Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C12

Advanced Subsidiary

Tuesday 13 January 2015 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 4 5 0 5 7 A 0 1 4 4

PEARSON

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1. Simplify the following expressions fully.

(a) $(x^6)^{\frac{1}{3}}$

(1)

(b) $\sqrt{2}(x^3) \div \sqrt{\dfrac{32}{x^2}}$

(2)

January 2015
International A Level WMA01/01 Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1.	(a) x^2	B1 **[1]**
	(b) $\frac{1}{4}x^4$ or $\frac{1}{2^2}x^4$ or $0.25x^4$	B1, B1 **[2]**
		3 marks
	Notes	

(a) **B1**: This answer only

(b) **B1:** For $\frac{1}{4}x^k$ **as final answer**, k can even be 0. Also accept $\frac{1}{2^2}$ for B1 but 2^{-2} is not simplified and is B0

B1: for x to power 4 (independent mark) so kx^4 with k a constant (could even be 1) **as final answer**
n.b. Can score B0B1 or B1B0 or B0B0 or B1B1
Mark the final answer on this question

Also note : Candidates who misread question as $\sqrt{2x^3} \div \sqrt{\frac{32}{x^2}}$ should get $\frac{1}{4}x^{\frac{5}{2}}$ This is awarded B1B0

Special case: The answer $\left(\frac{1}{\sqrt{2}}x\right)^4$ is awarded B0 B1 as x may be in a bracket with power 4 outside.

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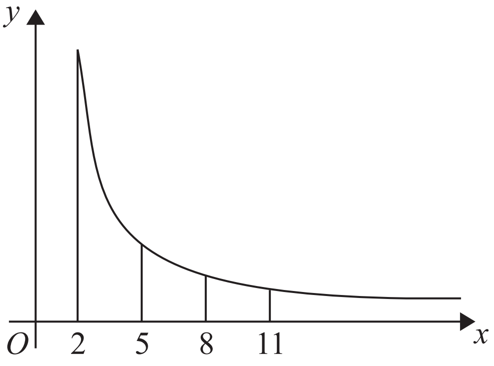

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \dfrac{12}{\sqrt{(x^2 - 2)}}, \ x \geqslant 2$

The table below gives values of y rounded to 3 decimal places.

x	2	5	8	11
y	8.485	2.502	1.524	1.100

(a) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for

$$\int_2^{11} \frac{12}{\sqrt{(x^2 - 2)}}\,\mathrm{d}x$$

(4)

(b) Use your answer to part (a) to estimate a value for

$$\int_2^{11} \left(1 + \frac{6}{\sqrt{(x^2 - 2)}}\right)\mathrm{d}x$$

(3)

Question Number	Scheme	Marks
2.	<table><tr><td>x</td><td>2</td><td>5</td><td>8</td><td>11</td></tr><tr><td>y</td><td>8.485</td><td>2.502</td><td>1.524</td><td>1.100</td></tr></table>	
(a)	State $h = 3$, or use of $\frac{1}{2}\times 3$	B1 aef
	$\{8.485+1.100+2(\ 2.502+1.524\ )\}$ For structure of $\{..............\}$	M1A1
	$\frac{1}{2}\times 3\times\{17.637\}$ $(=26.4555)$ $=$ awrt 26.46	A1
		[4]
(b)	Adds 9+……	M1
	half of **their answer from (a)** seen (allow use of half of 26.4555)	M1
	So required estimate = 9 + 13.23 = 22.23	A1
		[3]
		7 marks
(b)	Way 2: **Begins again with trapezium rule**	
	<table><tr><td>x</td><td>2</td><td>5</td><td>8</td><td>11</td></tr><tr><td>y</td><td>5.2425</td><td>2.251</td><td>1.762</td><td>1.550</td></tr></table>	M1
	Uses $\frac{1}{2}\times 3\times\{5.2425+1.550+2(2.251+1.762)\}$	M1
	= 22.23	A1 [3]
	Notes	

(a) **B1**: for using $\frac{1}{2}\times 3$ or 1.5 or equivalent or just states $h = 3$

M1: requires the correct $\{......\}$ bracket structure. It needs the first bracket to contain first y value plus last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed
(An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values
A1**: for the completely correct bracket** $\{......\}$

A1: for answer which rounds to 26.46 after attempt at trapezium rule
NB: Separate trapezia may be used: B1 for 1.5, M1 for 1/2 $h(a + b)$ used 2 or 3 times (and A1 if it is all correct) Then A1 for 26.46.
Special case: Bracketing mistake $1.5\times(8.485+1.1)+2(2.502+1.524)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).

(b) **Way 1:**

M1: Adds Area of Rectangle $=1\times 9$ or $\int 1\mathrm{d}x=[x]_2^{11}$ to their "13.23" or to their "26.46" or to their "52.92"

M1: Half answer to part (a) seen
A1: Accept awrt 22.23

Or Way 2: (If they **begin again with a trapezium rule)**
M1: for correct table **M1:** for correct use of trapezium rule
A1: awrt 22.23

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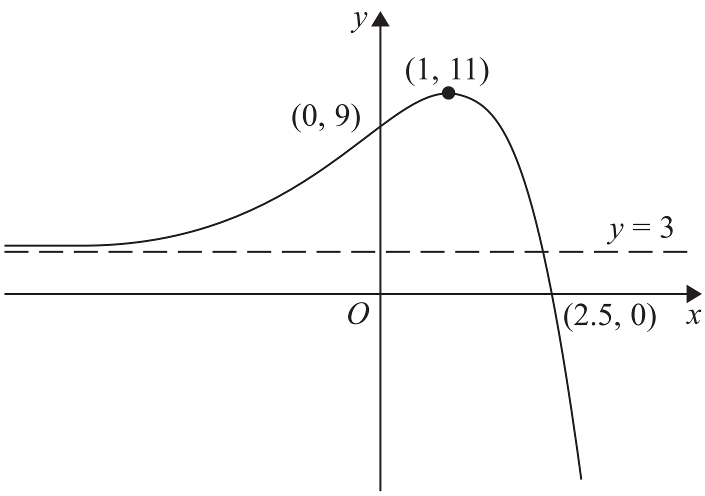


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.
The curve crosses the coordinate axes at the points (2.5, 0) and (0, 9), has a stationary point at (1, 11), and has an asymptote $y = 3$

On **separate** diagrams, sketch the curve with equation

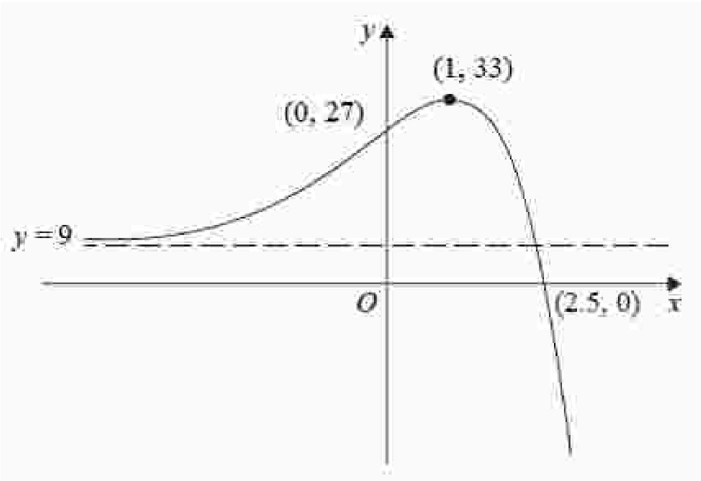
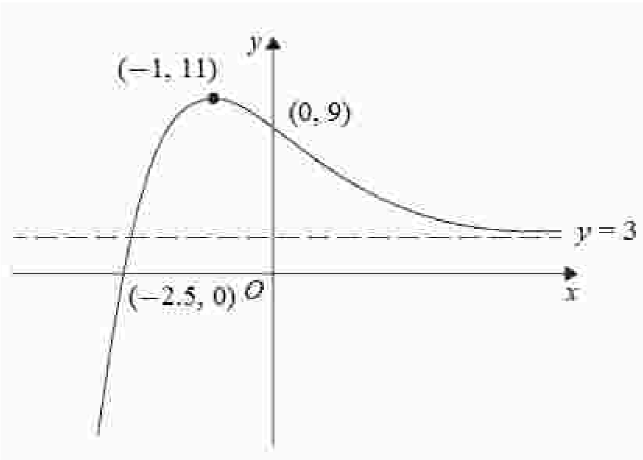
(a) $y = 3\mathrm{f}(x)$

(3)

(b) $y = \mathrm{f}(-x)$

(3)

On each diagram show clearly the coordinates of the points of intersection of the curve with the two coordinate axes, the coordinates of the stationary point, and the equation of the asymptote.

Question Number	Scheme		Marks
3.			
(a)		Shape- similar to before but with indication of **stretch** in y direction by at least **one correct** from the three traits: *y* intercept, (0, 27) maximum point (1, 33) **or** asymptote indicated at 9	B1
		Intercept (0,27), max (1,33) and *x* intercept (2.5,0) **all three of these seen**	B1 B1
			[3]
(b)		Shape (reflection in *y* axis)	B1
		(–1,11), (0,9) and (–2.5,0) seen	B1
		y = 3 (must be equation)	B1
			[3]
			6 marks
	Notes		

(a) **B1**: Correct shape with curve crossing x axis and one label correct from the three listed (i.e. a correct new y value). Condone "slight" imperfections in the curvature of the sketches.

B1: All three specified labels given to indicate the three new point positions. Do not need coordinates if clearly labelled on the axes. Accept 27 and accept 2.5 and even allow (27, 0) and (0, 2.5) on y and x axes respectively.

B1: Equation of asymptote correct (asymptote on figure takes precedence) Asymptote does not need to be drawn dotted.

(b) **B1**: Correct shape (maximum in 2nd quadrant, intercept on negative x - axis and approaches asymptote for large positive x) Condone "slight" imperfections in the curvature of the sketches.

B1: All three specified labels given to indicate the three new point positions. Accept 9 and accept -2.5 and even allow (9, 0) and (0, -2.5) on y and x axes respectively.

B1: Equation of asymptote correct (asymptote on figure takes precedence) Do not award this mark if they merely copy the original graph.

If there is no sketch – the maximum mark in part (a) is B0B1B1 and in part (b) is B0B1B0 so 3/6

Special case: Stretch in y direction of scale factor 1/3. If there is a graph of the correct shape with (0,3), (1, 11/3), (2.5,0) and asymptote $y = 1$ then award B0B0B1

Leave blank

4. (a) Find the first 4 terms in ascending powers of x of the binomial expansion of

$$\left(2 + \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Use your expansion to find an estimated value for 2.025^{10}, stating the value of x which you have used and showing your working.

(3)

Question Number	Scheme	Marks
4.	(a) $\left(2+\frac{x}{4}\right)^{10} = 2^{10} + \binom{10}{1}2^9.\left(\frac{x}{4}\right) + \binom{10}{2}2^8.\left(\frac{x}{4}\right)^2 + \binom{10}{3}2^7.\left(\frac{x}{4}\right)^3 ...$	M1
	$= 1024, +1280x + 720x^2 + 240x^3...$	B1, A1 A1
		[4]
	(b) State or Use $x = 0.1$	B1
	Estimate = $1024 + "1280" \times 0.1 + 720 \times (0.1)^2 + 240 \times (0.1)^3 ...$	M1
	= 1159.44 or 1159.440 or 1159 or 1159.4 (after correct working)	A1
		[3]
		7 marks
	Notes	

(a) **M1**: The **method** mark is awarded for an attempt at Binomial to get one or more of the terms in x – need **correct** binomial coefficient multiplied by the correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 4 or bracket errors. Accept any notation for ${}^{10}C_1$, ${}^{10}C_2$ and ${}^{10}C_3$, e.g. $\binom{10}{1}$, $\binom{10}{2}$ and $\binom{10}{3}$ (unsimplified) or 10. 45 and 120 from Pascal's triangle This mark may be given if no working is shown, but any of the terms including x is correct.

B1: must be simplified to 1024 (writing just 2^{10} is **B0). If miscopied later then isw**

A1: is cao and is for **two correct** from 1280 x, $720x^2$ and $240x^3$

A1: is c.a.o and is for all of 1280 x, $720x^2$ and $240x^3$ correct (ignore extra terms) if divided by 2 or 4 then isw
Allow terms given separately without + signs and with commas. Ignore extra terms. Ignore subsequent work once correct answer is seen in simplified form.

N.B. If the series is given **in Descending Order** the first M mark may be awarded and if the whole expansion is given (all 11 terms) then full marks is possible.

(b) **B1:** States or Uses $x = 0.1$

M1: Uses their solution of $\frac{x}{4} = 0.025$ substituted in to their series expansion – If no equation stated could see evidence of use of 0.1 or
0.01 (not 0.025) substituted consistently for example

A1: This is cao and must follow M1.

NB 1159.45 or 1159.44533 is A0 (used 2.025^{10}) But correct working followed by an answer 1159 or 1159.4 can be awarded A1

Leave blank

5. (a) Prove that the sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term of the series and d is the common difference between the terms.

(4)

(b) Find the sum of the integers which are divisible by 7 and lie between 1 and 500

(3)

Question Number	Scheme	Marks
5. (a)	$S_n = a + (a+d) + (a+2d) + ... + (a+(n-1)d)$	M1
	$S_n = (a+(n-1)d) + (a+(n-2)d) + ... + (a+d) + a$	M1
	$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + ... + (2a+(n-1)d)$	M1
	$S_n = \frac{n}{2}[2a+(n-1)d]$* See notes below for those who use triangle numbers in their proof	A1* **[4]**
(b)	**Uses** either $\frac{n}{2}(2\times a+(n-1)7)$ or $\frac{n}{2}(a+497)$ or $7\times\sum_{i=1}^{71} i$	M1
	i.e $\frac{71}{2}(2\times 7+70\times 7)$ or $\frac{72}{2}(2\times 0+71\times 7)$ or $\frac{71}{2}(7+497)$ or $7\times\frac{71}{2}(72)$	A1
	= 17892	A1 **[3]**
		7 marks
	Notes	

(a) **M1**: List terms including **at least first two** and **a last term which may be *a* + *nd* or *a* + (*n* – 1)*d* or *L***
M1: List terms in reverse including **at least their last term (or correct last term) and finally their first term**
M1: The LHS should be 2*S*. The RHS must follow from at least two terms correctly matching in the addition and should include at least two terms which are each **correctly** $\{2a+(n-1)d\}$ or $(a + L)$ **or should** be $n\{2a+(n-1)d\}$ or $n(a + L)$
A1: Need some indication of at least three terms being added (i.e at least three terms and their pairs listed with terms correctly matching or three additions seen) and also need to achieve final answer with no errors and if *L* was used need to state that $L = a + (n-1)d$

NB: Some candidates use a variation of

$$\sum_{r=1}^{n}(a+(r-1)d = \sum_{r=1}^{n} a + d\sum_{r=1}^{n}(r-1) = na + d\frac{n}{2}(n+1) - dn \text{ or } na + d\frac{(n-1)}{2}(n)$$

And conclude that $S_n = \frac{n}{2}[2a+(n-1)d]$. This gains the full 4 marks M1M1M1A1, but must be completely correct.

(b) **M1:Uses** correct formula (with their *a* and *n*) with *d* =7 or with last term correct
A1: Uses consistent and correct *a* and *n*
A1: Correct answer

Leave blank

6. Given that

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(a) show that

$$4x^2 - 16x - 9 = 0$$

(5)

(b) Hence solve the equation

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(2)

Question Number	Scheme	Marks
6.	(a) Use or state $2\log_4(2x+3)=\log_4(2x+3)^2$	M1
	Use or state $\log_4 4=1$ or $4^1=4$	M1
	Use or state $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\dfrac{(2x+3)^2}{x}$ etc	M1
	$(2x+3)^2=4x(2x-1)$ or equivalent including correct rational equations	A1
	Then $4x^2+12x+9=8x^2-4x$ and so $4x^2-16x-9=0$ *	A1*
		[5]
	(b) $(2x+1)(2x-9)=0$ so $x=$ (or use other method e.g formula or completion of square)	M1
	$x=(-\frac{1}{2}$ or $)\ \frac{9}{2}$	A1
		[2]
		7 marks
	Notes	

(a) M1**:** Uses power law for logs

M1: Connects 1 with 4 correctly

M1: Uses addition (or subtraction) law correctly

e.g. $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\dfrac{(2x+3)^2}{x}$ or

$\log_4(2x+3)^2-\log_4 x-\log_4(2x-1)=\log_4\dfrac{(2x+3)^2}{x(2x-1)}$ or even $\log_4 x+\log_4 4=\log_4 4x$ or

$\log_4(2x-1)+\log_4 4=\log_4 4(2x-1)$ or $\log_4(2x-1)+\log_4 4+\log_4 x=\log_4 4x(2x-1)$ etc…

A1: Correct equation (unsimplified) after correct work. e.g. $\dfrac{(2x+3)^2}{x(2x-1)}=4$

A1: Obtains printed answer correctly (This is a given answer so needs previous A mark to have been awarded and needs correct expansion)

Special case :

$$\log_4(2x+3)^2=1+\log_4 x(2x-1)\quad so\quad \frac{\log_4(2x+3)^2}{\log_4 x(2x-1)}=1\quad so\quad \frac{4x^2+12x+9}{2x^2-x}=4$$

This can have M1, M1, M1, A0, A0 so 3/5 losing accuracy because of the error in the second step.

(b) Some candidates who did not achieve marks in part (a) begin the log work again and make more progress here. Mark the better work. So credit for (a) may be given in (b). Credit for (b) should not be given in (a)

M1: Uses solution of their quadratic or of printed quadratic(see notes). This must be in part (b)

A1: x = 4.5 and discards x = - 0.5 (any equivalent form) Giving $x=-\frac{1}{2}$, $\frac{9}{2}$ is A0 This must be in part (b)

Leave blank

7. The circle C has equation

$$x^2 + y^2 + 10x - 6y + 18 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C.

(2)

The circle C meets the line with equation $x = -3$ at two points.

(c) Find the exact values for the y coordinates of these two points, giving your answers as fully simplified surds.

(4)

Question number	Scheme	Marks
7 (a)	Obtain $\underline{(x\pm5)^2}$ **and** $\underline{\underline{(y\pm3)^2}}$	M1
	Centre is $(-5, 3)$.	A1 [2]
(b)	See $\underline{(x\pm5)^2}+\underline{\underline{(y\pm3)^2}}=16(=r^2)$ or $(r^2=)"25"+"9"-18$	M1
	$r=4$	A1 [2]
(c)	Use $x=-3$ in either form of equation of circle to obtain simplified quadratic in y	M1
	e.g $x=-3\Rightarrow(-3+5)^2+(y-3)^2=16\Rightarrow(y-3)^2=12$ *or* $(-3)^2+y^2+10\times(-3)-6y+18=0\Rightarrow y^2-6y-3=0$	
	solve resulting quadratic to give $y=$	M1
	$y=3\pm2\sqrt{3}$	A1, A1 [4]
		8 marks
Alternatives (a)	*Method 2:* From $x^2+y^2+2gx+2fy+c=0$ centre is $(\pm g,\ \pm f)$ Centre is $(-g,\ -f)$ **, and so centre is** $(-5, 3)$.	M1 A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "-5" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "3" (Centre – chord theorem) . (-5, 3) is M1A1	M1 A1 (2)
(b)	*Method 2*: Using $\sqrt{g^2+f^2-c}$ or $(r^2=)"25"+"9"-18$ $r=4$	M1 A1 (2)
(c)	*Method 2*: Divide triangle PTQ and use Pythagoras with $r^2-(-3-"-5")^2=h^2$, then evaluate $"3\pm h"$ - then get $3\pm2\sqrt{3}$	M1 M1 A1 A1 (4)
	Notes	

Mark (a) and (b) together

(a) **M1** as in scheme and can be implied by $(\pm 5, \pm 3)$ **A1:** for correct centre and **(-5, 3) (without working) implies M1A1**

(b) **M1** for a complete and correct method leading to $r^2 = "25" + "9" - 18$ or $r = \sqrt{"25" + "9" - 18}$

or for using equation of circle in $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ form to identify $r = k$

N.B. $r^2 = k$ or $r = k^2$ is M0 Also $-$ "25" $-$ "9" $- 18$ is M0 and $r^2 = "25" + "9"$ (without the 18)is M0

A1 r **=** 4 (only and not with r = -4) Again **correct answer with no working implies M1A1**

Special case: if centre is given as $(5, -3)$ **or (5, 3) or (-5, -3)** allow **M1A1** for r = 4 worked correctly as $(r^2 =) "25" + "9" - 18$ i.e if they obtain r =4 after sign error give final A1 (So M1A0M1A1)

(c) M1 For substituting x = -3 into an equation for the circle and attempt to simplify to 3 term quadratic or to $(y - a)^2 = b$

M1 For attempting to solve their quadratic (following usual rules – see notes)

A1, A1 Answers must be given as surds – A1 for each correct answer. To earn both A marks, answers must be simplified.

Leave blank

8. A sequence is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 12, \qquad n \geqslant 1$$

where k is a constant.

(a) Write down fully simplified expressions for u_2, u_3 and u_4 in terms of k.
(4)

Given that $u_4 = 15$

(b) find the value of k,
(2)

(c) find $\sum_{i=1}^{4} u_i$, giving an exact numerical answer.
(3)

Question Number	Scheme	Marks
8.		
(a)	$u_2 = 3k - 12,\ u_3 = 3(u_2) - 12$	M1
	$u_2 = 3k - 12,\ u_3 = 9k - 48$	A1
	$u_4 = 3(9k - 48) - 12 = 27k - 156$ (ft their u_3) .	M1 A1ft
		[4]
(b)	$27k - 156 = 15$ so $k =$	M1
	$k = 6\frac{1}{3}$ or $\frac{19}{3}$ or 6.33 (3sf)	A1
		[2]
(c)	$\sum_{i=1}^{4} u_i = 6\frac{1}{3} + 7 + 9 + 15$ or $\sum_{i=1}^{4} u_i = k + 3k - 12 + 9k - 48 + 27k - 156$	M1
	$= 40k - 216,\ = 37\frac{1}{3}$ or $\dfrac{112}{3}$	A1ft, A1cao
		[3]
		9 marks
	Notes	

(a) **M1:** Attempt to use formula twice to find u_2 and u_3

A1: two correct simplified answers

M1: Attempt again to find u_4

A1ft: 4th term correct and simplified - follow through their u_3

(b) **M1**: Put their 4th term (not 5th) equal to 15 and attempt to find k =

A1: accept any correct fraction or decimal answer (allow 6.33 or better here)

(c) **M1:** Uses 1st term and their following 3 terms **with plus signs** (either numerical or in terms of k). Must be using terms from iteration and not formula for an AP or GP. May make a copying slip.

A1ft: for $40k - 216$ or follow through on their k so check $40k$ -216 for their k

A1: obtains $37\frac{1}{3}$ (must be exact) if exact answer given, then isw

Those who use 6.3 will obtain 36 They should have M1A1ftA0 – should have used exact k to give exact answer here.

Those who use 6.33 will obtain 37.2 This should have M1A1ftA0 – should have used exact k to give exact answer here.

Those who use 6.333 will obtain 37.32 This should have M1A1ftA0 – should have used exact k to give exact answer here.

6.3333 will obtain 37.332 This should have M1A1ftA0 – should have used exact k to give exact answer here.

6.33333 will obtain 37.3332 etc All these answers should have M1A1ftA0 – should have used exact k to give exact answer here. Etc

Special case: Those who use k = 6 will obtain 6 + 6 + 6 + 6 = 24 This is M1 A0 A0 in part (c) – as over simplified

Leave blank

9.

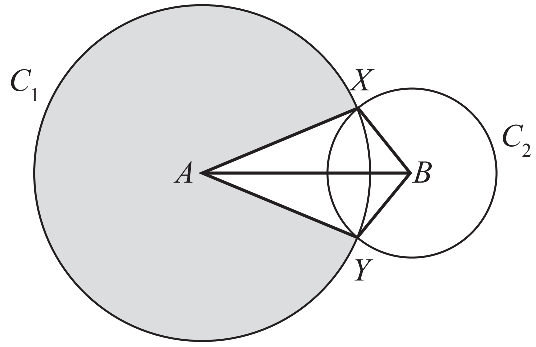


Figure 3

In Figure 3, the points A and B are the centres of the circles C_1 and C_2 respectively. The circle C_1 has radius 10 cm and the circle C_2 has radius 5 cm. The circles intersect at the points X and Y, as shown in the figure.

Given that the distance between the centres of the circles is 12 cm,

(a) calculate the size of the acute angle XAB, giving your answer in radians to 3 significant figures,

(2)

(b) find the area of the major sector of circle C_1, shown shaded in Figure 3,

(3)

(c) find the area of the kite $AYBX$.

(3)

Question Number	Scheme	Marks
9. (a)	$5^2 = 10^2 + 12^2 - 2\times10\times12\cos\angle XAB$, or $\cos\angle XAB = \frac{10^2+12^2-5^2}{2\times10\times12}$ or $\frac{219}{240}$ or 0.9125 or $\frac{73}{80}$	M1
	$\angle XAB = 0.421$ or 0.134π	A1
		[2]
(b)	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times10^2\times\theta$	M1
	Area of major sector is $\frac{1}{2}\times r^2(2\pi - 2\times\text{"}0.421\text{"})$ or $\pi\times r^2 - \frac{1}{2}\times r^2\times2\times\text{"}0.421\text{"})$	M1
	= 272	A1
		[3]
(c)	area of triangle $AXB = \frac{1}{2}10\times12\times\sin XAB$ **Way 2**: Find angle XBA and hence area XBY	M1
	area of kite $= 2\times$ triangleAXB Area of kite = area of XBY + Area XAY	dM1
	= awrt 49 = 37.298 + 11.76 = 49	A1
		[3]
	Way 3: Finds length XY by cosine rule or elementary trigonometry (8.173)	M1
	Uses area of kite = $\frac{1}{2}\text{"}8.173\text{"}\times12$	dM1
	= awrt 49	A1
		[3]
		8 marks
	Notes	

(a) **M1**: Uses cosine rule – must be a correct statement , allow statement $5^2 = 10^2 + 12^2 - 2\times10\times12\cos\angle XAB$

A1: accept awrt 0.421 (answers in degrees gain M1 A0). Also 0.42 is A0

(b) **M1**: Uses area formula with r = 10 and any angle in radians. If they use degrees they must use the formula $\frac{\theta}{360}\times\pi10^2$

M1: Finds angle in major sector ft their angle from (a) and uses sector formula **or** subtracts minor area from circle (allow work in degrees) Must use $(2\pi - 2\times\text{"}0.421\text{"})$ but r may be 5 instead of 10 for this mark

A1: Accept awrt 272 (may reach this using degrees)

(c) Way 1: **M1**: Finds area of triangle AXB, using 10, 12 and their angle XAB

dM1: Doubles area of triangle AXB

Way 2: **M1**: Finds angle XBA (0.958..) by valid method (cosine rule) (**NOT 90 – *XAB***) and hence area XBY $= \frac{1}{2}5\times5\times\sin1.9163$

dM1: Adds areas of triangles XBY and XAY (37.298 and 11.76)

Way 3: **M1**: Finds length XY by cosine rule or elementary trigonometry (8.173)

dM1: Uses area of kite = $\frac{1}{2}\text{"}8.173\text{"}\times12$

For each method A1: awrt 49- do not need units

Leave blank

10.

$$f(x) = 6x^3 + ax^2 + bx - 5$$

where a and b are constants.

When f(x) is divided by (x + 1) there is no remainder.

When f(x) is divided by (2x – 1) the remainder is –15

(a) Find the value of a and the value of b.

(5)

(b) Factorise f(x) completely.

(4)

Question Number	Scheme	Marks
10.	f $(x) =$ $6x^3+ax^2+bx-5$	
(a)	Attempts $\text{f}(\pm 1)$ **or** Attempts $\text{f}(\pm\frac{1}{2})$ Or Use long division as far as remainder*	M1
	Obtains $6(-1)^3+a(-1)^2+b(-1)-5=0$ or -6 + a – b – 5 = 0 or a – b = 11 or equivalent	A1
	Obtains $6(\frac{1}{2})^3+a(\frac{1}{2})^2+b(\frac{1}{2})-5=-15$ or $\frac{6}{8}+\frac{a}{4}+\frac{b}{2}-5=-15$ or a + 2b = - 43 or equivalent	A1
	Solve simultaneous equations to obtain a = -7 and b = -18	M1 A1
		[5]
(b)	$6x^3+ax^2+bx-5=(x+1)(6x^2+...x+...)$	M1
	$6x^3-7x^2-18x-5=(x+1)(6x^2-13x-5)$	A1
	$(6x^2-13x-5)=(ax+b)(cx+d)$ where $ac=$"6"and $bd=$"± 5"	M1
	$=(x+1)(2x-5)(3x+1)$	A1
		[4]
		9 marks
	Notes	

(a) **M1**: Using remainder theorem: As on scheme. One of these is sufficient do not need to equate to 0 and to -15

*Using Long division: need at least $6x^2+(a-6)x+....$ as quotient, and get as far as remainder **or for the other division reaches** $3x^2+(\frac{a+3}{2})x+....$ as quotient, and get as far as remainder .

A1: Any equivalent form *e.g. -11 – *b* + *a* = 0 (using remainder after division) The mark is earned for *a* – *b* = 11 even if "=0" not explicitly seen

A1: Any equivalent form *e.g. $-5+\frac{b}{2}+\frac{a+3}{4}=-15$ (using remainder after division) Must be accurate but may be unsimplified. NB Using 15 instead of -15 is A0

M1: Solves their **linear** equations to obtain *a* or *b*

A1: Both *a* and *b* correct. Correct answers without working can earn M1A1.

(b) M1: Recognises (x+1) is factor and obtains quadratic expression with correct first term by any method. Use of (x -1) is M0. NB Starting with (x + 1) (2x – 1) *(ax + b)* is also M0

A1: Correct quadratic $(6x^2-13x-5)$

M1: Attempt to factorise quadratic where $ac=\text{"}6\text{"}$ and $bd=\text{"}\pm5\text{"}$

A1: any correct combination e.g. $=2(x+1)(x-\frac{5}{2})(3x+1)$ or $=6(x+1)(x-\frac{5}{2})(x+\frac{1}{3})$ etc… (on one line)

Following a correct value for *a* and for *b:*
They may just write the factorised answer down.
For a correct answer this is M1A1M1A1
For $=(x+1)(x-2.5)(x+\frac{1}{3})$ award M1A0M1A0
For **correct answer** following **incorrect quadratic** give M1 A0 M1 A0 – fortuitous
If the correct answer follows incorrect *a* and *b*, it is fortuitous and again M1A0M1A0 should be given.

Leave blank

11.

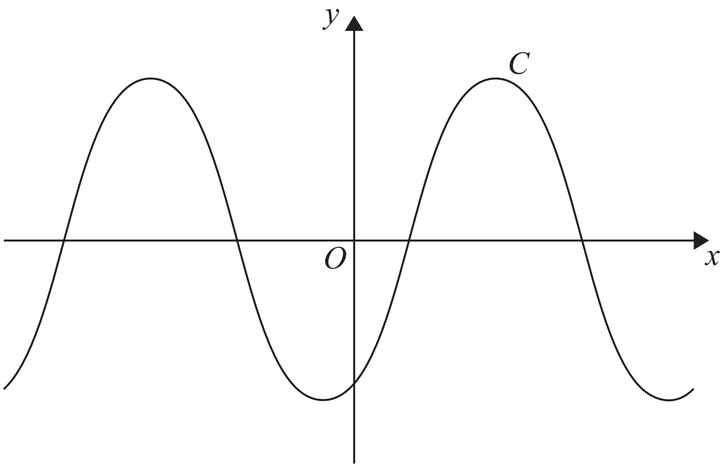


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \sin(x - 60°)$, $-360° \leqslant x \leqslant 360°$

(a) Write down the exact coordinates of the points at which C meets the two coordinate axes.

(3)

(b) Solve, for $-360° \leqslant x \leqslant 360°$,

$$4\sin(x - 60°) = \sqrt{6} - \sqrt{2}$$

showing each stage of your working.

(5)

Question Number	Scheme	Marks
11 (a)	$\left(0, -\frac{\sqrt{3}}{2}\right)$	B1
	and (60°, 0) and (240°, 0) and (-120°, 0) and (-300°, 0)	B1 B1
		[3]
(b)	$\sin(x-60^{o}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **(**= 0.2588)	M1
	x – 60° = 15° (or 165° or – 195° or – 345°) or 0.262 or $\frac{\pi}{12}$ radians	A1
	So x = 75° or 225° or – 135° or –285° (allow awrt)	M1 A1 A1
		[5]
		8 marks
	Notes	

(a) **B1** : Correct exact y intercept (not decimal) – allow on the diagram or in the text. Allow $y = -\frac{\sqrt{3}}{2}$

B1 for 2 correct x intercepts then **third B1** for all 4 correct x intercepts (may or may not be given as coordinates – may be given on graph) Must be in degrees. (Extra answers in the range lose the **third B1**)

(b) **M1**: Divides by 4 first giving correct statement $\sin(x-60^{o}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ but $(x-60^{o}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is M0 and** $\sin x - \sin 60^{o} = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is also M0 and** $\sin(x-60^{o}) = \frac{\sqrt{4}}{4}$ **is M0** if not preceded by correct statement

A1: Obtains 15° (or 165° or – 195° or – 345°)

M1: Adds 60° to their previous answer which should have been in degrees and obtained by using inverse sine

A1: Two correct answers second **A1:** All four correct answers Extra answers outside range are ignored. Lose final A mark for extra wrong answers in the range.

If they approximate too early allow awrt answers given for full marks. (e.g. 75.01 etc)

Answers in mixture, degrees and radians: Allow first M A1 only so M1A1M0A0A0 for 60.262 for example

Leave blank

12. A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1

(a) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(b) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(c) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)

Question Number	Scheme	Marks
12.(a)	Uses $275000\times(1.1)^5$ or finds £442890.25 or uses $275000\times(1.1)^4$ or finds £402627.50	M1
	Finds both of the above and subtracts to give £40 262.75 and concludes approx. £40300*	M1 A1*
	Or	**[3]**
	Uses $275000\times(1.1)^5 - 275000\times(1.1)^4$, $= awrt\,40260 = 40300\ (3sf)$ *	M1 M1,A1*
		[3]
(b)	Puts $275000\times(1.1)^{n-1} > 1000000$ or $275000\times(1.1)^{n-1} = 1000000$	M1
	$(1.1)^{n-1} > \frac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64). Or $(1.1)^{n-1} = \frac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64)	M1
	$n-1 > \frac{\log\left(\frac{40}{11}\right)}{\log 1.1}$ or $n-1 = \frac{\log\left(\frac{40}{11}\right)}{\log 1.1}$	M1
	($n > 14.5$ or $n > 14.6$ or $n = 15$) so the year is 2030	A1
		[4]
(c)	Uses $S = \frac{275000(1.1^n - 1)}{1.1-1}$ or uses $S = \frac{275000(1-1.1^n)}{1-1.1}$	M1
	Uses n = 11 in formula	A1
	Awrt £5 096 100	A1
	Or: adds 11 terms £275000 + 302500 + 332750 + 366025 + 402627.5 + 442890.25 + 487179.275 + 535897.2025 + 589486.9228 + 648435.615 + 713279.1765 = awrt 5096100 (see notes below)	**[3]**
		10 marks
	Notes	

(a) **M1**: for correct expression for profit in 2021 **or** in 2020, by any method (including subtracting the sums S_{n+1} - S_n) to give a term

M1: for finding both correct expressions and subtracting

A1: answers wrt£442900 and wrt£402600 subtracted **or** wrt£40260 obtained **then rounded to** £40300 (answer given)

(b) **M1**: Correct inequality – or allow equality . N.B. $250000\times(1.1)^n$ or $302500\times(1.1)^{n-2}$ on LHS are also correct.

M1: Division – isw if initial fraction is correct. Not dependent on previous mark. It could follow wrong combination of *a* and *n* for example, which would give M0 M1

M1: Correct use of logs to give *n* or $n-1>\frac{\log(k)}{\log 1.1}$ or $\log_{1.1} k$ after $(1.1)^{n-1}>k$ Allow equality for this mark

(3.63 is truncated value of $\frac{40}{11}$ and 3.64 is rounded value – allow either of these if used in place of fraction)

A1: 2030 is required . If inequalities are used and errors are seen, then this mark is A0 (even for 2030)

(Trial and improvement or listing can have full marks for the correct answer, need to see both 14th and 15th term – otherwise zero)

Special case: If *n* is used instead of $n-1$ and they reach 2029 then mark profile is likely to be M0 M1 M1 A0 **unless they recover to the correct answer when full marks may be earned**

If an equals sign is used throughout and then correct answer is obtained allow 4/4

Special case: Uses Sum formula – Can earn M0 M0 M1 A1 for "correct work"

Uses $S=\frac{275000(1.1^n-1)}{1.1-1}>1000000$ (M0) $1.1^n>1+\frac{1000000}{2750000}$ (M0) $n>\frac{\log(15/11)}{\log 1.1}$ (M1) $n>3.254....$ so 2019 (A1)

Using this method with errors can earn M0M0M1A0 for proceeding from $1.1^n>k$ with $k>0$ to $n>\frac{\log(k)}{\log 1.1}$

(c) **M1**: Correct *a* and *r* but *n* may be wrong

A1: Correct use of formula with $n = 11$

A1: awrt £5 096 100 (again – this answer implies all 3 marks)

Or M1: adds 11 terms (mostly correct)

A1: lists 11 correct terms £275000 + £302500 + £332750 + £366025 + £402627.5 + £442890.25 + £487179.275 + £535897.2025 + £589486.9228 + £648435.615 + £713279.1765

A1: correct answer = awrt £5096100 (this implies two previous marks)

Leave blank

13. The curve C has equation

$$y = 3x^2 - 4x + 2$$

The line l_1 is the normal to the curve C at the point $P(1, 1)$

(a) Show that l_1 has equation

$$x + 2y - 3 = 0$$

(5)

The line l_1 meets curve C again at the point Q.

(b) By solving simultaneous equations, determine the coordinates of the point Q.

(4)

Another line l_2 has equation $kx + 2y - 3 = 0$, where k is a constant.

(c) Show that the line l_2 meets the curve C once only when

$$k^2 - 16k + 40 = 0$$

(4)

(d) Find the two exact values of k for which l_2 is a tangent to C.

(2)

Question Number	Scheme	Marks
13.	$y = 3x^2 - 4x + 2$	
(a)	$\frac{dy}{dx} = 6x - 4 + \{0\}$	M1A1
	At (1, 1) gradient of curve is 2 and so gradient of normal is $-\frac{1}{2}$	M1
	$\therefore (y-1) = -\frac{1}{2}(x-1)$ and so $x + 2y - 3 = 0$*	M1 A1*
		[5]
(b)	Eliminate x or y to give $2(3x^2 - 4x + 2) + x - 3 = 0$ or $y = 3(3-2y)^2 - 4(3-2y) + 2$	M1
	Solve three term quadratic e.g $6x^2 - 7x + 1 = 0$ or $12y^2 - 29y + 17 = 0$ to give $x =$ or $y =$	M1
	$x = \frac{1}{6}$ **or** $y = 1\frac{5}{12}$	A1
	Both $x = \frac{1}{6}$ and $y = 1\frac{5}{12}$ i.e. $(\frac{1}{6}, 1\frac{5}{12})$ *or* (0.17,1.42) { Ignore (1 , 1) listed as well }	A1
		[4]
(c)	When this line meets the curve $2(3x^2 - 4x + 2) + kx - 3 = 0$	M1
	So $6x^2 + (k-8)x + 1 = 0$	dM1
	Uses condition for equal roots "$b^2 = 4ac$" on their three term quadratic to get expression in k	ddM1
	So obtain $(k-8)^2 = 24$ i.e. $k^2 - 16k + 40 = 0$ *	A1 *
	If they use **gradient of tangent** to do part (c) see the end of the notes below*.	**[4]**
(d)	Solve the given quadratic or their quadratic by formula or completion of the square to give	M1A1
	$k = 8 \pm \sqrt{24}$ or $8 \pm 2\sqrt{6}$ or $\frac{16 \pm \sqrt{96}}{2}$	**[2]**
		15 marks
	Notes	

(a) **M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once
A1: Both terms correct
M1: Substitutes x = 1 into their derivative and uses perpendicular property
M1: Correct method for Linear equation, using (1,1) and their changed gradient
A1: Should conclude with printed answer (this answer is given in the question)
(b) M1: May make sign slips in their algebra; {e.g. substitute 3 + 2y }– does not need to be simplified so isw.
But putting $3(3-2y)^2 - 4(3-2y) + 2 = 0$ instead of = y is M0
M1: Solve three term quadratic to give one of the two variables
A1: One Correct coordinate – accept any equivalent
A1: Both correct – any equivalent form. Allow decimals if correct awrt (0.17, 1.42) (ignore (1,1) given as well)
(c) M1: Eliminate y (condone small copying errors)
dM1: Collect into 3 term quadratic in x or identifies "a", " b" and " c" clearly (may be implied by later work).
ddM1: Uses condition "$b^2 = 4ac$" on quadratic in x (dependent on both previous M marks)
NB **M0** for $b^2 > 4ac$ or $b^2 \geq 4ac$ or $b^2 < 4ac$ or $b^2 \leq 4ac$
A1: Need $(k-8)^2 = 24$ or equivalent before stating printed answer
*Alternative method for part (c)
M1: Use gradient of line = gradient of curve so "$6x - 4$" = "$-\frac{k}{2}$"
M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use line equation to get y = $\frac{3}{2} - \frac{1}{3}k + \frac{k^2}{24}$ (these equations do not need to be simplified)
M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use curve equation to get y = $\frac{2}{3} + \frac{k^2}{48}$ (these equations do not need to be simplified)
A1: Puts two correct expressions for y equal and obtains printed answer without error.
(d) M1: Solve by formula or completion of the square to give k = (Attempt at factorization is M0)
A1: Correct answer – should be one of the forms given in the main scheme or equivalent exact form
Answers only with no working 2 marks (exact and correct) or 0 marks (approximate or wrong)

Leave blank

14. *In this question, solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve, for $0 \leqslant x < 360°$,

$$3\sin x + 7\cos x = 0$$

Give each solution, in degrees, to one decimal place.

(4)

(ii) Solve, for $0 \leqslant \theta < 2\pi$,

$$10\cos^2\theta + \cos\theta = 11\sin^2\theta - 9$$

Give each solution, in radians, to 3 significant figures.

(6)

Question Number	Scheme	Marks
14. (i)	Way 1: Use $\frac{\sin x}{\cos x} = \tan x$ to give $\tan x =$ Way2: complete method to find sinx = or cosx =	M1
	$\tan x = -\frac{7}{3}$ or $\sin x = \pm\frac{7}{\sqrt{58}}$ or $\cos x = \pm\frac{3}{\sqrt{58}}$	A1
	So x = 113.2, 293.2	M1 A1 **[4]**
(ii)	$10\cos^2\theta + \cos\theta = 11(1-\cos^2\theta) - 9$	M1
	Solves their three term quadratic "$21\cos^2\theta + \cos\theta - 2 = 0$" to give $\cos\theta = ...$	M1
	So $(\cos\theta =)\ -\frac{1}{3}$ or $\frac{2}{7}$	A1
	$\theta = 1.91,\ 4.37,\ 1.28$ or 5.00 (allow 5 instead of 5.00)	M1 A1 A1 **[6]**
		10 marks
	Notes	

(i) M1: (Way 1) Attempts to use $\frac{\sin x}{\cos x} = \tan x$ (there may be a sign error or may omit x and write tan =)

(Way 2) $3\sin x = -7\cos x$ so $9\sin^2 x = 49\cos^2 x$ and uses $\sin^2 x + \cos^2 x = 1$ to find sin$x$ = or cos$x$ =

A1: must be $\tan x = -\frac{7}{3}$ (way 1) or allow $\sin x = \pm\frac{7}{\sqrt{58}}$ or $\cos x = \pm\frac{3}{\sqrt{58}}$ (way 2). Ignore cos$x$ = 0 as extra answer.

M1: One correct angle in degrees in range – so need either 113.2 or 293.2 in most cases

But If they had $\tan x = -\frac{3}{7}$, then obtaining 156.8 or 336.8 is equivalent work and gains M1

If however they had $\tan x = +\frac{7}{3}$, then obtaining an answer in the range is not equivalent work – so is M0

A1: These two answers - accept awrt 113.2 and 293.2 Extra answers in range – lose this mark
Working in radians gives a maximum of M1A1M0A0

(ii) M1: Replaces $\sin^2\theta$ by $(1-\cos^2\theta)$

M1: Collects terms and solves their three term quadratic by usual methods (see notes)
A1: Both correct answers needed , but isw if one then rejected. Allow awrt -0.333 and 0.286
M1 Uses inverse cosine to obtain at least two correct answers for their values of cosine (check with calculator if they have followed wrong values)
A1: Any two completely correct answers (allow awrt)
A1: All four correct (awrt) Allow 0.608π, 1.39 π, 0.408 π, or 1.59 π
Extra answers outside range – ignore **Extra answers in the range** – lose final mark. Inaccurate answers to 3sf lose final A mark
Answers in degrees lose final two marks
So two of awrt 73, 287, 109 (or 109.5), 251 (or 250.5) would earn M1A0A0

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15.

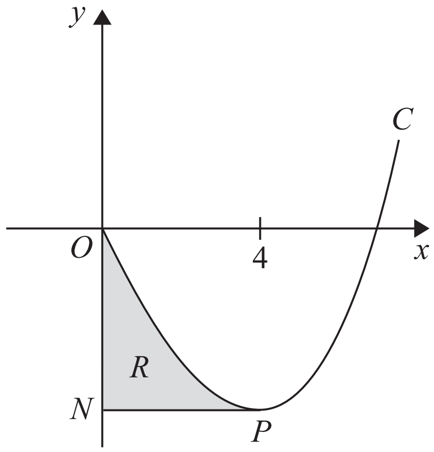

Figure 5

Figure 5 shows a sketch of part of the curve C with equation

$$y = x^3 + 10x^{\frac{3}{2}} + kx, \qquad x \geqslant 0$$

where k is a constant.

(a) Find $\dfrac{dy}{dx}$

(2)

The point P on the curve C is a minimum turning point.
Given that the x coordinate of P is 4

(b) show that $k = -78$

(2)

The line through P parallel to the x-axis cuts the y-axis at the point N.

The finite region R, shown shaded in Figure 5, is bounded by C, the y-axis and PN.

(c) Use integration to find the area of R.

(7)

Question Number	Scheme	Marks
15.	$y = x^3 + 10x^{\frac{3}{2}} + kx$	
(a)	$\frac{dy}{dx} = 3x^2 + 10 \times \frac{3}{2}x^{\frac{1}{2}} + k$	M1 A1 [2]
(b)	Substitutes $x = 4$ and $\frac{dy}{dx} = 0$ to give $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0 \quad \Rightarrow k = -78$ *	M1 A1* **[2]**
(c)	When $x = 4$, $y = -168$ (see this stated – or see rectangle has height 168)	B1
	$\int x^3 + 10x^{\frac{3}{2}} - 78x\ (+168)dx = \frac{1}{4}x^4 + \frac{10}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{78}{2}x^2 (+168x + c)$	M1 A1
	Use limits 0 and 4 to give ±432 or if 168x included to give ±240	dB1
	Rectangle area is 4 × "168" (= 672) or see 168x in integrated answer with limits	M1
	So R has area "672 – 432" or see +168 in original integrand	M1
	= 240	A1 **[7]**
		11 marks
	Notes	
(a)	**M1:** Fractional power dealt with correctly so becomes $\frac{3}{2}x^{\frac{1}{2}}$ (may be implied by simplification to 15) **A1:** All terms correct, may not be simplified	
(b)	**M1: Substitutes** $x = 4$ and $\frac{dy}{dx} = 0$ Must see $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0$ or $48 + 30 + k = 0$ ***A1:** This is a printed answer so all must be correct in the working and conclusion k = -78 is needed.	
(c)	**B1:** Substitute into y = to find y (This may appear anywhere in the answer) **M1:** Attempt to integrate so at least one power increases **A1:** Accept unsimplified correct answer and allow with or without their +168x, or even with their -168x **dB1**: Use limit 4 to give 432 but may be implied by later answer 240- needs to follow M1A1 for integration **M1:** Calculates rectangle area (may be by integration). Must be rectangle and not triangle area **M1:** Subtracts (either way round) numerical areas – should be (+) – (+) or (-) - (-) (subtraction may be in their original integral but penalize wrong sign here eg -168x instead of +168x) (Again use of triangle is M0) **A1:** 240 only (Can recover from -240 to 240) Common error: If 168x (instead of 168) is integrated this may only gain a maximum of B1 M1 A1 dB1 (for seeing 432 calculated if integrals are separated) M0 M0 A0 4/7	