi}{4}$</td></tr><tr><td>y</td><td>0</td><td>0.5</td><td>0.866</td><td>1</td></tr></table></td><td></td></tr>
<tr><td colspan="2">Uses $\frac{1}{2}\times\frac{\pi}{12}$
May be implied by use of e.g. $\frac{1}{2}h=\frac{1}{2}\left(\frac{\pi}{6}-\frac{\pi}{12}\right)=\frac{1}{2}(0.261....)$</td><td>B1</td></tr>
<tr><td colspan="2">$....\{(0+1)+2(0.5+0.866)\}$</td><td>M1</td></tr>
<tr><td colspan="2">0.4885176576… awrt 0.49</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>[3]</td></tr>
<tr><td></td><td colspan="2"></td><td>5 marks</td></tr>
<tr><td></td><td colspan="3">Notes</td></tr>
<tr><td>(a)</td><td colspan="3">Notes as above
B1: Correct shape with positive gradient through O
B1: Need not see endpoints labelled. Ignore any part of the curve to the left of the origin but if the curve extends beyond $x=\frac{3\pi}{2}$ then then $x=\frac{3\pi}{2}$ must be labelled on the diagram. Labels for $\frac{\pi}{2}$ and π may be on the diagram or in the text but not just in a table of values and must be in radians not degrees. (Allow awrt 1.57 and 3.14)
The amplitudes must not be significantly different above and below the x-axis.</td></tr>
<tr><td>(b)</td><td colspan="3">B1: Need ½ of $\frac{\pi}{12}$ or to see $\frac{\pi}{24}$ or ½ of 0.261….
M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the two in the table.
If values used in brackets are x values instead of y values this scores M0.
A1: for awrt 0.49
Separate trapezia may be used: B1 for $\frac{\pi}{24}$ and M1 for ½$h(a+b)$ used 3 times
Special Case: Bracketing mistake: i.e. $\frac{\pi}{24}(0+1)+2(0.5+0.866)\ (=2.86...)$
scores B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).
Need to see trapezium rule used so answer only (with no working) is 0/3</td></tr>
</table>

Leave blank

6. $$f(x) = x^3 + x^2 - 12x - 18$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.
(2)

(b) Factorise $f(x)$.
(2)

(c) Hence find exact values for all the solutions of the equation $f(x) = 0$
(3)

Question Number	Scheme	Marks
6.	$\mathrm{f}(x) = x^3 + x^2 - 12x - 18$	
(a)	Attempts $\mathrm{f}(\pm 3)$	M1
	$\{\mathrm{f}(-3)=\}$ 0 so $(x + 3)$ is a factor of $\mathrm{f}(x)$.	A1
		[2]
(b)	$x^3 + x^2 - 12x - 18 = (x+3)(x^2 + ...$	M1
	$x^3 + x^2 - 12x - 18 = (x+3)(x^2 - 2x - 6)$ or $x^3 + x^2 - 12x - 18 = (x+3)(x - 1 + \sqrt{7})(x - 1 - \sqrt{7})$ oe	A1
		[2]
(c)	$(x =)\ -3$	B1
	$x = \dfrac{2 \pm \sqrt{4+24}}{2} = 1 \pm \sqrt{7}$ or by completion of square $(x-1)^2 = 7$ so $x = 1 \pm \sqrt{7}$ or $\left(x - 1 + \sqrt{7}\right)\left(x - 1 - \sqrt{7}\right) = 0 \Rightarrow x = 1 \pm \sqrt{7}$	M1 A1
		[3]
		7 marks
	Notes	
(a)	**M1**: As on scheme – must use the <u>factor theorem</u> **A1:** for seeing 0 and conclusion which may be in a preamble and may be minimal e.g. QED, proven, true, tick etc. There must be no obvious errors but need to see at least $(-3)^3 + (-3)^2 - 12(-3) - 18 = 0$ for A1 but allow invisible brackets e.g. $-3^3 + -3^2 - 12(-3) - 18 = 0$ provided there are no obvious errors.	
(b)	**M1**: Uses $(x + 3)$ as a factor and obtains correct first term of quadratic factor by division or any other method e.g. comparing coefficients or finding roots and factorising **A1 :** Correct quadratic and writes $(x+3)(x^2 - 2x - 6)$ or $(x+3)(x - 1 + \sqrt{7})(x - 1 - \sqrt{7})$ oe Note that this work may be done in part (a) and the result re-stated here.	
(c)	**B1:** States -3 **M1:** Method for finding their roots. Allow the usual rules applied to their quadratic. **This mark is for finding the roots and not for just finding factors. You may need to check their roots if no working is shown e.g. if they give decimal answers (3.645…, -1.645…)** **A1:** need both roots. Correct answer implies M mark. Allow $x = \dfrac{2 \pm \sqrt{28}}{2}$ If they give extra roots e.g. $x = -3,\ -1,\ \dfrac{2 \pm \sqrt{28}}{2}$, lose the final A mark (B1M1A0)	

Leave blank

7. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^8$, where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1512

(b) find the value of k.

(3)

Question Number	Scheme	Marks
7(a)	$(1+kx)^8 = 1+\binom{8}{1}(kx)+\binom{8}{2}(kx)^2+\binom{8}{3}(kx)^3 ...$	M1
	$=1+8kx, +28k^2x^2, +56k^3x^3+...$	B1, A1, A1
		[4]
(b)	Sets $"56k^3"=1512$ and obtains $k^3=\frac{1512}{56}$	M1 A1
	So $k = 3$	A1
		[3]
		7 marks
	Notes	
(a)	**M1**: The **method** mark is awarded for an attempt at the Binomial expansion to get the third **and/or** fourth term. The **correct** binomial coefficient needs to be combined with the correct power of *x*. Ignore bracket errors and omission of or incorrect powers of *k*. Accept any notation for ${}^8C_2$ or ${}^8C_3$, e.g. $\binom{8}{2}$ or $\binom{8}{3}$ or 28 or 56 from Pascal's triangle. This mark may be given if no working is shown, but either or both of $28k^2x^2$ and $56k^3x^3$ is found. **B1:** This is for 1 + 8*kx* and not for just $1+\binom{8}{1}(kx)$ **A1:** is cao and is for $28k^2x^2$ or for $28(kx)^2$ **A1:** is cao and is for $56k^3x^3$ or for $56(kx)^3$ Any extra terms in higher powers of *x* should be ignored. Allow terms separated by commas or given as a list for all the marks.	
(b)	**M1:** Sets their coefficient of $x^3=1512$ and obtains $k^n = ..$ where *n* is 1 or 3 **A1:** $k^3=\frac{1512}{56}$ or equivalent e.g. 27 (May be implied by their final answer) **A1:** *k* = 3 cao (±3 is A0) **Note (b) can be marked independently of part (a) so part (a) might be incorrect or not attempted but they have $56k^3=1512$ etc. in (b)**	

8. (a) Given that $7\sin x = 3\cos x$, find the exact value of $\tan x$.

(1)

(b) Hence solve for $0 \leqslant \theta < 360°$

$$7\sin(2\theta + 30°) = 3\cos(2\theta + 30°)$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Marks
	$7\sin x = 3\cos x$	
8(a)	$\left(\tan x =\right)\frac{3}{7}$	B1
		[1]
(b)	$\tan\left(2\theta+30\right)=\frac{3}{7}$	B1ft
	$\tan^{-1}"\frac{3}{7}"\ \left(\alpha\right)$	M1
	One of $\theta =$ awrt 87 or awrt 177 or awrt 267 or awrt 357	A1
	Follow through any of their final θ's for $\theta \pm 90n$ within range	A1ft
	All of $\theta =$ 86.6, 176.6, 266.6, 356.6	A1
		[5]
		6 marks
	Notes	
(a)	**B1: (** $\tan x =)\frac{3}{7}$ or exact equivalent so accept recurring decimal ($0.428571...$) but not rounded answer	
(b)	**B1ft:** Correct equation as shown or follow through their value for tan x from part (a). Must be $\tan\left(2\theta+30\right)=\ldots$ but $2\theta+30$ may be implied later by an attempt to subtract 30 and then divide by 2. If the processing is unclear or incorrect and $2\theta+30$ is never seen, score B0 here. **M1:** Finds arctan of their $\frac{3}{7}$. Could be implied by their value e.g. 23.19.. or just $\tan^{-1}"\frac{3}{7}"$ **A1:** For **one** of either $\theta =$ awrt 87 or awrt 177 or awrt 267 or awrt 357 **A1ft:** Follow through any of their **final** answers to which an integer multiple of 90 has been added or subtracted to give another solution in range **but not for adding a multiple of 90 to just α.** **A1: For all 4 correct answers to the required accuracy as stated in the scheme**. Ignore extra answers outside range but lose last A mark for extra answers inside range.	

9. The resident population of a city is 130 000 at the end of Year 1

A model predicts that the resident population of the city will increase by 2% each year, with the populations at the end of each year forming a geometric sequence.

(a) Show that the predicted resident population at the end of Year 2 is 132 600

(1)

(b) Write down the value of the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year which will end with the resident population of the city exceeding 260 000

(c) Show that

$$N > \frac{\log_{10} 2}{\log_{10} 1.02} + 1$$

(4)

(d) Find the value of N.

(1)

Question Number	Scheme	Marks
9.(a)	$130000\times(1.02)=132600*$ or $2\%=2600$ and $130000+2600=132600*$	B1
		[1]
(b)	$(r=)\ 1.02$	B1
		[1]
(c)	Uses $130000\times(1.02)^{N-1}>260000$ **or** $130000\times(1.02)^{N-1}=260000$	M1
	So $(1.02)^{N-1}>2$	A1
	$(N-1)\log_{10}(1.02)>\log_{10}2$ or $(N-1)\log_{10}(1.02)=\log_{10}2$ or $(N-1)>\log_{1.02}2$ or $(N-1)=\log_{1.02}2$	M1
	$N>\dfrac{\log_{10}2}{\log_{10}(1.02)}+1*$	A1cso
		[4]
(d)	$(N=)\ \ 37$	B1
		[1]
		7 marks
	Notes	
(a)	**B1:** A reason must be provided for this mark as the answer is printed. **Allow both** $130000\times(1+2\%)$ **and** $130000\times(102\%)$ **as both give the correct answer when entered this way on a calculator. But not** $130000\times1+2\%$	
(b)	**B1:** For 1.02 oe e.g. allow $\dfrac{51}{50}$	
(c)	**M1: Correct** inequality or equality – may use *r* or their *r* or 1.02 and may use *N* or *n*. **A1:** $(1.02)^{N-1}>2$ cao. Allow $(1.02)^{n-1}>2$ **M1:** Correct use of logs power rule on their previous line which must have come from using the n^{th} term of a GP. Condone missing brackets for this mark e.g. $N-1\log_{10}(1.02)>\log_{10}2$. (May follow use of $=$ instead of $>$ or use of *r* instead of 1.02 or use of *N* instead of *N* - 1). These cases can get M0A0M1. Allow the base to be absent or just 'ln' for this mark. If the inequality sign is reversed at this point, still allow the M1. **A1*:** Answer is **exactly** as printed (**including the bases**) and **all** inequality work should be correct and all previous marks scored and **no missing brackets earlier.** Allow this mark to score from a correct previous line provided the power rule is used. So fully correct work leading to $(N-1)\log_{10}(1.02)>\log_{10}2\Rightarrow N>\dfrac{\log_{10}2}{\log_{10}(1.02)}+1$ **scores the final M1A1 but** $(1.02)^{N-1}>2\Rightarrow N>\dfrac{\log_{10}2}{\log_{10}(1.02)}+1$ **scores M0A0 (no explicit use of power rule)**	
(d)	**B1:** Only need $N=37$ – may follow trial and error or uses logs to a different base. Do not allow $N\geq37$ or $N>37$ or $N=37.0$	

10. The curve C has equation

$$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000, \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(b) Hence find the coordinates of the stationary point on C.

(5)

(c) Use $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to determine the nature of this stationary point.

(3)

Question Number	Scheme	Marks
	$y = 12x^{\frac{5}{4}} - \dfrac{5}{18}x^2 - 1000$	
10.(a)	$\dfrac{dy}{dx} = 12\times\dfrac{5}{4}x^{\frac{1}{4}} - \dfrac{10}{18}x$	M1 A1
		[2]
(b)	Put $12\times\frac{5}{4}x^{\frac{1}{4}} - \frac{10}{18}x = 0$ so $x^n = k$ $\left(n \in \mathbb{R},\ k \neq 0\right)$	M1
	$\therefore\ x = (\ \ )^{\frac{4}{3}}$	dM1
	$\therefore\ x = 81$ **(Ignore *x* = 0 if given as a second solution)**	A1
	So $y = 12(81)^{\frac{5}{4}} - \frac{5}{18}(81)^2 - 1000$ i.e. $y = 93.5$	dM1A1
		[5]
(c)	$\dfrac{d^2y}{dx^2}$ **=** $= \dfrac{15}{4}x^{-\frac{3}{4}} - \dfrac{5}{9}$	B1ft
	Substitutes their non-zero *x* (positive or negative) into their second derivative.	M1
	Obtains maximum after correctly substituting 81 into correct second derivative to give correct negative quantity $-\dfrac{15}{36}$ o.e. or decimal e.g. -0.4…. (see note below) and considers negative sign deducing maximum. Note that a correct second derivative followed by $x = 81 \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{15}{4}81^{-\frac{3}{4}} - \dfrac{5}{9} = -\dfrac{5}{12}$ therefore maximum scores B1M1A0 here.	A1
		[3]
		10 marks
	Notes	
(a)	**M1:** Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just $1000 \to 0$) **A1:** Two correct terms and no extra terms. Terms may be un-simplified.	
(b)	**M1:** Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where *n* is real and *k* is non-zero **dM1:** Correct processing to obtain a value for *x*. (Dependent on the first method mark). This mark can only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must have the **correct powers of *x*.** E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}}\left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow x = \sqrt[3]{k}$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. **A1:** cao **dM1:** Substitutes their **positive** value for *x* into *y* = … **and not into** $\dfrac{dy}{dx} = \ldots$ (Dependent on the first method mark) **A1:** cao If *x* = 81 appears from **no working** following a correct derivative score M1M0A0 then allow full recovery.	
(c)	**B1ft:** Correct **follow through** second derivative **M1:** Substitutes their non-zero *x* (**positive or negative**) into their second derivative. **Note**: Solving $\dfrac{d^2y}{dx^2} = 0$ is M0 **A1cso:** Completely correct work ($-\dfrac{5}{12}$ o.e.). Note that o.e. could be $= \dfrac{15}{4}\times\dfrac{1}{27} - \dfrac{5}{9}$ or $\dfrac{15}{108} - \dfrac{5}{9}$ or $\dfrac{5}{36} - \dfrac{5}{9}$ or -0.4….**but it has to be correct for the final mark.**	

11.

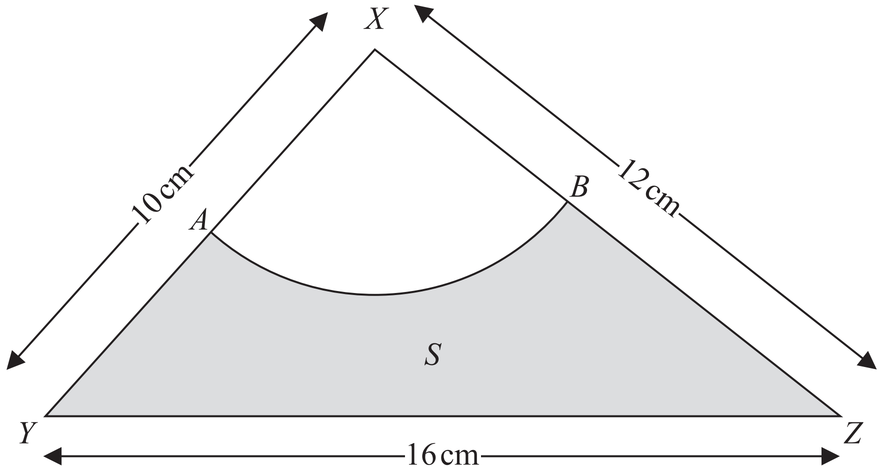


Figure 1

Figure 1 shows a triangle *XYZ* with *XY* = 10 cm, *YZ* = 16 cm and *ZX* =12 cm.

(a) Find the size of the angle *YXZ*, giving your answer in radians to 3 significant figures.

(3)

The point *A* lies on the line *XY* and the point *B* lies on the line *XZ* and *AX* = *BX* = 5 cm. *AB* is the arc of a circle with centre *X*.

The shaded region *S*, shown in Figure 1, is bounded by the lines *BZ*, *ZY*, *YA* and the arc *AB*.

Find

(b) the perimeter of the shaded region to 3 significant figures,

(4)

(c) the area of the shaded region to 3 significant figures.

(4)

Question Number	Scheme	Marks
11(a)	$16^2=10^2+12^2-2\times10\times12\cos\angle YXZ$	M1
	$\cos\angle YXZ=\frac{10^2+12^2-16^2}{2\times10\times12}$ or $\frac{-12}{240}$ or -0.05	A1
	$\angle BOC=1.62(08..)$ (N.B. 92.87 degrees is A0)	A1
		[3]
(b)	Uses $s=5\theta$ with their θ from part (a)	M1
	awrt 8.1	A1
	Perimeter $=r\theta+28$, $=$ 28 + their arc length	M1
	awrt 36.1	A1
		[4]
(c)	area of sector $=\frac{1}{2}(5)^2\theta$	B1ft
	area of triangle $=\frac{1}{2}10\times12\sin\theta$ ($=59.92$ *or* 59.93)	B1ft
	Area of shaded region = $\frac{1}{2}\times10\times12\sin\theta-\frac{1}{2}(5)^2\theta$ = 59.9… - 20.2… = 39.7 (cm^2)	M1 A1
		[4]
		(11 marks)
	Notes	
(a)	**M1:** Uses cosine rule – must be a correct statement **A1:** Correct value or correct numerical expression for $\cos\angle YXZ$ **A1:** accept awrt 1.62 and must be seen in part (a) (answer in degrees is A0 (92.865…))	
(b)	**M1:** Uses $s=5\theta$ with their θ in radians, or correct formula for degrees if working in degrees **A1:** Accept awrt 8.1 (may be implied by their perimeter) **M1:** Adds their arc length to 28 or (16 + 7 + 5) **A1:** Accept awrt 36.1 do not need units (ignore any given)	
(c)	**B1ft:** This formula **used** with their θ in radians or correct formula for degrees **B1ft:** Correct formula for area **used** – may use half base times height (may be implied by a correct answer (59.9…)) **M1:** Subtracts their sector area from their triangle area **this way round**. **A1:** awrt 39.7 – do not need units (ignore any given)	
	<u>Alternative approach to finding angle YXZ and area of triangle:</u> Let foot of perpendicular from X to YZ be W and $XW=h$ and $YW=x$ so $WZ=16-x$: $h^2+x^2=100,\quad h^2+(16-x)^2=144\Rightarrow x=\frac{53}{8},h=\frac{3\sqrt{399}}{8}$ M1: Correct work leading to values of x and h $\angle YXZ=\sin^{-1}\left(\frac{53}{80}\right)+\sin^{-1}\left(\frac{25}{32}\right)=1.62$ A1: Correct expression for $\angle YXZ$, A1: awrt 1.62 The B1 for the triangle area in (c) can then score for $\frac{1}{2}\times16\times"\frac{3\sqrt{399}}{8}"$. Note this is $3\sqrt{399}$	

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12.

$$\mathrm{f}(x) = \frac{(4 + 3\sqrt{x})^2}{x}, \qquad x > 0$$

(a) Show that $\mathrm{f}(x) = Ax^{-1} + Bx^{k} + C$, where A, B, C and k are constants to be determined.

(4)

(b) Hence find $\mathrm{f}'(x)$.

(2)

(c) Find an equation of the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = 4$

(4)

Question Number	Scheme	Marks
	(a) and (b) can be marked together	
12(a)	$\mathrm{f}(x)=\dfrac{16+24\sqrt{x}+9x}{x}$	M1
	$\mathrm{f}(x)=16x^{-1}+24x^{-\frac{1}{2}}+9$	M1A1A1
		[4]
(b)	$\mathrm{f}'(x)=-16x^{-2}-12x^{-\frac{3}{2}}$	M1 A1
		[2]
(c)	When $x = 4$, $y = 25$	B1
	$\mathrm{f}'(4)=-1-\dfrac{12}{8}=-2\frac{1}{2}$	M1
	Equation of tangent is $y-25=-\dfrac{5}{2}(x-4)$	M1 A1
		[4]
		10 marks
	Notes	
(a)	**M1:** expands numerator into a three (or four) term quadratic in $\sqrt{x}$ (allow $\left(\sqrt{x}\right)^2$ for x) **M1:** Divides at least one term in numerator by x correctly following an attempt at expansion. May just be $\dfrac{16}{x}$. **A1:** Two correct terms **A1:** All terms correct	
(b)	**M1:** Evidence of differentiation $x^n \to x^{n-1}$ of an expression of the form Ax^{-1} **or** Bx^k so $x^{-1}\to x^{-2}$ **or** $x^k \to x^{k-1}$ $(k\neq 1)$ and not just $C \to 0$. Differentiating top and bottom separately is M0. Note this is a hence and so attempts at e.g. use of the quotient rule scores M0. **A1:** cao and cso (May be un-simplified) **Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the same derivative so scores M1A0 if otherwise correct.**	
(c)	**B1:** 25 only **M1:** Substitute $x = 4$ into their derived function **M1:** Uses their "25" and their "gradient" which has come from calculus (**not the normal gradient**) and $x = 4$ to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value for c **A1:** any correct form e.g. $y=-\dfrac{5}{2}x+35,\qquad 5x+2y-70=0$ **BUT NOT JUST** $\dfrac{y-25}{x-4}=-\dfrac{5}{2}$ **, this scores M1A0** **Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the correct answer in (c) and will lose the final A mark if otherwise correct.**	

13. The equation $k(3x^2 + 8x + 9) = 2 - 6x$, where k is a real constant, has no real roots.

(a) Show that k satisfies the inequality

$$11k^2 - 30k - 9 > 0$$

(4)

(b) Find the range of possible values for k.

(4)

Question Number	Scheme	Marks
13(a)	$3kx^2 + (8k+6)x + 9k - 2 = 0$ or $3kx^2 + 8kx + 6x + 9k - 2 = 0$	B1
	Uses $b^2 - 4ac$ with $a = 3k,\ b = 8k \pm 6$ and $c = 9k \pm 2$	M1
	$-44k^2 + 120k + 36 < 0$ or $36 < 44k^2 - 120k$ o.e. **Reached with no errors**	A1
	$11k^2 - 30k - 9 > 0$ *	A1*
		[4]
(b)	Attempts to solve $11k^2 - 30k - 9 = 0$ to give $k =$	M1
	$\Rightarrow$ Critical values, $k = 3, -\dfrac{3}{11}$	A1
	$k > 3 \text{(or) } k < -\dfrac{3}{11}$	M1 A1cao
		[4]
		8 marks
	Notes	
(a)	**B1:** Multiplies by k and collects terms to one side in any order. Allow the x terms not to be combined and the '= 0' may be implied by use of a **correct** discriminant. **M1:** Attempts $b^2 - 4ac$ with $a = 3k,\ b = 8k \pm 6$ and $c = 9k \pm 2$ or uses quadratic formula with $b^2 - 4ac$ seen to solve their equation or uses $b^2 = 4ac$ or e.g. $b^2 < 4ac$. **There must be no x's.** **A1: Obtains a correct three term quadratic inequality** that is not the printed answer with **no errors seen.** **A1:** Correct answer with **no errors**	
(b)	**M1:** Uses factorisation, formula, or completion of square method to find **two values** for k or finds two **correct** answers with no obvious method for <u>**the given**</u> three term quadratic **A1:** Obtains $k = 3, -\dfrac{3}{11}$ accept awrt - 0.272 **M1: Chooses outside region (**$k <$ Their Lower Limit $k >$ Their Upper Limit) for a 3 term quadratic inequality. Do not award simply for diagram or table. **A1:** $k > 3 \text{(or) } k < -\dfrac{3}{11}$ must be exact here but allow $-0.\dot{2}\dot{7}$ for $-\dfrac{3}{11}$. Allow other notation such as $\left(-\infty, -\dfrac{3}{11}\right) \cup (3, \infty)$ $k > 3$ **and** $k < -\dfrac{3}{11}$ and $-\dfrac{3}{11} > k > 3$ score M1A0 ISW if possible e.g. $k > 3,\ k < -\dfrac{3}{11}$ followed by $-\dfrac{3}{11} > k > 3$ can score M1A1 $k > 3,\ k > -\dfrac{3}{11}$ followed by $k > 3 \text{(or) } k < -\dfrac{3}{11}$ can score M1A1 Allow (b) to be solved in terms of x for the first 3 marks but the final A mark needs the regions in terms of k. **Fully correct answer with no working scores full marks.** **Answers that are otherwise correct but use $\le, \ge$ lose final mark.**	

14. (i) Given that

$$\log_a x + \log_a 3 = \log_a 27 - 1, \text{ where } a \text{ is a positive constant}$$

find, in its simplest form, an expression for x in terms of a.

(4)

(ii) Solve the equation

$$(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$$

showing each step of your working.

(4)

Question Number	Scheme	Marks
14(i)	$\log_a x+\log_a 3=\log_a 27-1$ so $\log_a \frac{3x}{27}=-1$ Or $\log_a x+\log_a 3=\log_a 27-\log_a a$ so $\log_a 3x=\log_a \frac{27}{a}$ Or $\log_a x+1=\log_a 27-\log_a 3=\log_a 9$ so $\log_a ax=\log_a 9$	M1 A1
	$\frac{3x}{27}=a^{-1}$	M1
	$x=9a^{-1}$ or $\frac{9}{a}$	A1
		[4]
(ii)	$x^2-7x+12=0$ and attempt to solve to give $x=\ldots$ or $\log_5 y=\ldots$ (implied by correct answers)	M1
	x (or $\log_5 y$) = 3 **and** 4	A1
	$y=5^3$ **or** 5^4	dM1
	y = 125 **and** 625	A1
		[4]
		8 marks
	Notes	
(i)	**M1**: Uses sum or difference of logs correctly e.g. $\log x+\log 3=\log 3x$ or $\log 27-\log 3=\log 9$ or $\log 27-\log x=\log\frac{27}{x}$ etc. or writes 1 as $\log_a a$ **A1:** Uses **two** rules correctly to obtain correct log equation **M1:** Removes logs correctly to obtain an equation connecting x and a **A1:** Correct simplified answer Note that some candidates interpret $\log_a 27-1$ as $\log_a(27-1)$. This can score a maximum of 1 out of 4 if they have $\log x+\log 3=\log 3x$ Note that $\log_a x+\log_a 3=\log_a 27-1$ so $\frac{\log_a 3x}{\log_a 27}=-1\Rightarrow\frac{3x}{27}=a^{-1}$ etc. scores **M1A0M0A0** Note that $\log_a x+\log_a 3=\log_a 27-1$ so $\frac{\log_a x\log_a 3}{\log_a 27}=-1\Rightarrow\frac{3x}{27}=a^{-1}$ etc. scores **no marks**	
(ii)	**M1:** Recognise and attempt to solve quadratic **A1:** Obtain both 3 and 4 (Both correct implies M1A1) **dM1:** Uses powers **correctly** to find a value for y **(Dependent on first method mark)** **A1:** Both values correct	

15. The points A and B have coordinates $(-8, -8)$ and $(12, 2)$ respectively.
AB is the diameter of a circle C.

(a) Find an equation for the circle C.

(6)

The point $(4, 8)$ also lies on C.

(b) Find an equation of the tangent to C at the point $(4, 8)$, giving your answer in the form $ax + by + c = 0$

(4)

Question Number	Scheme	Marks
15 (a)	Mid-point of AB = (2, - 3)	M1 A1
	$(r^2) = (12-"2")^2 + (2-"-3")^2$ or $(r^2) = (-8-"2")^2 + (-8-"-3")^2$ or $(d^2) = (-8-12)^2 + (-8-2)^2$	M1
	$r^2 = 125$	A1
	$"125" = (\boldsymbol{x} \pm "2")^2 + (\boldsymbol{y} \pm "-3")^2$	M1
	$125 = (x-2)^2 + (y+3)^2$	A1
		[6]
(b)	gradient from "(2, −3)" to (4, 8) $= \dfrac{8-"-3"}{4-"2"}, \left(=\dfrac{11}{2}\right)$	M1
	ZM has gradient $-\dfrac{1}{m}$ $\left(=-\dfrac{2}{11}\right)$	M1
	Either : $y - 8 = "-\frac{2}{11}"(x-4)$ **or**: $y = "-\frac{2}{11}"x + c$ and $8 = "-\frac{2}{11}"(4) + c \Rightarrow c = "8\frac{8}{11}"$	ddM1
	$2x + 11y - 96 = 0$	A1
		[4]
		(10marks)
	Notes	
(a)	**M1:** Uses midpoint formula, or implied by y coordinate of -3 or x coordinate of 2 **A1:** cao **M1:** Finds radius or radius^2, diameter or diameter^2 using any valid method – probably distance from centre to one of the points. **Need not state *r* =… so ignore lhs – you are just looking for correct use of Pythagoras with or without the square root so ignore how they reference it for this mark.** **A1:** for any equivalent $r^2 = 125$ or $r = \sqrt{125}$ (11.18...) etc . Their numeric answer must be identified here as either r or r^2 (may be implied by their equation). If they halve it or double it, this is M1 A0. **M1:** Attempt to use a true equation for circle with their centre and radius or the letter r, allow sign slips in brackets but do not allow use or r instead of r^2 in the equation. So must be using $r^2 = (\boldsymbol{x} \pm ...)^2 + (\boldsymbol{y} \pm ...)^2$ **A1:** correct answer only (Allow $\left(5\sqrt{5}\right)^2$ instead of 125 but not $5\sqrt{5}^{\ 2}$)	
(b)	**M1:** States or uses gradient equation correctly **with their centre and (4, 8).** Must be using their centre and (4, 8). If no method is shown and gradient incorrect for their values score M0. **M1:** Finds negative reciprocal. Follow through their gradient **ddM1:** Correct straight line method with (4, 8) and perpendicular gradient. Dependent on **both** previous method marks having been scored. **A1:** cao – accept multiples of this equation **(Note integer coefficients not required)** A common error here is to use the diameter to find the gradient. This usually scores M0M1ddM0A0 i.e. just one mark for the perpendicular gradient rule.	
	(b) Alternative uses implicit differentiation: e.g. $125 = (x-2)^2 + (y+3)^2 \Rightarrow 2(x-2) + 2(y+3)\dfrac{dy}{dx} = 0$ M1(correct implicit differentiation) oe $\Rightarrow \dfrac{dy}{dx} = \dfrac{2-x}{y+3} = \dfrac{2-4}{8+3}$ M1(Substitution) Then follow the scheme.	

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16.

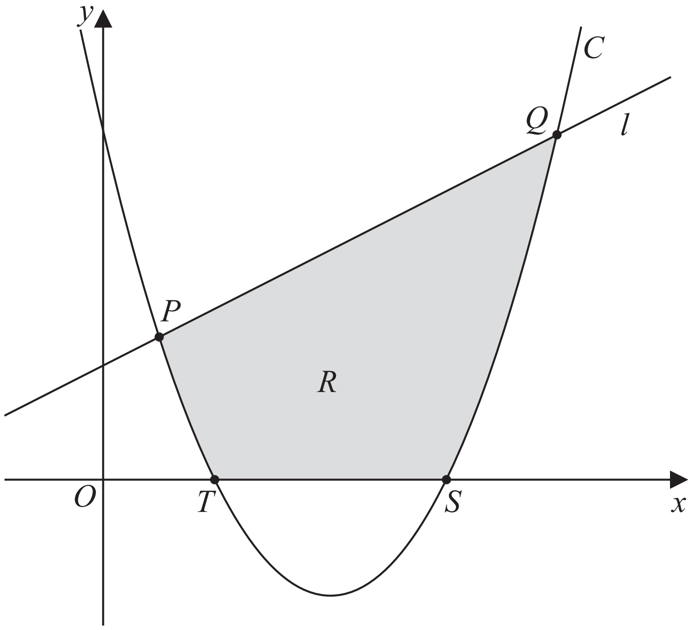

Figure 2

The straight line l with equation $y = \frac{1}{2}x + 1$ cuts the curve C, with equation $y = x^2 - 4x + 3$, at the points P and Q, as shown in Figure 2

(a) Use algebra to find the coordinates of the points P and Q.

(5)

The curve C crosses the x-axis at the points T and S.

(b) Write down the coordinates of the points T and S.

(2)

The finite region R is shown shaded in Figure 2. This region R is bounded by the line segment PQ, the line segment TS, and the arcs PT and SQ of the curve.

(c) Use integration to find the exact area of the shaded region R.

(8)

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Question Number	Scheme	Marks
16(a)	$\frac{1}{2}x+1=x^2-4x+3$	M1
	$2x^2-9x+4=0 \Rightarrow x=½$ or $x=4$	dM1 A1
	$y=5/4$ or $y=3$	dM1 A1
		[5]
(b)	Curve meets x-axis at $x=3$ and at $x=1$ **(No need to see $y = 0$)**	M1 A1
		[2]
	NOTE that the subscripted A's refer to areas on the diagram given at the end of the scheme. **All the method marks are for <u>their</u> x = 1/2 , 4, 1 and 3**	
(c) **Way 1**	$\int x^2-4x+3\,\mathrm{d}x=\frac{1}{3}x^3-2x^2+3x$	M1 A1
	Use limits 1 and ½ $[(\frac{1}{3}(1)^3-2(1)^2+3\times1)-(\frac{1}{3}(\frac{1}{2})^3-2.(\frac{1}{2})^2+3\times(\frac{1}{2}))]$ A_1	M1
	Use limits 4 and 3 $[(\frac{1}{3}(4)^3-2(4)^2+3\times(4))-(\frac{1}{3}(3)^3-2.(3)^2+3\times(3))]$ A_2	M1
	Area of trapezium = $\frac{1}{2}(a+b)\times h=\frac{1}{2}(\frac{5}{4}+3)\times(4-\frac{1}{2})=...$ or $\int_{\frac{1}{2}}^{4}(\frac{1}{2}x+1)\mathrm{d}x=\left[\frac{1}{4}x^2+x\right]_{\frac{1}{2}}^{4}=(4+4)-(\frac{1}{16}+\frac{1}{2})=...$	M1
	7.4375 $(7\frac{7}{16})$ $\left(\frac{119}{16}\right)$ (may be implied by correct final answer)	A1
	Uses correct combination of correct areas. Area of region = Area of trapezium – A_1-A_2 **Dependent on all previous method marks**	ddddM1
	$=7.4375-\frac{7}{24}-\frac{4}{3}=\frac{93}{16}$ or 5.8125	A1
		[8]
(c) **Way 2**	**Alternative method using "line – curve"** and subtracting area below x- axis	
	$\int -x^2+\frac{9}{2}x-2\mathrm{d}x=-\frac{x^3}{3}+\frac{9}{4}x^2-2x$ or $\int x^2-\frac{9}{2}x+2\mathrm{d}x=\frac{x^3}{3}-\frac{9}{4}x^2+2x$	M1A1
	Use limits ½ and 4 on this *subtracted* integration $\left(A_3+A_4+A_5+A_6\right)=6\frac{2}{3}+\frac{23}{48}=...$	M1
	$\pm\int x^2-4x+3\mathrm{d}x=\pm\left(\frac{1}{3}x^3-2x^2+3x\right)$	M1
	Use limits 1 and 3 on their integrated curve to obtain $A_6=\pm\frac{4}{3}$	M1A1
	Uses correct combination of correct areas. Area of region $=\left(A_3+A_4+A_5+A_6\right)-A_6$ **Dependent on all previous method marks**	ddddM1
	$6\frac{2}{3}+\frac{23}{48}-\frac{4}{3}=\frac{93}{16}$	A1
		[8]
(c) **Way 3**	**Alternative method using "line – curve"** for areas A_3 and A_4 and adding smaller trapezium	
	$\int -x^2+\frac{9}{2}x-2\mathrm{d}x=-\frac{x^3}{3}+\frac{9}{4}x^2-2x$ or $\int x^2-\frac{9}{2}x+2\mathrm{d}x=\frac{x^3}{3}-\frac{9}{4}x^2+2x$	M1A1
	Use limits 1 and ½ $[(-\frac{1}{3}(1)^3+\frac{9}{4}(1)^2-2\times1)-(-\frac{1}{3}(\frac{1}{2})^3+\frac{9}{4}(\frac{1}{2})^2-2\times\frac{1}{2}]$ A_3	M1
	Use limits 4 and 3 $[(-\frac{1}{3}(4)^3+\frac{9}{4}(4)^2-2\times4)-(-\frac{1}{3}(3)^3+\frac{9}{4}(3)^2-2\times3]$ A_4	M1
	Area of trapezium = $\frac{1}{2}(a+b)\times h=\frac{1}{2}(\frac{3}{2}+\frac{5}{2})\times(3-1)=...$ or $\int_{1}^{3}(\frac{1}{2}x+1)\mathrm{d}x=\left[\frac{1}{4}x^2+x\right]_{1}^{3}=(\frac{9}{4}+3)-(\frac{1}{4}+1)=...$	M1
	$=4$	A1
	Uses correct combination of correct areas. Area of region $=A_3+A_4+A_5$ **Dependent on all previous method marks**	ddddM1
	$\frac{19}{48}+\frac{17}{12}+4=\frac{93}{16}$	A1
		[8]

(c) **Way 4**	**Alternative method: Finds area of larger trapezium and subtracts A₁+ A₂ which is found by integrating quadratic between ½ and 4 and adding area below *x*-axis**	
	$\int x^2-4x+3\ \mathrm{d}x=\tfrac{1}{3}x^3-2x^2+3x$	M1 A1
	Use limits 4 and ½ $[(\tfrac{1}{3}(4)^3-2(4)^2+3\times4)-(\tfrac{1}{3}(\tfrac{1}{2})^3-2.(\tfrac{1}{2})^2+3\times(\tfrac{1}{2}))]$ $A_1+A_2-A_6$ **AND** Use limits 3 and 1 $\pm[(\tfrac{1}{3}(3)^3-2(3)^2+3\times3)-(\tfrac{1}{3}(1)^3-2.(1)^2+3\times(1))]$ $\pm A_6$	M2
	Area of trapezium = $\tfrac{1}{2}(a+b)\times h=\tfrac{1}{2}(\tfrac{5}{4}+3)\times(4-\tfrac{1}{2})=...$ or $\int_{\frac{1}{2}}^{4}(\tfrac{1}{2}x+1)\mathrm{d}x=\left[\tfrac{1}{4}x^2+x\right]_{\frac{1}{2}}^{4}=(4+4)-(\tfrac{1}{16}+\tfrac{1}{2})=...$	M1
	7.4375 $(7\tfrac{7}{16})$ (may be implied by correct final answer)	A1
	Uses correct combination of correct areas. Area of region $=7.4375-\left(A_1+A_2-A_6+A_6\right)$ **Dependent on all previous method marks**	ddddM1
	$=7.4375-\left(\tfrac{7}{24}+\tfrac{4}{3}\right)=\dfrac{93}{16}$	A1
		[8]
		15 marks
	Notes	
(a)	**M1:** Puts equations equal or finds *x* in terms of *y* and substitutes or substitutes for *x* **dM1:** Solves three term quadratic in *x* to obtain *x* = … or in *y* to obtain *y* = … (Dependent on **first** M) **A1:** Both answers correct **dM1:** Obtains at least one value for *y* or *x* (Dependent on **first** M) **A1:** Both correct **Note:** Allow candidates to obtain $x^2-\dfrac{9}{2}x+2=0$ and solve as $\left(2x-1\right)\left(x-4\right)=0\Rightarrow x=\dfrac{1}{2},4$ **The coordinates do not need to be 'paired'**	
(b)	**M1:** Attempts to solve $0=x^2-4x+3$ according to the usual rules **A1:** cao Attempts by T&I can score both marks for *x* = 1 and *x* = 3. If one solution is obtained by this, score M1A0	
	For (c) do not allow 'mixed' methods. For their strategy, they must be finding the appropriate areas but apply the method for the scheme that gives the most credit for the candidate.	
(c) **Way 1**	**M1:** Attempt at integration of the given quadratic expression $\left(x^n\to x^{n+1}\text{ at least once}\right)$ **A1:** Correct integration of the given quadratic expression **M1:** Finds area of A_1 **M1:** Finds area of A_2 **M1:** Finds area of appropriate trapezium **A1:** Correct area of trapezium 7.4375 $(7\tfrac{7}{16})$ **ddddM1:** correct final combination **A1:** any correct form of this exact answer	
(c) **Way 2**	**M1:** Attempt at integration of ±(the given quadratic expression – the given line) $\left(x^n\to x^{n+1}\text{ at least once}\right)$ **A1:** Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0 **M1:** Uses the limits ½ and 4 on their *subtracted* integration **M1:** Attempts to integrate curve **M1:** Uses the limits 1 and 3 on the integrated curve *C* **A1:** Obtains $A_6=\pm\tfrac{4}{3}$ **ddddM1:** correct final combination **A1:** any correct form of this exact answer Note: A common error with this method is to use the limits ½ and 4 on their *subtracted* integration and then stop (this should give an area of $\dfrac{343}{48}$). This will usually score 3/8 in (c)	

(c) **Way 3**	**M1:** Attempt at integration of ±(the given quadratic expression – the given line) $\left(x^n \to x^{n+1} \text{ at least once}\right)$ **A1:** Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0 **M1:** Uses the limits ½ and 1 on their *subtracted* integration **M1:** Uses the limits 4 and 3 on their *subtracted* integration **M1:** Finds area of appropriate trapezium **A1:** Correct area of trapezium 4 **ddddM1:** correct final combination **A1:** any correct form of this exact answer
(c) **Way 4**	**M1:** Attempt at integration of the given quadratic expression $\left(x^n \to x^{n+1} \text{ at least once}\right)$ **A1:** Correct integration of the given quadratic expression **M2:** Finds area of $A_1 + A_2 - A_6$ by using the limits ½ and 4 **and** finds area of A_6 by using the limits 1 and 3 **M1:** Finds area of appropriate trapezium **A1:** Correct area of trapezium $7.4375 \ (7\tfrac{7}{16})$ **ddddM1:** correct final combination **A1:** any correct form of this exact answer

Diagram for Question 16