Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Core Mathematics C12

Advanced Subsidiary

Monday 13 January 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Find the first 3 terms in ascending powers of x of

$$\left(2 - \frac{x}{2}\right)^6$$

giving each term in its simplest form.

(4)

Question Number	Scheme	Marks
1.	$\left(2-\frac{x}{2}\right)^6 = 2^6 + \binom{6}{1}2^5.\left(-\frac{x}{2}\right)+\binom{6}{2}2^4.\left(\frac{-x}{2}\right)^2+...$	M1
	$= 64, -96x, +60x^2 + ...$	B1, A1, A1
		[4]
	Special case $= 64, -192\left(\frac{x}{2}\right), +240\left(\frac{x}{2}\right)^2 + ..$ This is correct but unsimplified M1B1A1A0	
		4 marks
Alternative method	$[2^6]\left(1-\frac{x}{4}\right)^6 = [2^6]\left(1+\binom{6}{1}\left(-\frac{x}{4}\right)+\binom{6}{2}\left(\frac{-x}{4}\right)^2+...\right)$	M1
	$= 64, -96x, +60x^2 + ...$	B1, A1, A1
	Notes	
	M1: The **method** mark is awarded for an attempt at Binomial to get the second **and/or** third term – need **correct** binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or sign or bracket errors. Accept any notation for ${}^6C_1$ and ${}^6C_2$, e.g. $\binom{6}{1}$ and $\binom{6}{2}$ (unsimplified) or 6 and 15 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including x is correct. **B1:** must be simplified to 64 (writing just 2^6 is **B0).** This must be the only constant term (do not isw here) **A1:** is cao and is for **–**96 x. The x is required for this mark. Allow +(-96x) **A1:** is cao and is for $60x^2$ (can follow omission of negative sign in working) Any extra terms in higher powers of x should be ignored **Isw** if this is followed by $=16, -24x, +15x^2 + ...$ Allow terms separated by commas and given as list Alternative Method M1: Does not require power of 2 to be accurate B1: If answer is left as $64\left(1+\binom{6}{1}\left(-\frac{x}{4}\right)+\binom{6}{2}\left(\frac{-x}{4}\right)^2+...\right)$ Allow M1 B1 A0 A0	

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2.

$$f(x) = \frac{8}{x^2} - 4\sqrt{x} + 3x - 1, \qquad x > 0$$

Giving your answers in their simplest form, find

(a) $f'(x)$

(3)

(b) $\int f(x)\,dx$

(4)

Question Number	Scheme	Marks
2.(a)	$\text{f}'(x) = -16x^{-3} - 2x^{-\frac{1}{2}} + 3 \quad or \quad \text{f}'(x) = -\frac{16}{x^3} - \frac{2}{\sqrt{x}} + 3$	M1 A1 A1
		[3]
(b)	$\int \text{f}(x)\text{d}x = -8x^{-1} - \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^2}{2} - x + (c)$	M1 A1 A1
	$\int \text{f}(x)\text{d}x = -8x^{-1} - \frac{8x^{\frac{3}{2}}}{3} + \frac{3x^2}{2} - x + c \quad or \quad \frac{-8}{x} - \frac{8x\sqrt{x}}{3} + \frac{3x^2}{2} - x + c$	A1
		[4]
		7 marks
	Notes	
(a)	**M1:** Attempt to differentiate – power reduced $x^n \to x^{n-1}$ or $3x$ becomes 3 **A1:** two correct terms (of the three shown). They may be unsimplified **A1**: fully correct and **simplified** then isw (any equivalent simplified form acceptable)	
(b)	**M1:** Attempt to integrate original f(x)– one power increased $x^n \to x^{n+1}$ **A1:** Two of the four terms in x correct unsimplified – (ignore lack of constant here) **A1**: Three terms correct unsimplified – (ignore lack of constant here) A1: All correct **simplified** with constant – allow **-**1x for –x N.B Integrating answer to part (a) is M0	

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3.

$$f(x) = 10x^3 + 27x^2 - 13x - 12$$

(a) Find the remainder when $f(x)$ is divided by

(i) $x - 2$

(ii) $x + 3$

(3)

(b) Hence factorise $f(x)$ completely.

(4)

Question Number	Scheme	Marks
3.	f $(x) = 10x^3 + 27x^2 - 13x - 12$	
(a)	Attempts $f(\pm 2)$ or $f(\pm 3)$ Or Uses long division as far as a remainder	M1
	(i) {f(2) =} 150	A1
	(ii) {f(-3)=} 0	A1
		[3]
(b)	$10x^3 + 27x^2 - 13x - 12 = (x+3)(10x^2 + ...$	M1
	$10x^3 + 27x^2 - 13x - 12 = (x+3)(10x^2 - 3x - 4)$	A1
	$"(10x^2 - 3x - 4)" = (ax+b)(cx+d)$ where $\lvert ac \rvert = 10$ and $\lvert bd \rvert = 4$	dM1
	$= (x+3)(5x-4)(2x+1)$	A1
		[4]
		7 marks
	Notes	
(a)	**M1**: As on scheme **A1:** for 150**, next A1**: for 0 Both cao (If division has been used it should be clear that they know these values are the remainders)	
(b)	**M1**: Recognises (x+3) is factor and obtains correct first term of quadratic factor by division or any other method **A1**: Correct quadratic [may have been done in part (a)] **dM1**: Attempt to factorise **their quadratic** **A1:** Need all three factors together, accept any **correct** equivalent e.g. $10(x+3)(x-\frac{4}{5})(x+\frac{1}{2})$ If the three roots of f(x) = 0 are given after correct factorisation then isw Special case. Just writes down the three factors $= (x+3)(5x-4)(2x+1)$ with no working : Full marks Allow trial and error or use of calculator for completely correct answer – so 4 marks or 0 marks if "hence" is not used.	

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4. Answer this question without the use of a calculator and show all your working.

(i) Show that

$$\frac{4}{2\sqrt{2}-\sqrt{6}} = 2\sqrt{2}(2+\sqrt{3})$$

(4)

(ii) Show that

$$\sqrt{27}+\sqrt{21}\times\sqrt{7}-\frac{6}{\sqrt{3}} = 8\sqrt{3}$$

(3)

Question Number	Scheme	Marks
4. (i)	$\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{2}+\sqrt{6}\right)}$	M1
	$\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{2}+\sqrt{6}\right)=8-6=2$	B1
	$\sqrt{6}=\sqrt{2}\sqrt{3}$ used in numerator **-** may be implied by a **correct** factorisation of numerator	B1
	Concludes $\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{2}=2\sqrt{2}\left(2+\sqrt{3}\right)$ *	A1 *
		[4]
(ii)	1st two terms $\sqrt{27}=3\sqrt{3}$ **and** $\sqrt{21}\times\sqrt{7}=7\sqrt{3}$	B1
	3rd term See $2\sqrt{3}$ or $\dfrac{6\sqrt{3}}{3}$	B1
	$3\sqrt{3}+7\sqrt{3}-2\sqrt{3}=8\sqrt{3}$ or $3\sqrt{3}+7\sqrt{3}-\dfrac{6\sqrt{3}}{3}=8\sqrt{3}$ *	B1 *
		[3]
Alternative for (i)	Assume result and multiply both sides by $\left(2\sqrt{2}-\sqrt{6}\right)$	M1
	$\left(2\sqrt{2}-\sqrt{6}\right)\left(4\sqrt{2}+2\sqrt{6}\right)=16-12=4$	B1 B1
	So LHS = RHS and result is true	A1
		[4]
Alternative for (ii)	$\dfrac{\sqrt{81}+\sqrt{21\times7\times3}-6}{\sqrt{3}}$ *Or* $\sqrt{81}+\sqrt{21\times7\times3}-6=8\sqrt{3}\sqrt{3}$	B1
	$\dfrac{9+21-6}{\sqrt{3}}$ $9+21-6=$	B1
	$\dfrac{24}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=8\sqrt{3}$ $9+21-6=24$ **so equation is true**	B1
		[3] **(7 marks)**
	Notes	

(i) M1: Multiplies numerator and denominator by $\pm\left(2\sqrt{2}+\sqrt{6}\right)$

B1: correct treatment of denominator to give 2 (may be implied by answer obtained with no errors seen)

B1: Splits $\sqrt{6}=\sqrt{2}\sqrt{3}$ - may be implied , but **B0 for** $2\sqrt{6}=2\sqrt{2}(2\sqrt{3}...)$ **A1** cao reaches result and no errors should be seen

N.B. $\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{2}=2\sqrt{2}\left(2+\sqrt{3}\right)$ may be awarded B1 A1 as there is an implication that $\sqrt{6}=\sqrt{2}\sqrt{3}$

(ii) B1: expresses both of first two terms as multiple of root 3 correctly

B1: rationalises denominator in second term -may not see working

B1: has used $3\sqrt{3}+7\sqrt{3}-2\sqrt{3}=8\sqrt{3}$ N.B. $3\sqrt{3}+7\sqrt{3}-\dfrac{6}{\sqrt{3}}=8\sqrt{3}$ is B1B0B0

(i) Alternative	**M1:** Assume result and multiply both sides by $\left(2\sqrt{2}-\sqrt{6}\right)$ **2nd B1**: Uses $\sqrt{2}\sqrt{3}=\sqrt{6}$ **1st B1**: Multiplies out these two brackets to give 4 **A1**: conclusion
(ii) Alternatives	**B1:** Uses common denominator **or** multiplies **both sides** by root 3 and obtains correct unsimplified equation **B1: LHS** numerator correctly simplified or just see 9 + 21 – 6 B1: In the first alternative must see multiplication of numerator and denominator by $\sqrt{3}$ to give $8\sqrt{3}$ In the second need statement LHS = RHS and so true

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5. A sequence is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}, \quad n \geqslant 1$$

Find the exact values of

(a) u_2, u_3 and u_4 **(3)**

(b) u_{61} **(1)**

(c) $\sum_{i=1}^{99} u_i$ **(3)**

Question Number	Scheme	Marks
5.		
(a)	$u_2 = 2 - \frac{4}{3} = \frac{2}{3},\ u_3 = 2 - \frac{4}{\frac{2}{3}} = -4,\ u_4 = 2 - \frac{4}{-4} = 3$	M1 A1 A1
		[3]
(b)	$u_{61} = 3.$	B1
		[1]
(c)	$\sum_{i=1}^{99} u_i = (3 + \frac{2}{3} - 4) + (3 + \frac{2}{3} - 4) + (3 + \frac{2}{3} - 4) + ...$	M1
	$\sum_{i=1}^{99} u_i = 33 \times (... + ... + ...)\quad ,\quad = -11$	A1, A1
		[3]
(c)	**Alternative method** for part (c) Adds $n \times "3" + n \times "-4" + n \times "\frac{2}{3}"$	M1
	Uses n = 33	A1
	-11	A1
		[3]
		7 marks
	Notes	
(a)	**M1:** Attempt to use formula correctly (implied by first term correct, or given as 0.67, or third term following through from their second etc) **A1**: two correct answers **A1**: 3 correct answers (allow 0.6 recurring but not 0.667) Look for the values. Ignore the u_r label	
(b)	**B1**: cao (NB Use of AP is B0)	
(c)	**M1:** Uses sum of at least 3 terms found from part (a)) (may be implied by correct answer). Attempt to sum an AP here is M0. **A1**: obtains $33 \times$(sum of three adjacent terms) or $11 \times$(sum of nine adjacent terms) **A1**: - 11 cao (-11 implies both A marks) N.B. Use of n = 99 is M1A0A0	

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6. Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25$$

$$\log_4 a - \log_4 b = 3$$

Show each step of your working, giving exact values for a and b.

(6)

Question Number	Scheme	Marks
6.	$\log_4 \frac{a}{b} = 3$ or $\log_4 a + \log_4 b = \log_4 25$ or $\log_4 \frac{a}{\frac{25}{a}} = 3$ or $\log_4 \frac{\frac{25}{b}}{b} = 3$ (If this is preceded by wrong algebra (e.g. b = 25 –a) M1 can still be given if their b is used	M1
	$\log_4 64 = 3$ or $4^3 = 64$ (may be implied by the use of 64) or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ (these latter two statements will be implied by correct answers)	B1
	Correct algebraic elimination of a variable to obtain expression in *a* or *b* without logs	dM1
	$a = 40$ **or** $b = \frac{5}{8}$	A1
	Substitutes to give second variable or solves again from start	dM1
	$a = 40$ **and** $b = \frac{5}{8}$ and no other answers.	A1
		[6]
		6 marks
	Notes	
	M1: Uses addition or subtraction law correctly for logs (N.B. $\log_4 a + \log_4 b = 25$ **is M0)** **B1**: See number 64 used (independent of M mark) or or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ **dM1**: Dependent on first M mark. Eliminates *a* or *b* (with appropriate algebra) and eliminates logs **A1**: Either *a* or *b* correct **dM1**: Dependent on first M mark . Attempts to find second variable **A1**: Both *a* and *b* correct – allow *b* = 0.625 If *a* = -40 and *b* = -5/8 are **also** given as answers **lose the last A mark**. **NB** Log *a* + log *b* = 2.3219..will not yield exact answers If they round their answers to 40 and 0.625 after decimal work, do not give final A mark. NB: Some will change the base of the log and use loga – log b = 3log4	

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7. (a) Show that

$$12\sin^2 x - \cos x - 11 = 0$$

may be expressed in the form

$$12\cos^2 x + \cos x - 1 = 0$$

(1)

(b) Hence, using trigonometry, find all the solutions in the interval $0 \leqslant x \leqslant 360°$ of

$$12\sin^2 x - \cos x - 11 = 0$$

Give each solution, in degrees, to 1 decimal place.

(4)

Question Number	Scheme	Marks
7. (a)	$12\sin^2 x - \cos x - 11 = 0$	
	$12(1-\cos^2 x) - \cos x - 11 = 0$ and so $12\cos^2 x + \cos x - 1 = 0$ *	B1 * [1]
(b)	Solve quadratic to obtain (cos x) $= \frac{1}{4}$ or $-\frac{1}{3}$	M1 A1
	x = 75.5, 109.5, 250.5, 284.5	M1 A1cao
	Answers in radians (see notes)	**[4]**
		5 marks
	Notes	
(a)	**B1:** Replaces $\sin^2 x$ by $(1-\cos^2 x)$ - or replace 11 by $11(\sin^2 x + \cos^2 x)$ and no errors seen to give printed answer including = 0	
(b)	**M1**: Solving the correct quadratic equation (allow sign errors), by the usual methods (see notes) – implied by correct answers **A1**: Both answers needed – allow 0.25 and awrt – 0.33 **M1** Uses inverse cosine to obtain two correct values for x for their values of cos$x$ e.g. (75.5 and 109.4 or 109.5) or (75.5 and 284.5) or (109.5 and 250.5) – allow truncated answers or awrt here. **A1:** All four correct – allow awrt. Ignore extra answers outside range but lose last A mark for extra answers inside range Answers in radians are 1.3, 5.0, 1.9 and 4.4 Allow M1A0 for two or more correct asnwers	

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8. Find the range of values of k for which the quadratic equation

$$kx^2 + 8x + 2(k + 7) = 0$$

has no real roots.

(7)

Question Number	Scheme	Marks
8.	$kx^2 + 8x + 2(k+7) = 0$	
	Uses $b^2 - 4ac$ with $a = k$, $b = 8$ and attempt at $c = 2(k + 7)$	M1
	$b^2 - 4ac = 64 - 56k - 8k^2$ *or* $64 = 56k + 8k^2$ *o.e.*	A1
	Attempts to solve "$k^2 + 7k - 8 = 0$" to give $k =$	dM1
	$\Rightarrow$ Critical values, $k = 1, -8$.	A1cso
	Uses $b^2 - 4ac < 0$ or $b^2 < 4ac$ or $4ac - b^2 > 0$	M1
	$k^2 + 7k - 8 > 0$ gives $k > 1$ (or) $k < -8$	M1 A1
		[7]
		7 marks
	Notes	
	M1: Attempts $b^2 - 4ac$ for $a = k$, $b = 8$ and $c = 2(k+7)$ or attempt at c from quadratic = 0 (may omit bracket or make sign slip or lose the 2, so $2k + 7$ or $k + 7$ for example) or uses quadratic formula to solve equation or uses on two sides of an equation or inequation **A1: Correct three term quadratic** expression for $b^2 - 4ac$ - (may be under root sign) **dM1:** Uses factorisation, formula, or completion of square method to find two values for k, or finds two **correct** answers with no obvious method for **their** three term quadratic **A1:** Obtains 1 and -8 **M1:** states $b^2 - 4ac < 0$ *or* $b^2 < 4ac$ anywhere (may be implied by the following work) **M1: Chooses outside region (** $k <$ Their Lower Limit $k >$ Their Upper Limit) for appropriate 3 term quadratic inequality . Do not award simply for diagram or table. **A1:** $k > 1$ or $k < -8$ - allow anything which clearly indicates these regions e.g. $(-\infty, -8)$ or $(1, \infty)$ $k > 1$, $k < -8$ is A1 but $k > 1$ and $k < -8$ is A0 but $x > 1$, $x < -8$ is A0 (only lose 1 mark for using x instead of k) and $k \geq 1$ (or) $k \leq -8$ is A0 Also $1 < k < -8$ is M1 A0 N.B. Lack of working: If there is no mention of $b^2 - 4ac < 0$ or $b^2 < 4ac$ then just the correct answer $k > 1$, $k < -8$ can imply the last M1M1A1 $k \geq 1$, $k \leq -8$ can imply M0M1A0 $k > 1$, $k < -8$ can imply M1M1A0 Anything else needs to apply scheme	

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9. In the first month after opening, a mobile phone shop sold 300 phones. A model for future sales assumes that the number of phones sold will increase by 5% per month, so that 300×1.05 will be sold in the second month, 300×1.05^2 in the third month, and so on.

Using this model, calculate

(a) the number of phones sold in the 24th month,

(2)

(b) the total number of phones sold over the whole 24 months.

(2)

This model predicts that, in the Nth month, the number of phones sold in that month exceeds 3000 for the first time.

(c) Find the value of N.

(3)

Question Number	Scheme	Marks
9.(a)	Uses $300 \times (1.05)^{23}$	M1
	Obtains 921 or 922 or 920	A1
		[2]
(b)	Uses $S = \frac{300(1.05^{24} - 1)}{1.05 - 1}$ Must have correct *r* and *n* but can use their *a* (e.g. 315)	M1
	13351 (accept awrt 13400)	A1
		[2]
(c)	Uses $300(1.05)^{n-1} > 3000$ Or $300(1.05)^{n-1} = 3000$	M1
	$(n-1)\log 1.05 > \log 10$ Or $(n-1)\log 1.05 = \log 10$ Or $(n-1) = \log_{1.05} 10$ Or correct equivalent log work ft	M1
	$n > 48.19$ $N = 49$	A1
		[3]
		7 marks
	Notes	
(a)	M1: for correct statement of formula with correct *a, r* and *n* A1: cao (This answer implies the M1)	
(b)	M1: Correct formula with r = 1.05 and *n* = 24 ft their *a* (If they list all the terms – correct answer implies method mark) A1: answers which round to 13400 are acceptable	
(c)	M1: Correct inequality or uses equality and interprets correctly later (ft their a) M1: Correct algebra then correct use of logs on their previous line (may follow use of =, or use of *n* instead of *n* -1) Can get M0M1A0 A1: need to see 49 or 49th month **Special case**: Uses sum formula: If they reach $(1.05)^n > 1\frac{1}{2}$ and then use logs correctly to give $n\log(1.05) > \log 1\frac{1}{2}$ then give M0M1A0 If trial and error is used then the correct answer implies the method. So 49 is M1M1A1 and 48 scores M1M0A0. Similar marks follow answer only with no working.	

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10. The curve C has equation $y = \cos\left(x - \frac{\pi}{3}\right)$, $0 \leqslant x \leqslant 2\pi$

(a) In the space below, sketch the curve C.

(2)

(b) Write down the exact coordinates of the points at which C meets the coordinate axes.

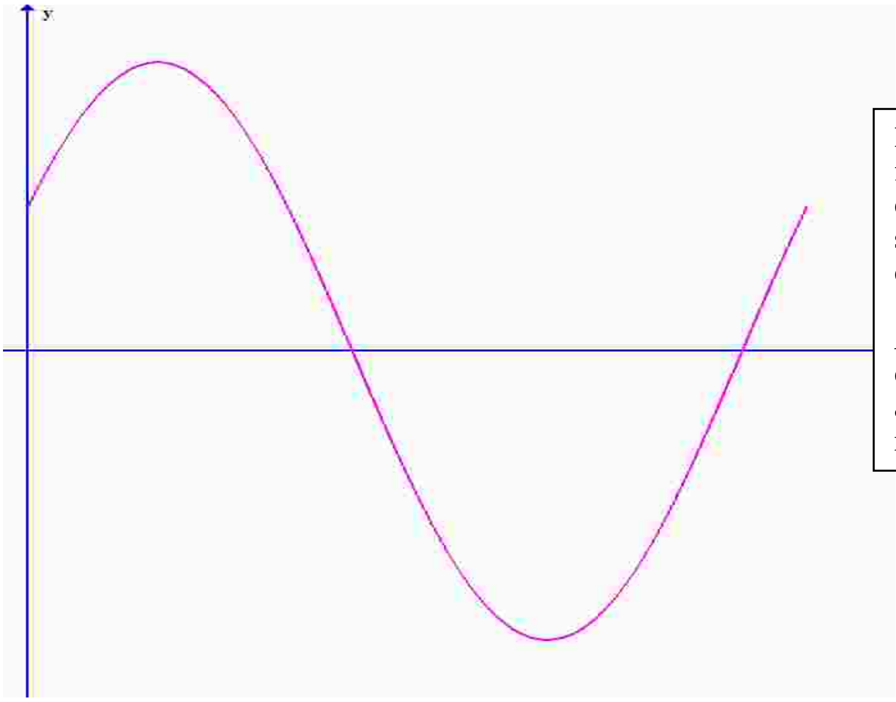
(3)

(c) Solve, for x in the interval $0 \leqslant x \leqslant 2\pi$,

$$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answers in the form $k\pi$, where k is a rational number.

(4)

Question Number	Scheme	Marks
10. (a)	 Method mark for harmonic curve i.e. any sine or cosine curve. Accuracy for correct section and position relative to the	M1 A1 **[2]**
(b)	(0 , ½) ; $\left(\frac{5\pi}{6}, 0\right)$ or (150, 0) and $\left(\frac{11\pi}{6}, 0\right)$ or (330,0)	B1; B1 B1 **[3]**
(c)	$\left(x-\frac{\pi}{3}\right)=\frac{\pi}{4}$ or $-\frac{\pi}{4}$ $x=\frac{7\pi}{12}$ or $\frac{\pi}{12}$	M1 M1 A1 A1 **[4]**
		9 marks
	Notes	

(a) M1: Could be part of a cycle, or several cycles but needs at least one max and one min
A1: Needs to be **only** one cycle. Needs to be positive y intercept and positive gradient at start and finish (not zero gradient). Needs to be solely $x \geq 0$ and to finish at the same y value as it started.

(b) Each answer is cao Need coordinates with zeroes (as given) unless points are indicated correctly on the graph e.g. ½ or (1/2 , 0) on the y axis may be given credit etc Allow degrees on x axis. Extra in range lose last B1. If answers are given in text and on diagram, text takes precedence.

(c) M1: Uses inverse cos correctly to obtain at least one correct answer (may be in degrees)
M1: Adds $\frac{\pi}{3}$ to their previous answer, which must have been in radians but may add 60 to answer in degrees
A1: one correct answer A1: Both answers correct
Extra answers in range lose final A1 Extra answers outside range isw

Special case: All answers given as decimals (b) B1 B0 B0 (c) M1 M1 A0 A0
All answers in degrees:
(b) 150 and 330 then (c) 15 and 105 (just lose final two A marks) so B1, B1 in (b) then M1M1A0A0

Leave blank

11. The first three terms of an arithmetic series are 60, $4p$ and $2p - 6$ respectively.

(a) Show that $p = 9$

(2)

(b) Find the value of the 20th term of this series.

(3)

(c) Prove that the sum of the first n terms of this series is given by the expression

$$12n\,(6 - n)$$

(3)

Question Number	Scheme	Marks
11. (a)	Uses $(2p-6)-4p = 4p-60$ or $4p=\frac{60+(2p-6)}{2}$ or 60+2(4*p*-60) =2*p*-6 or etc… or two correct equations with *d*	M1
	So *p* = 9 *	A1 * **[2]**
Alternative to (a)	Use *p*=9 to give 60, 36 and 12 and deduce *d* = -24 so conclude AP when *p* = 9	M1 A1 [2]
(b)	Uses *a* + 19*d* with *a* = 60	M1
	Finds *d* = 36 – 60 = -24	B1
	So obtains -396	A1 **[3]**
(c)	Uses $\frac{n}{2}(2\times 60+(n-1)d)$	M1
	Uses $\frac{n}{2}(2\times 60-24(n-1))$	A1
	= 12*n* (6-*n*) *	A1* **[3]**
		8 marks
	Notes	
(a)	M1**: Correct** equation to enable *p* to be found or two correct equations if *d* introduced and solving simultaneous equations to eliminate *d* and enable *p* to be found NB May add three terms and use sum formula giving e.g. $60+4p+2p-6=\frac{3}{2}(60+2p-6)$ A1: cso (Do not need intermediate step)	
(b)	M1: Correct formula with their value for *d* B1: *d* = -24 seen in (a) or (b) A1: -396 If all terms are found and added 60 + 36 + 12 + -12 + .. Need 20 terms for M1, need -24 implied by first 4 terms for B1 and correct answer for A1	
(c)	M1:Uses correct formula with their value for *d* A1: Correct value for *d* A1: given answer – must be no errors to award this mark Special case: Proves formula for sum of AP M1: Correct method of proof using their d A1: For *d* = -24 A1: given answer – must be no errors to award this mark	

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12.

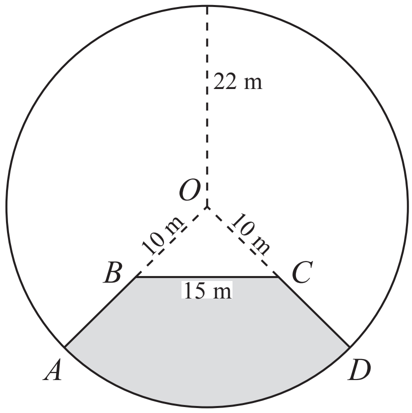

Diagram **NOT** drawn to scale

Figure 1

Figure 1 shows the plan for a pond and platform. The platform is shown shaded in the figure and is labelled *ABCD*.

The pond and platform together form a circle of radius 22 m with centre *O*.

OA and *OD* are radii of the circle. Point *B* lies on *OA* such that the length of *OB* is 10 m and point *C* lies on *OD* such that the length of *OC* is 10 m. The length of *BC* is 15 m.

The platform is bounded by the arc *AD* of the circle, and the straight lines *AB*, *BC* and *CD*.

Find

(a) the size of the angle *BOC*, giving your answer in radians to 3 decimal places, **(3)**

(b) the perimeter of the platform to 3 significant figures, **(4)**

(c) the area of the platform to 3 significant figures. **(4)**

Question Number	Scheme	Marks
12. (a)	$15^2 = 10^2 + 10^2 - 2\times 10\times 10\cos\angle BOC$	M1
	$\cos\angle BOC = \dfrac{10^2+10^2-15^2}{2\times 10\times 10}$ *or* $\dfrac{-25}{200}$ *or* -0.125	A1
	$\angle BOC = 1.696$ (N.B. 97.2 degrees is A0)	A1 **[3]**
(b)	Uses $s = 22\theta$ with their θ from part (a) not $-(2\pi-\theta)$	M1
	$r\theta = 22\times 1.696 = 37.3(15)$	A1
	Perimeter $= r\theta + 15 + x + x, = 39 + \textit{their arc length}$ [76.3 (m)]	M1 A1ft **[4]**
(c)	area of sector $= \frac{1}{2}(22)^2\theta$ -not $-(2\pi-\theta)$	B1
	area of triangle $= \frac{1}{2}(10)^2\sin\theta$	B1
	Area of paved area = $\frac{1}{2}(22)^2\theta - \frac{1}{2}(10)^2\sin\theta$ = 410.432 – 49.6 or $410.432 - \dfrac{75\sqrt{7}}{4}$ = 360.8 or awrt 361 (m^2)	M1 A1 **[4]**
		(11 marks)
	Notes	
(a)	M1: Uses cosine rule – must be correct or other correct trigonometry e.g. $2\times\theta$ where $\sin\theta = \dfrac{7.5}{10}$ A1: makes cos subject of formula correctly or uses $2\times\sin^{-1}\left(\dfrac{7.5}{10}\right)$ A1: accept awrt 1.696 (answer in degrees is A0). If answer is given as 1.70 (3sf) then A0 but remaining As are available (special case below)	
(b)	M1: Uses $s = 22\theta$ with their θ in radians, or correct formula for degrees if working in degrees A1: Accept awrt 37.3 (may be implied by their perimeter) M1: Adds arc length to 15 to two further equal lengths for Perimeter A1ft: Accept awrt 76.3 do not need metres ft on their arc length—so 39 + arc length	
(c)	B1: This formula **used** with their θ in radians or correct formula for degrees - allow miscopy of angle B1: Correct formula for area – may use half base times height M1: Subtracts correct triangle (two sides of length 10) from their sector A1: awrt 361 – do not need units Special case – uses 3 sf instead of 3 dp in part (a) Loses final A mark in part (a) but can have A marks in part (b) for 37.4 and 76.4 and can have A mark in part (c) for awrt 362	

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13. The curve C has equation

$$y = \frac{(x-3)(3x-25)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in a fully simplified form.
(3)

(b) Hence find the coordinates of the turning point on the curve C.
(4)

(c) Determine whether this turning point is a minimum or maximum, justifying your answer.
(2)

The point P, with x coordinate $2\frac{1}{2}$, lies on the curve C.

(d) Find the equation of the normal at P, in the form $ax + by + c = 0$, where a, b and c are integers.
(5)

Question Number	Scheme	Marks
13. (a)	So $y = 3x - 34 + \frac{75}{x}$	B1
	$\frac{dy}{dx} = 3 - 75x^{-2} + \{0\}$ $(x > 0)$ Accept $\frac{dy}{dx} = \frac{3x^2 - 75}{x^2}$ or equivalent	M1 A1
		[3]
(b)	Put $\frac{dy}{dx} = 3 - 75x^{-2} = 0$	M1
	$x = 5$	A1
	Substitute to give y = - 4	M1 A1
		[4]
(c)	Consider $\frac{d^2y}{dx^2} = 150x^{-3} > 0$	M1
	So minimum	A1
		[2]
(d)	When x = 2.5, y = 3.5	B1
	Also gradient of curve found by substituting 2.5 into their $\frac{dy}{dx}$ (= -9)	M1
	So gradient of normal is $-\frac{1}{m}$ $(= \frac{1}{9})$	dM1
	Either : $y - "3.5" = "\frac{1}{9}"(x - 2.5)$ **or**: $y = "\frac{1}{9}"x + c$ and $"3.5" = "\frac{1}{9}"(2.5) + c \Rightarrow c = "3\frac{2}{9}"$	dM1
	So $\underline{x - 9y + 29 = 0}$ or $\underline{9y - x - 29 = 0}$ or any multiple of these answers	A1
		[5]
		14 marks

	Notes
(a)	**B1:** any correct equivalent 3 or 4 term polynomial **M1:** Evidence of differentiation following attempt at division, or at multiplication by x^{-1}, so $x^n \rightarrow x^{n-1}$ at least once so $x^1 \rightarrow 1$ or x^0 or $x^{-1} \rightarrow x^{-2}$ not just $-34 \rightarrow 0$ **A1:** $3 - 75x^{-2}$ Both terms correct, and simplified. Allow even if 34 was incorrect. Do not need to include domain $x > 0$
(b)	**M1:** Puts $\frac{dy}{dx} = 0$ **A1:** Ignore extra answer x = -5 **M1:** Substitute into their y = to find y **A1:** Ignore extra answer -64
(c)	**M1:** Considers second derivative (by reducing by 1 a power of their $\frac{dy}{dx}$) and consider its sign, or considers gradient either side, or considers shape of curve **A1:** Has correct second derivative*, has positive value for x (may not be used) and has stated >0 **or equivalent** and concludes "minimum" * Allow even if 3 was incorrect in first derivative.
(d)	**B1:** cao **M1:** Substitutes 2.5 into their gradient function (may not get -9) **dM1:** Finds perpendicular gradient **dM1**: Equation of normal using their normal gradient , using x = 2.5 and their value for y. This depends on both previous method marks (Use of (5, -4) here is M0) **A1:** Must have = 0 and integer coefficients

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14.

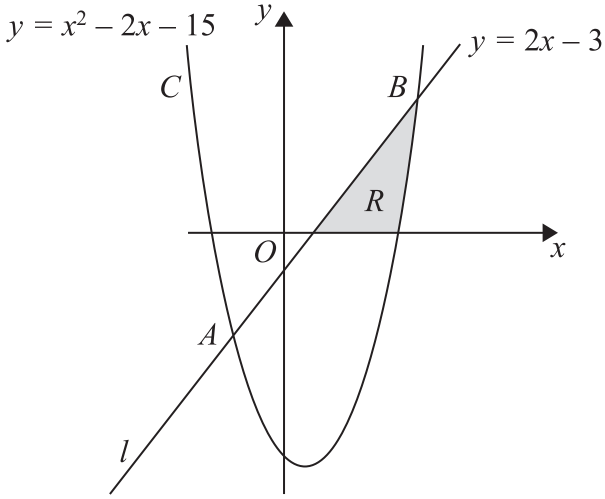

Diagram **NOT** drawn to scale

Figure 2

Figure 2 shows part of the line l with equation $y = 2x - 3$ and part of the curve C with equation $y = x^2 - 2x - 15$

The line l and the curve C intersect at the points A and B as shown.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

In Figure 2, the shaded region R is bounded by the line l, the curve C and the positive x-axis.

(b) Use integration to calculate an exact value for the area of R.

(7)

Question Number	Scheme	Marks
14. (a)	$2x-3=x^2-2x-15$ so $x^2-4x-12=0$	M1
	$x=6$ or $x=-2$	dM1 A1
	$y=9$ or $y=-7$	dM1 A1
		[5]
(b)	$\int x^2-2x-15\mathrm{d}x=\frac{1}{3}x^3-x^2-15x$	B1
	Line meets x-axis at $x=1\frac{1}{2}$ (may be implied by use in limits or in triangle area) and curve meets axis at x = 5. These numbers may appear on the diagram.	B1 B1
	Uses correct combination of correct areas. Area of region = Area of large triangle MINUS $[\frac{1}{3}x^3-x^2-15x]_5^6$	M1
	Area of large triangle = $\frac{1}{2}\times(6-1\frac{1}{2})\times 9$ (may use rectangle – trapezium)	dM1
	$=\frac{1}{2}\times(6-1\frac{1}{2})\times 9-[(\frac{1}{3}6^3-6^2-15\times 6)-(\frac{1}{3}(5)^3-(5)^2-15\times(5))]$	M1
	$=20.25-(-54-(-58\frac{1}{3}))=\frac{191}{12}=15\frac{11}{12}$	A1
		[7]
		(12 marks)
	First Alternative method using "line – curve" and adding small triangle	
	$\int -x^2+4x+12\mathrm{d}x=-\frac{x^3}{3}+2x^2+12x$ or $\int x^2-4x-12\mathrm{d}x=\frac{x^3}{3}-2x^2-12x$	B1
	Line meets x-axis at $x=1\frac{1}{2}$ and curve meets axis at x = 5	B1 B1
	Uses correct combination of correct areas. Area of region = Area of small triangle PLUS $[-\frac{1}{3}x^3+2x^2+12x]_5^6$	M1
	Area of small triangle = $\frac{1}{2}\times(5-1\frac{1}{2})\times 7$	dM1
	$\frac{1}{2}\times(5-1\frac{1}{2})\times 7+[(-\frac{1}{3}6^3+2\times 6^2+12\times 6)-(-\frac{1}{3}(5)^3+2\times(5)^2+12\times(5))]$	M1
	$=12.25+(72-(68\frac{1}{3}))=\frac{191}{12}=15\frac{11}{12}$	A1
		[7]
	Alternative method using "line – curve" (long method here and unlikely)	
	First three B marks as in First Alternative	B1 B1 B1
	Then $\int_{1\frac{1}{2}}^{6} -x^2+4x+12\mathrm{d}x \pm \int_{1\frac{1}{2}}^{5} x^2-2x-15\,\mathrm{d}x$	M1
	$\int_{1\frac{1}{2}}^{5} x^2-2x-15\,\mathrm{d}x$	dM1
	Uses limits correctly	M1
	$50\frac{5}{8}-34\frac{17}{24}=15\frac{11}{12}$	A1

	Notes for Question 14	
(a)	M1: Puts equations equal dM1 Solves quadratic to obtain x = A1: both answers correct dM1: finds y = A1: both correct	
(b)	B1: Correct integration of one of the quadratic expression (given in the mark scheme) to give one of the given cubic expression (ignore limits). Allow correct answer even if terms not collected nor simplified. Sign errors subtracting in alternative methods before integration gain B0 B1: Line intersection correct (see 1.5) B1: curve intersection correct (see 5) M1: Uses correct combination of correct areas (allow numerical slips) so (i)Area of triangle using their "6" – their "1.5" times their "9" MINUS area beneath curve between their 5 and their 6 (ii) Area of triangle using their "5" – their "1.5" times their "7" PLUS area between curves between their 5 and their 6 (iii) Subtracts area below axis from area between curves THEIR 1.5 must NOT BE ZERO! M1: Attempts second area (so area of a triangle **relevant** to the method- or integral of **the** linear function with relevant limits- or integral of original quadratic in second alternative method) M1: Uses their limits (even zero) correctly on any cubic expression (subtracting either way round) Can be given for wrong limits or for wrong areas. No evidence of substitution of limits is M0 A1: Final answer – not decimal – cso	

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15.

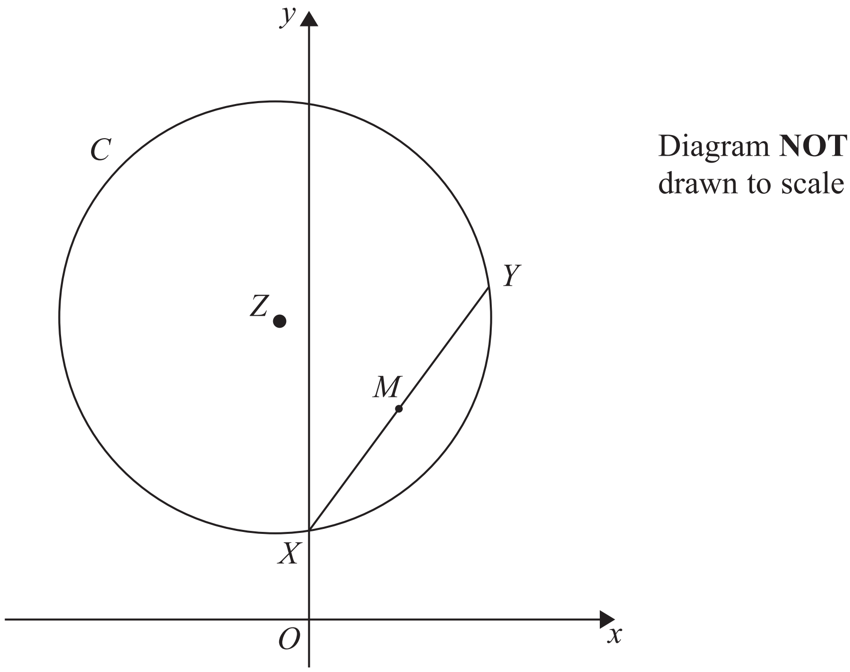

Figure 3

The points X and Y have coordinates (0, 3) and (6, 11) respectively. XY is a chord of a circle C with centre Z, as shown in Figure 3.

(a) Find the gradient of XY.

(2)

The point M is the midpoint of XY.

(b) Find an equation for the line which passes through Z and M.

(5)

Given that the y coordinate of Z is 10,

(c) find the x coordinate of Z,

(2)

(d) find the equation of the circle C, giving your answer in the form

$$x^2 + y^2 + ax + by + c = 0$$

where a, b and c are constants.

(5)

Question Number	Scheme	Marks
15. (a)	gradient $= \frac{11-3}{6-0}, \; = \frac{4}{3}$	M1 A1 **[2]**
(b)	Mid-point of XY = (3, 7)	M1 A1
	ZM has gradient $-\frac{1}{m}$ $\left(= -\frac{3}{4}\right)$	B1ft
	Either : $y - "7" = "-\frac{3}{4}"(x - "3")$ **or**: $y = "-\frac{3}{4}"x + c$ and $"7" = "-\frac{3}{4}"("3") + c \Rightarrow c = "9\frac{1}{4}"$	M1
	$4y + 3x - 37 = 0$ or $y - 7 = -\frac{3}{4}(x - 3)$ Or $y = -\frac{3}{4}x + 9\frac{1}{4}$	A1 **[5]**
(c)	Substitute y = 10 into their line equation to give x =	M1
	x = -1	A1 **[2]**
(d)	$(r^2) = (-1-0)^2 + (10-3)^2$ or $(r^2) = (-1-6)^2 + (10-11)^2$	M1
	$r^2 = 50$	A1
	$"50" = (x \pm "(-1)")^2 + (y \pm "10")^2$	M1
	$"50" = (x - "(-1)")^2 + (y - "10")^2$	A1ft
	$x^2 + y^2 + 2x - 20y + 51 = 0$	A1 **[5]** **(14 marks)**
	Alternative methods to part (d)	
	(i)Use equation $x^2 + y^2 + ax + by + c = 0$ and substitute three points, usually (0,3), (6,11) and another point on the circle maybe (-2,17) or (-8,9) - **not** point Z	M1
	Solves simultaneous equations	dM1
	a = 2, b = -20 and c = 51	A1,A1,A1
	(ii) Uses centre to write a = and b = (doubles x coordinate and y coordinate respectively, $\pm"2"$ *and* $\pm"20"$)	M1
	Obtains a= 2 and b = -20 (or just writes these values down so these answers imply M1A1)	A1
	Completes method to find c, (could substitute one of the points on the circle) or could find r	dM1
	Accurate work e.g. $r^2 = 50$ or e.g. $x^2 + y^2 + 2x - 20y = (-8)^2 + 9^2 + 2\times -8 - 20\times 9 =$	A1
	c = 51	A1

	Notes for Question 15	
(a)	M1: States gradient equation or uses correctly A1: 4/3 or 8/6 or decimal equivalent	
(b)	M1: Uses midpoint formula, or implied by y coordinate of 7. A1: (3, 7) cao B1: : Uses negative reciprocal follow through their gradient M1: Line equation with their midpoint and perpendicular gradient A1: correct at any stage **may be unsimplified**, isw. Should be linear.	
(c)	M1: Substitute y = 10 into line equation to give x = A1: cao (Answer only with no working may have M1A1)	
(d)	M1:Finds radius or diameter or r^2 using any valid method – probably distance from centre to one of the points. Need not state r = A1: for any equivalent $r^2 = 50$ or $r = \sqrt{50}$ etc . Their numeric answer must be identified. If they halve it or double it, this is M1 A0. M1: Attempt to use a true equation for circle with their centre and their radius or the letter r - allow sign slips in brackets. Do not allow use of r instead of r^2 in the equation A1ft: correct work ft their centre and genuine attempt at radius A1: correct and given in this form **Alternative methods** **Do not need to write out equation at the end** a = 2, b = -20 and c = 51 is sufficient.	