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Pearson Edexcel	Centre Number	Candidate Number
Advanced Level	nemati	<u>cs C12</u>
Advanced Subsidia	ry	
Advanced Subsidia Wednesday 20 May 2015 – Time: 2 hours 30 minutes	Morning	Paper Reference WMA01/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.





Turn over 🕨



WMA01

Past Paper This resource was created and owned by Pearson Edexcel Leave blank **1.** The line l_1 has equation 10x - 2y + 7 = 0(a) Find the gradient of l_1 (1) The line l_2 is parallel to the line l_1 and passes through the point $\left(-\frac{1}{3}, \frac{4}{3}\right)$. (b) Find the equation of l_2 in the form y = mx + c, where *m* and *c* are constants. (3) 2

Question Number	Scheme	Marks
1.	The line l_1 has equation $10x - 2y + 7 = 0$	
(a) (b)	Gradient is 5 Gradient of parallel line is equal to their previous gradient Equation is $y - \frac{4}{3} = "5"(x - (-\frac{1}{3}))$ So $y = 5x + 3$	B1 [1] M1 M1 A1 [3] (4 marks)

(a)

B1 Gradient given as 5 or 10/2 or exact equivalent. Do not accept if embedded within an equation. You must see 5 (or equivalent)

(b)

- M1 Gradient of lines are the same. This may be implied by sight of their '5' in a gradient equation. For example you may see y = '5'x + c or equivalent as the equation of their ''parallel'' line.
- M1 For an attempt to find an equation of a line using $\left(-\frac{1}{3}, \frac{4}{3}\right)$ and a numerical gradient (which may be different to the gradient used in part (a). For example they may try to find a normal!) It must be a full attempt to find an equation.

Accept $y - \frac{4}{3} =$ "numerical $m'\left(x - \frac{1}{3}\right)$ or equivalent. Allow one sign slip for the coordinate. If y = mx + c is used it must proceed as far as finding the value of "c"

A1 Correct answer only (no equivalents) y = 5x + 3, but do allow y = 5x + c followed by c = 3.

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		Leave
2.	$f(x) = x^4 - x^3 + 3x^2 + ax + b$	blank
where <i>a</i> an	nd b are constants.	
When $f(x)$	is divided by $(x - 1)$ the remainder is 4	
When $f(x)$	is divided by $(x + 2)$ the remainder is 22	
Find the va	alue of <i>a</i> and the value of <i>b</i> .	
		(5)
4		

Question Number	Scheme	Marks
2.	$f(x) = x^4 - x^3 + 3x^2 + a x + b$	
	Attempts to set $f(\pm 1) = 4$ or attempts to set $f(\pm 2) = 22$	M1
	Obtains $3+a+b=4$ or equivalent	A1
	Obtains $16+8+12-2a+b=22$ or equivalent	A1
	Solve simultaneous equations to obtain $a = 5$ and $b = -4$	M1 A1
		[5]
		(5 marks)

M1 Attempts to set either $f(\pm 1) = 4$ or $f(\pm 2) = 22$ Alternatively dividing f(x) by (x-1) or dividing f(x) by (x+2) and setting the remainder equal to 4 or 22 For division look for a minimum of

$$x-1)\overline{x^{4}-x^{3}+3x^{2}+ax+b} \quad \text{OR} \qquad x+2)\overline{x^{4}-x^{3}+3x^{2}+ax+b}$$

(a) + (b)	(a) + (b)

followed by the remainder (involving both a and b) set equal to 4 or 22

A1 3+a+b=4 or equivalent.

The powers must be multiplied out but there is no requirement to collect terms. A1 16+8+12-2a+b=22

The powers must be multiplied out but there is no requirement to collect terms.

M1 Solves a pair of simultaneous equations in both *a* and *b* For your information the correct simplified equations are a + b = 1 and -2a + b = -14Minimal evidence is required for this as it could be done on a GC. Just accept values of both *a* and *b*

A1 a = 5 and b = -4

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2			Leave blank
3.	Given that		
	1 3		
	$y = \frac{1}{27}x^3$		
	express each of the following in the form kx^n where k and n are constants.		
	(a) $v_{3}^{\frac{1}{3}}$		
	(u) y ²	(1)	
	(b) $3y^{-1}$		
		(1)	
	(c) $\sqrt{(27y)}$	(1)	
		(1)	

Question Number	Scheme	Marks
3. (a)	$\frac{1}{3}x$ as the final answer.	B1
(b)	$81x^{-3}$ as the final answer	B1
(c)	$x^{\frac{3}{2}}$ as the final answer	B1
		[3] (3 marks)

In all parts of this question candidates do not have to explicitly state the values of k and n. Award the mark(s) as above. If they go on to give incorrect values of k and n you may isw, but do not isw on incorrect index work Eg. $\frac{1}{3}x = x^{-3}$. If candidates make two different attempts and give two (or more) different answers then please put these in review

B1
$$\frac{1}{3}x$$
 but accept equivalent such as $\frac{x}{3}$, $\frac{1}{3} \times x$, $\frac{1}{3}x^1$ or $3^{-1}x$ etc.

(b)

B1 $81x^{-3}$. Accept exact equivalents such as $81 \times x^{-3}$ Do not accept $\frac{81}{x^3}$ as the final answer unless it is preceded by $81x^{-3}$ as it is not in the form required by the question.

(c)

B1

 $x^{\frac{3}{2}}$ but accept exact equivalents such as $1 \times x^{1.5}$ Do not accept $x x^{\frac{1}{2}}$ or $x \sqrt{x}$ as the final answer unless preceded by $x^{\frac{3}{2}}$ as they are not in the form required by the question.

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(2)

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blank

4. (a) Sketch the graph of $y = \frac{1}{x}$, x > 0

The table below shows corresponding values of x and y for $y = \frac{1}{x}$, with the values for y rounded to 3 decimal places where necessary.

x	1	1.5	2	2.5	3
У	1	0.667	0.5	0.4	0.333

(b) Use the trapezium rule with all the values of y from the table to find an

approximate value, to 2 decimal places, for $\int_{1}^{3} \frac{1}{x} dx$ (4)



Question Number		Scheme					Mark	s		
4 (a)					M pa A1 an	1: General sha rt of rectangu 1: Accuracy on d no crossing o	pe correct so llar hyperbola asymptotes of axes	ı	M1 A1	
			1							[2]
(b)	x	1	1.5	2		2.5	3	_		
(0)	У	1	0.667	0.5		0.4	0.333			
	State $h = 0.5$, or	use of $\frac{1}{2} \times 0$.	5;						B1 aef	
	${1+0.3}$	33+2(0.667	+0.5+0.4)			For structure o	f {	};	M1A1	
	$\frac{1}{2} \times 0.5 \times \{ 4.467 \}$	$\frac{n}{2}$ = awrt 1.12							A1cao	
									(6 ma	[4] arks)

(a)

- M1 General shape correct showing **part of a rectangular hyperbola in the first quadrant**. Condone for this mark the curve meeting or intersecting either axis. Condone incorrect asymptotes, for example at y = 1. See Practice, examples on the following page and Qualification for clarification
- A1 Do not allow for intersections with axes. Curve must appear to approach/ be asymptotic to both the x and y axes. Ignore sections where x < 0

(b)

- B1 For using a strip width of 0.5. This may appear in a trapezium rule as $\frac{1}{2} \times 0.5$ or 0.25 or equivalent
- M1 Scored for the correct $\{\dots, \}$ bracket structure. It needs to contain the first *y* value +last *y* value with 2 times the sum of the remaining *y* values in the table (with no additional values). If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). It is M0 if values used in brackets are *x* values instead of *y* values
- A1 For the correct bracket $\{\dots, \}$. Accept $\{1+0.33+2(0.67+0.5+0.4)\}$ where candidate rounds to 2dp
- A1 For awrt 1.12.

NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 h(a + b) used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Special case: Bracketing mistake $0.25 \times (1+0.333) + 2(0.667+0.5+0.4)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).



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		Leav
5. (i) Find, g	iving your answer to 3 significant figures, the value of y for which	blan
	21, 12	
	$3^{y} = 12$	(2)
(11) Solve, g	giving an exact answer, the equation	
	$\log_2(x+3) - \log_2(2x+4) = 4$	
(You sł	hould show each step in your working.)	
		(4)

Question Number	Scheme	Marks
5. (i)	Use or state $y \log 3 = \log 12$ or $\log_3 12$	M1
	Obtain $y = 2.26$ or awrt 2.26	A1 [2]
(ii)	Use or state $\log_2 16 = 4$ or $2^4 = 16$	B1
	Use $\log_2(x+3) - \log_2(2x+4) = \log_2\frac{(x+3)}{(2x+4)}$	M1
	Then $\frac{x+3}{(2x+4)} = 16$, and so $x = -\frac{61}{31}$	A1 , A1 [4]
		6 marks
Alt (ii)	$\log_2(x+3) - \log_2(2x+4) = \log_2 16$	B1
	$\log_2(x+3) = \log_2 16 + \log_2(2x+4)$	
	$\log_2(x+3) = \log_2 16(2x+4)$	M1
	(x+3) = 16(2x+4)	
	$x = -\frac{61}{31}$	A1

(i)

M1 Uses power law for logs. This method may be implied by sight of $y \log 3 = \log 12$ or $\log_3 12$ The method may be scored for using trial and error provided you see $3^{2.26} = 11.9...$ and $3^{2.27} = 12.1...$ A1 awrt 2.26.

Just the answer with no (incorrect) working is 2 marks.

(ii)

B1 Connects 16 with '4' correctly. Evidence would be $\log_2 16=4$ or $2^4 = 16$ and could be scored at any time

M1 For the correct use of either the addition or subtraction law of logs

A1 Correct equation not involving logs

A1 cso $x = -\frac{61}{31}$

Special case:

Candidates who recover from "incorrect " log work and achieve the correct answer can score special case 1010.

$$\log_2(x+3) - \log_2(2x+4) = 4 \Longrightarrow \frac{\log_2(x+3)}{\log_2(2x+4)} = \log_2 16 \Longrightarrow \frac{x+3}{2x+4} = 16 \Longrightarrow x = -\frac{61}{31}$$

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6. (a) Find the	e first 3 terms in ascending powers of x of the binomial expansion o	f
	$(2+ax)^{\circ}$	
where <i>a</i>	is a non-zero constant. Give each term in its simplest form.	
		(4)
Given that,	in the expansion, the coefficient of x is equal to the coefficient of x	2
(b) find the	value of <i>a</i> .	
		(2)
12		

Summer 2015 Past Paper (Mark Scheme)

Question Number	Scheme	Marks
6.(a)	$(2+ax)^6 = 2^6 + {6 \choose 1} 2^5 \cdot (ax) + {6 \choose 2} 2^4 \cdot (ax)^2 + \dots$	M1
	$=64,+192ax+240a^2x^2+$	B1, A1, A1
		[4]
(b)	$192a = 240a^2$	M1
	$a = \frac{192}{240} = 0.8$ or equivalent	A1
		[2] 6 marks
Alt 6.(a)	$(2+ax)^{6} = 2^{6} \left(1+\frac{a}{2}x\right)^{6} = 2^{6} \left(1+6\times\frac{a}{2}x+\frac{6\times5}{2}\left(\frac{a}{2}x\right)^{2}+\dots\right)$	M1
	$=64,+192ax+240a^2x^2+$	B1, A1, A1
		[4]

(a)

M1 The method mark is awarded for an attempt at a Binomial expansion to get an unsimplified second or third term – Look for a correct binomial coefficient multiplied by a correct power of x. Eg ${}^{6}C_{1}...x$ or ${}^{6}C_{2}..x^{2}$ Condone bracket errors or errors (or omissions) in the powers of 2. Accept any notation for ${}^{6}C_{1}$, ${}^{6}C_{2}$, e.g. as on scheme or 6, and 15 from Pascal's triangle. This mark may be given if no working is shown, if either or both of the terms including x is correct. If the candidate attempts the expansion in descending powers allow ${}^{6}C_{5}...x^{5}$ or ${}^{6}C_{4}..x^{4}$ oe.

In the alternative it is for the correct form inside the bracket accepting either $1 + 6 \times \frac{a}{2}x + \frac{6 \times 5}{2} \left(\frac{a}{2}x\right)^2$

or
$$1 + 6 \times \frac{a}{2}x + \frac{6 \times 5}{2} \frac{a}{2}x^2$$

B1 Must be simplified to 64 (writing just 2^6 is B0).

A1 Score for either of 192a x or $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket

A1 Score for both of 192a x and $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket Allow listing of terms 64, 192ax, $240a^2x^2$ for all 4 marks.

(b)

M1 Score for setting the coefficients of their x and x^2 terms equal. They must reach an equation not involving x's. A1 This is cso for any equivalent fraction or decimal to 0.8. Ignore any reference to a = 0.

www.mystudybro.com This resource was created and owned by Pearson Edexcel Past Paper WMA01 Leave blank 7. В A -9cm-Figure 1 Figure 1 shows a circle with centre O and radius 9 cm. The points A and B lie on the circumference of this circle. The minor sector OAB has perimeter 30 cm and the angle between the radii OA and OB of this sector is θ radians. Find (a) the length of the arc AB, (1) (b) the value of θ , (2) (c) the area of the minor sector OAB, (2) (d) the area of triangle OAB, giving your answer to 3 significant figures. (2)



	Γ	1
Question Number	Scheme	Marks
7. (a)	Length of the arc is 12 (cm)	B1
(b)		[1]
(0)	States or uses arc length $12 = 9 \times \theta$	IVI I
	$A = \frac{4}{3}$ or awrt 1.33	A1
	$\frac{0}{3}$	[2]
(c)	Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times \theta'$	M1
	Area = awrt 54 (cm^2)	A1
		[2]
(d)	Area of triangle $QAB = 10.0$ with $A = -$ owner 20.4 cm ²	M1, A1
	Area of triangle $OAD = \frac{1}{2}9 \times 9 \times \sin \theta$, = awit 39.4 cm	[2]
		7 marks

Mark parts (a) and (b) together. For example the arc length may be scored in part (b)

B1 12 (cm) (b) Uses or states arc length $s = 9 \times \theta$ with their value of 's' from part (a) M1 Allow degree calculations for the M mark. Score for equivalents of $\frac{\theta}{12} = \frac{360}{2\pi \times 9}$ FYI $\theta \approx 76.4^{\circ}$ For $\theta = \frac{4}{3}$ or equivalent such as $\theta = \frac{12}{9}, 1\frac{1}{3}$. Allow decimals awrt 1.33 A1 Accept answers in terms of π but the accuracy must be either awrt 0.424π or 0.425π (c) Uses or states area $A = \frac{1}{2}9^2 \times \theta$ with their value of ' θ ' in radians from part (b) **M**1 Allow for a correct formula in degrees. $A = \frac{\theta}{360} \times \pi 9^2$ with their value of ' θ ' in degrees from part (b) Area = awrt 54 (cm^2) A1 (d) Correct method to find the area of triangle ABC, usually by using $\frac{1}{2} \times 9 \times 9 \times \sin \theta$ with their value of ' θ ' M1 from part (b). Alternative methods are acceptable using the fact that triangle ABC is isosceles but it must be a full

method involving the calculation of both base and perpendicular height by trigonometry twice or trigonometry once and Pythagoras once followed by $\frac{1}{2}bh$ The segment formula $\frac{1}{2} \times 9 \times 9 \times \theta - \frac{1}{2} \times 9 \times 9 \times \sin \theta$ will score M0 unless the candidate subtracts this from the

area of a sector.

A1 Awrt = 39.4 cm^2

(a)

		T
3. A 25-year programme for building new houses began in Core Town in the year 1986 and finished in the year 2010.		
The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find		
(a) the number of houses built in 1986, the first year of the building		
programme,	(5)	
(b) the total number of houses built in the 25 years of the programme.	(2)	

Question Number	Scheme	Marks
8. (a)	238 = a + k d or $108 = a + k d$ with any values for k	M1
	$238 = a + (14)d$ or $108 = a + (24)d$ or " d " = ± 13	A1
	238 = a + (14)d and $108 = a + (24)d$	A1
	Solves their simultaneous equations to obtain $a = ;$ (so $a = 420$	M1; A1
(b)	Uses $\frac{25}{2}(2 \times a + (25 - 1) \times "-13")$ or Uses $\frac{25}{2}(a + 108)$, to obtain = 6600	M1 , A1 [2]
		7 marks

(a)

M1 Score for 238 = a + k d or 108 = a + k d with any non-zero integer value for k

- A1 One of 238 = a + (14)d, or 108 = a + (24)d or "d" = ± 13 The "d" = ± 13 can be achieved from equations such as 238 = a + (13)d, or 108 = a + (23)d
- A1 Both 238 = a + (14)d, and 108 = a + (24)d
- M1 Finds the value of *a* by solving a pair of simultaneous equations in *a* and *d*
- A1 Achieves (a =) 420

In an alternative, working by a method of differences, you may see very few formulae: The scheme can be easily applied, 238-108 = 108-238

1st M1 Seeing $\frac{238 - 108}{10}$ or $\frac{108 - 238}{10}$

1st A1 For ± 13

- 2^{nd} A1 For $238+13\times14$ or $108+13\times24$. Note that this is achieved after the award of the next mark. It is scored as the third mark of (a) on e -pen.
- 2^{nd} M1 Sight of $238 + k' \times d$ or $108 + k' \times d$ with any non-zero integer values for k and their d
- 3rd A1 Achieves 420

(b)

M1 Uses a correct sum formula $S = \frac{n}{2} (2a + (n-1)d)$ with n = 25 and their values of a and d

Alternatively uses $S = \frac{n}{2}(a+l)$ with n = 25, l = 108 and their value of a.

A1 cao 6600

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		L
9. The equ	action $x^2 + (6k + 4)x + 3 = 0$, where k is a constant, has no real roots.	b
(a) Sho	\mathbf{w} that k satisfies the inequality	
(a) 5110	w that k satisfies the inequality	
	$9k^2 + 12k + 1 < 0$	(3)
(b) Find simp	I the range of possible values for <i>k</i> , giving your boundaries as fully plified surds.	
		(4)

Question Number	Scheme	Marks
9(a)	Uses $b^2 - 4ac = (6k + 4)^2 - 4 \times 1 \times 3$	B1
	Multiplies out and uses $b^2 - 4ac < 0$	M1
	to give $36k^2 + 48k + 16 - 12 < 0$ so $9k^2 + 12k + 1 < 0^*$	A1 *
(b)	Solves quadratic by formula or completion of square to give $k =$	M1
	Obtains $k = \frac{-12 \pm \sqrt{108}}{18}$ or accept $k = awrt - 1.24, -0.09$	A1
	Chooses region between two values and deduces $\frac{-2-\sqrt{3}}{3} < k < \frac{-2+\sqrt{3}}{3}$	M1 A1cao
		[4]
		/ marks

B1 Uses $b^2 - 4ac = (6k+4)^2 - 4 \times 1 \times 3$ or equivalent

M1 Attempt to multiply out and use their
$$b^2 - 4ac < 0$$
 or $b^2 < 4ac$

Alternatively they can find $b^2 - 4ac$ and then state $b^2 - 4ac < 0$ or $b^2 < 4ac$

A1* Achieves $9k^2 + 12k + 1 < 0$

This is a given answer and you must check that all aspects are correct. In all cases you should expect to see an intermediate line(s) (including <0) before the final answer.

$$b^2 - 4ac = 4(9k^2 + 12k + 1)$$
. As $b^2 - 4ac < 0 \Rightarrow 9k^2 + 12k + 1 < 0$ is fine for B1 M1 A1*
 $b^2 - 4ac = 4(9k^2 + 12k + 1)$. $\Rightarrow 9k^2 + 12k + 1 < 0$ is B1 M0 A0*

M1 Solves the equation $9k^2 + 12k + 1 = 0$ by formula or completing the square. Factorisation is not a suitable method in this case and scores M0. The answers could just appear from a graphical calculator.

Sight of either root,
$$\frac{-12-\sqrt{108}}{18}$$
 or $\frac{-12+\sqrt{108}}{18}$ is evidence that the formula has been used.
Sight of $-\frac{6}{9} + \sqrt{\frac{27}{81}}$ oe could be evidence that completion of square has been used.

A1 Accept
$$k = \frac{-12 - \sqrt{108}}{18}$$
 and $\frac{-12 + \sqrt{108}}{18}$ or decimal equivalents awrt 2dp -1.24 and -0.09

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation. This can be awarded for an inside region appearing in the form $\frac{-2-\sqrt{3}}{3} \le k \le \frac{-2+\sqrt{3}}{3}$

A1 cao
$$\frac{-2-\sqrt{3}}{3} < k < \frac{-2+\sqrt{3}}{3}$$
. Accept $-\frac{2}{3} - \frac{1}{\sqrt{3}} < k < -\frac{2}{3} + \frac{1}{\sqrt{3}}$
Accept equivalents such as $\left(\frac{-2-\sqrt{3}}{3}, \frac{-2+\sqrt{3}}{3}\right)$ $k > \frac{-2-\sqrt{3}}{3}$ and $k < \frac{-2+\sqrt{3}}{3}$ even $\left\{\frac{-2-\sqrt{3}}{3}, \frac{-2+\sqrt{3}}{3}\right\}$
Do not accept $\left[\frac{-2-\sqrt{3}}{3}, \frac{-2+\sqrt{3}}{3}\right]$ $k > \frac{-2-\sqrt{3}}{3}$ or $k < \frac{-2+\sqrt{3}}{3}$ and $k > \frac{-2-\sqrt{3}}{3}$, $k < \frac{-2+\sqrt{3}}{3}$

including the version of the final one without the coma

If the candidate writes $\frac{-2-\sqrt{3}}{3} < x < \frac{-2+\sqrt{3}}{3}$ this can be awarded M1 A0

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blank

This resource was created and owned by Pearson Edexcel **10.** A sequence is defined by $u_1 = 4$ $u_{n+1} = \frac{2u_n}{3}, \qquad n \ge 1$ (a) Find the exact values of u_2 , u_3 and u_4 (2) (b) Find the value of u_{20} , giving your answer to 3 significant figures. (2) $12 - \sum_{i=1}^{16} u_i$ (c) Evaluate giving your answer to 3 significant figures. (3) (d) Explain why $\sum_{i=1}^{N} u_i < 12$ for all positive integer values of *N*. (1)



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Question Number	Scheme	Marks
10 (a)	$u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$	M1, A1
(b)	$u_{20} = 4 \times \left(\frac{2}{3}\right)^{19}$; = 0.00180 or 0.0018 or exact equivalent	[2] M1; cao A1
(c)	Use $\sum_{i=1}^{16} u_i = \frac{4(1-(\frac{2}{3})^{16})}{1-\frac{2}{3}}$	[2] M1
	Find 12 - their $\sum_{i=1}^{16} u_i$	dM1
	= 12 - 11.9817 = awrt 0.0183	A1 [3]
(d)	12 is the sum to infinity (and all terms are positive) so sum is less than 12 Or $\sum_{i=1}^{n} u_i = \frac{4(1-(\frac{2}{3})^n)}{1-\frac{2}{3}} = 12-12(\frac{2}{3})^n$ and $(\frac{2}{3})^n > 0$ so is less than 12	B1 [1]
		[8 marks]

(a)

M1	Any one term is 2/3 the previous term. Accept for example	$u_2 = awrt 2.67$	
	8	16	

A1 All 3 terms correct. Accept exact equivalents $u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$ (b)

M1 Uses correct nth term formula ar^{n-1} with a = 4, n = 20 and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7 Condone for the M mark use of ar^{n-1} with $a = \frac{8}{3}$ (awrt 2.67), n = 20 and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7 Expressions such as $4 \times (\frac{2}{3})^{19}$, $\frac{8}{3} \times (\frac{2}{3})^{18}$ and $\frac{2^{n+1}}{3^{n-1}} \rightarrow \frac{2^{21}}{3^{19}}$ are correct and sufficient for M1 A1 Accept any of 0.0018, 0.00180, 1.80×10⁻³ or 1.8×10⁻³

(c)

M1 Uses the correct sum formula
$$S = \frac{a(r^n - 1)}{(r - 1)}$$
 or $S = \frac{a(1 - r^n)}{(1 - r)}$ with $a = 4, r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$
Condone the sum formula $S = \frac{a(r^n - 1)}{(r - 1)}$ or $S = \frac{a(1 - r^n)}{(1 - r)}$ with $a = \frac{8}{3}$ (awrt 2.67), $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

dM1 Dependent upon the previous M mark. Score for an attempt at finding $12 - \sum_{i=1}^{n} u_i$ A1 awrt 0.0183

Note: Some candidates may list all 16 terms which is acceptable provided the answer is accurate

(d)

B1 Need a reason + a minimal conclusion. Eg The sum to infinity =12 **and** sum is less than 12 Allow sum to infinity is 12, hence true.

Leave blank

11. The curve *C* has equation y = f(x), x > 0, where

$$f'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$$

Given that the point P(9, 14) lies on C,

(a) find f(x), simplifying your answer,

(6)

(b) find an equation of the normal to *C* at the point *P*, giving your answer in the form ax + by + c = 0 where *a*, *b* and *c* are integers.

(5)



Ques Num	stion nber	Scheme	Marks
11	1.	$f'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$	
(8	a)	$3\sqrt{x} - \frac{9}{\sqrt{x}} + 2 = 3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$	B1
		$f(x) = \int (3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2)dx = \left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x\right) (+c)$	M1 A1 A1
		When $x = 9$, $y = 14$ so $c =$ $\Rightarrow f(x) = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$	M1 A1 [6]
(t))	Gradient of curve at P is $f'(9) = 8$	B1
		Gradient of normal is $k = -\frac{1}{f'(9)}$	M1
		Equation of normal is $(y-14) = k(x-9)$ or $y = kx + c'$ with use of (9, 14) to find c' This is $(y-14) = -\frac{1}{8}(x-9)$ or $y = -\frac{1}{8}x + 15\frac{1}{8}$	dM1 A1
		So $x + 8y - 121 = 0$	A1 [5] 11 marks
(a) B1 M1 A1 A1 M1	Expre Evide Two to All the Puts <i>x</i>	ession written as $3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$ (May be implied by correct integration) ence of integration, so $x^n \to x^{n+1}$ at least once. Accept on the 2 and follow through on incom- erms of $\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct. There is no need to simplify or have $+ c$ ree terms of $c \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct with no need to simplify or have $+ c$. x = 9 and $y = 14$ into a changed expression (with $+ c$) to obtain $c =$	rect powers.
A1	Needs each term simplified and <i>c</i> correct. Allow simplified equivalents of $f(x) = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$		
	For example allow $y = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$ and $y = 2x\sqrt{x} - 18\sqrt{x} + 2x - 4$. Remember to isw after a correct answer		
(b) B1	f'(9) = 8		
M1	Uses or states that the gradient of the normal is $-\frac{1}{f'(9)}$		
dM1	A1 Finds the equation of a line using $(9,14)$ and their $-\frac{1}{f'(9)}$ as the numerical gradient.		

It is dependent upon the previous M

Allow for
$$(y - 14) = k(x - 9)$$
 or $y = kx + c'$ where $k = -\frac{1}{f'(9)}$

A1 A correct unsimplified answer such as
$$(y-14) = -\frac{1}{8}(x-9)$$
 or $y = -\frac{1}{8}x + \frac{121}{8}$

A1 For any multiple of x + 8y - 121 = 0





Question Number	Scheme	Marks
12.(a)	y = -f(x) y = -f(x) (6,0) (3,-2) (5,-6) W shape Crosses the x axis at (0,0) and (6,0) All 3 turning points correct	B1 B1 B1 [3]
(b)	Same shape passing through (or starting at) the origin $ \begin{array}{c} x \\ (6,2) \\ (12,0) \\ \end{array} $ Same shape passing through (or starting at) the origin Cuts the x axis at (12,0) All 3 turning points correct	B1 B1 B1 [3]
(c)	(-3, 6) $(-3, 6)$ $(-4, 0)$ $(-4,$	B1 B1 B1 [3]

Note: There are candidates who sketch graphs in which the curve just sits on (and does not pass through) the x –axis. Applying the scheme would mean that such a candidate would not gain the second mark in each case. We are ruling that you are only to withhold the mark **the first time** it occurs, so potentially a candidate could score 8 out of 9.

- (a)
- B1 For a W shape
- B1 For a curve **crossing** the *x* axis at (0, 0) and (6,0) **only**. Accept if the curve passes through the origin and (6,0) with only '6' marked. Condone (0,6) being marked on *x* axis. Accept if the coordinates of the points are given in the body of the script as say P = (6,0) and P being marked in the correct place on the diagram. Do not award if the curve meets the *x* axis at an additional point.
- B1 All three turning points correctly marked; maximum = (3, -2) and minima (1, -6) and (5, -6). Do not allow the coordinates being transposed.

Special case: If the candidate sketches y = f(-x) correctly with all coordinates correct they can score B1B0B0



(b)

- B1 Score for the same 'shape' passing through/ or starting at the origin. The two maxima and one minimum must be in Quadrant 1
- B1 Curve **crosses** the *x* axis at the origin and (12, 0) only. See part (a) for allowable alternatives.
- B1 All three turning points correctly marked; minimum = (6, 2) and maxima (2, 6) and (10, 6). Do not allow the coordinates being transposed.

Note: There is no special case as a candidate sketching $y = \frac{1}{2}f(x)$ will score B1B0B0 under the scheme anyway.

(c)

- B1 For an attempted translation of the whole curve left or right. Award if the *y* coordinates of the turning points have stayed at 6 and 2 at least one *x* coordinate has changed. The curve does not need to meet or cross the *x* axis.
- B1 Curve **crosses** the x axis at (2,0) and (-4,0) only
- B1 All 3 turning points correct and curve positioned correctly; Maximum point in quadrant one at (1, 6)Maximum point in quadrant two at (-3, 6) and minimum point in quadrant two at (-1, 2)

Special case for candidate sketching y = f(x) + 4 with all coordinates correct can score B1B0B0



Maximum points = (1,10) and (5,10)

Crosses *x* - axis either side of the origin with neither being given values.

Minimum point = (3, 6)

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		L
13. (i) Showing	each step in your reasoning prove that	b
	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	(2)
		(3)
(ii) Solve, for	$0 \leqslant heta < 360^{\circ},$	
	$3\sin\theta = \tan\theta$	
giving your a	nswers in degrees to 1 decimal place, as appropriate.	
		(6)
(Solutions bas	sed entirely on graphical or numerical methods are not accen	otable)
(Solutions du	sea entirely on graphical or numerical methods are not accep	

Questi Numb	ion ber	Scheme	Marks
13. ((i)	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x$ $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - (1 - \cos^2 x) \cos x - (1 - \sin^2 x) \sin x$	1 st M1 2 nd M1
		$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	A1 *
Alt I (i	.)	Use LHS = $(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$ = $\sin^3 x + \sin x \cos^2 x - \sin^2 x \cos x + \sin^2 x \cos x - \cos^2 x \sin x + \cos^3 x$ $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	[3] 2 nd M1 1 st M1 A1 * [3]
Alt II ((i)	Use RHS $\equiv \sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$ = $(\sin x + \cos x)(1 - \sin x \cos x)$	M1 M1 A1 [3]
(ii))	Use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to give $3\sin \theta = \frac{\sin \theta}{\cos \theta}$ $\cos \theta = \frac{1}{2}$ and $\sin \theta = 0$	M1 A1 A1
		So θ = 70.5, 289.5, or <u>0 and 180 (</u> do not require degrees symbol)	M1 A1, <u>B1</u> [6] 9 marks
(i) M1	Expa Allo	nds bracket to form 4 terms - condone sign slips but terms must be correct. w $\sin x \sin x \cos x$ for $\sin^2 x \cos x$ and condone $\cos x^2$ for $\cos^2 x$	
M1	Repla	aces $\sin^2 x$ by $(1-\cos^2 x)$ and $\cos^2 x$ by $(1-\sin^2 x)$	
A1*	Conc Com varia	lone $\cos x^2$ for $\cos^2 x$. This mark could be seen proceeding the line $\sin x(1 - \cos^2 x) + \cos x(1 - \sin^2 x)$ pletes proof with no errors. This is a given answer. Withhold this mark if poor notation or mixed bles are seen. Examples would be $\sin x + \cos x - (1 - \cos^2 x) \cos x - (1 - \sin^2 \theta) \sin x$	$n^{2} x$).
	Don'	t accept for the A1* $\cos x^2$ for $\cos^2 x$ unless it is clearly bracketed.	
(ii) M1	Uses	$\theta \tan \theta = \frac{\sin \theta}{\cos \theta}$ or equivalent to give $3\sin \theta = \frac{\sin \theta}{\cos \theta}$ or $3\cos \theta = 1$	
A1 .	Achi	eves $\cos \theta = \frac{1}{3}$	
A1 .	Achi	eves $\sin \theta = 0$	

M1 For
$$\operatorname{arccos}\left(\operatorname{their}\frac{1}{3}\right)$$
 leading to one value of θ to the nearest degree. (You may need a calculator to check this)

A1 For $\theta = a \operatorname{wrt} 70.5^\circ, 289.5^\circ$. This mark is withheld for extra solutions of $\operatorname{arccos}\left(\frac{1}{3}\right)$ in the range. Ignore extra solutions outside the range.

B1 $\theta = 0,180^{\circ}$ This mark is withheld for extra solutions arising from $\sin \theta = 0$ in the range. Ignore extra solutions outside the range.

Note: Students who proceed from $\frac{\sin \theta}{\tan \theta} = \cos \theta$ can score M1A1A0M1A1B0 for 4 out of 6 Radian solutions, withhold only the final A1 mark. For your information solutions are awrt 2dp 1.23, 5.05, 0, 3.14 = (pi)

Mathematics C12



Question Number	Scheme		Marks
14. (a)	$4x + 3 = 2x^{\frac{3}{2}} - 2x + 3$ so $6x = 2x^{\frac{3}{2}}$		M1
	x = 9 Points are (0, 3) and (9, 39)		A1 B1, B1 [4]
	Method 1	Method 2	
(b)	$\int 2x^{\frac{3}{2}} - 2x + 3dx = \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^{2} + 3x$	$\pm \left(\int 6x - 2x^{\frac{3}{2}} dx = 3x^2 - \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} \right)$	M1A1
	Uses their upper limit (and subtracts lower limit, often 0) to obtain an area	Uses their upper limit (and subtracts lower limit, often 0) to obtain an area	M1
	Area of trapezium = $\frac{1}{2} \times (3+39) \times 9$	Implied by final answer of 48.6 or -48.6	B1
	OR $2 \times 9^2 + 3 \times 9$		
	Use = Area of trapezium – Area beneath curve	Implied by subtraction in the integration	M1
	= 189 - 140.4 = 48.6	=48.6	A1cso
			[6] (10 marks)

(a)

M1 Set $4x + 3 = 2x^{\frac{3}{2}} - 2x + 3$, collects terms and forms an equation equivalent to $Ax = Bx^{\frac{3}{2}}$, $A, B \neq 0$

A1 Obtains x = 9 from a correct equation. You may ignore any reference to x = 0

B1 One of (0,3), (9,39)

B1 Both (0,3) and (9,39)

Note: This question requires the use of algebra to find (0,3), (9,39). Just the answers with no equation scores 0,0,1,1.

Similarly $2x^{\frac{3}{2}} - 2x + 3 = 4x + 3$ must be simplified before M1A1 can be scored.

Once $2x^{\frac{3}{2}} = 6x$ has been reached you can accept x = 9 without any real working as it could be done in your head.

(b)

M1 Attempted integration on either
$$\int 2x^{\frac{3}{2}} - 2x + 3dx$$
 or $\int (4x+3) - (2x^{\frac{3}{2}} - 2x + 3)dx$ either way around

Accept as evidence $x^n \to x^{n+1}$ on any term

A1 Correct integration-may be unsimplified.

Accept
$$\frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x$$
 or $\left(4\frac{x^2}{2} + 3x\right) - \left(\frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x\right)$ either way around.

M1 Uses their upper limit (and subtracts the lower limit or vice versa) in their integrated function. If the lower limit is 0 you do not have to see the subtraction.

B1 Area of trapezium = 189 or
$$\frac{1}{2} \times (3+39) \times 9$$
 or $2 \times 9^2 + 3 \times 9$ from $\int 4x + 3dx$

This may be implied by the correct answer of 48.6 or (-48.6) in the alternative method.

- M1 Uses area of trapezium area under curve (either way around). It is usually implied by line 1 in the alternative method. The mark is scored for a correct method of finding the shaded area,
- A1 48.6.

Note -48.6 is A0 even if candidate loses the sign

Mathematics C12





Question	Scheme	Marks
number		
15 (a)	$\sqrt{(12-5)^2 + (7-6)^2}$,= $\sqrt{50}$ or $5\sqrt{2}$	M1, A1
		[2]
(b)	See $(x \pm 5)^2 + (y \pm 6)^2 = (\text{their numerical } r)^2$	M1
	$(x-5)^2 + (y-6)^2 = ,50$	B1, A1 [3]
(c)	Gradient of $AP = \frac{1}{7}$	B1
	So gradient of tangent is -7	M1
	Equation of tangent is $(y-7) = -7(x-12)$	dM1 A1 [4]
(d)	$AB = \sqrt{180} = (6\sqrt{5}), BC = \sqrt{160} = (4\sqrt{10}), AC = 10$	M1 A1 A1
	$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{160 + 100 - 180}{20\sqrt{160}}$	M1
	So $C = awrt 71.6$	A1
		14 marks

(a)
M1 Sight of
$$\sqrt{(12-5)^2 + (7-6)^2}$$
 or $r^2 = 7^2 + 1^2$ or equivalent. It may be implied by $\sqrt{50}$ or $5\sqrt{2}$ or awrt 7.1
A1 $\sqrt{50}$ or $5\sqrt{2}$
(b)
M1 For the form $(x \pm 5)^2 + (y \pm 6)^2 = (\text{their numerical } r)^2$
Accept a form $x^2 + y^2 \pm 10x \pm 12y + c = 0$ where $c = 61 - (\text{their numerical } r^2)$
B1 For $(x - 5)^2 + (y - 6)^2 = ...$ or $x^2 + y^2 - 10x - 12y.... = 0$ oe
This can be awarded from $(x - 5)^2 + (y - 6)^2 = r^2$ with an algebraic 'r'
A1 Completely correct answer $(x - 5)^2 + (y - 6)^2 = 50$
 $(x - 5)^2 + (y - 6)^2 = (5\sqrt{2})^2$ and $x^2 + y^2 - 10x - 12y + 11 = 0$ are acceptable alternatives.
(c)
B1 Gradient $AP = \frac{1}{7}$
M1 For using the negative reciprocal of the gradient for AP in finding the gradient of the tangent
dM1 Dependent upon the previous M1. It is for a linear equation through (12,7) with their negative reciprocal
gradient. If the form $y = mx + c$ it must be a full method to find 'c'
Accept any unsimplified form and remember to isw. Accept $y - 7 = -7(x - 12), y = -7x + 91,$
(d)
M1 A correct attempt to find the length of **any** line using the 'difference' between the coordinates.
Accept any of $AB = \sqrt{12^2 + 6^2} (\sqrt{180}), BC = \sqrt{12^2 + 4^2} (\sqrt{160}), AC = \sqrt{8^2 + 6^2} (\sqrt{100} = 10)$
This would be implied by the sight of $AB = awrt 13.4, BC = awrt 12.6, AC = 10$
A1 Two lengths 'correct' either exact or awrt 3 sf $AB = \sqrt{180} = 13.4, BC = \sqrt{160} = 12.6, AC = 10$
A1 All three lengths 'correct' either exact or awrt 3 sf $AB = \sqrt{180} = 13.4, BC = \sqrt{160} = 12.6, AC = 10$
A1 All three lengths 'correct' either exact or awrt 3 sf $AB = \sqrt{180} = 13.4, BC = \sqrt{160} = 12.6, AC = 10$
A1 Correct application of the cosine rule with their AB, AC, BC . Look for $cosC = \frac{BC^2 + AC^2 - AB^2}{2 \times BC \times AC}$ with their
lengths in the correct positions.

Allow the method of finding $\cos C$ from $AB^2 = AC^2 + BC^2 - 2 \times AC \times AB \cos C$ with their lengths in the correct positions. You may condone rearrangement errors in this method.

A1 Accept awrt C = 71.6

WMA01 Leave

blank

16. [*In this question you may assume the formula for the area of a circle and the following formulae:*

a sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$

a cylinder of radius r and height h has volume $V = \pi r^2 h$ and curved surface area $S = 2\pi rh$]





Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius R cm. The walls are modelled by the curved surface of a circular cylinder of radius R cm and height H cm. The floor is modelled by a circular disc of radius R cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm³ and that 0 < R < 10.6

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(b) Show that the surface area, $A \text{ cm}^2$, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(c) Use calculus to find the value of *R*, to 3 significant figures, for which *A* is a minimum.

(5)

(2)

- (d) Prove that this value of R gives a minimum value for A.
- (e) Find, to 3 significant figures, the value of *H* which corresponds to this value for *R*.

(1)



Question Number	Scheme	Marks
16. (a)	$\pi R^2 H + \frac{2}{3}\pi R^3 = 800\pi$ so $H = \frac{800}{R^2} - \frac{2}{3}R^*$	M1 A1*
(b)	$A = \pi R^2 + 2\pi R H + 2\pi R^2$	B1
	$A = 3\pi R^{2} + 2\pi R \left(\frac{800}{R^{2}} - \frac{2}{3}R\right) \text{so} A = \frac{5\pi R^{2}}{3} + \frac{1600\pi}{R}$	M1 A1 *
		[5]
(c)	Find $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$	M1 A1
	Put derivative equal to zero and obtain $R^3 = 480$	dM1 A1
	So $R = 7.83$	A1
		[5]
(d)	Consider $\frac{d^2 A}{d^2 + 3200\pi R^{-3}} > 0$ so minimum	M1A1
	$dR^2 = 3$	[2]
(e)	H = awrt 7.83	B1
		[1]
		13 marks

(a)

M1 Sets up volume equation with $800\pi = \pi R^2 H + \frac{2}{3}\pi R^3$ and attempts to make *H* the subject. Condone 800 instead of 800π . Accept for this mark lower case letters $800\pi = \pi r^2 H + \frac{2}{3}\pi r^3$ and a lack of consistency in lettering. A1* This is a show that question and there must be an intermediate line showing (or implying) a division of

A1* This is a show that question and there must be an intermediate line showing (or implying) a division of $\pi r^2 / \pi R^2$. Lettering must be correct and consistent from the point where you see $800\pi = \dots$. Examples of an intermediate line are;

$$800\pi = \pi R^{2}H + \frac{2}{3}\pi R^{3} \Longrightarrow H = \frac{800\pi - \frac{2}{3}\pi R^{3}}{\pi R^{2}} \Longrightarrow H = \frac{800}{R^{2}} - \frac{2}{3}R$$

$$800\pi = \pi R^{2}H + \frac{2}{3}\pi R^{3} \Longrightarrow \frac{800}{R^{2}} = H + \frac{2}{3}R \Longrightarrow H = \frac{800}{R^{2}} - \frac{2}{3}R$$

(b)

B1 A correct expression for the surface area containing three separate correct elements

Allow either
$$A = \pi R^2 + 2\pi RH + 2\pi R^2$$
 or $A = \pi R^2 + 2\pi RH + \frac{4\pi R^2}{2}$

Allow lower case lettering for this mark

M1 Score for replacing $H = \frac{800}{R^2} - \frac{2}{3}R$ in their expression for A which must be of the form, $A = B\pi R^2 + C\pi RH$, $B, C \in \mathbb{N}$, condoning missing brackets.

A1* This is a show that question and all aspects must be correct. Lettering in (b) must be consistent and correct from the point at which $\frac{800}{R^2} - \frac{2}{3}R$ is substituted. Do not, however, withhold a second mark for using lower case letters if it has been withheld in part (a) for mixed lettering. Accept $A = 2\pi R^2 + \pi R^2 + 2\pi R \left(\frac{800}{2} - \frac{2}{2}R\right) \Rightarrow A = \frac{5\pi R^2}{2} + \frac{1600\pi}{2}$ with little or no evidence

Accept
$$A = 2\pi R^2 + \pi R^2 + 2\pi R \left(\frac{800}{R^2} - \frac{2}{3}R\right) \Longrightarrow A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$
 with little or no evidence

(c)

- M1 Evidence of differentiation so sight of $R^2 \to R$ or $R^{-1} \to R^{-2}$
- A1 Both terms correct. Does not need to be simplified. $\frac{dA}{dR} = \frac{10}{3}\pi R \frac{1600\pi}{R^2}$

Do not be concerned by inconsistent lettering and condone incorrect notation on the lhs such as $\frac{dy}{dx} =$

dM1 Sets
$$\frac{dA}{dR} = 0$$
 and proceeds to $R^n = C$ This is dependent upon the previous M

A1 A correct intermediate answer. $R^3 = 480$. This may be implied by a correct answer following a correct derivative. A1 Allow awrt R = 7.83

(d)

M1 Achieves a correct second derivative $\frac{d^2 A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$

Or attempts to substitute their positive R in their second derivative (usual rules for differentiation having been applied to at least one term)

Or alternatively attempts to find the numerical value of the gradient either side of their value of ROr alternatively attempts to find the numerical value of V at R and either side of R

A1 This requires (1) a correct function, (2) a correct statement and (3) a correct conclusion

Either (1)
$$\frac{d^2 A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$$
 (2) At their $R = + \dots, \frac{d^2 A}{dR^2} = +\dots > 0$ (3) hence minimum.

Or (1)
$$\frac{d^2 A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$$
, (2) as R>0, $\frac{d^2 A}{dR^2} > 0$ (3) hence minimum

Or (1) $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$ (2) at R = 7 $\frac{dA}{dR} = -29 < 0$ at R = 8 $\frac{dA}{dR} = 5 > 0$ (3)Hence minimum

The table below can be used to check most calculations

Value of <i>R</i>	7	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
Gradient at R	-29	-25	-22	-18	-14	-11	-7	-4	-1	2	5

Or (1) $A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ (2), $A_{7.82} > A_{7.83} < A_{7.84}$ with evidence (see below), (3) Hence minimum

Value of <i>R</i>	7.8	7.81	7.82	7.83	7.84	7.85	7.86	7.87	7.88	7.89	7.9
Value of A	962.50	962.49	962.49	962.48	962.49	962.49	962.50	962.51	962.52	962.54	962.56

(e)

B1 *H* =awrt 7.83