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Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Core Mathematics C12

Advanced Subsidiary

Monday 19 May 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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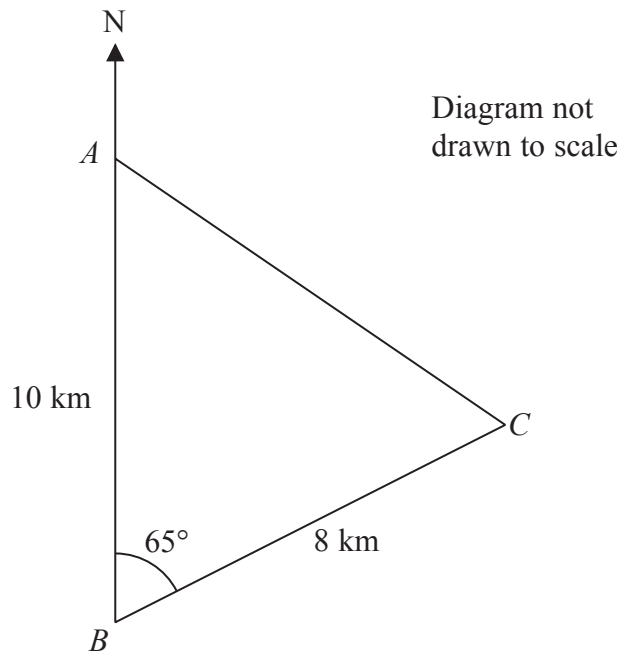


Figure 1

Figure 1 shows the position of three stationary fishing boats A , B and C , which are assumed to be in the same horizontal plane.

Boat A is 10 km due north of boat B .

Boat C is 8 km on a bearing of 065° from boat B .

(a) Find the distance of boat C from boat A , giving your answer to the nearest 10 metres.

(3)

(b) Find the bearing of boat C from boat A , giving your answer to one decimal place.

(3)



Question Number	Scheme	Marks
1(a)	$AC^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \cos 65^\circ \Rightarrow AC = ..$ $AC = 9.8\dots, \Rightarrow AC = 9.82 \text{ km (9820m) (to nearest 10 m)}$	M1 A1,A1 (3)
(b)	$\frac{\sin A}{8} = \frac{\sin 65^\circ}{9.817\dots} \Rightarrow A =$ $\angle A = \text{awrt } 47.6^\circ$ $\frac{\sin C}{10} = \frac{\sin 65^\circ}{9.817\dots} \Rightarrow C =$ $\angle C = \text{awrt } 67.4^\circ$ $\Rightarrow \text{Bearing} = \text{awrt } 132.4^\circ$	M1 A1 A1ft (3) (6 marks)
Alt (b)	$\cos A = \frac{10^2 + AC^2 - 8^2}{2 \times 10 \times AC} \Rightarrow A = \dots \text{ OR } \cos C = \frac{8^2 + AC^2 - 10^2}{2 \times 8 \times AC} \Rightarrow C = \dots$	M1

(a)

M1

Uses the cosine rule, or otherwise to find AC . The rule, if stated, must be correct. If it is not stated it must be of the correct form. Accept $AC^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \cos 65^\circ \Rightarrow AC = ..$

It is possible to find AC by other methods, eg dropping a perpendicular from A to a point X on BC . For M1 to be scored it must be a full method. Eg A full method could be; find AX by using $\sin 65^\circ$, BX by $\cos 65^\circ$, and CX by subtraction of BX from 8. The M1 is finally scored after an application of Pythagoras' theorem to find AC .

A1

Accept answers rounding or truncating to $AC = 9.8\dots$ (km)

A1

Accept $AC = 9.82 \text{ km}$ or 9820m . Both the accuracy and the units are necessary.

(b)

M1

Uses the sine rule (or cosine rule) with their answer for AC to find angle A or angle C .

$$\text{Accept } \frac{\sin A}{8} = \frac{\sin 65^\circ}{9.817\dots} \Rightarrow A = \dots \text{ or } \cos A = \frac{10^2 + 9.817^2 - 8^2}{2 \times 10 \times 9.817} \Rightarrow A = \dots$$

$$\text{Accept } \frac{\sin C}{10} = \frac{\sin 65^\circ}{9.817\dots} \Rightarrow C = \dots \text{ or } \cos C = \frac{8^2 + 9.817^2 - 10^2}{2 \times 8 \times 9.817} \Rightarrow C = \dots$$

In the sine rule the sides and angles need to be correctly matched

A1

Accept $\angle A = \text{awrt } 47.6^\circ$ or $\angle C = \text{awrt } 67.4^\circ$

Don't be overly concerned with the labelling of the angle

A1ft

Awrt 132.4° or follow through on their $(180 - A)^\circ$ if they found A or $(65 + C)^\circ$ if they found C .

Question Number	Scheme	Marks
2	$\sqrt{27} = 3\sqrt{3}, \frac{6}{\sqrt{3}} = 2\sqrt{3}$ $x\sqrt{27} + 21 = \frac{6x}{\sqrt{3}} \Rightarrow 3\sqrt{3}x + 21 = 2\sqrt{3}x$ $\Rightarrow \sqrt{3}x = -21$ $\Rightarrow x = -\frac{21}{\sqrt{3}} \Rightarrow x = -7\sqrt{3}$	<p>M1 A1</p> <p>M1 A1</p> <p>(4 marks)</p>

M1 Simplify either $\sqrt{27} = 3\sqrt{3}$ or $\frac{6}{\sqrt{3}} = 2\sqrt{3} = \left(\frac{6\sqrt{3}}{3}\right)$

A1 Uses both $\sqrt{27} = 3\sqrt{3}$ and $\frac{6}{\sqrt{3}} = 2\sqrt{3}$ to rewrite equation in a form equivalent to $3\sqrt{3}x + 21 = 2\sqrt{3}x$

M1 Collects x terms on one side of the equation, simplifies and divides reaching $x = \dots$

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
Alt 2	$x\sqrt{27} + 21 = \frac{6x}{\sqrt{3}} \quad (\times\sqrt{3}) \Rightarrow \sqrt{3}\sqrt{27}x + 21\sqrt{3} = 6x$ $\Rightarrow 9x + 21\sqrt{3} = 6x$ $\Rightarrow 3x = -21\sqrt{3} \Rightarrow x = -7\sqrt{3}$	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>(4 marks)</p>

M1 Multiply equation by $\sqrt{3}$, seen in at least two terms.

A1 $9x + 21\sqrt{3} = 6x$ or equivalent but the $\sqrt{81}$ must have been dealt with

M1 Collects terms in x , and proceeds to $x = \dots$

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
3 (a)	$4^a = 20 \Rightarrow a \log 4 = \log 20 \Rightarrow a = \dots$ $= \frac{\log 20}{\log 4} = \text{awrt } 2.16$	M1 A1 (2)
(b)	$3 + 2 \log_2 b = \log_2 30b \Rightarrow 3 + \log_2 b^2 = \log_2 30b$ $\Rightarrow 3 = \log_2 30b - \log_2 b^2$ $\Rightarrow 3 = \log_2 \left(\frac{30b}{b^2} \right)$ $\Rightarrow 2^3 = \frac{30b}{b^2} \Rightarrow b = \dots$ $b = 3.75$	M1 M1A1 dM1 A1 (5) (7 marks)
Alt (b)	$3 + 2 \log_2 b = \log_2 (30b) \Rightarrow 3 + 2 \log_2 b = \log_2 30 + \log_2 b$ $\Rightarrow 3 + \log_2 b = \log_2 30$ $\Rightarrow \log_2 8 + \log_2 b = \log_2 30$ $\Rightarrow \log_2 8b = \log_2 30$ $\Rightarrow 8b = 30 \Rightarrow b = \dots$ $\Rightarrow b = 3.75$	2nd M1 1st M1 A1 dM1 A1 (5)

(a)

M1 Takes logs of both sides leading to $a = \dots$ Accept for this mark $\log_4 20, \frac{\log 20}{\log 4}$ or $a \log 4 = \log 20 \Rightarrow a = \dots$

A1 awrt 2.16. Just the answer with no incorrect working scores both marks.

Do not accept answers from trial and error.

(b) Note that this part is B1M1M1dM1A1 on e pen. We are scoring it M1M1A1dM1A1

M1 Score for a correct use of the power law for logs

Accept either $2 \log_2 b = \log_2 b^2$, $2 \log b = \log b^2$ or $3 = \log_2 8$

M1 Score for a use of the addition or subtraction law of logs.

Examples of this would be $\log(30b) = \log 30 + \log b$ and $\log(30b) - \log(b^2) = \log\left(\frac{30b}{b^2}\right)$ Do not accept attempts such as $\log(30b) - 2 \log(b) = \log\left(\frac{30b}{2b}\right)$ A1 Achieving a correct intermediate line of the form $\log_2 (\dots) = \dots$ or $\log_2 \dots = \log_2 \dots$ Accept exact equivalents of $3 = \log_2 \left(\frac{30b}{b^2}\right)$, $\log_2 8b = \log_2 30$ and $\log_2 8b^2 = \log_2 30b$, $b = 2^{\log_2(30)-3}$ dM1 Dependent upon both previous M's it is for correctly undoing the logs and solving to get a value for b A1 $b = 3.75$ or exact equivalent. Ignore any reference to $b = 0$

4.

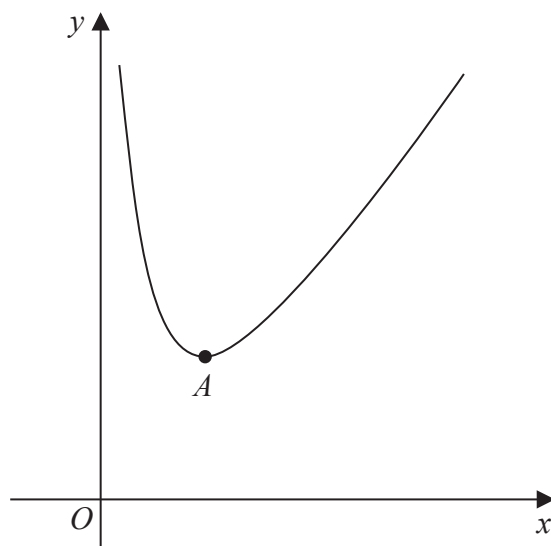


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = x^2 + \frac{16}{x}, \quad x > 0$$

The curve has a minimum turning point at A .

(a) Find $f'(x)$. (2)

(b) Hence find the coordinates of A . (4)

(c) Use your answer to part (b) to write down the turning point of the curve with equation

(i) $y = f(x + 1)$,

(ii) $y = \frac{1}{2}f(x)$. (2)



Question Number	Scheme	Marks
4(a)	$f(x) = x^2 + \frac{16}{x} \Rightarrow f'(x) = 2x - \frac{16}{x^2}$	M1A1 (2)
(b)	Setting $2x - \frac{16}{x^2} = 0 \Rightarrow x = ..$ $x^3 = 8 \Rightarrow x = 2$ $A = (2, 12)$	M1 dM1A1 A1 (4)
(c)(i)	$A' = (1, 12)$	B1ft
(ii)	$A' = (2, 6)$	B1ft (2)
		(8 marks)

(a)

M1 $x^n \rightarrow x^{n-1}$ for either term. Accept $x^2 \rightarrow x$ or $\frac{1}{x} \rightarrow \frac{1}{x^2}$ ($x^{-1} \rightarrow x^{-2}$)A1 A correct unsimplified $f'(x) = 2x - \frac{16}{x^2}$. Accept versions such as $f'(x) = 2 \times x + 16 \times -1x^{-2}$

(b)

M1 Sets their $f'(x) = 0$ and proceeds to $x = ..$. Don't be overly concerned with how they get to $x = ..$ dM1 Dependent upon the previous M mark. It is scored for $\times x^2$ to reach $x^3 = k \Rightarrow x = \sqrt[3]{k}$
This may be implied by the correct answer to their equation.A1 Correctly achieving $x = 2$. Ignore any additional solutions.A1 Correctly achieving $A = (2, 12)$. Accept $x = 2, y = 12$.If any additional solutions are given ($x > 0$) this mark will be withheld.Accept $y = 12$ appearing in part (c) as long as you are convinced that it is for $y = f(x)$

(c)(i)

B1 ft $A' = (1, 12)$. Accept this on a sketch graph or as $x = 1, y = 12$ If $A = (p, q)$ was incorrect follow through on their value from part (b), $A' = (p-1, q)$

If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

(c)(ii)

B1 ft $A' = (2, 6)$ Accept this on a sketch graph or as $x = 2, y = 6$ If $A = (p, q)$ was incorrect follow through on their value from part (b), $A' = (p, \frac{1}{2}q)$

If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

Do Not allow multiple attempts, mark in the order given if not clearly labelled.

The isw rule is suspended for this part of the question.

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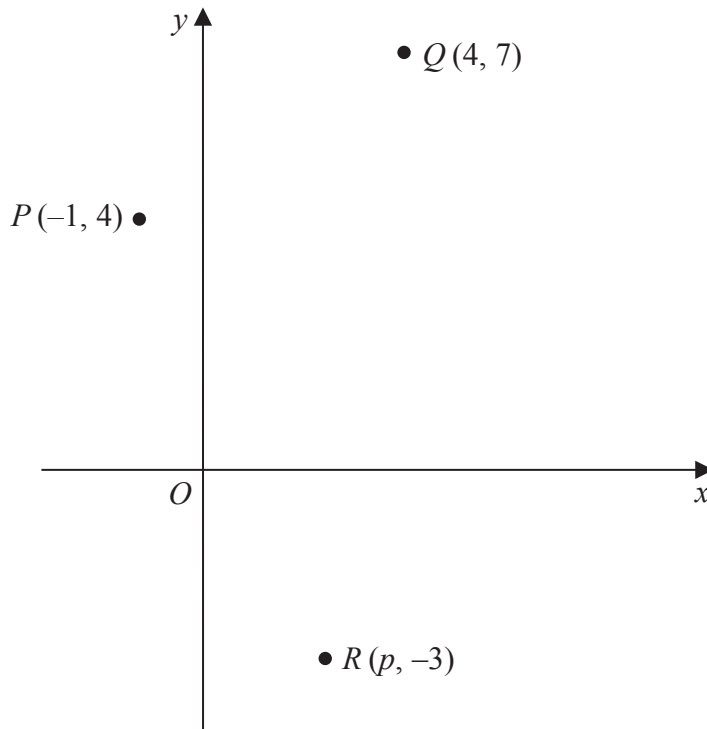


Diagram not drawn to scale

Figure 3

Figure 3 shows the points P , Q and R .

Points P and Q have coordinates $(-1, 4)$ and $(4, 7)$ respectively.

(a) Find an equation for the straight line passing through points P and Q .

Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

The point R has coordinates $(p, -3)$, where p is a positive constant.

Given that angle $QPR = 90^\circ$,

(b) find the value of p .

(3)



Question Number	Scheme	Marks
5(a)	Gradient $PQ = \frac{y_1 - y_2}{x_1 - x_2} = \frac{7 - 4}{4 - (-1)} = \frac{3}{5}$ Equation of line PQ $\frac{3}{5} = \frac{y-7}{x-4}$, $\frac{3}{5} = \frac{y-4}{x-1}$ oe $\pm k(3x - 5y + 23 = 0)$ k an integer	M1A1 M1 A1 (4)
(b)	Uses gradient $PR = -\frac{5}{3} \Rightarrow \frac{-7}{p+1} = -\frac{5}{3}$ $5p + 5 = 21 \Rightarrow p = \frac{16}{5}$ oe.	M1 dM1, A1 (3) (7 marks)
Alt 5(a)	Sub $(-1,4)$ and $(4,7)$ into $y = mx + c \Rightarrow 7 = 4m + c$ and $4 = -1m + c$ Solve simultaneously to get $m = \frac{3}{5}$ Sub in either to get $c = \dots \left(\frac{23}{5}\right)$ Rearranges to $\pm k(3x - 5y + 23 = 0)$	M1, A1 M1 A1 (4)

(a)

M1 Attempts gradient of $PQ = \frac{\Delta y}{\Delta x}$ using both $(4, 7)$ and $(-1, 4)$. Accept as evidence $\frac{7-4}{4-(-1)}$ or

$\frac{7-4}{4-(-1)} = \frac{y-7}{x-4}$ or similar. You may allow only one error on the sign, there must be an attempt to find

a difference. For the M accept $\frac{7-4}{4-1}$ BUT NOT $\frac{7+4}{4-1}$ or $\frac{4--1}{7-4}$

A1 Achieves the gradient of $\frac{3}{5}$ or its equivalent 0.6

This may be embedded within an equation. Eg $\frac{3}{5} = \frac{y-7}{x-4}$

M1 Attempts equation of line using their gradient and a point.

Accept their ' $\frac{3}{5}$ ' = $\frac{y-7}{x-4}$ and versions such as $y-7 = \text{'their' } \frac{3}{5}(x-4)$ with both signs correct

Accept their ' $\frac{3}{5}$ ' = $\frac{y-4}{x--1}$ and versions such as $y-4 = \text{'their' } \frac{3}{5}(x--1)$ with both signs correct

If $y = mx + c$ is used then it must be a full attempt to find c .

Accept $7 = \frac{3}{5} \times 4 + c \Rightarrow c = ..$

A1 $\pm k(3x - 5y + 23 = 0)$ where k is an integer

Note 1: that candidates who calculate the eqn of a line as $\frac{y-7}{7-4} = \frac{x-4}{4--1}$ oe score M1A1M1 straight away

Note 2: An alternative method can be seen when candidates express their equation in the form $y = mx + c$ or $ax + by + c = 0$ or any such linear form.

M1 is scored for subbing both $(-1, 4)$ and $(4, 7)$ into their linear expression forming two simultaneous equations AND solving.

A1 is scored for $m = \frac{3}{5}$ in $y = mx + c$ or $\frac{a}{b} = -\frac{3}{5}$ in $ax + by + c = 0$

M1 is scored for subbing their $m = \frac{3}{5}$ into $y = mx + c$ to find $c = ..$ or $\frac{a}{b} = -\frac{3}{5}$ into $ax + by + c = 0$ to find $\frac{c}{a}$

A1 $\pm k(3x - 5y + 23 = 0)$ where k is an integer

(b)

M1 Attempts to use the fact that the gradients are perpendicular to set up an equation in p .

Possible ways this can be done are

Attempting to use $m_1 \times m_2 = -1$ So accept $\frac{3}{5} \times \frac{4--3}{-1-p} = -1$, $\frac{7-4}{4--1} = -\frac{1}{\frac{4--3}{-1-p}}$

Or attempting to find the equation of line PR using the perpendicular gradient and $P(-1, 4)$ followed by a substitution of $(p, -3)$. Accept p being interchanged with x .

Or attempting to use Pythagoras $PQ^2 + PR^2 = QR^2 \Rightarrow (p+1)^2 + 7^2 + 5^2 + 3^2 = (p-4)^2 + 10^2$

In all cases it must be evident that a difference in the coordinates is attempted.

dM1 Dependent upon the previous M.

It is scored for proceeding to (and solving) a linear equation in p (or x).

In both methods look for $. Ap + Q = R \Rightarrow p = ..$

A1 $p = \frac{16}{5}$ or equivalents such as 3.2, $3\frac{1}{5}$

Accept written as $x =$

Leave blank

6. (a) Show that

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} \equiv 1 - \tan^2 x, \quad x \neq (2n + 1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \tag{2}$$

(b) Hence solve, for $0 \leq x < 2\pi$,

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0$$

Give your answers in terms of π . (5)

Question Number	Scheme	Marks
6(a)	Uses $1 - \sin^2 x = \cos^2 x$ $\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \frac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x$	M1 A1* (2)
(b)	$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0 \Rightarrow 1 - \tan^2 x + 2 = 0$ $\tan^2 x = 3$ $\tan x = (\pm)\sqrt{3} \Rightarrow x = ..$ $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$	M1 A1 dM1 A1,A1 (5)

(a)

M1 Uses $1 - \sin^2 x = \cos^2 x$ in the denominator of their expression.The usual case will be sight of $\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} \rightarrow \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$ If candidate starts on the right hand side $1 - \tan^2 x = 1 - \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x}$ then

M1A1 is scored at the end

If the candidate multiplies both sides by $1 - \sin^2 x$ and multiples out then

$$(1 - \tan^2 x)(1 - \sin^2 x) = 1 - \tan^2 x - \sin^2 x + \sin^2 x \tan^2 x$$

they will not get any credit until they start to use $1 - \sin^2 x = \cos^2 x$

A1* Completes proof with no errors. This is a show that question.

Look for a minimum of ALL 3 steps shown below

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \frac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x \quad \text{OR} \quad \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x$$

(b)

M1 Scored for using part (a) $\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0 \Rightarrow \tan^2 x = k$ A1 $\tan^2 x = 3$ M1 Correct order $\tan^2 x = k \Rightarrow \tan x = (\pm)\sqrt{k}$, no need for negative, leading to at least one value of x (which may not be correct). Evidence could be the answer in degrees $x = 60^\circ$ or in radians 1.047
If they achieve $\tan^2 x = k$, with $k < 0$ this mark cannot be scored.A1 Two correct answers as multiples of π . Accept versions such as $\frac{\pi}{1.5}$.

Degrees and answers like 1.047 etc score A0 although recovery is allowed.

A1 All four correct with no extras inside range. Ignore any extra solutions outside the range.

Special Case Although this question clearly states 'Hence' we will allow a maximum of 1,1,1,0,0 for a solution where candidates re start as follows

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0 \Rightarrow \frac{1 - \sin^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0 \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$$

AND similarly for

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0 \Rightarrow \cos^2 x = \frac{1}{4} \Rightarrow x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$$

First '1' for either $\sin^2 x = A$ OR $\cos^2 x = A$ where $0 \leq A \leq 1$ followed by '1' for two correct solutions and '1' for two additional correct solutions.

Question Number	Scheme	Marks
<p>7(i)</p> <p>(ii)</p>	$f'(x) = \frac{4}{x^3} + 2x - 1 \Rightarrow f(x) = \frac{4}{-2}x^{-2} + x^2 - 1x (+c)$ $\text{Sub (2, 3)} \quad 3 = -\frac{1}{2} + 4 - 2 + c \Rightarrow c = \frac{3}{2}$ $f(1) = -\frac{2}{1^2} + 1^2 - 1 + \frac{3}{2} = -\frac{1}{2}$ <p>cs0</p> $\int_1^4 (3\sqrt{x} + A) dx = \left[\frac{3x^{1.5}}{1.5} + Ax \right]_1^4 = 21$ $\left(\frac{3 \times 4^{1.5}}{1.5} + 4A \right) - \left(\frac{3 \times 1^{1.5}}{1.5} + 1A \right) = 21$ $16 + 4A - 2 - A = 21$ $\Rightarrow 3A = 7 \Rightarrow A = \frac{7}{3}$	<p>M1A1</p> <p>dM1A1</p> <p>A1</p> <p>(5)</p> <p>M1A1</p> <p>M1</p> <p>dM1A1</p> <p>(5)</p> <p>(10 marks)</p>

(i)

M1 Score for raising the power by 1 in any of the 3 terms. Accept $x^{-3} \rightarrow x^{-2}$, $x \rightarrow x^2$ or $1 \rightarrow x$ A1 A correct and unsimplified form. Accept $f(x) = 4\frac{x^{-2}}{-2} + 2\frac{x^2}{2} - 1x (+c)$. There is no requirement for $+c$ dM1 Substitutes (2, 3) into their $f(x)$, which must have a $+c$ and proceeds to find a numerical value to c
It is dependent upon the previous M having been scored.A1 $c = \frac{3}{2}$ or equivalent.A1 cso $f(1) = -\frac{1}{2}$ Accept exact equivalents such as -0.5

(ii)

M1 Score for either $3\sqrt{x} \rightarrow \dots x^{\frac{3}{2}}$ or $A \rightarrow Ax$ A1 Fully correct (unsimplified) integral- There is no need to set $=21$ or sub in any limits or have $+c$.Accept versions of $3\frac{x^{1.5}}{1.5} + Ax (+c)$ M1 Sub in the limits 4 and 1 into a 'changed' function, subtract (either way around) and set the result $=21$.Accept versions of $\left(\frac{3 \times 4^{1.5}}{1.5} + 4A\right) - \left(\frac{3 \times 1^{1.5}}{1.5} + 1A\right) = 21$ with their integrand with or without brackets

dM1 Proceed to solve equation in A.

This is dependent upon **both** previous M's **and having achieved** $A \rightarrow Ax$

It must end with a numerical value for A.

A1 $A = \frac{7}{3}$ or equivalents such as $2\frac{1}{3}$, $2.\dot{3}$. Do not accept 2.3 or 2.33 but remember to isw.

8. Given that

$$1 + 12x + 70x^2 + \dots$$

is the binomial expansion, in ascending powers of x of $(1 + bx)^n$, where $n \in \mathbb{N}$ and b is a constant,

(a) show that $nb = 12$ **(1)**

(b) find the values of the constants b and n . **(6)**



Question Number	Scheme	Marks
<p>8 (a)</p> <p>(b)</p>	$(1+bx)^n = 1 + nbx + \left(\frac{n(n-1)(bx)^2}{2!} \right) = 1 + 12x + (70x^2)$ <p>Compares x terms $nb = 12$</p> <p>Comparing x^2 terms $\frac{n(n-1)b^2}{2} = 70 (\Rightarrow n(n-1)b^2 = 140)$</p> <p>Substitutes $b = \frac{12}{n}$ into $n(n-1)b^2 = 140 \Rightarrow \frac{n(n-1)144}{n^2} = 140$</p> $\Rightarrow 144(n-1) = 140n \Rightarrow n = 36$ <p>cso</p> <p>Substitutes $n = 36$ into $nb = 12 \Rightarrow b = \frac{1}{3}$</p> <p style="text-align: right;">cso</p>	<p>B1* (1)</p> <p>M1A1</p> <p>M1</p> <p>dM1A1</p> <p>A1 (6)</p> <p>(7 marks)</p>

(a) For the purpose of marking parts (a) and (b) can be scored together

B1* Use the binomial series expansion with ' x ' = bx and proceeding as far as the x term.

This is a show that question and you cannot just accept the given statement $nb = 12$

Accept minimal evidence however:

Accept either (including the 1) $(1 + bx)^n = 1 + nbx + \dots \Rightarrow nb = 12$ OR

$$(1 + bx)^n = 1 + {}^n C_1 bx + \dots \Rightarrow nb = 12$$

or a statement to the effect that 'comparing the x /second terms ${}^n C_1 1^{n-1} (bx) = 12x \Rightarrow nb = 12$ '

Note: $nbx = 12x \Rightarrow nb = 12$ does not score B1 unless you see the accompanying statement 'comparing x or second terms'

(b)

M1 Attempts to set the x^2 term of the binomial expansion equal to 70. The form of the binomial must be correct so accept $\frac{n(n-1)(bx)^2}{2} = 70x^2$, ${}^n C_2 (bx)^2 = 70x^2$ or $\frac{n!(bx)^2}{2!(n-2)!} = 70x^2$ with or without brackets.

It may be implied by a correct equation. It could be scored in part (a)

Condone invisible brackets, so accept for M1 just the statement $\frac{n(n-1)b}{2} = 70$

A1 Compares terms in x^2 to achieve a second correct equation in b and n .

Accept equivalent versions of $\frac{n(n-1)b^2}{2} = 70$ or $n(n-1)b^2 = 140$

Do not accept $\frac{n!b^2}{2!(n-2)!} = 70$ or ${}^n C_2 (b)^2 = 70$ unless one of the above could be implied by later work.

M1 Substitutes either $b = \frac{12}{n}$ or $n = \frac{12}{b}$ or $nb = 12$ into their second equation involving both b and n to produce an equation in a single variable.

dM1 A valid method of solving their equation allowing for slips. See practice items for examples
This is dependent upon **both** previous M's

A1 Correct solution only $n = 36$ or $b = \frac{1}{3}$

A1 Correct solution only $n = 36$ and $b = \frac{1}{3}$ **and** no other solutions such as $n = 0$

Allow solutions just written down after two correct equations provided you don't see incorrect working.

Question Number	Scheme	Marks
<p>9(i)</p> <p>(ii)</p>	$\sum_{r=1}^{20} (3+5r) = 8+13+18+\dots+103$ <p>Use of $S_n = \frac{n}{2}(2a+(n-1)d)$ or $S_n = \frac{n}{2}(a+l)$ with $a=3$ or 8, $n=19$ or 20, $d=5$ and $l=103$</p> $S_{20} = \frac{20}{2}(8+103) = 1110$ $\sum_{r=0}^{\infty} \frac{a}{4^r} = 16 \Rightarrow \frac{a}{1} + \frac{a}{4} + \frac{a}{16} \dots = 16 \quad r = \frac{1}{4} \text{ oe}$ <p>Use of $S_{\infty} = \frac{a}{1-r}$ with $0 < r < 1$ and $S_{\infty} = 16$</p> $16 = \frac{a}{1-r} \Rightarrow a = ..$ $a = 12$	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>B1</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>(4)</p> <p>(7 marks)</p>

(i)

- M1 Minimal evidence of the sum of an arithmetic sequence .
 Accept as evidence the first 3 terms written out as $8+13+18..$ or $8+13+..+103$
 or $8,13,18$ followed by the sum formula $S_n = \frac{n}{2}(2a + (n-1)d)$
 or $8,13,..103$ followed by the sum formula $S_n = \frac{n}{2}(a+l)$
 Do not accept on its own however $3+5 \times 20$ or 103 without any reference to a sum
- M1 Uses $S_n = \frac{n}{2}(2a + (n-1)d)$ with $a = 3$ or $8, d = 5$ and $n = 19$ or 20
 or $S_n = \frac{n}{2}(a+l)$ with $n = 19$ or $20, a = 3$ or $8,$ and $l=103$
 Accept a list of 20 terms as long as all terms are written out.
- A1 1110. Accept this for all 3 marks as long as no incorrect working is seen.

Alt (i)

- M1 Splits the sum into two separate parts, and uses/states $\sum 1 = n$ and $\sum r = \frac{n(n+1)}{2}$ both
- $$\sum 3 + 5r = \sum 3 + \sum 5r = 3 \times n + 5 \times \frac{n(n+1)}{2}$$
- M1 sub 20 or 19 into the above
 A1 1110

(ii)

- B1 For stating or implying that $r = \frac{1}{4}$.
- You may see a series or sequence of terms with $\times \frac{1}{4}$ or $\times 0.25$.
- Accept variations on $a + \frac{a}{4} + \frac{a}{4^2}$ or even $\frac{a}{4}, \frac{a}{16}, \frac{a}{64}, \dots$
- M1 For using the formula $S_\infty = \frac{a}{1-r}$ with $S_\infty = 16$ and $0 < |r| < 1$
- dM1 Dependent upon the previous M. For proceeding to $a = ..$
 A1 12

Question Number	Scheme	Marks
10(a)	$kx^2 + 4x + k = 2 \Rightarrow kx^2 + 4x + k - 2 = 0$ Attempts to calculate $b^2 - 4ac$ with $a = k, b = 4$ and $c = k \pm 2$ $b^2 - 4ac = 4^2 - 4 \times k \times (k - 2)$ Sets their $b^2 - 4ac > 0 \Rightarrow 16 - 4k(k - 2) > 0$ $4k^2 - 8k - 16 < 0$ $k^2 - 2k - 4 < 0$	M1 A1 dM1 A1* (4)
(b)	Solves $(k - 1)^2 - 5 = 0 \Rightarrow k = 1 \pm \sqrt{5}$ ‘Insides’ $1 - \sqrt{5} < k < 1 + \sqrt{5}$	M1 M1A1 (3) (7 marks)

- (a) Note that this is M1M1A1A1 on e pen. Mark in the order seen below as M1A1M1A1.
- M1 Attempts to calculate $b^2 - 4ac$ with $a = k, b = 4$ and $c = k \pm 2$. Condone poor/incomplete bracketing. Alternatively they may set $b^2 \dots 4ac$ with ... being =, >, <, \leq or \geq with $a = k, b = 4$ and $c = k \pm 2$.
- A1 Correct (unsimplified) $b^2 - 4ac$. Accept $4^2 - 4k(k - 2)$ oe. The bracketing must be correct. In the alternative it is for $4^2 \dots 4k(k - 2)$ with ... being =, >, <, \leq or \geq The bracketing must be correct
- dM1 Sets their $b^2 - 4ac > 0$ with $a = k, b = 4$ and $c = k \pm 2 \Rightarrow 4^2 - 4k(k \pm 2) > 0$
 In the alternative it is for just $b^2 > 4ac$ with $a = k, b = 4$ and $c = k \pm 2 \Rightarrow 4^2 > 4k(k \pm 2)$
- A1* Proceeds correctly to the given answer $k^2 - 2k - 4 < 0$. You should expect to see the terms moved over to the other side of the inequality and a division of 4. Alternatively a division of -4 should be preceded by the reversal of the inequality. There is no requirement for these steps to be explained but you need to check for sign errors which would be A0.
- Special case: If they start with $a = kx^2, b = 4x$ and $c = k \pm 2$ and proceed correctly they will score a maximum of M1A1dM1A0*. Treat candidates who start with $\sqrt{b^2 - 4ac}$ in a similar way.
- (b)
- M1 For an attempt by either the formula or completing the square to solve $3TQ=0$. Do not accept factorisation. If the formula is quoted it must be correct. If it is not quoted only accept expressions of the form $\frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -4}}{2 \times 1}$ with or without either bracket. Accept awrt 3.24, -1.24 from a GC.
- If completing the square is attempted accept $k^2 - 2k - 4 = 0 \Rightarrow (k \pm 1)^2 \pm 1 \pm 4 = 0 \Rightarrow k = \dots$
- M1 Chooses the inside values to their solution to the $3TQ=0$. This is not dependent upon the previous M so for this mark you can accept the inside region from their roots obtained factorisation. If α is the smaller root and β is the larger root look for $\alpha < k < \beta$ or $k > \alpha$ and $k < \beta$ or $k > \alpha$ or $k < \beta$ or (α, β)
- A1 $1 - \sqrt{5} < k < 1 + \sqrt{5}$. Accept also $k > 1 - \sqrt{5}$ and $k < 1 + \sqrt{5}$, $k > 1 - \sqrt{5}$ $k < 1 + \sqrt{5}$, $(1 - \sqrt{5}, 1 + \sqrt{5})$
 Also allow $k > 1 - \sqrt{5}$ $k < 1 + \sqrt{5}$ with a comma between.
 Accept exact equivalents like $\frac{2 - \sqrt{20}}{2} < k < \frac{2 + \sqrt{20}}{2}$ or $1 - \frac{\sqrt{20}}{2} < k < 1 + \frac{\sqrt{20}}{2}$
Do not accept $k > 1 - \sqrt{5}$ or $k < 1 + \sqrt{5}$, $[1 - \sqrt{5}, 1 + \sqrt{5}]$ or decimals
 Do not accept x in place of k for the final mark.
- Special case for candidate who achieves $1 - \sqrt{5} \leq k \leq 1 + \sqrt{5}$ or $[1 - \sqrt{5}, 1 + \sqrt{5}]$ score M1M1A0

11.

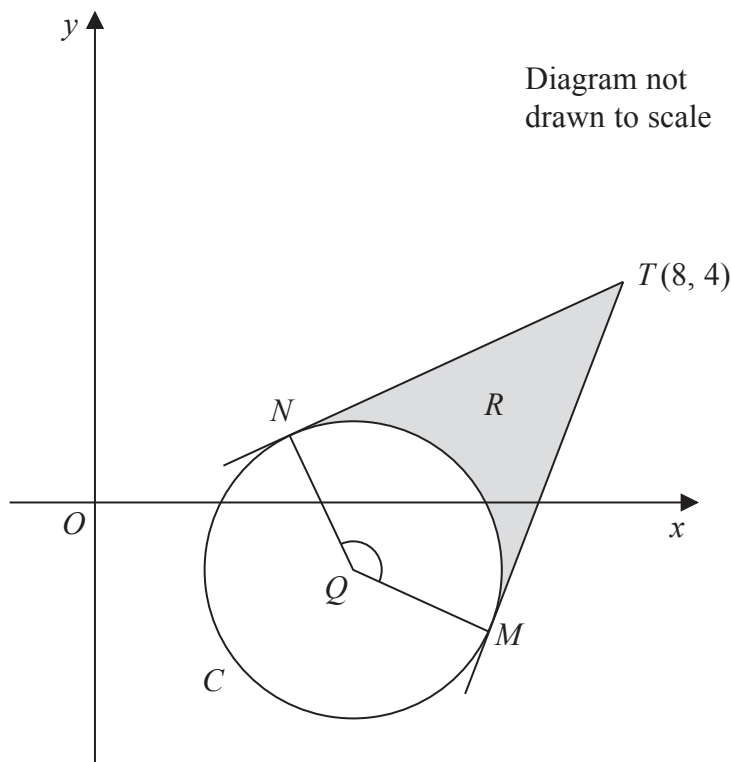


Figure 4

Figure 4 shows a sketch of the circle C with centre Q and equation

$$x^2 + y^2 - 6x + 2y + 5 = 0$$

(a) Find

- (i) the coordinates of Q ,
- (ii) the exact value of the radius of C .

(5)

The tangents to C from the point $T(8, 4)$ meet C at the points M and N , as shown in Figure 4.

(b) Show that the obtuse angle MQN is 2.498 radians to 3 decimal places.

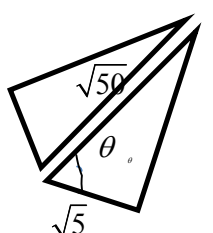
(5)

The region R , shown shaded in Figure 4, is bounded by the tangent TN , the minor arc NM , and the tangent MT .

(c) Find the area of region R .

(5)



Question Number	Scheme	Marks
11(a)(i)	$x^2 + y^2 - 6x + 2y + 5 = 0$	Obtains $(x \pm 3)^2$ and $(y \pm 1)^2$
	$(x - 3)^2 - 9 + (y + 1)^2 - 1 + 5 = 0$ Centre = (3, -1)	Obtains $(x - 3)^2$ and $(y + 1)^2$
(ii)	Radius ² = '3' ² + '-1' ² - 5 = 5 $\Rightarrow r = \sqrt{5}$	M1 A1 A1 M1A1 (5)
(b)	Calculates $TQ = \sqrt{(8-3)^2 + (4-(-1))^2} = \sqrt{50}$	M1A1
(c)		M1A1
	Uses $\cos \theta = \frac{\sqrt{5}}{\sqrt{50}} \Rightarrow \theta = 1.249\dots$ angle MQN IS 2.498 radians to 3 decimal places	A1*
(c)	Area of sector = ' $\frac{1}{2}r^2\theta$ ' = $\frac{1}{2} \times (\sqrt{5})^2 \times 2.498$ (= awrt 6.24 / 6.25) Area of triangle = ' $\frac{1}{2}ab \sin C$ ' = $\frac{1}{2} \times \sqrt{5} \times \sqrt{50} \times \sin 1.249$ = (7.50) Shaded Area = 15.0 - 6.245 = 8.76 or 8.75	M1A1 M1 dM1,A1 (5)
		(15 marks)

Mark (a)(i) and (a)(ii) together as one part.

(a)

M1 Obtains $(x \pm 3)^2$ and $(y \pm 1)^2$. This could be implied by the candidate writing down a centre of $(\pm 3, \pm 1)$

A1 Obtains $(x - 3)^2$ **and** $(y + 1)^2$

A1 Centre = (3, -1). Accept this without any working for M1A1A1

M1 Uses $r^2 = '3'^2 + '-1'^2 - 5 = \dots \Rightarrow r = \dots$ The value of r^2 must be positive for this to be scored. Condone invisible bracketing. This may be implied by $r =$ awrt 2.24

A1 $r = \sqrt{5}$. Do not accept 2.24 on its own but remember to isw.

Note that the M1 A1 can be scored for correct radius following a centre of (-3, +1)

Alternative to part (a) using $x^2 + y^2 + 2gx + 2fy + c = 0$, Centre = $(-g, -f)$ Radius = $\sqrt{g^2 + f^2 - c}$

M1 States that $2g = \pm 6$ and $2f = \pm 2$. This could be implied by the candidate writing down a centre of $(\pm 3, \pm 1)$

A1 States that $2g = -6$ **and** $2f = 2$

A1 Centre = (3, -1) Accept this without any working for M1A1A1

M1 Uses Radius = $\sqrt{'g'^2 + 'f'^2 - c}$

- A1 $r = \sqrt{5}$. Do not accept 2.24 on its own but remember to isw.
- (b)
- M1 Calculates the distance TQ from points (8,4) and their Q using Pythagoras.
Look for $TQ^2 = (8 - '3')^2 + (4 - '(-1)')^2 \Rightarrow TQ = ..$
There must be an attempt seen to find the difference between the coordinates.
- A1 Calculates the length of $TQ = \sqrt{50}$. Accept $5\sqrt{2}$, awrt 7.07
- M1 Uses a correct method to find the half angle. Usually $\cos \theta = \frac{\text{their } r}{\text{their } TQ} \Rightarrow \theta = ...$
- A1 Correct half angle calculated or implied. Accept $\theta = \text{awrt } 1.25\dots, 71.6^\circ$ or $\theta = \arcsin\left(\frac{\sqrt{5}}{\sqrt{50}}\right)$
- A1* cso Angle MQN IS 2.498 radians to 3 decimal places.
This is a show that questions and all elements including the accuracy must be correct.
Therefore this must follow accuracy of at least 1.249 for the half angle.
- (c)
- M1 Uses area of a sector formula $A = \frac{1}{2}r^2\theta$ with their value of r and $\theta = 2.498$ or 1.249 rads
If the formula is quoted it must be correct. It can be embedded within the area of a segment
- A1 For the area of the sector. Accept $\frac{1}{2} \times (\sqrt{5})^2 \times 2.498$ or $\frac{1}{2} \times 5 \times 2.498$ or awrt 6.24 or awrt 6.25
This may be scored on the penultimate line for $2 \times \frac{1}{2} \times 5 \times 1.249$
- M1 A correct method of finding the area of the triangle TQN or triangle TQM or the kite $TNQM$.
Accept use of $\frac{1}{2}ab \sin C$ with their r , their TQ and 1.249
Alternatively candidates could find TN from $\sqrt{TQ^2 - r^2}$ followed by $\frac{TN \times r}{2}$
The kite $TNQM$ could be found by $\frac{1}{2}NM \times QT$ with NM being found by the cosine rule
- dM1 A correct method to find shaded area R by finding
 $2 \times \text{Triangles} - \text{Whole sector}$ **or** $2 \times (\text{Triangle} - \text{Half sector})$
or Kite - Sector **or** Kite - Segment - Isosceles triangle
- It is dependent upon having scored **both** previous method marks.
- A1 Accept awrt 8.76 or 8.75

12.

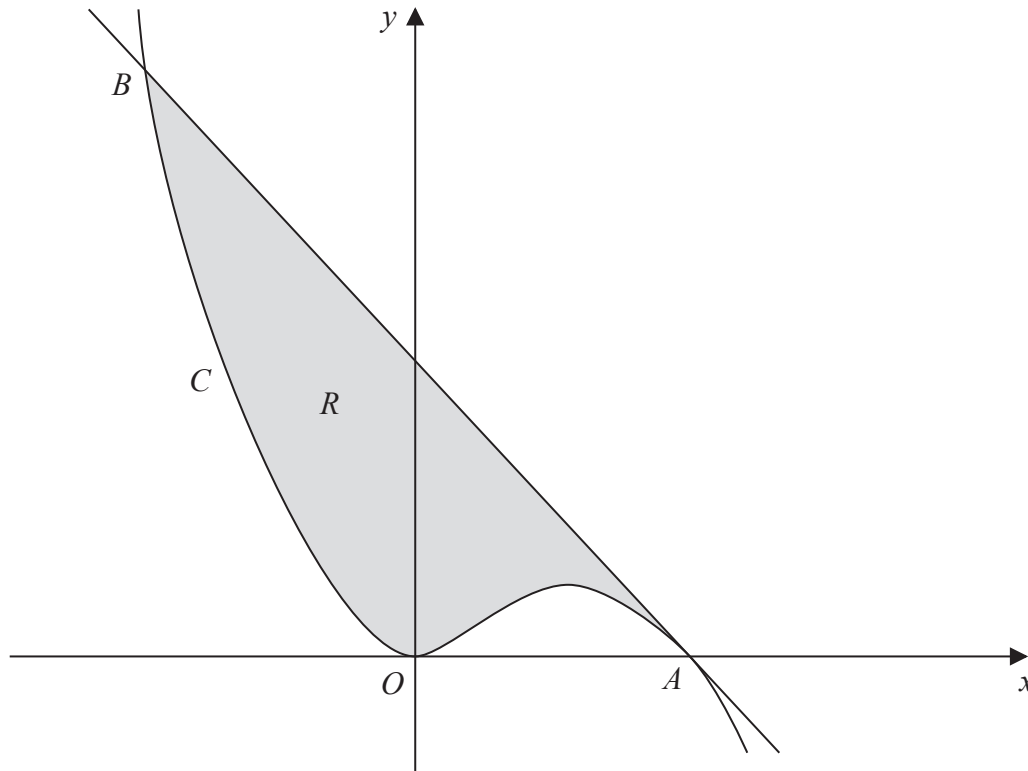


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = x^2 - \frac{1}{3}x^3$

C touches the x -axis at the origin and cuts the x -axis at the point A .

(a) Show that the coordinates of A are $(3, 0)$. (1)

(b) Show that the equation of the tangent to C at the point A is $y = -3x + 9$ (5)

The tangent to C at A meets C again at the point B , as shown in Figure 5.

(c) Use algebra to find the x coordinate of B . (4)

The region R , shown shaded in Figure 5, is bounded by the curve C and the tangent to C at A .

(d) Find, by using calculus, the area of region R . (5)
(Solutions based entirely on graphical or numerical methods are not acceptable.)



Question Number	Scheme	Marks
12(a)	Sub $x = 3$ in $y = x^2 - \frac{1}{3}x^3 = 9 - 9 = 0$	B1* (1)
(b)	$y = x^2 - \frac{1}{3}x^3 \Rightarrow \frac{dy}{dx} = 2x - x^2$ Subs $x = 3$ in $\frac{dy}{dx} = 2x - x^2 = 6 - 9 = (-3)$ Equation of tangent is $-3 = \frac{y-0}{x-3} \Rightarrow y = -3x + 9$	M1A1 dM1 ddM1A1* (5)
(c)	Sets $x^2 - \frac{1}{3}x^3 = -3x + 9 \Rightarrow x^3 - 3x^2 - 9x + 27 = 0$ oe Solves $x^3 - 3x^2 - 9x + 27 = 0 \Rightarrow (x-3)^2(x+3) = 0 \Rightarrow x = -3$	M1, A1 dM1, A1 (4)
(d)	Area under curve = $\int x^2 - \frac{1}{3}x^3 dx = \left[\frac{1}{3}x^3 - \frac{1}{12}x^4 \right]$ Area of triangle = $\frac{1}{2} \times (3 - x_B) \times y_B = (54)$ Shaded area = Triangle - area under curve = '54' - $\left(\left(\frac{1}{3} \times 3^3 - \frac{1}{12} \times 3^4 \right) - \left(\frac{1}{3} \times (-3)^3 - \frac{1}{12} \times (-3)^4 \right) \right) = 36$	M1A1 M1 dM1A1 (5)
	Alternative to (d) by integration Area = $\int_{x=-3}^{x=3} (-3x+9) - \left(x^2 - \frac{1}{3}x^3 \right) dx$ Either way around $\left[-3 \frac{x^2}{2} + 9x - \frac{1}{3}x^3 + \frac{1}{12}x^4 \right]_{x=-3}^{x=3}$ $= \left(-3 \times \frac{3^2}{2} + 9 \times 3 - \frac{1}{3} \times 3^3 + \frac{1}{12} \times \frac{3^4}{4} \right) - \left(-3 \times \frac{(-3)^2}{2} + 9 \times (-3) - \frac{1}{3} \times (-3)^3 + \frac{1}{12} \times \frac{(-3)^4}{4} \right)$ $= 36$	

(a)

B1* Substitutes $x = 3$ in $y = x^2 - \frac{1}{3}x^3$ to get $y = 9 - \frac{1}{3} \times 27 = 0$

Alternatively substitutes $y = 0$ in $y = x^2 - \frac{1}{3}x^3$ to get $0 = x^2 - \frac{1}{3}x^3 \Rightarrow 0 = x^2 \left(1 - \frac{1}{3}x \right) = 0 \Rightarrow x = 3$

Or substitutes $x = 3$ and $y = 0$ in $y = x^2 - \frac{1}{3}x^3$ to get $0 = 9 - 9$

In all cases you should expect to see a minimum of one intermediate line

(b)

M1 For any term correct (unsimplified). Look for either $x^2 \rightarrow 2x$ or $-\frac{1}{3}x^3 \rightarrow -\frac{1}{3} \times 3x^2$

A1 Gradient completely correct (unsimplified). Accept $\left(\frac{dy}{dx}\right) = 2 \times x - \frac{1}{3} \times 3x^2$

dM1 substitutes $x = 3$ into their derivative. The previous M must have been awarded.

ddM1 Full method to find a numerical equation of the tangent at (3, 0). It is dependent upon **both** previous Method marks having been scored. Accept as evidence ' -3 ' = $\frac{y-0}{x-3}$

If the form $y = mx + c$ is used it must be a full method to find the numerical value of c .

So (3,0) must be subbed into $y = -3x + c$ to find a value for c

A1* cso $y = -3x + 9$. This is a given answer and all aspects must be correct.

Note: It is possible to gain marks in part (b) by using roots of equations. It must be clearly part (b) and not part

(c) Minimal evidence is required to part (c) but the requirement of the question is to use algebra

M1 Setting equations equal to each other. Look for $x^2 - \frac{1}{3}x^3 = -3x + 9$

It may be implied by $\frac{1}{3}x^3 - x^2 - 3x + 9 = 0$

A1 Correct cubic equation = 0. Accept either $\frac{1}{3}x^3 - x^2 - 3x + 9 = 0$ or multiples of $x^3 - 3x^2 - 9x + 27 = 0$

The '= 0' may be implied by subsequent working such as factorisation etc

dM1 Solving by factorisation to find another value of x other than 3.

This is dependent upon the previous M.

Accept as evidence division of cubic by $(x-3)^2$ - check first and last terms only

Accept as evidence division of cubic by $(x-3)$ followed by factorisation or use of formula on resulting quadratic. Accept the solution 'appearing' from a graphical calculator

A1 cso $x = -3$.

Note: We are going to allow a special case, scored 1,1,0,0 where the candidate writes down

$x^2 - \frac{1}{3}x^3 = -3x + 9$ followed by $x = -3$

(d)

M1 Attempts the integral of $x^2 - \frac{1}{3}x^3$. One term must be correct, (un simplified) accept either $\frac{1}{3}x^3$ or $-\frac{x^4}{12}$

A1 Correct simplified expression $\frac{1}{3}x^3 - \frac{1}{12}x^4 (+c)$.

However the correct answer of 18 for this area, or 36 for the required area would imply this mark.

M1 Correct method for the area of the triangle. Look for $\frac{1}{2} \times (3 - x_B) \times y_B = 54$

dM1 For subtracting the area under the curve (with their 'correct' limits used) from area of the triangle. It is dependent upon both M's and the subtraction of the areas can be either way around.

A1 cso 36

The alternative version can be marked in an equivalent way. M1A1 scored for integrating the $\int x^2 - \frac{1}{3}x^3$ part correctly. Condone sign errors made before the integration takes place. 2nd M1 scored for using integration to find the area of the triangle. The integration must be correct $\left[-\frac{3x^2}{2} + 9x\right]$ and evaluated with limits of 3 and their '-3' dM1 For subbing in their 'correct' limits and subtracting the functions (either way around). It is dependent upon both M's. A1 cso 36 Subtracting the wrong way around would score A0

13. The height of sea water, h metres, on a harbour wall at time t hours after midnight is given by

$$h = 3.7 + 2.5 \cos(30t - 40)^\circ, \quad 0 \leq t < 24$$

- (a) Calculate the maximum value of h and the exact time of day when this maximum first occurs.

(4)

Fishing boats cannot enter the harbour if h is less than 3

- (b) Find the times during the morning between which fishing boats cannot enter the harbour.

Give these times to the nearest minute.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)



Question Number	Scheme	Marks
13(a)	$h = 3.7 + 2.5 \cos(30t - 40)^\circ, \quad 0 \leq t < 24$ $\text{Max} = 3.7 + 2.5 = 6.2\text{m}$ <p>Occurs when $30t - 40 = 0 \Rightarrow t = \frac{40}{30} = 1:20\text{am} (01:20)$</p>	B1 M1A1,A1 (4)
(b)	$3.7 + 2.5 \cos(30t - 40)^\circ = 3 \Rightarrow \cos(30t - 40)^\circ = -\frac{0.7}{2.5} = (-0.28)$ $30t - 40 = 106.3, (253.7)$ $t = \text{awrt } 4.9 \text{ or } 9.8$ <p>2nd Value $30t - 40 = 253.7 \Rightarrow t = \dots$ $t = \text{awrt } 4.9 \text{ and } 9.8$</p> <p>Boat cannot enter the harbour between 04:53 and 09:47</p>	M1 A1 A1 M1 A1 A1 (6) (10 marks)

(a)

B1 $h_{\max} = 6.2 (m)$. The units are not important.M1 Solves either $30t - 40 = 0 \Rightarrow t = \dots$ or $30t - 40 = 360 \Rightarrow t = \dots$. It may be implied by $t = \frac{40}{30}, \frac{400}{30}$ oeA1 $t = \frac{40}{30}$, or exact equivalents like $\frac{4}{3}, 1.\bar{3}$.

A1 1:20am or (01:20) The exact time of day is required. 1:20 or 1:20pm is incorrect.

Special case: Candidates who solve this part by differentiation can be allowed full marks

B1 $h_{\max} = 6.2 (m)$. The units are not importantM1 For $h' = A \sin(30t - 40)^\circ = 0 \Rightarrow t = \dots$ A1 $t = \frac{40}{30}$, or exact equivalents like $\frac{4}{3}, 1.\bar{3}$.

A1 1:20am Accept 01:20 or 0120 The exact time of day is required. 1:20 or 1:20pm is incorrect

(b)

M1 Sets $h=3$ and proceeds to $\cos(30t - 40)^\circ = \dots$. Accept inequalities in place of = signA1 Proceeds by taking invcos to reach either $30t - 40 = \text{awrt } 106.3$, or $30t - 40 = \text{awrt } 253.7$ Accept inequalities in place of = sign. This may be implied by a correct answer for $t = \text{awrt } 4.9, (9.8)$ A1 One value for t correct. Accept either awrt 1 dp $t = 4.9$ or 9.8 M1 For the correct method of finding a second value of t .Accept $30t - 40 = (360 - \alpha) \Rightarrow t = \dots$ where α is their principal valueA1 Both values of t correct awrt 1 dp. $t = 4.9$ and 9.8 . Ignore any values where $t \geq 12$ but withhold this mark for extra values in the range. These may be implied by 293 minutes and 587 minutes

A1 cso Both 04:53 and 09:47. Accept both 0453 and 0947

Accept 4:53 and 9:47 without the am as the question requires morning times.

Accept 293 and 587 minutes

If they state between 0 and 293 minutes and 587 and ... minutes it is A0

14.

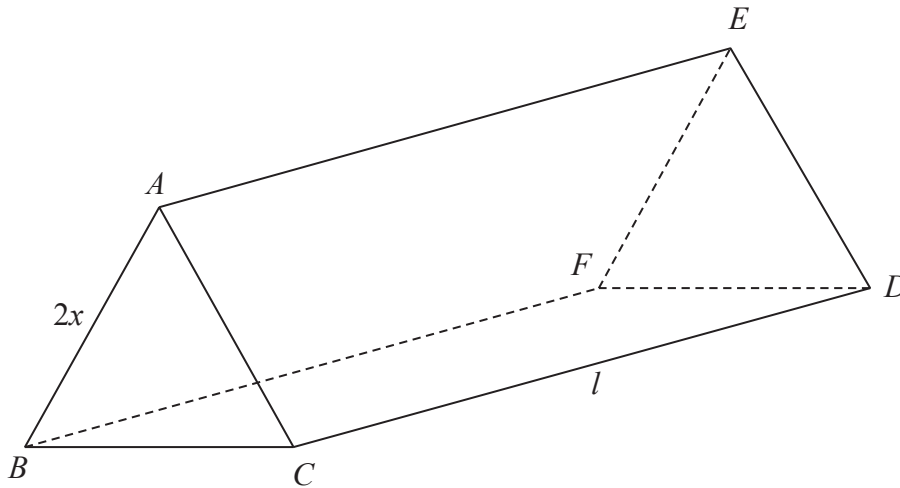


Figure 6

Figure 6 shows a solid triangular prism $ABCDEF$ in which $AB = 2x$ cm and $CD = l$ cm.

The cross section ABC is an equilateral triangle.

The rectangle $BCDF$ is horizontal and the triangles ABC and DEF are vertical.

The total surface area of the prism is S cm² and the volume of the prism is V cm³.

(a) Show that $S = 2x^2\sqrt{3} + 6xl$ (3)

Given that $S = 960$,

(b) show that $V = 160x\sqrt{3} - x^3$ (5)

(c) Use calculus to find the maximum value of V , giving your answer to the nearest integer. (5)

(d) Justify that the value of V found in part (c) is a maximum. (2)



Question Number	Scheme	Marks
14(a)	Area of triangle = ' $\frac{1}{2}ab \sin C$ ' = $\frac{1}{2} \times 2x \times 2x \times \sin 60 = \sqrt{3}x^2$ $S = 2 \times \sqrt{3}x^2 + 3 \times 2xl = 2x^2\sqrt{3} + 6xl$	M1 dM1A1* (3)
(b)	$960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \frac{960 - 2x^2\sqrt{3}}{6x}$ $V = x^2\sqrt{3}l$ Substitute $l = \frac{960 - 2x^2\sqrt{3}}{6x}$ into $V = x^2\sqrt{3}l$ $\Rightarrow V = x^2\sqrt{3} \times \left(\frac{960 - 2x^2\sqrt{3}}{6x} \right) = 160x\sqrt{3} - x^3$	M1A1 B1 dM1A1* (5)
(c)	$\frac{dV}{dx} = 160\sqrt{3} - 3x^2 = 0$ $\Rightarrow x = \text{awrt } 9.6$ $\Rightarrow V = 160 \times 9.611 \times \sqrt{3} - 9.611^3 = 1776$	M1A1 A1 dM1 A1 (5)
(d)	$\frac{d^2V}{dx^2} = -6x < 0 \Rightarrow \text{Maximum}$	M1A1 (2)
		(15 marks)

(a) E pen has this part marked M1 A1 A1*. We are going to score it M1dM1A1*

M1 Score for an acceptable method for finding the area of the triangle. You may have to look at the diagram for evidence.

Using $\frac{1}{2}ab \sin C$ sight of $\frac{1}{2} \times 2x \times 2x \times \sin 60$ is sufficient or $\frac{1}{2} \times 2x \times 2x \times \frac{\sqrt{3}}{2}$ with 60° on the diagram.

Use of $\frac{1}{2}bh$ is fine with Pythagoras' theorem being used to find h using $h^2 + x^2 = (2x)^2$

Accept $\frac{1}{2} \times 2x \times \sqrt{3}x$ with $\sqrt{3}x$ marked as a perpendicular height on the diagram

dM1 For adding the area of 3 rectangles and two triangles. It is dependent upon the previous M mark.

Look for something like $3 \times 2xl + 2 \times \frac{1}{2} \times 2x \times 2x \times \sin 60$ before the given answer

A1* For completing the proof with no errors. Accept it written in a different order. Eg $S = 2\sqrt{3}x^2 + 6xl$

This is a proof so the expectation is that they show that the area of the triangle is $\sqrt{3}x^2$

But accept for special case 1,0,0 $S = 3 \times 2xl + 2 \times \frac{1}{2} \times 2x \times 2x \times \frac{\sqrt{3}}{2}$ or $S = 3 \times 2xl + 2 \times \frac{1}{2} \times 2x \times \sqrt{3}x$

(b)

M1 Substitutes $S = 960$ into $S = 2x^2\sqrt{3} + 6xl$ and attempts to make l or lx the subject of the formula.

Accept as evidence $960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \dots$ or $960 = 2x^2\sqrt{3} + 6xl \Rightarrow lx = \dots$

A1 $l = \frac{960 - 2x^2\sqrt{3}}{6x}$ oe $l = \frac{160}{x} - \frac{x\sqrt{3}}{3}$ or $lx = \frac{960 - 2x^2\sqrt{3}}{6}$ oe $lx = 160 - \frac{x^2\sqrt{3}}{3}$

B1 States that the volume V is $V = x^2\sqrt{3}l$. The order of the terms is not important.

It may be implied by $V = x^2\sqrt{3} \times \text{their } l$ or $V = x\sqrt{3} \times \text{their } lx$

dM1 Substitute their expression for $l = \dots$ (or $lx = \dots$) into their expression for $V = \dots$ to obtain an expression for V just in terms of x . It is dependent upon the previous M having been awarded.

A1* This is a given answer. All aspects of the proof must be correct. $V = 160x\sqrt{3} - x^3$

(c) For the purpose of marking parts(c) and (d) can be scored together

M1 For differentiating with at least one term correct and setting $=0$.

A1 $160\sqrt{3} - 3x^2 = 0$. Accept awrt $277 - 3x^2 = 0$

A1 $x = \text{awrt } 9.6$. This appearing on its own is A0. Accept versions of $x = \sqrt{\frac{160\sqrt{3}}{3}}$

The question states that calculus must be used and a correct expression for $\frac{dV}{dx}$ must be seen.

dM1 Dependent upon the previous M, this is for subbing their answer to $\frac{dV}{dx} = 0$ in $V = 160x\sqrt{3} - x^3$ to find

V .

A1 $V = \text{awrt } 1776$

(d)

M1 Either attempts to find the value of $\frac{d^2V}{dx^2} = -6x$ (or kx following a slip), **and** substitutes the value of x found in part (c),

Or attempts the statement $\frac{d^2V}{dx^2} = -6x$ **and** states it is less than 0

A1 Either $\frac{d^2V}{dx^2} = -6x \Rightarrow \frac{d^2V}{dx^2} < 0$ when $x = \dots$, hence a maximum. Allow on any (positive) value of x found in (c). For this to be scored $\frac{d^2V}{dx^2} = -6x$ must be correct, it must be calculated (or implied to be calculated) at the value of x found in part (c), and there must be a reason $\left(\frac{d^2V}{dx^2} < 0\right)$ and a conclusion, hence a maximum.

Or also allow $\frac{d^2V}{dx^2} = -6x = -6 \times \text{positive} = \text{negative}$ followed by a reason $\left(\frac{d^2V}{dx^2} < 0\right)$ and a conclusion, hence a maximum.

There are alternatives using values of the gradient and values of the function.

Using the gradient

M1 Attempts to find the numerical value of the gradient $\frac{dV}{dx}$ either side of the x value found in part (c)

A1 $\frac{dV}{dx} = 160\sqrt{3} - 3x^2$ must be correct, it must be calculated either side, there must be a reason referring to gradient being +ve, 0 and -ve, followed by a conclusion, hence maximum

Using the function

M1 Attempts to find the value of V either side of the value of x found in part (c), **and** starts to compare these with the value of V found in part (c).

A1 Correct numerical values Eg $V_{9.60} = 1775.694$, $V_{9.61} = 1775.698$, $V_{9.62} = 1775.695$ followed by a reason which could include a sketch and a conclusion, hence maximum.