Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Core Mathematics C12

Advanced Subsidiary

Wednesday 20 May 2015 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The line l_1 has equation

$$10x - 2y + 7 = 0$$

(a) Find the gradient of l_1

(1)

The line l_2 is parallel to the line l_1 and passes through the point $\left(-\frac{1}{3}, \frac{4}{3}\right)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

Question Number	Scheme	Marks
1.	The line l_1 has equation $10x - 2y + 7 = 0$	
(a)	Gradient is 5	B1 **[1]**
(b)	Gradient of parallel line is equal to their previous gradient	M1
	Equation is $y - \frac{4}{3} = "5"\left(x - \left(-\frac{1}{3}\right)\right)$	M1
	So $y = 5x + 3$	A1 **[3]** **(4 marks)**

(a)
B1 Gradient given as 5 or 10/2 or exact equivalent. Do not accept if embedded within an equation. You must see 5 (or equivalent)

(b)
M1 Gradient of lines are the same. This may be implied by sight of their '5' in a gradient equation. For example you may see $y = '5'x + c$ or equivalent as the equation of their ‘’parallel’’ line .

M1 For an attempt to find an equation of a line using $\left(-\frac{1}{3}, \frac{4}{3}\right)$ and a numerical gradient (which may be different to the gradient used in part (a). For example they may try to find a normal!)
It must be a full attempt to find an equation.
Accept $y - \frac{4}{3} = \text{"numerical } m\text{"}\left(x - -\frac{1}{3}\right)$ or equivalent. Allow one sign slip for the coordinate.
If $y = mx + c$ is used it must proceed as far as finding the value of "c"

A1 Correct answer only (no equivalents) $y = 5x + 3$, but do allow $y = 5x + c$ followed by $c = 3$.

Leave blank

2. $$f(x) = x^4 - x^3 + 3x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$ the remainder is 4

When $f(x)$ is divided by $(x + 2)$ the remainder is 22

Find the value of a and the value of b.

(5)

Question Number	Scheme	Marks
2.	$f(x) = x^4 - x^3 + 3x^2 + ax + b$	
	Attempts to set $f(\pm 1) = 4$ **or a**ttempts to set $f(\pm 2) = 22$	M1
	Obtains $3 + a + b = 4$ or equivalent	A1
	Obtains $16 + 8 + 12 - 2a + b = 22$ or equivalent	A1
	Solve simultaneous equations to obtain $a = 5$ and $b = -4$	M1 A1
		[5] **(5 marks)**

M1 Attempts to set either $f(\pm 1) = 4$ or $f(\pm 2) = 22$

Alternatively dividing $f(x)$ by $(x - 1)$ or dividing $f(x)$ by $(x + 2)$ and setting the remainder equal to 4 or 22

For division look for a minimum of

$$x-1\overline{\big)x^4 - x^3 + 3x^2 + ax + b}\quad x^3 + \ldots\ldots\ldots \qquad \text{OR} \qquad x+2\overline{\big)x^4 - x^3 + 3x^2 + ax + b}\quad x^3 + \ldots\ldots\ldots$$

$$\overline{(\ldots a) + (\ldots b)} \qquad\qquad \overline{(\ldots a) + (\ldots b)}$$

followed by the remainder (involving both a and b) set equal to 4 or 22

A1 $3 + a + b = 4$ or equivalent.

The powers must be multiplied out but there is no requirement to collect terms.

A1 $16 + 8 + 12 - 2a + b = 22$

The powers must be multiplied out but there is no requirement to collect terms.

M1 Solves a pair of simultaneous equations in both a and b

For your information the correct simplified equations are $a + b = 1$ and $-2a + b = -14$

Minimal evidence is required for this as it could be done on a GC. Just accept values of both a and b

A1 $a = 5$ and $b = -4$

Leave blank

3. Given that

$$y = \frac{1}{27}x^3$$

express each of the following in the form kx^n where k and n are constants.

(a) $y^{\frac{1}{3}}$ **(1)**

(b) $3y^{-1}$ **(1)**

(c) $\sqrt{(27y)}$ **(1)**

Question Number	Scheme	Marks
3.(a)	$\frac{1}{3}x$ as the final answer.	B1
(b)	$81x^{-3}$ as the final answer	B1
(c)	$x^{\frac{3}{2}}$ as the final answer	B1
		[3] **(3 marks)**

In all parts of this question candidates do not have to explicitly state the values of k and n. Award the mark(s) as above.
If they go on to give incorrect values of k and n you may isw, but do not isw on incorrect index work Eg. $\frac{1}{3}x = x^{-3}$.
If candidates make two different attempts and give two (or more) different answers then please put these in review

(a)

B1 $\frac{1}{3}x$ but accept equivalent such as $\frac{x}{3}$, $\frac{1}{3}\times x$, $\frac{1}{3}x^{1}$ or $3^{-1}x$ etc.

(b)

B1 $81x^{-3}$. Accept exact equivalents such as $81\times x^{-3}$
Do not accept $\frac{81}{x^3}$ as the final answer unless it is preceded by $81x^{-3}$ as it is not in the form required by the question.

(c)

B1 $x^{\frac{3}{2}}$ but accept exact equivalents such as $1\times x^{1.5}$
Do not accept $x\,x^{\frac{1}{2}}$ or $x\sqrt{x}$ as the final answer unless preceded by $x^{\frac{3}{2}}$ as they are not in the form required by the question.

Leave blank

4. (a) Sketch the graph of $y = \frac{1}{x}$, $x > 0$

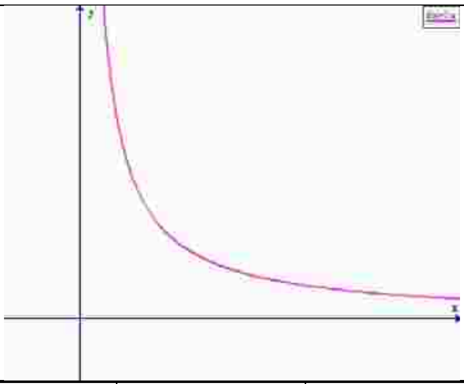
(2)

The table below shows corresponding values of x and y for $y = \frac{1}{x}$, with the values for y rounded to 3 decimal places where necessary.

x	1	1.5	2	2.5	3
y	1	0.667	0.5	0.4	0.333

(b) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for $\int_1^3 \frac{1}{x}\,dx$

(4)

Question Number	Scheme	Marks
4 (a)	 M1: General shape correct so **part of rectangular hyperbola** A1: Accuracy on asymptotes and no crossing of axes	M1 A1 [2]
(b)	x: 1, 1.5, 2, 2.5, 3; y: 1, 0.667, 0.5, 0.4, 0.333	
	State h = 0.5, or use of $\frac{1}{2}\times 0.5$;	B1 aef
	$\underline{\{1+0.333+2(\ 0.667+0.5+0.4\ )\}}$ For structure of $\{................\}$;	M1A1
	$\frac{1}{2}\times 0.5\ \times\underline{\{4.467\}}$ = awrt 1.12	A1cao
		[4] **(6 marks)**

x	1	1.5	2	2.5	3
y	1	0.667	0.5	0.4	0.333

(a)

M1 General shape correct showing **part of a rectangular hyperbola in the first quadrant**. Condone for this mark the curve meeting or intersecting either axis. Condone incorrect asymptotes, for example at y = 1. See Practice, examples on the following page and Qualification for clarification

A1 Do not allow for intersections with axes. Curve must appear to approach/ be asymptotic to both the x and y axes. Ignore sections where $x < 0$

(b)

B1 For using a strip width of 0.5. This may appear in a trapezium rule as $\frac{1}{2}\times 0.5$ or 0.25 or equivalent

M1 Scored for the correct $\{......\}$ bracket structure. It needs to contain the first y value +last y value with 2 times the sum of the remaining y values in the table (with no additional values). If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however).
It is M0 if values used in brackets are x values instead of y values

A1 For the correct bracket $\{......\}$. Accept $\{1+0.33+2(\ 0.67+0.5+0.4\ )\}$ where candidate rounds to 2dp

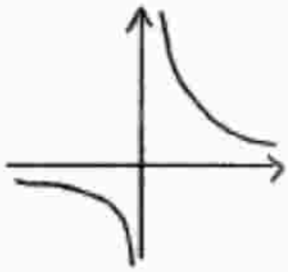
A1 For awrt 1.12.

NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 $h(a + b)$ used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Special case: Bracketing mistake $0.25\times(1+0.333)+2(0.667+0.5+0.4)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).

Q4 All score M1 A1

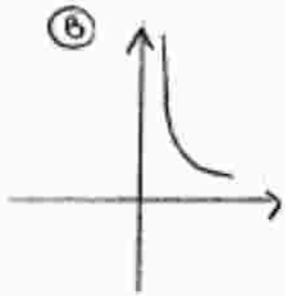
(A)



M1 Shape ✓

A1 Accuracy ✓

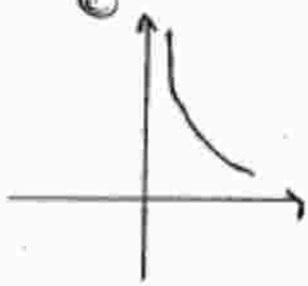
(B)



M1 Shape ✓

A1 Condone lack of symmetry

(C)



M1 Shape ✓

A1 – curvature going in the right direction

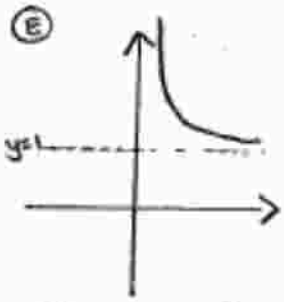
All score M1 A0

(D)

M1 part of hyperbola

A0 hits y axis and curvature incorrect at right hand end x

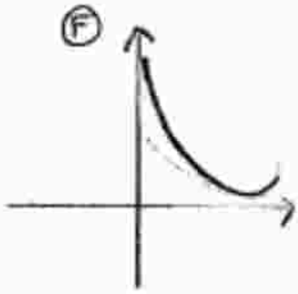
(E)



M1 Shape ✓

A0 'too' far away from x axis (or asymptote y=1 shown)

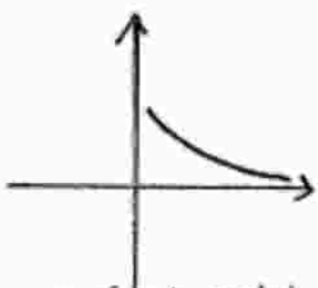
(F)



M1 – part of hyperbola

A0 – curvature at rhs incorrect.

(G)

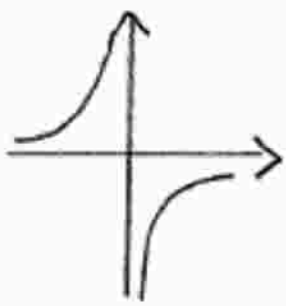


M1 – part of a hyperbola

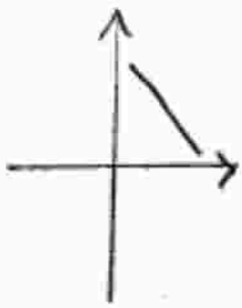
A0 – curvature lacking accuracy at left hand end

Both score M0 A0

(H)



(I)



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5. (i) Find, giving your answer to 3 significant figures, the value of y for which

$$3^y = 12$$

(2)

(ii) Solve, giving an exact answer, the equation

$$\log_2(x + 3) - \log_2 (2x + 4) = 4$$

(You should show each step in your working.)

(4)

Question Number	Scheme	Marks
5. (i)	Use or state $y\log 3 = \log 12$ or $\log_3 12$	M1
	Obtain y = 2.26 or awrt 2.26	A1
		[2]
(ii)	Use or state $\log_2 16 = 4$ or $2^4 = 16$	B1
	Use $\log_2(x+3) - \log_2(2x+4) = \log_2 \dfrac{(x+3)}{(2x+4)}$	M1
	Then $\dfrac{x+3}{(2x+4)} = 16$, and so $x = -\dfrac{61}{31}$	A1 , A1
		[4]
		6 marks
Alt (ii)	$\log_2(x+3) - \log_2(2x+4) = \log_2 16$	B1
	$\log_2(x+3) = \log_2 16 + \log_2(2x+4)$	
	$\log_2(x+3) = \log_2 16(2x+4)$	M1
	$(x+3) = 16(2x+4)$	A1
	$x = -\dfrac{61}{31}$	A1

(i)

M1 Uses power law for logs. This method may be implied by sight of $y\log 3 = \log 12$ **or** $\log_3 12$

The method may be scored for using trial and error provided you see $3^{2.26} = 11.9\ldots$ **and** $3^{2.27} = 12.1\ldots$

A1 awrt 2.26.

Just the answer with no (incorrect) working is 2 marks.

(ii)

B1 Connects 16 with '4' correctly. Evidence would be $\log_2 16 = 4$ or $2^4 = 16$ and could be scored at any time

M1 For the correct use of either the addition or subtraction law of logs

A1 Correct equation not involving logs

A1 cso $x = -\dfrac{61}{31}$

Special case:

Candidates who recover from "incorrect " log work and achieve the correct answer can score special case 1010.

$$\log_2(x+3) - \log_2(2x+4) = 4 \Rightarrow \frac{\log_2(x+3)}{\log_2(2x+4)} = \log_2 16 \Rightarrow \frac{x+3}{2x+4} = 16 \Rightarrow x = -\frac{61}{31}$$

Leave blank

6. (a) Find the first 3 terms in ascending powers of x of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in the expansion, the coefficient of x is equal to the coefficient of x^2

(b) find the value of a.

(2)

Question Number	Scheme	Marks
6.(a)	$(2+ax)^6 = 2^6 + \binom{6}{1}2^5.(ax) + \binom{6}{2}2^4.(ax)^2 + ...$	M1
	$= 64, +192ax + 240a^2x^2 + ...$	B1, A1, A1
		[4]
(b)	$192a = 240a^2$	M1
	$a = \dfrac{192}{240} = 0.8$ or equivalent	A1
		[2]
		6 marks
Alt 6.(a)	$(2+ax)^6 = 2^6\left(1+\dfrac{a}{2}x\right)^6 = 2^6\left(1+6\times\dfrac{a}{2}x+\dfrac{6\times5}{2}\left(\dfrac{a}{2}x\right)^2+...\right)$	**M1**
	$= 64, +192ax + 240a^2x^2 + ...$	**B1, A1, A1**
		[4]

(a)

M1 The method mark is awarded for an attempt at a Binomial expansion to get an unsimplified second or third term – Look for a correct binomial coefficient multiplied by a correct power of *x*. Eg ${}^6C_1...x$ or ${}^6C_2..x^2$
Condone bracket errors or errors (or omissions) in the powers of 2.
Accept any notation for ${}^6C_1$, ${}^6C_2$, e.g. as on scheme or 6, and 15 from Pascal's triangle.
This mark may be given if no working is shown, if either or both of the terms including *x* is correct.
If the candidate attempts the expansion in descending powers allow ${}^6C_5...x^5$ or ${}^6C_4..x^4$ oe.

In the alternative it is for the correct form inside the bracket accepting either $1+6\times\dfrac{a}{2}x+\dfrac{6\times5}{2}\left(\dfrac{a}{2}x\right)^2$

or $1+6\times\dfrac{a}{2}x+\dfrac{6\times5}{2}\dfrac{a}{2}x^2$

B1 Must be simplified to 64 (writing just 2^6 is B0).

A1 Score for either of $192a\,x$ or $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket

A1 Score for both of $192a\,x$ and $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket
Allow listing of terms $64,\ 192ax,\ 240a^2x^2$ for all 4 marks.

(b)

M1 Score for setting the coefficients of their *x* and x^2 terms equal. They must reach an equation not involving *x*'s.
A1 This is cso for any equivalent fraction or decimal to 0.8. Ignore any reference to *a* =0.

Leave blank

7.

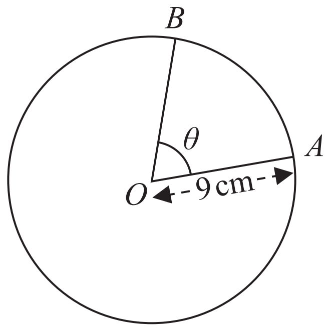


Figure 1

Figure 1 shows a circle with centre O and radius 9 cm. The points A and B lie on the circumference of this circle. The minor sector OAB has perimeter 30 cm and the angle between the radii OA and OB of this sector is θ radians.

Find

(a) the length of the arc AB,
(1)

(b) the value of θ,
(2)

(c) the area of the minor sector OAB,
(2)

(d) the area of triangle OAB, giving your answer to 3 significant figures.
(2)

Question Number	Scheme	Marks
7. (a)	Length of the arc is 12 (cm)	B1 **[1]**
(b)	States or uses arc length '12' = $9\times\theta$	M1
	$\theta=\frac{4}{3}$ or awrt 1.33	A1 **[2]**
(c)	Area of sector = $\frac{1}{2}r^2\theta=\frac{1}{2}\times 9^2\times '\theta'$	M1
	Area = awrt 54 (cm^2)	A1 **[2]**
(d)	Area of triangle OAB = $\frac{1}{2}9\times 9\times\sin\theta$, = awrt 39.4 cm^2	M1, A1 [2]
		7 marks

Mark parts (a) and (b) together. For example the arc length may be scored in part (b)

(a)

B1 12 (cm)

(b)

M1 Uses or states arc length $s=9\times\theta$ with their value of $'s'$ from part (a)

Allow degree calculations for the M mark. Score for equivalents of $\frac{\theta}{12}=\frac{360}{2\pi\times 9}$ FYI $\theta\approx 76.4°$

A1 For $\theta=\frac{4}{3}$ or equivalent such as $\theta=\frac{12}{9},1\frac{1}{3}$. Allow decimals awrt 1.33

Accept answers in terms of π but the accuracy must be either awrt 0.424π or 0.425π

(c)

M1 Uses or states area $A=\frac{1}{2}9^2\times\theta$ with their value of $'\theta'$ in radians from part (b)

Allow for a correct formula in degrees. $A=\frac{\theta}{360}\times\pi 9^2$ with their value of $'\theta'$ in degrees from part (b)

A1 Area = awrt 54 (cm^2)

(d)

M1 Correct method to find the area of triangle *ABC,* usually by using $\frac{1}{2}\times 9\times 9\times\sin\theta$ with their value of $'\theta'$ from part (b). Alternative methods are acceptable using the fact that triangle *ABC* is isosceles but it must be a full method involving the calculation of both base and perpendicular height by trigonometry twice or trigonometry once and Pythagoras once followed by $\frac{1}{2}bh$

The segment formula $\frac{1}{2}\times 9\times 9\times\theta-\frac{1}{2}\times 9\times 9\times\sin\theta$ will score M0 unless the candidate subtracts this from the area of a sector.

A1 Awrt = 39.4 cm^2

Leave blank

8. A 25-year programme for building new houses began in Core Town in the year 1986 and finished in the year 2010.

The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find

(a) the number of houses built in 1986, the first year of the building programme,

(5)

(b) the total number of houses built in the 25 years of the programme.

(2)

Question Number	Scheme	Marks
8. (a)	$238 = a + k\,d$ **or** $108 = a + k\,d$ with any values for k	M1
	$238 = a + (14)d$ or $108 = a + (24)d$ or $"d" = \pm 13$	A1
	$238 = a + (14)d$ **and** $108 = a + (24)d$	A1
	Solves their simultaneous equations to obtain $a =$; (so a) = 420	M1; A1 [5]
(b)	Uses $\frac{25}{2}(2\times a+(25-1)\times"-13")$ **or** Uses $\frac{25}{2}(a+108)$, to obtain = 6600	M1 , A1 [2]
		7 marks

(a)

M1 Score for $238 = a + k\,d$ **or** $108 = a + k\,d$ with any non- zero integer value for k

A1 One of $238 = a + (14)d$, or $108 = a + (24)d$ or $"d" = \pm 13$
The $"d" = \pm 13$ can be achieved from equations such as $238 = a + (13)d$, or $108 = a + (23)d$

A1 Both $238 = a + (14)d$, **and** $108 = a + (24)d$
M1 Finds the value of a by solving a pair of simultaneous equations in a and d
A1 Achieves (a =) 420

In an alternative, working by a method of differences, you may see very few formulae: The scheme can be easily applied,
1st M1 Seeing $\frac{238-108}{10}$ or $\frac{108-238}{10}$
1st A1 For ± 13
2nd A1 For $238+13\times 14$ or $108+13\times 24$.
Note that this is achieved after the award of the next mark. It is scored as the third mark of (a) on e –pen.
2nd M1 Sight of $238+'k'\times d$ or $108+'k'\times d$ with any non- zero integer values for k and their d
3rd A1 Achieves 420

(b)

M1 Uses a correct sum formula $S=\frac{n}{2}\left(2a+(n-1)d\right)$ with $n=25$ and their values of a and d

Alternatively uses $S=\frac{n}{2}\left(a+l\right)$ with $n = 25, l = 108$ and their value of a.

A1 cao 6600

Leave blank

9. The equation $x^2 + (6k + 4)x + 3 = 0$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$9k^2 + 12k + 1 < 0$$

(3)

(b) Find the range of possible values for k, giving your boundaries as fully simplified surds.

(4)

Question Number	Scheme	Marks
9(a)	Uses $b^2-4ac=(6k+4)^2-4\times1\times3$	B1
	Multiplies out and uses $b^2-4ac<0$	M1
	to give $36k^2+48k+16-12<0$ so $9k^2+12k+1<0$ *	A1 * **[3]**
(b)	Solves quadratic by formula or completion of square to give $k=$	M1
	Obtains $k=\dfrac{-12\pm\sqrt{108}}{18}$ or accept $k=$ awrt $-1.24, -0.09$	A1
	Chooses region between two values and deduces $\dfrac{-2-\sqrt{3}}{3}<k<\dfrac{-2+\sqrt{3}}{3}$	M1 A1**cao**
		[4] **7 marks**

(a)

B1 Uses $b^2 - 4ac = (6k+4)^2 - 4\times 1\times 3$ or equivalent

M1 Attempt to multiply out and use their $b^2 - 4ac < 0$ **or** $b^2 < 4ac$

Alternatively they can find $b^2 - 4ac$ and then state $b^2 - 4ac < 0$ **or** $b^2 < 4ac$

A1* Achieves $9k^2 + 12k + 1 < 0$

This is a given answer and you must check that all aspects are correct. **In all cases you should expect to see an intermediate line(s) (including <0) before the final answer.**

$b^2 - 4ac = 4(9k^2 + 12k + 1)$. As $b^2 - 4ac < 0 \Rightarrow 9k^2 + 12k + 1 < 0$ is fine for B1 M1 A1*

$b^2 - 4ac = 4(9k^2 + 12k + 1)$. $\Rightarrow 9k^2 + 12k + 1 < 0$ is B1 M0 A0*

(b)

M1 Solves the equation $9k^2 + 12k + 1 = 0$ by formula or completing the square. Factorisation is not a suitable method in this case and scores M0. The answers could just appear from a graphical calculator.

Sight of either root, $\dfrac{-12-\sqrt{108}}{18}$ *or* $\dfrac{-12+\sqrt{108}}{18}$ is evidence that the formula has been used.

Sight of $-\dfrac{6}{9} + \sqrt{\dfrac{27}{81}}$ oe could be evidence that completion of square has been used.

A1 Accept $k = \dfrac{-12-\sqrt{108}}{18}$ **and** $\dfrac{-12+\sqrt{108}}{18}$ oe or decimal equivalents awrt 2dp -1.24 and -0.09

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation.

This can be awarded for an inside region appearing in the form $\dfrac{-2-\sqrt{3}}{3} \leqslant k \leqslant \dfrac{-2+\sqrt{3}}{3}$

A1 cao $\dfrac{-2-\sqrt{3}}{3} < k < \dfrac{-2+\sqrt{3}}{3}$. Accept $-\dfrac{2}{3} - \dfrac{1}{\sqrt{3}} < k < -\dfrac{2}{3} + \dfrac{1}{\sqrt{3}}$

Accept equivalents such as $\left(\dfrac{-2-\sqrt{3}}{3}, \dfrac{-2+\sqrt{3}}{3}\right)$ $k > \dfrac{-2-\sqrt{3}}{3}$ and $k < \dfrac{-2+\sqrt{3}}{3}$ even $\left\{\dfrac{-2-\sqrt{3}}{3} \cap \dfrac{-2+\sqrt{3}}{3}\right\}$

Do not accept $\left[\dfrac{-2-\sqrt{3}}{3}, \dfrac{-2+\sqrt{3}}{3}\right]$ $k > \dfrac{-2-\sqrt{3}}{3}$ or $k < \dfrac{-2+\sqrt{3}}{3}$ and $k > \dfrac{-2-\sqrt{3}}{3}, k < \dfrac{-2+\sqrt{3}}{3}$

including the version of the final one without the coma

If the candidate writes $\dfrac{-2-\sqrt{3}}{3} < x < \dfrac{-2+\sqrt{3}}{3}$ this can be awarded M1 A0

Leave blank

10. A sequence is defined by

$$u_1 = 4$$
$$u_{n+1} = \frac{2u_n}{3}, \qquad n \geqslant 1$$

(a) Find the exact values of u_2, u_3 and u_4

(2)

(b) Find the value of u_{20}, giving your answer to 3 significant figures.

(2)

(c) Evaluate

$$12 - \sum_{i=1}^{16} u_i$$

giving your answer to 3 significant figures.

(3)

(d) Explain why $\sum_{i=1}^{N} u_i < 12$ for all positive integer values of N.

(1)

Question Number	Scheme	Marks
10 (a)	$u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$	M1, A1
		[2]
(b)	$u_{20} = 4\times\left(\frac{2}{3}\right)^{19}$;= 0.00180 or 0.0018 or exact equivalent	M1; cao A1
		[2]
(c)	Use $\sum_{i=1}^{16} u_i = \frac{4(1-\left(\frac{2}{3}\right)^{16})}{1-\frac{2}{3}}$	M1
	Find 12 - *their* $\sum_{i=1}^{16} u_i$	dM1
	= 12 – 11.9817 = awrt 0.0183	A1
		[3]
(d)	12 is the **sum to infinity** (and all terms are **positive)** so sum is less than 12	B1
	Or $\sum_{i=1}^{n} u_i = \frac{4(1-\left(\frac{2}{3}\right)^{n})}{1-\frac{2}{3}} = 12 - 12\left(\frac{2}{3}\right)^n$ *and* $\left(\frac{2}{3}\right)^n > 0$ so is less than 12	**[1]**
		[8 marks]

(a)

M1 Any one term is 2/3 the previous term. Accept for example $u_2 = \text{awrt } 2.67$

A1 All 3 terms correct. Accept exact equivalents $u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$

(b)

M1 Uses correct nth term formula ar^{n-1} with $a = 4, n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Condone for the M mark use of ar^{n-1} with $a = \frac{8}{3}$ (awrt 2.67), $n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Expressions such as $4\times\left(\frac{2}{3}\right)^{19}$, $\frac{8}{3}\times\left(\frac{2}{3}\right)^{18}$ and $\frac{2^{n+1}}{3^{n-1}} \rightarrow \frac{2^{21}}{3^{19}}$ are correct and sufficient for M1

A1 Accept any of $0.0018, 0.00180$, 1.80×10^{-3} or 1.8×10^{-3}

(c)

M1 Uses the correct sum formula $S = \frac{a\left(r^n - 1\right)}{(r-1)}$ or $S = \frac{a\left(1-r^n\right)}{(1-r)}$ with $a = 4, r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

Condone the sum formula $S = \frac{a\left(r^n - 1\right)}{(r-1)}$ or $S = \frac{a\left(1-r^n\right)}{(1-r)}$ with $a = \frac{8}{3}$(awrt 2.67), $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

dM1 Dependent upon the previous M mark. Score for an attempt at finding $12 - \sum_{i=1}^{16} u_i$

A1 awrt 0.0183

Note: Some candidates may list all 16 terms which is acceptable provided the answer is accurate

(d)

B1 Need a reason + a minimal conclusion. Eg The sum to infinity =12 **and** sum is less than 12
Allow sum to infinity is 12, hence true.

Leave blank

11. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$$

Given that the point P (9, 14) lies on C,

(a) find $f(x)$, simplifying your answer,

(6)

(b) find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

Question Number	Scheme	Marks
11.	$\text{f}'(x) = 3\sqrt{x} - \dfrac{9}{\sqrt{x}} + 2$	
(a)	$3\sqrt{x} - \dfrac{9}{\sqrt{x}} + 2 = 3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$	B1
	$\text{f}(x) = \int (3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2)\text{d}x \quad = \left(\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x\right) \ (+c)$	M1 A1 A1
	When $x = 9$, $y = 14$ so $c =$	M1
	$\Rightarrow \text{f}(x) \ = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$	A1 [6]
(b)	Gradient of curve at P is $\text{f}'(9) = 8$	B1
	Gradient of normal is $k = -\dfrac{1}{\text{f}'(9)}$	M1
	Equation of normal is $(y-14) = k(x-9)$ *or* $y = kx + c'$ with use of (9, 14) to find c'	dM1
	This is $(y-14) = -\frac{1}{8}(x-9)$ *or* $y = -\frac{1}{8}x + 15\frac{1}{8}$	A1
	So $x + 8y - 121 = 0$	A1 **[5]**
		11 marks

(a)

B1 Expression written as $3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$ (May be implied by correct integration)

M1 Evidence of integration, so $x^n \to x^{n+1}$ at least once. Accept on the 2 and follow through on incorrect powers.

A1 Two terms of $\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct. There is no need to simplify or have $+ c$

A1 All three terms of c $\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct with no need to simplify or have $+ c$.

M1 Puts $x = 9$ and $y = 14$ into a changed expression (with $+ c$) to obtain $c =$

A1 Needs each term simplified and c correct. Allow simplified equivalents of $\text{f}(x) = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$

For example allow $y = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$ and $y = 2x\sqrt{x} - 18\sqrt{x} + 2x - 4$.

Remember to isw after a correct answer

(b)

B1 $\text{f}'(9) = 8$

M1 Uses or states that the gradient of the normal is $-\dfrac{1}{\text{f}'(9)}$

dM1 Finds the equation of a line using $(9,14)$ and their $-\dfrac{1}{\text{f}'(9)}$ as the numerical gradient.

It is dependent upon the previous M

Allow for $(y-14) = k(x-9)$ *or* $y = kx + c'$ where $k = -\dfrac{1}{\text{f}'(9)}$

A1 A correct unsimplified answer such as $(y-14) = -\dfrac{1}{8}(x-9)$ *or* $y = -\dfrac{1}{8}x + \dfrac{121}{8}$

A1 For any multiple of $x + 8y - 121 = 0$

Leave blank

12.

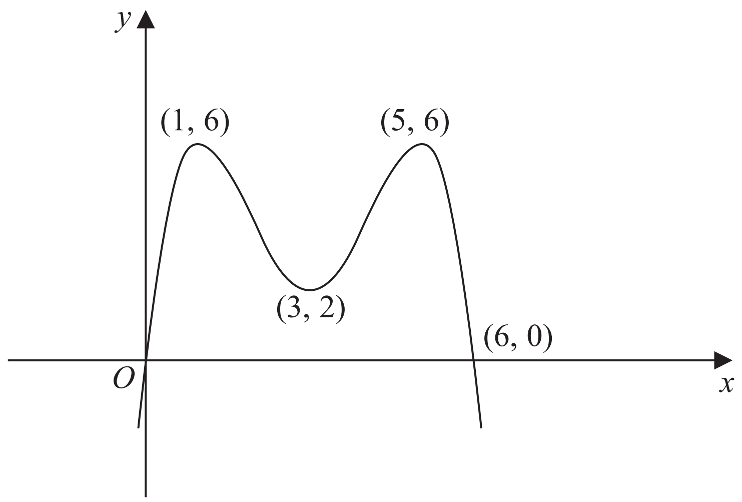


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

The curve crosses the x-axis at the origin and at the point (6, 0). The curve has maximum points at (1, 6) and (5, 6) and has a minimum point at (3, 2).

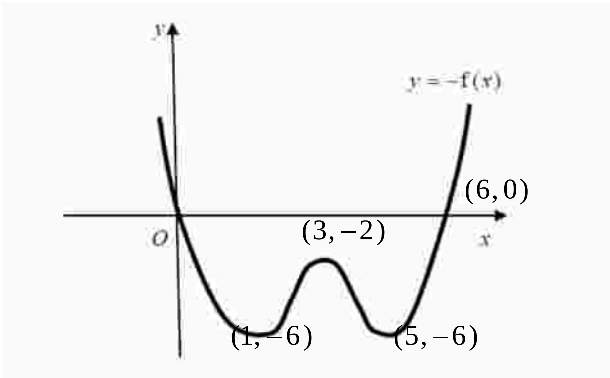
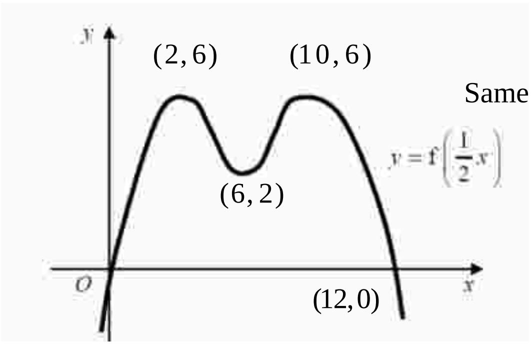
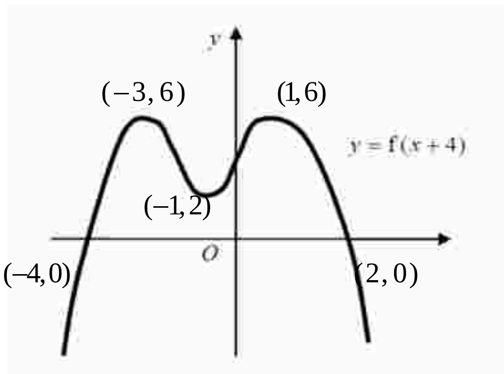
On **separate** diagrams sketch the curve with equation

(a) $y = -f(x)$ **(3)**

(b) $y = f\left(\frac{1}{2}x\right)$ **(3)**

(c) $y = f(x + 4)$ **(3)**

On each diagram show clearly the coordinates of the maximum and minimum points, and the coordinates of the points where the curve crosses the x-axis.

Question Number	Scheme		Marks
12.(a)		W shape	B1
		Crosses the x axis at (0,0) and (6,0)	B1
		All 3 turning points correct	B1
			[3]
(b)		Same shape passing through (or starting at) the origin	B1
		Cuts the x axis at (12,0)	B1
		All 3 turning points correct	B1
			[3]
(c)		Translation left or right	B1
		Cuts the x axis at (2,0) and (-4,0)	B1
		All 3 turning points in correct position with correct coordinates	B1
			[3]
			9 marks

Note: There are candidates who sketch graphs in which the curve just sits on (and does not pass through) the x –axis. Applying the scheme would mean that such a candidate would not gain the second mark in each case. We are ruling that you are only to withhold the mark **the first time** it occurs, so potentially a candidate could score 8 out of 9.

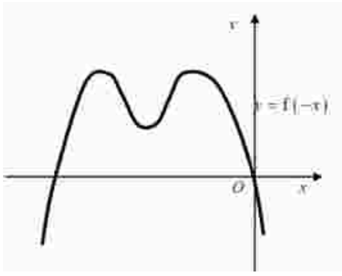
(a)

B1 For a W shape

B1 For a curve **crossing** the x - axis at (0, 0) and (6,0) **only**. Accept if the curve passes through the origin and (6,0) with only '6' marked. Condone (0,6) being marked on x axis. Accept if the coordinates of the points are given in the body of the script as say P = (6,0) and P being marked in the correct place on the diagram. Do not award if the curve meets the x axis at an additional point.

B1 All three turning points correctly marked; maximum = $(3,-2)$ and minima $(1,-6)$ and $(5,-6)$. Do not allow the coordinates being transposed.

Special case: If the candidate sketches $y = \mathrm{f}(-x)$ correctly with all coordinates correct they can score B1B0B0



Maximum points = $(-1,6)$ and $(-5,6)$
Crosses x - axis at the origin and $(-6,0)$
Minimum point = $(-3,2)$

(b)

B1 Score for the same 'shape' passing through/ or starting at the origin. The two maxima and one minimum must be in Quadrant 1

B1 Curve **crosses** the x - axis at the origin and (12, 0) only. See part (a) for allowable alternatives.

B1 All three turning points correctly marked; minimum = $(6,2)$ and maxima $(2,6)$ and $(10,6)$. Do not allow the coordinates being transposed.

Note: There is no special case as a candidate sketching $y = \frac{1}{2}\mathrm{f}(x)$ will score B1B0B0 under the scheme anyway.

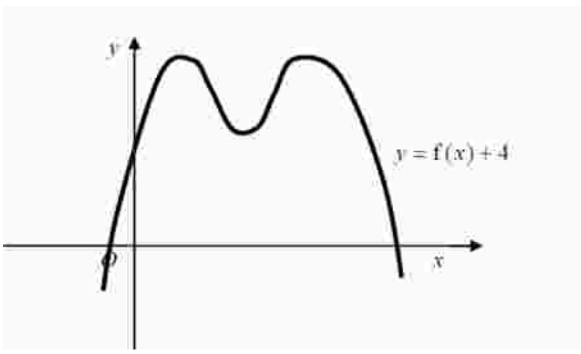
(c)

B1 For an attempted translation of the whole curve left or right. Award if the y coordinates of the turning points have stayed at 6 and 2 at least one x coordinate has changed. The curve does not need to meet or cross the x axis.

B1 Curve **crosses** the x - axis at (2,0) and (-4,0) only

B1 All 3 turning points correct and curve positioned correctly; Maximum point in quadrant one at $(1,6)$
Maximum point in quadrant two at $(-3,6)$ and minimum point in quadrant two at $(-1,2)$

Special case for candidate sketching $y = \mathrm{f}(x)+4$ with all coordinates correct can score B1B0B0



Maximum points = $(1,10)$ and $(5,10)$

Crosses x - axis either side of the origin with neither being given values.

Minimum point = $(3,6)$

Leave blank

13. (i) Showing each step in your reasoning, prove that

$$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$$

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$,

$$3\sin\theta = \tan\theta$$

giving your answers in degrees to 1 decimal place, as appropriate.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Question Number	Scheme	Marks
13. (i)	$(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin x+\cos x-\sin^2 x\cos x-\cos^2 x\sin x$	1st M1
	$(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin x+\cos x-(1-\cos^2 x)\cos x-(1-\sin^2 x)\sin x$	2nd M1
	$(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin^3 x+\cos^3 x$ *	A1 *
		[3]
Alt I (i)	Use LHS = $(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)$	2nd M1
	$\equiv \sin^3 x+\sin x\cos^2 x-\sin^2 x\cos x+\sin^2 x\cos x-\cos^2 x\sin x+\cos^3 x$	1st M1
	$(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin^3 x+\cos^3 x$	A1 *
		[3]
Alt II (i)	Use RHS $\equiv \sin^3 x+\cos^3 x$ = $(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)$	M1
	= $(\sin x+\cos x)(1-\sin x\cos x)$	M1 A1
		[3]
(ii)	Use $\tan\theta=\frac{\sin\theta}{\cos\theta}$ to give $3\sin\theta=\frac{\sin\theta}{\cos\theta}$	M1
	$\cos\theta=\frac{1}{3}$ and $\sin\theta=0$	A1 A1
	So θ = 70.5, 289.5, or <u>0 and 180</u> (do not require degrees symbol)	M1 A1, <u>B1</u>
		[6]
		9 marks

(i)

M1 Expands bracket to form 4 terms - condone sign slips but terms must be correct.

Allow $\sin x\sin x\cos x$ for $\sin^2 x\cos x$ and condone $\cos x^2$ for $\cos^2 x$

M1 Replaces $\sin^2 x$ by $(1-\cos^2 x)$ **and** $\cos^2 x$ by $(1-\sin^2 x)$

Condone $\cos x^2$ for $\cos^2 x$. This mark could be seen proceeding the line $\sin x(1-\cos^2 x)+\cos x(1-\sin^2 x)$.

A1* Completes proof with no errors. This is a given answer. Withhold this mark if poor notation or mixed variables are seen. Examples would be $\sin x+\cos x-(1-\mathbf{cos^2})\cos x-(1-\sin^2\boldsymbol{\theta})\sin x$

Don't accept for the A1* $\cos x^2$ for $\cos^2 x$ unless it is clearly bracketed.

(ii)

M1 Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ or equivalent to give $3\sin\theta=\frac{\sin\theta}{\cos\theta}$ *or* $3\cos\theta=1$

A1 Achieves $\cos\theta=\frac{1}{3}$

A1 Achieves $\sin\theta=0$

M1 For $\arccos\left(their\frac{1}{3}\right)$ leading to one value of θ to the nearest degree. (You may need a calculator to check this)

A1 For $\theta=$ awrt $70.5°, 289.5°$. This mark is withheld for extra solutions of $\arccos\left(\frac{1}{3}\right)$ in the range.

Ignore extra solutions outside the range.

B1 $\theta=0, 180°$ This mark is withheld for extra solutions arising from $\sin\theta=0$ in the range.
Ignore extra solutions outside the range.

Note: Students who proceed from $\frac{\sin\theta}{\tan\theta}=\cos\theta$ can score M1A1A0M1A1B0 for 4 out of 6

Radian solutions, withhold only the final A1 mark. For your information solutions are awrt 2dp 1.23, 5.05, 0, 3.14 = (pi)

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14.

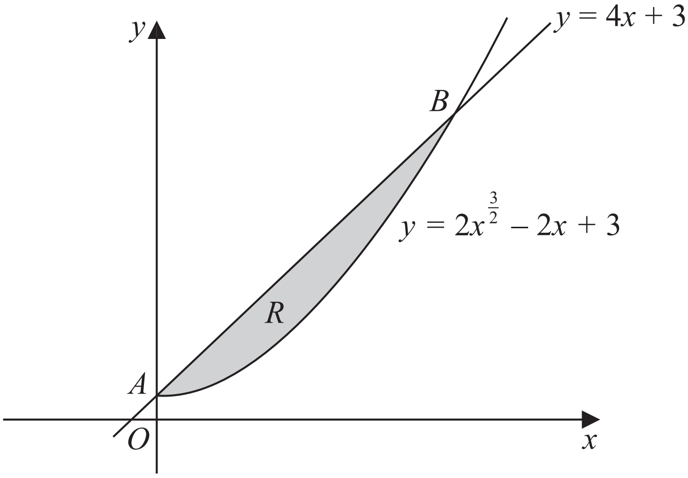

Figure 3

The finite region R, which is shown shaded in Figure 3, is bounded by the straight line l with equation $y = 4x + 3$ and the curve C with equation $y = 2x^{\frac{3}{2}} - 2x + 3,\ x \geqslant 0$

The line l meets the curve C at the point A on the y-axis and l meets C again at the point B, as shown in Figure 3.

(a) Use algebra to find the coordinates of A and B.

(4)

(b) Use integration to find the area of the shaded region R.

(6)

Question Number	Scheme		Marks
14. (a)	$4x+3=2x^{\frac{3}{2}}-2x+3$ so $6x=2x^{\frac{3}{2}}$		M1
	$x=9$		A1
	Points are (0, 3) and (9, 39)		B1, B1
			[4]
	Method 1	**Method 2**	
(b)	$\int 2x^{\frac{3}{2}}-2x+3\,\mathrm{d}x=\frac{2}{\frac{5}{2}}x^{\frac{5}{2}}-x^2+3x$	$\pm\left(\int 6x-2x^{\frac{3}{2}}\mathrm{d}x=3x^2-\frac{2}{\frac{5}{2}}x^{\frac{5}{2}}\right)$	M1A1
	Uses their upper limit (and subtracts lower limit, often 0) to obtain an area	Uses their upper limit (and subtracts lower limit, often 0) to obtain an area	M1
	Area of trapezium = $\frac{1}{2}\times(3+39)\times9$ OR $2\times9^2+3\times9$	Implied by final answer of 48.6 or -48.6	B1
	Use = Area of trapezium – Area beneath curve	Implied by subtraction in the integration	M1
	= 189 – 140.4 = 48.6	=48.6	A1cso
			[6] **(10 marks)**

(a)

M1 Set $4x+3=2x^{\frac{3}{2}}-2x+3$, collects terms and forms an equation equivalent to $Ax=Bx^{\frac{3}{2}}$, $A,B\neq0$

A1 Obtains $x=9$ from a correct equation. You may ignore any reference to $x=0$

B1 One of $(0,3)$, $(9,39)$

B1 Both $(0,3)$ and $(9,39)$

Note: This question requires the use of algebra to find $(0,3)$, $(9,39)$. Just the answers with no equation scores 0,0,1,1.

Similarly $2x^{\frac{3}{2}}-2x+3=4x+3$ must be simplified before M1A1 can be scored.

Once $2x^{\frac{3}{2}}=6x$ has been reached you can accept $x=9$ without any real working as it could be done in your head.

(b)

M1 Attempted integration on either $\int 2x^{\frac{3}{2}}-2x+3\,\mathrm{d}x$ or $\int(4x+3)-\left(2x^{\frac{3}{2}}-2x+3\right)\mathrm{d}x$ either way around

Accept as evidence $x^n\rightarrow x^{n+1}$ on any term

A1 Correct integration-may be unsimplified.

Accept $\frac{2}{\frac{5}{2}}x^{\frac{5}{2}}-x^2+3x$ or $\left(4\frac{x^2}{2}+3x\right)-\left(\frac{2}{\frac{5}{2}}x^{\frac{5}{2}}-x^2+3x\right)$ either way around.

M1 Uses their upper limit (and subtracts the lower limit or vice versa) in their integrated function. If the lower limit is 0 you do not have to see the subtraction.

B1 Area of trapezium = 189 or $\frac{1}{2}\times(3+39)\times9$ or $2\times9^2+3\times9$ from $\int 4x+3\,\mathrm{d}x$

This may be implied by the correct answer of 48.6 or (-48.6) in the alternative method.

M1 Uses area of trapezium - area under curve (either way around). It is usually implied by line 1 in the alternative method. The mark is scored for a correct method of finding the shaded area,

A1 48.6.

Note -48.6 is A0 even if candidate loses the sign

Leave blank

15.

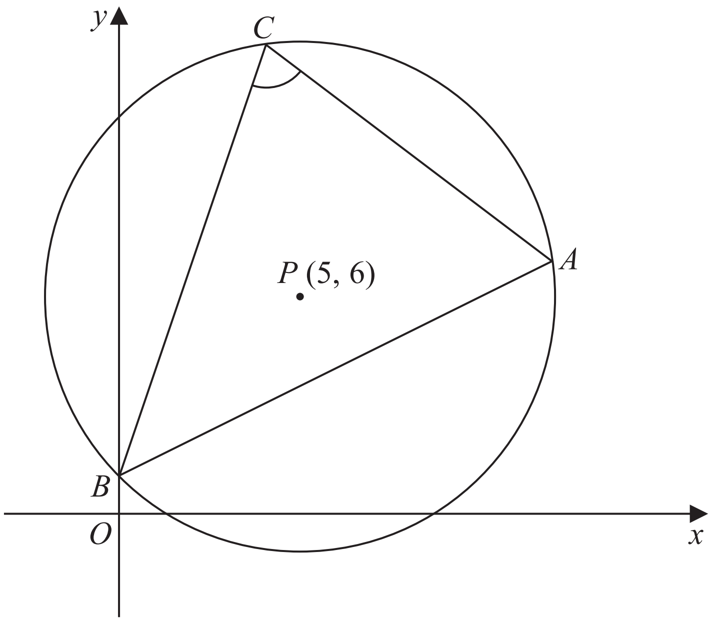

Figure 4

The circle shown in Figure 4 has centre P (5, 6) and passes through the point A (12, 7).

Find

(a) the exact radius of the circle,

(2)

(b) an equation of the circle,

(3)

(c) an equation of the tangent to the circle at the point A.

(4)

The circle also passes through the points B (0, 1) and C (4, 13).

(d) Use the cosine rule on triangle ABC to find the size of the angle BCA, giving your answer in degrees to 3 significant figures.

(5)

Question number	Scheme	Marks
15 (a)	$\sqrt{(12-5)^2+(7-6)^2}$,= $\sqrt{50}$ or $5\sqrt{2}$	M1, A1
		[2]
(b)	See $(x\pm5)^2+(y\pm6)^2=(\text{their numerical } r)^2$	M1
	$(x-5)^2+(y-6)^2=,50$	B1, A1
		[3]
(c)	Gradient of AP = $\frac{1}{7}$	B1
	So gradient of tangent is -7	M1
	Equation of tangent is $(y-7)=-7(x-12)$	dM1 A1
		[4]
(d)	$AB=\sqrt{180}=\left(6\sqrt{5}\right)$, $BC=\sqrt{160}=\left(4\sqrt{10}\right)$, $AC=10$	M1 A1 A1
	$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{160+100-180}{20\sqrt{160}}$	M1
	So C = awrt 71.6	A1
		[5]
		14 marks

(a)

M1 Sight of $\sqrt{(12-5)^2+(7-6)^2}$ or $r^2 = 7^2+1^2$ or equivalent. It may be implied by $\sqrt{50}$ or $5\sqrt{2}$ or awrt 7.1

A1 $\sqrt{50}$ or $5\sqrt{2}$

(b)

M1 For the form $(x\pm5)^2+(y\pm6)^2=(\text{their numerical } r)^2$

Accept a form $x^2+y^2\pm10x\pm12y+c=0$ where $c=61-(\text{their numerical } r^2)$

B1 For $(x-5)^2+(y-6)^2=..$ or $x^2+y^2-10x-12y....=0$ oe

This can be awarded from $(x-5)^2+(y-6)^2=r^2$ with an algebraic 'r'

A1 Completely correct answer $(x-5)^2+(y-6)^2=50$

$(x-5)^2+(y-6)^2=\left(5\sqrt{2}\right)^2$ and $x^2+y^2-10x-12y+11=0$ are acceptable alternatives.

(c)

B1 Gradient $AP=\frac{1}{7}$

M1 For using the negative reciprocal of the gradient for AP in finding the gradient of the tangent

dM1 Dependent upon the previous M1. It is for a linear equation through (12,7) with their negative reciprocal gradient. If the form $y=mx+c$ it must be a full method to find 'c'

A1 Accept any unsimplified form and remember to isw. Accept $y-7=-7(x-12)$, $y=-7x+91$,

(d)

M1 A correct attempt to find the length of **any** line using the 'difference' between the coordinates.

Accept any of $AB=\sqrt{12^2+6^2}\left(\sqrt{180}\right)$, $BC=\sqrt{12^2+4^2}\left(\sqrt{160}\right)$, $AC=\sqrt{8^2+6^2}\left(\sqrt{100}=10\right)$

This would be implied by the sight of. $AB=awrt\,13.4$, $BC=awrt\,12.6$, $AC=10$

A1 Two lengths 'correct' either exact or awrt 3 sf $AB=\sqrt{180}=13.4$, $BC=\sqrt{160}=12.6$, $AC=10$

A1 All three lengths 'correct' either exact or awrt 3 sf $AB=\sqrt{180}=13.4$, $BC=\sqrt{160}=12.6$, $AC=10$

M1 Correct application of the cosine rule with their AB, AC, BC. Look for $cosC=\frac{BC^2+AC^2-AB^2}{2\times BC\times AC}$ with their lengths in the correct positions.

Allow the method of finding cos C from $AB^2=AC^2+BC^2-2\times AC\times AB\cos C$ with their lengths in the correct positions. You may condone rearrangement errors in this method.

A1 Accept awrt C = 71.6

Leave blank

16. [*In this question you may assume the formula for the area of a circle and the following formulae:*

a ***sphere*** *of radius r has volume* $V = \frac{4}{3}\pi r^3$ *and surface area* $S = 4\pi r^2$

a ***cylinder*** *of radius r and height h has volume* $V = \pi r^2 h$ *and curved surface area* $S = 2\pi rh$]

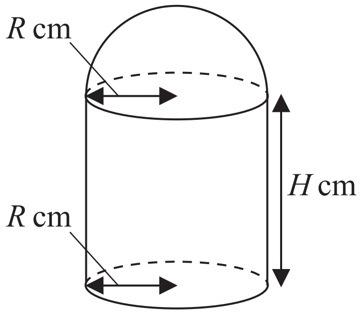

Figure 5

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius R cm. The walls are modelled by the curved surface of a circular cylinder of radius R cm and height H cm. The floor is modelled by a circular disc of radius R cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm^3 and that $0 < R < 10.6$

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(b) Show that the surface area, A cm^2, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(c) Use calculus to find the value of R, to 3 significant figures, for which A is a minimum.

(5)

(d) Prove that this value of R gives a minimum value for A.

(2)

(e) Find, to 3 significant figures, the value of H which corresponds to this value for R.

(1)

Question Number	Scheme	Marks
16. (a)	$\pi R^2 H + \frac{2}{3}\pi R^3 = 800\pi$ so $H = \dfrac{800}{R^2} - \dfrac{2}{3}R$ *	M1 A1*
(b)	$A = \pi R^2 + 2\pi RH + 2\pi R^2$	B1
	$A = 3\pi R^2 + 2\pi R\left(\dfrac{800}{R^2} - \dfrac{2}{3}R\right)$ so $A = \dfrac{5\pi R^2}{3} + \dfrac{1600\pi}{R}$ *	M1 A1 *
		[5]
(c)	Find $\dfrac{dA}{dR} = \dfrac{10}{3}\pi R - \dfrac{1600\pi}{R^2}$	M1 A1
	Put derivative equal to zero and obtain $R^3 = 480$	dM1 A1
	So $R = 7.83$	**A1**
		[5]
(d)	Consider $\dfrac{d^2A}{dR^2} = \dfrac{10\pi}{3} + 3200\pi R^{-3} > 0$ so minimum	M1A1
		[2]
(e)	H = awrt 7.83	B1
		[1]
		13 marks

(a)

M1 Sets up volume equation with $800\pi = \pi R^2 H + \dfrac{2}{3}\pi R^3$ and attempts to make H the subject. Condone 800 instead of 800π. Accept for this mark lower case letters $800\pi = \pi r^2 H + \dfrac{2}{3}\pi r^3$ and a lack of consistency in lettering.

A1* This is a show that question and there must be an intermediate line showing (or implying) a division of $\pi r^2 / \pi R^2$. Lettering must be correct and consistent from the point where you see $800\pi = ...$.
Examples of an intermediate line are;

$$800\pi = \pi R^2 H + \frac{2}{3}\pi R^3 \Rightarrow H = \frac{800\pi - \frac{2}{3}\pi R^3}{\pi R^2} \Rightarrow H = \frac{800}{R^2} - \frac{2}{3}R$$

$$800\pi = \pi R^2 H + \frac{2}{3}\pi R^3 \Rightarrow \frac{800}{R^2} = H + \frac{2}{3}R \Rightarrow H = \frac{800}{R^2} - \frac{2}{3}R$$

(b)

B1 A correct expression for the surface area containing three separate correct elements

Allow either $A = \pi R^2 + 2\pi RH + 2\pi R^2$ or $A = \pi R^2 + 2\pi RH + \dfrac{4\pi R^2}{2}$

Allow lower case lettering for this mark

M1 Score for replacing $H = \dfrac{800}{R^2} - \dfrac{2}{3}R$ in their expression for A which must be of the form,

$A = B\pi R^2 + C\pi RH, \quad B, C \in \mathbb{N}$,condoning missing brackets.

A1* This is a show that question and all aspects must be correct. Lettering in (b) must be consistent and correct from the point at which $\dfrac{800}{R^2} - \dfrac{2}{3}R$ is substituted. Do not, however, withhold a second mark for using lower case letters if it has been withheld in part (a) for mixed lettering.

Accept $A = 2\pi R^2 + \pi R^2 + 2\pi R\left(\dfrac{800}{R^2} - \dfrac{2}{3}R\right) \Rightarrow A = \dfrac{5\pi R^2}{3} + \dfrac{1600\pi}{R}$ with little or no evidence

(c)

M1 Evidence of differentiation so sight of $R^2 \to R$ or $R^{-1} \to R^{-2}$

A1 Both terms correct. Does not need to be simplified. $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$

Do not be concerned by inconsistent lettering and condone incorrect notation on the lhs such as $\frac{dy}{dx} =$

dM1 Sets $\frac{dA}{dR} = 0$ and proceeds to $R^n = C$ This is dependent upon the previous M

A1 A correct intermediate answer. $R^3 = 480$ **.** This may be implied by a correct answer following a correct derivative.

A1 Allow awrt *R*= 7.83

(d)

M1 Achieves a correct second derivative $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$

Or attempts to substitute their positive *R* in their second derivative (usual rules for differentiation having been applied to at least one term)
Or alternatively attempts to find the numerical value of the gradient either side of their value of *R*
Or alternatively attempts to find the numerical value of *V* at *R* and either side of *R*

A1 This requires (1) a correct function, (2) a correct statement and (3) a correct conclusion

Either (1) $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$ (2) At their $R = + \ldots$, $\frac{d^2A}{dR^2} = +\ldots > 0$ (3) hence minimum.

Or (1) $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$, (2) as R>0, $\frac{d^2A}{dR^2} > 0$ (3) hence minimum

Or (1) $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$ (2) at $R = 7$ $\frac{dA}{dR} = -29 < 0$ at $R = 8$ $\frac{dA}{dR} = 5 > 0$ (3)Hence minimum

The table below can be used to check most calculations

Value of *R*	7	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
Gradient at *R*	-29	-25	-22	-18	-14	-11	-7	-4	-1	2	5

Or (1) $A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ (2), $A_{7.82} > A_{7.83} < A_{7.84}$ with evidence (see below), (3) Hence minimum

Value of *R*	7.8	7.81	7.82	7.83	7.84	7.85	7.86	7.87	7.88	7.89	7.9
Value of *A*	962.50	962.49	962.49	962.48	962.49	962.49	962.50	962.51	962.52	962.54	962.56

(e)

B1 *H* =awrt 7.83