Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 7 November 2018

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Core Mathematics C34

Advanced

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. (a) Write $\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < \pi$, the equation

$$\cos 2\theta + 4\sin 2\theta = 1.2$$

giving your answers to 2 decimal places.

(5)

Question Number	Scheme	Marks
1(a)	$\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$	
	$R = \sqrt{1^2 + 4^2} = \sqrt{17}$	B1
	$\alpha = \arctan 4 = \text{awrt } 1.326$	M1A1
		(3)
(b)	$\sqrt{17}\cos(2\theta - 1.326) = 1.2 \Rightarrow \cos(2\theta - 1.326) = \frac{1.2}{\sqrt{17}}$	M1
	$\Rightarrow (2\theta - 1.326) = \pm 1.275.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 1.30 \text{ or awrt } 0.03$	A1
	$2\theta - 1.326 = '1.275...'..$ **and** $'-1.275...'$	ddM1
	$\Rightarrow \theta = \text{awrt } 1.30$ **and** 0.03	A1
		(5)
		[8 marks]

(a)

B1 For $R = \sqrt{17}$. Condone $R = \pm\sqrt{17}$

M1 For $\alpha = \arctan(\pm 4)$ or $\alpha = \arctan\left(\pm\frac{1}{4}\right)$ leading to a solution of α

It is implied by $\alpha = \text{awrt } 76°$ or awrt 1.3 rads

Condone any solutions coming from $\cos\alpha = 1, \sin\alpha = 4$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{1}{'R'}\right)$ $\alpha = \arcsin\left(\pm\frac{4}{'R'}\right)$

A1 $\alpha = \text{awrt } 1.326$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their}1.326) = \frac{1.2}{\text{their } R}$.

Condone slips on the 1.2 and miscopying their 1.326

This may be implied by $(2\theta \pm \text{their}1.326) = \arccos\left(\frac{1.2}{\text{their } R}\right)$

Condone for this mark $\cos(\theta \pm \text{their}1.326) = \frac{1.2}{\text{their } R}$ or $\cos(2\theta \pm 2\times\text{their}1.326) = \frac{1.2}{\text{their } R}$

but $2\cos(\theta \pm \text{their}1.326) = \frac{1.2}{\text{their } R}$ is M0 and hence dM0…etc

dM1 Dependent upon the first M1. It is for a full method to find one value of θ within the range 0 to π from their principal value. Look for the correct order of operations, that is dealing with the "1.326" before the "2". Condone adding 1.326 instead of subtracting.

$$\cos(2\theta \pm \text{their}1.326) = ... \Rightarrow 2\theta \pm \text{their}1.326 = \beta \Rightarrow \theta = \frac{\beta \pm \text{their}1.326}{2}$$

A1 awrt $\theta = 1.30$ or $\theta =$ awrt 0.03 Only allow 1.3 if it is preceded by an answer that rounds to 1.30

ddM1 For a correct method to find a second value of θ (**for their** α) in the range 0 to π.
Eg $2\theta \pm 1.326 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 1.326 = 2\pi + '\beta' \Rightarrow \theta =$ THEN MINUS π

A1 awrt $\theta = 1.30$ and $\theta =$ awrt 0.03 . Only allow 1.3 if it is preceded by an answer that rounds to 1.30
Withhold this mark if there are extra solutions **in the range**.

Degree solution: Only lose the first time it occurs.

FYI. In degrees only lose the first A mark awrt (a) $\alpha = 75.964°$ and (b) $\theta_1 = 74.52°, \theta_2 = 1.44°$
Mixing degrees and radians only scores the first M in part (b)

Answers without working.
If $\sqrt{17}\cos(2\theta - 1.326) = 1.2$ is written down then all marks are available. (3 marks for one correct answer)
If there is no initial statement then score SC B1 B1 then 0,0, 0 for a maximum of 2, 1 for each solution.

Leave blank

2. A curve C has equation

$$x^3 - 4xy + 2x + 3y^2 - 3 = 0$$

Find an equation of the normal to C at the point $(-3, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(7)

Question Number	Scheme	Marks
2.	$x^3-4xy+2x+3y^2-3=0 \Rightarrow 3x^2-4x\frac{dy}{dx}-4y+2+6y\frac{dy}{dx}=0$	B1 M1 A1
	Substitute $(-3,2)$ $\Rightarrow \frac{dy}{dx}=\left(-\frac{7}{8}\right)$	M1
	Uses gradient of normal = $-\frac{1}{\left.\frac{dy}{dx}\right\vert_{x=-3}}$	dM1
	$y-2=\frac{8}{7}(x+3) \Rightarrow 8x-7y+38=0$	M1, A1
		[7 marks]

B1 Applies the product rule to $-4xy$ $-4xy \to -4x\frac{dy}{dx}-4y$

Accept exact alternatives such as $-4xy \to -4\left(x\frac{dy}{dx}+y\right)$ and allow if recovered from poor bracketing. You may see $-4xy \to -4x\,dy-4y\,dx$

M1 Attempts the chain rule to $3y^2 \to Ay\frac{dy}{dx}$

You may see $3y^2 \to Ay\,dy$

A1 For correct differentiation on $x^3+2x+3y^2-3 \Rightarrow 3x^2+2+6y\frac{dy}{dx}$

You may see $x^3+2x+3y^2-3 \Rightarrow 3x^2dx+2dx+6ydy$

M1 Substitutes $(-3,2)$ into a differentiated form and attempts to find a numerical value of $\frac{dy}{dx}$

It is dependent upon the differentiated form having **exactly** two terms in $\frac{dy}{dx}$, **one** from $-4xy$ and **one** from $3y^2$ If the candidate attempts to rearrange and collect terms before substituting you may condone poor algebra.

dM1 Attempts to find a numerical value to the gradient of the normal. It is dependent upon the previous method mark and finding the negative reciprocal of the value of $\left.\frac{dy}{dx}\right|_{x=3}$

M1 Correct attempt at the form of the normal at $(-3,2)$ Eg. $y-2=$ their "$-\frac{1}{m}$"$(x+3)$

Condone one sign slip on the -2 or the +3. Condone, for this method, answers from poor differentiation (eg just having one $\frac{dy}{dx}$ term). This mark is for the method of finding the equation of a normal. If the form $y=mx+c$ is used it is for proceeding as far as $c=$ ...

A1 $8x-7y+38=0$ Allow $k(8x-7y+38=0)$ where k is an integer

Note that the error $-4xy \to 4x\frac{dy}{dx}-4y$ can lead to $\frac{dy}{dx}=\infty$ which in turn gives a normal of $y=2$

This can potentially score B0 M1 A1 M1 dM1 M1 A0 for 5 out of 7

Leave blank

3. Given

$$\cos\theta° = p\text{, where } p \text{ is a constant and } \theta° \text{ is acute}$$

use standard trigonometric identities to find, in terms of p,

(a) $\sec\theta°$ **(1)**

(b) $\sin(\theta - 90)°$ **(2)**

(c) $\sin 2\theta°$ **(3)**

Write each answer in its simplest form.

Question Number	Scheme	Marks
3.(a)	$\sec\theta° = \dfrac{1}{\cos\theta°} = \dfrac{1}{p}$ oe	B1
		(1)
(b)	$\sin(\theta-90)° = \sin\theta°\cos 90° - \cos\theta°\sin 90° = -p$	M1A1
		(2)
(c)	$\sin 2\theta° = 2\sin\theta°\cos\theta°$	B1
	Uses $\sin\theta° = \sqrt{1-\cos^2\theta°} = \sqrt{1-p^2}, \Rightarrow \sin 2\theta° = 2p\sqrt{1-p^2}$	M1, A1
		(3)
		[6 marks]

(a)

B1 $\sec\theta° = \dfrac{1}{p}$ Accept $\sec\theta° = p^{-1}$

(b)

M1 Attempts $\sin(\theta-90)° = \sin\theta°\cos 90° \pm \cos\theta°\sin 90°$ with $\cos\theta° = p$ used

Alternatively uses $\sin(\theta-90)° = -\sin(90-\theta)° = -(\sin 90°\cos\theta° - \cos 90°\sin\theta°)$ with $\cos\theta° = p$ used

Similarly $\sin(\theta-90)° = -\sin(90-\theta)° = -\cos\theta°$ with $\cos\theta° = p$ used

Or $\sin(\theta-90)° = \sin\theta°\cos{-90°} \pm \cos\theta°\sin{-90°}$ with $\cos\theta° = p$ used

A1 $\sin(\theta-90)° = -p$.

Allow $-p$ for both marks as long as no incorrect work is used to generate this answer.

(c)

B1 States $\sin 2\theta° = 2\sin\theta°\cos\theta°$ or $\sin 2\theta° = \sin\theta°\cos\theta° + \sin\theta°\cos\theta°$

M1 Attempts $\sin^2\theta + \cos^2\theta = 1$ in part (c) with $\cos\theta° = p$ to get $\sin\theta°$ in terms of p.

Only accept $\sin\theta° = 1-p$ if a version involving squares has been seen first.

Allow $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin\theta = \sqrt{1-p}$ as a slip. (We have seen the Pythagorean identity)

You may see an attempt using a right angled triangle. The same scheme may be applied.

A1 $\sin 2\theta° = 2p\sqrt{1-p^2}$ but NOT $\sin 2\theta° = \pm 2p\sqrt{1-p^2}$

or equivalent such as $\sin 2\theta° = 2\sqrt{p^2-p^4}$, $2p\sqrt{(1+p)(1-p)}$ or $\sqrt{4p^2-4p^4}$

Final answer (do not isw here).

$\sin 2\theta° = 2p\sqrt{1-p^2} = 2p(1-p)$ is B1 M1 A0

....

B1 Alternatively attempts to use $\sin^2 2\theta + \cos^2 2\theta = 1$ and $\cos 2\theta = 2\cos^2\theta - 1$

M1 $\sin 2\theta = \sqrt{1-\cos^2 2\theta} = \sqrt{1-(1-2p^2)^2}$

A1 Usually $\sqrt{4p^2-4p^4}$ via this method

....

Leave blank

4.

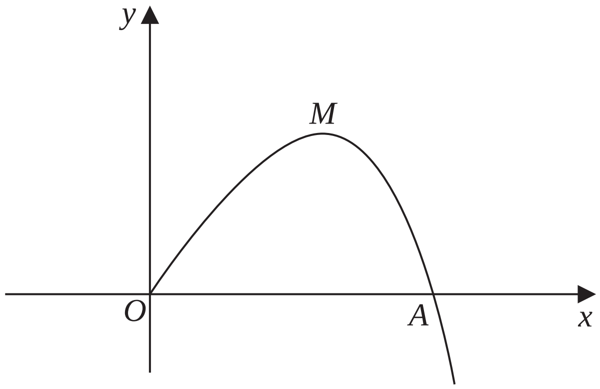


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 8x - xe^{3x}$, $x \geqslant 0$

The curve meets the x-axis at the origin and cuts the x-axis at the point A.

(a) Find the exact x coordinate of A, giving your answer in its simplest form.

(2)

The curve has a maximum turning point at the point M.

(b) Show, by using calculus, that the x coordinate of M is a solution of

$$x = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$$

(5)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\ln\left(\frac{8}{1+3x_n}\right)$$

with $x_0 = 0.4$ to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

Question Number	Scheme	Marks
4 (a)	Sets $8x - xe^{3x} = 0$	
	$\Rightarrow e^{3x} = 8 \Rightarrow 3x = \ln 8 \Rightarrow x = \frac{1}{3}\ln 8 = \ln 2$	M1A1
		(2)
(b)	$\frac{dy}{dx} = 8 - \left(\underline{3xe^{3x} + e^{3x}}\right)$	M1 A1
	Sets $\frac{dy}{dx} = 0 \Rightarrow (1+3x)e^{3x} = 8$	M1
	$\Rightarrow e^{3x} = \frac{8}{(1+3x)} \Rightarrow x = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$	dM1, A1*
		(5)
(c)	$x_1 = \frac{1}{3}\ln\left(\frac{8}{1+3\times 0.4}\right) =$ awrt 0.430	M1A1
	$x_2 =$ awrt 0.417, $x_3 =$ awrt 0.423	A1
		(3)
		[10 marks]

(a)

M1 Attempts to solve $e^{3x} = 8$ using a correct order of operations. (Eg. some may take $\sqrt[3]{}$ first)
Allow for $3x = \ln 8$ Condone a slip on the 8. It may be implied by answers awrt 0.693

A1 $x = \ln 2$ Note that $x = \frac{1}{3}\ln 8$ is M1 A0

(b)

M1 Attempts to differentiate the $-xe^{3x}$ term to $\pm Axe^{3x} \pm Be^{3x}$ using the product rule.
If the rule is quoted it must be correct.

A1 Correct derivative $\frac{dy}{dx} = 8 - \left(3xe^{3x} + e^{3x}\right)$ with correct bracketing or $\frac{dy}{dx} = 8 - 3xe^{3x} - e^{3x}$

M1 States or sets $\frac{dy}{dx} = 0$ (which may be implied) and takes out a common factor of e^{3x} reaching a form $(...\pm...)e^{3x} = ...$ or $e^{3x} = \frac{...}{(...\pm...)}$

dM1 Dependent upon the **both** previous M's, it is scored for using correct ln work, moving from
$(...\pm...)e^{3x} = ... \Rightarrow x =$ or $e^{3x} = \frac{...}{(...\pm...)} \Rightarrow x =$

A1* Reaches $x = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$, $x = \frac{1}{3}\ln\left(\frac{8}{3x+1}\right)$ or $x = \frac{1}{3}\ln\frac{8}{3x+1}$ oe with correct work and no errors or omissions (See scheme for necessary steps that need to be seen)

$\frac{dy}{dx} = 8 - \left(\underline{3xe^{3x} + e^{3x}}\right) \Rightarrow 3xe^{3x} + e^{3x} = 8$ is an example where there is missing step. (No $\frac{dy}{dx} = 0$)

$\frac{dy}{dx} = 0 \Rightarrow 3xe^{3x} + e^{3x} = 8 \Rightarrow e^{3x} = \frac{8}{3x+1}$ is also an example where there is missing step. (No attempt to show the factorised line $(1+3x)e^{3x} = 8$)

$\frac{dy}{dx} = 0 \Rightarrow 3xe^{3x} + e^{3x} = 8 \Rightarrow 3x+1.e^{3x} = 8 \Rightarrow e^{3x} = \frac{8}{3x+1}$ is also an example where there is a missing bracket (for factorisation)

……..

If the first M1 isn't scored for an attempt at the product rule a special case M0A0M1dM0 A0 may be awarded for setting their $\frac{dy}{dx} = 0$ and proceeding to a form $e^{3x} = ...$

For example, if $\frac{dy}{dx} = 8 - 3xe^{3x}$ it would be for proceeding to $e^{3x} = \frac{8}{3x}$

……….

(c)

M1 Calculates x_1 from the given iterative formula. May be implied by $\frac{1}{3}\ln\left(\frac{8}{1+3\times 0.4}\right)$ or awrt 0.43

A1 awrt 0.430 Allow answer written as 0.43

A1 awrt $x_2 = 0.417$, $x_3 = 0.423$. NB. The subscripts are not important.

Leave blank

5.

$$\mathrm{f}(x) = \frac{4x^2 + 5x + 3}{(x+2)(1-x)^2} \equiv \frac{A}{(x+2)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(ii) Find the exact value of $\int_0^{\frac{1}{2}} \mathrm{f}(x)\,\mathrm{d}x$, writing your answer in the form $p + \ln q$, where p and q are constants.

(6)

Question Number	Scheme	Marks
5. (a)	$4x^2+5x+3 = A(1-x)^2 + B(x+2)(1-x) + C(x+2)$	B1
	Sub $x=1$ $\quad C=4$	M1
	$x=-2 \quad \Rightarrow A=1$ $\quad$ any two constants correct	A1
	Coefficients of x^2	
	$4 = A-B \Rightarrow B=-3$ $\quad$ all three constants correct	A1
		(4)
(b)(i)	$\int\left(\frac{1}{(x+2)}-\frac{3}{(1-x)}+\frac{4}{(1-x)^2}\right)dx = \underline{\ln(x+2)+3\ln(1-x)}+4(1-x)^{-1} \quad (+c)$ oe	<u>M1</u> M1 A1ft
(ii)	$\int_0^{\frac{1}{2}} \frac{4x^2+5x+1}{(x+2)(1-x)^2}\,dx = \left[\ln(x+2)+3\ln(1-x)+4(1-x)^{-1}\right]_0^{\frac{1}{2}}$	
	$=\left(\ln\frac{5}{2}+3\ln\frac{1}{2}+8\right)-(\ln 2+3\ln 1+4)$	M1
	$=\ln\left(\frac{\frac{5}{2}\times\left(\frac{1}{2}\right)^3}{2}\right)+\ \ldots$	M1
	$=4+\ln\left(\frac{5}{32}\right)$	A1 **(6)**
		[10 marks]

(a)

B1 Writes $4x^2+5x+3=A(1-x)^2+B(x+2)(1-x)+C(x+2)$

This may be implied by the sight of two equivalent fractions or via work leading to the constants

M1 Substitutes $x=1$ or $x=-2$ or equivalent and attempts to find the value of one constant.
It can be scored after scoring B0.

Eg condone the use of $4x^2+5x+3=A(1-x)^2(1-x)+B(x+2)(1-x)+C(x+2)$ or similar

Alternatively attempts to equate coefficients of x^2, x and constant terms to produce and solve simultaneous equations to find the value of one constant.

A1 Any two constants correct

A1 All three constants correct

(b)(i)

M1 For $\int \frac{A}{x+2} \to ..\ln(x+2)$ and $\int \frac{B}{1-x} \to ..\ln(1-x)$

M1 For $\int \frac{C}{(1-x)^2} \to ..(1-x)^{-1}$

A1ft All three of their integrals correct, following through on incorrect constants (but not zero's)

There must be some attempt to write in the simplest form. (Cannot leave $--\ 4(1-x)^{-1}$ for instance)

(b)(ii)

M1 Substitutes both $x=\frac{1}{2}$ and $x=0$ into their answer for (b)(i) which involves lns and subtracts (either way around).

M1 Uses correct ln work to combine **their** ln terms

A1 cao $=4+\ln\left(\frac{5}{32}\right)$ Note that the decimal equivalent $4+\ln 0.15625$ is correct

Leave blank

6. (a) Use binomial expansions to show that, for $|x| < \frac{1}{2}$

$$\sqrt{\frac{1+2x}{1-x}} \approx 1 + \frac{3}{2}x + \frac{3}{8}x^2$$

(6)

(b) Find the exact value of $\sqrt{\frac{1+2x}{1-x}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{3}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into the expansion given in part (a) and hence find an approximate value for $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(2)

Question Number	Scheme	Marks
6. (a)	$\left\{\sqrt{\left(\frac{1+2x}{1-x}\right)}\right\} = (1+2x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$=\left(1+\left(\frac{1}{2}\right)(2x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(2x)^2+...\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+...\right)$	M1 A1
	$=\left(1+x-\frac{1}{2}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$	A1
	$=1+\frac{1}{2}x+\frac{3}{8}x^2+x+\frac{1}{2}x^2-\frac{1}{2}x^2+...$	M1
	$=1+\frac{3}{2}x+\frac{3}{8}x^2$.	A1 *
		(6)
(b)	$\sqrt{\left(\frac{1+(\frac{2}{10})}{1-(\frac{1}{10})}\right)}=\frac{2}{3}\sqrt{3}$	B1
		(1)
(c)	Sub $x=\frac{1}{10}\Rightarrow "k"\sqrt{3}=1+\frac{3}{2}\left(\frac{1}{10}\right)+\frac{3}{8}\left(\frac{1}{10}\right)^2$	M1
	so, $\sqrt{3}\approx\frac{2769}{1600}$	A1
		(2) **[9 marks]**
Alt 6. (a)	$\left\{\sqrt{\left(\frac{1+2x}{1-x}\right)}\right\}=1+\frac{3}{2}x+\frac{3}{8}x^2\Rightarrow(1+2x)^{\frac{1}{2}}=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$\Rightarrow\left(1+\left(\frac{1}{2}\right)(2x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(2x)^2+...\right)=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)\left(1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2\right.$ -	M1 A1 A1
	$\Rightarrow\left(1+x-\frac{1}{2}x^2+...\right)=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$\Rightarrow\left(1+x-\frac{1}{2}x^2+...\right)=1+x-\frac{1}{2}x^2$ Hence true	M1A1*
		(6)

(a)

B1 For writing the given expression in index form $(1+2x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ It may be implied by working but it must be a form that can lead to the answer.

Do not allow $\frac{(1+2x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$ for this mark unless the expanded $(1-x)^{\frac{1}{2}}$ is subsequently set with index -1

M1 Score for the form of the binomial expansion with index $\frac{1}{2}$ or $-\frac{1}{2}$

Eg $=\left[1+\left(\frac{1}{2}\right)(**x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(**x)^2+...\right]$ or $=\left[1+\left(-\frac{1}{2}\right)(**x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(**x)^2+...\right]$

A1 Correct unsimplified form for one expression

A1 The correct simplified form for both expressions seen.
This mark may be implied if a correct final expression is found following correct working.
There doesn't need to be an implication that these expressions are to be multiplied.

M1 For multiplying terms in the first expansion by terms in the second expansion.
Expect to see an attempt to find the six terms required to produce the given solution. Allow terms in x^3 and greater to be seen which don't need to be correct.
Follow through on their expansions but condone "changes" in an attempt to reach the given solution. (See Practice Items)

A1* Correct solution only $= 1 + \frac{3}{2}x + \frac{3}{8}x^2$

(b)

B1 $\frac{2}{3}\sqrt{3}$ or statement $k = \frac{2}{3}$ seen in (b)

(c)

M1 Sub $x = \frac{1}{10}$ into both sides of (a) $\Rightarrow$ "k"$\sqrt{3} = 1 + \frac{3}{2}\left(\frac{1}{10}\right) + \frac{3}{8}\left(\frac{1}{10}\right)^2$ or $\sqrt{\frac{1 + 2/10}{1 - 1/10}} = 1 + \frac{3}{2}\left(\frac{1}{10}\right) + \frac{3}{8}\left(\frac{1}{10}\right)^2$

Do not allow $k = 1$

A1 $\sqrt{3} \approx \frac{2769}{1600}$ or exact equivalent. Condone $\sqrt{3} \approx \frac{1600}{923}$ which follows (b) $= \frac{2}{\sqrt{3}}$

....

You may see a variety of solutions to part (a). Please consider carefully when marking.
Example: Mark in this order

B1: $\left\{\sqrt{\left(\frac{1 + 2x}{1 - x}\right)}\right\} = \sqrt{\left(\frac{1 - x + 3x}{1 - x}\right)} = \sqrt{\left(1 + \frac{3x}{1 - x}\right)} = \left(1 + \frac{3x}{1 - x}\right)^{\frac{1}{2}}$

M1: For one attempt at the binomial expansion $\left(1 + \frac{*x}{1 - x}\right)^{\frac{1}{2}} = 1 + \frac{1}{2} \times \frac{*x}{1 - x} + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{*x}{1 - x}\right)^2$

condoning slips on the bracketing

A1: Completely correct intermediate form. $\left(1 + \frac{3x}{1 - x}\right)^{\frac{1}{2}} = 1 + \frac{1}{2} \times \frac{3x}{1 - x} + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{3x}{1 - x}\right)^2$

M1: For a second use of the binomial expansion. It is dependent upon a correct first use.

$$\left(1 + \frac{*x}{1 - x}\right)^{\frac{1}{2}} = 1 + \frac{*x}{2} \times (1 - x)^{-1} - \frac{*x^2}{8}(1 - x)^{-2} = 1 + \frac{*x}{2} \times \left(1 + x + (x^2)\right) - \frac{*x^2}{8} \times \left(1 + (2x + 3x^2)\right)$$

Expect to see a correct use of the binomial expansion in both TERMS.

A1: $\left(1 + \frac{*x}{1 - x}\right)^{\frac{1}{2}} = 1 + \frac{3x}{2} \times (1 - x)^{-1} - \frac{9x^2}{8}(1 - x)^{-2} = 1 + \frac{3x}{2} \times (1 + x + ...) - \frac{9x^2}{8} \times (1 + ...)$

A1: $= 1 + \frac{3}{2}x + \frac{3}{8}x^2$

Leave blank

7. A curve has equation

$$y = \ln(1 - \cos 2x), \quad x \in \mathbb{R}, 0 < x < \pi$$

Show that

(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = k\cot x$, where k is a constant to be found.

(4)

Hence find the exact coordinates of the point on the curve where

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}$

(4)

Question Number	Scheme	Marks
7 (a)	$y=\ln(1-\cos 2x) \Rightarrow \dfrac{dy}{dx}=\dfrac{2\sin 2x}{1-\cos 2x}$	M1A1
	$\Rightarrow \dfrac{dy}{dx}=\dfrac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}, =\dfrac{4\sin x\cos x}{2\sin^2 x}=2\cot x$	M1, A1
		(4)
(b)	$2\cot x=2\sqrt{3} \Rightarrow \tan x=\dfrac{1}{\sqrt{3}}$	
	$x=\arctan\left(\dfrac{1}{\sqrt{3}}\right) \Rightarrow x=\dfrac{\pi}{6}$	M1A1
	$y=\ln\left(1-\cos\left(\dfrac{2\pi}{6}\right)\right)=\ln\dfrac{1}{2}$ or $-\ln 2$	M1A1
		(4) **[8 marks]**
7 (a)alt I	$y=\ln(1-\cos 2x) \Rightarrow y=\ln(2\sin^2 x) \Rightarrow \dfrac{dy}{dx}=\dfrac{4\sin x\cos x}{2\sin^2 x}$	M1A1
	$\Rightarrow \dfrac{dy}{dx}=2\dfrac{\cos x}{\sin x}=2\cot x$	M1, A1
		(4)
7 (a)alt II	$y=\ln(2\sin^2 x) \Rightarrow y=\ln 2+2\ln\sin x \Rightarrow \dfrac{dy}{dx}=0+\dfrac{2\cos x}{\sin x}$	M1A1
	$\Rightarrow \dfrac{dy}{dx}=2\dfrac{\cos x}{\sin x}=2\cot x$	M1, A1
		(4)
7 (a)alt III	$y=\ln(1-\cos 2x) \Rightarrow e^y=1-\cos 2x \Rightarrow e^y\dfrac{dy}{dx}=2\sin 2x$	M1A1
	Then as main scheme	M1, A1
		(4)

(a)

M1 For differentiating $y=\ln(1-\cos 2x)$ to $\dfrac{dy}{dx}=\dfrac{\pm A\sin 2x}{1-\cos 2x}$

A1 $\dfrac{dy}{dx}=\dfrac{2\sin 2x}{1-\cos 2x}$ oe

M1 Uses the double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$

The double angle for $\cos 2x$ may be implied by sight of "$1-1-2\sin^2 x$" on the denominator as we can condone the missing bracket.

A1 Simplifies to show that $\Rightarrow \dfrac{dy}{dx}=2\cot x$ showing at least one correct intermediate line between

$\dfrac{dy}{dx}=\dfrac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$ usually $\dfrac{4\sin x\cos x}{2\sin^2 x}$ or $\dfrac{2\cos x}{\sin x}$

In the alternative versions the double angle identity (or identities) are seen before the differentiation.

For example.
(i) This one shows incorrect differentiation and is scored M0 A0 M1 A0

$$y = \ln(1-\cos 2x) \Rightarrow y = \ln(2\sin^2 x) \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin^2 x}$$

(ii) This one can be awarded the method mark for differentiation and is scored M1 A0 M1 A0

$$y = \ln(1-\cos 2x) \Rightarrow y = \ln(2\sin^2 x) \Rightarrow \frac{dy}{dx} = \frac{2\sin x\cos x}{2\sin^2 x} = \cot x$$

The two accuracy marks are linked and cannot really be awarded separately M1 A1 M1 A0 would be difficult to score via this method.

(b)

M1 Uses $\cot x = \frac{1}{\tan x}$ and proceeds to find x

A1 $x = \frac{\pi}{6}$. Ignore additional (incorrect) values such as $x = \frac{5\pi}{6}$

Do not accept $30°$ for this mark. Do not allow this for candidates who guess $k = 2$

M1 Substitutes their value of x in $y = \ln(1-\cos 2x)$

A1 For $y = \ln\frac{1}{2}$ or $-\ln 2$ following $x = \frac{\pi}{6}$ or $30°$

Do not allow this for candidates who guess $k = 2$

Withhold this final mark if there is another value (x, y) given within the range

Leave blank

8. (i) Find $\int x \sin x \, \mathrm{d}x$

(3)

(ii) (a) Use the substitution $x = \sec\theta$ to show that

$$\int_1^2 \sqrt{1 - \frac{1}{x^2}} \, \mathrm{d}x = \int_0^{\frac{\pi}{3}} \tan^2\theta \, \mathrm{d}\theta$$

(3)

(b) Hence find the exact value of

$$\int_1^2 \sqrt{1 - \frac{1}{x^2}} \, \mathrm{d}x$$

(4)

Question Number	Scheme	Marks
8(i)	$\int x\sin x\,\mathrm{d}x = -x\cos x + \int \cos x\,\mathrm{d}x$	M1
	$= -x\cos x + \sin x\,(+c)$	dM1A1
		(3)
(ii)(a)	$\mathrm{d}x = \sec\theta\tan\theta\,\mathrm{d}\theta$	B1
	$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \sqrt{1-\cos^2\theta}\sec\theta\tan\theta\,\mathrm{d}\theta$	
	$= \int_0^{\frac{\pi}{3}} \sqrt{\sin^2\theta} \times \frac{1}{\cos\theta}\tan\theta\,\mathrm{d}\theta = \int_0^{\frac{\pi}{3}} \tan^2\theta\,\mathrm{d}\theta$	M1 A1*
		(3)
(b)	$\int \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int \tan^2\theta\,\mathrm{d}\theta = \int \left(\sec^2\theta - 1\right)\mathrm{d}\theta = \tan\theta - \theta$	M1A1
	$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \left[\tan\theta - \theta\right]_{\theta=0}^{\theta=\frac{\pi}{3}} = \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$	dM1A1
		(4) **[10 marks]**

(i)

M1 Attempts integration by parts the correct way $\int x\sin x\,\mathrm{d}x = \pm x\cos x \pm \int \cos x\,\mathrm{d}x$

dM1 Integrates again to $\int x\sin x\,\mathrm{d}x = \pm x\cos x \pm \sin x \quad (+c)$

A1 $= -x\cos x + \sin x\,(+c)$ with no need for + c
$= -\cos x.x + \sin x\,(+c)$ can be accepted

Note that international centres sometimes teach a "tabular method" "D-I method"

	Diff	Int
+	x	$\sin x$
-	1	$-\cos x$
+	0	$-\sin x$

If there is no evidence of an intermediate line, a tabular method, or integration by parts score
M1 dM1 A1 for a fully correct answer
AND M0 dM0 A0 for an incorrect answer

(ii)(a)

B1 $\mathrm{d}x = \sec\theta\tan\theta\,\mathrm{d}\theta$ or equivalent. Eg accept $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \sec\theta\tan\theta$

M1 Substitutes $x = \sec\theta$ into $\sqrt{1-\frac{1}{x^2}}$ and simplifies to $\sin\theta$, $\frac{1}{\operatorname{cosec}\theta}$, $\frac{\tan\theta}{\sec\theta}$ or $\sqrt{\sin^2\theta}$ $\sqrt{\frac{1}{\operatorname{cosec}^2\theta}}$ $\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}$ This may be implied if the $\sqrt{1-\frac{1}{x^2}}$ has been adapted

A1* Completes proof.

This is a show that question and you should expect the $\sec\theta$ to be either replaced by $\frac{1}{\cos\theta}$ or allow to be cancelled as seen below

$$\int \frac{\tan\theta}{\sec\theta} \times \sec\theta\tan\theta\,\mathrm{d}\theta = \int \tan^2\theta\,\mathrm{d}\theta$$

Expect to see the correct limits and correct notation **within their solution.** (Condone incorrect notation in jottings/working at the side of their solution)

The limits can just appear without the need to see calculations.

A notational error is $\tan^2\theta$ being written $\tan\theta^2$ without a bracket

It is possible to do this question from rhs to lhs but marks in a similar way.

(ii)(b)

M1 $\int \tan^2\theta\,\mathrm{d}\theta = \int \left(\pm\sec^2\theta \pm 1\right)\mathrm{d}\theta = \pm\tan\theta \pm \theta$

A1 $\int \tan^2\theta\,\mathrm{d}\theta = \tan\theta - \theta$

dM1 Uses the limit(s) $\frac{\pi}{3}$ (and 0) in a function of the form $\pm\tan\theta \pm \theta$

A1 cso $\sqrt{3} - \frac{\pi}{3}$

Leave blank

9. A rare species of mammal is being studied. The population P, t years after the study started, is modelled by the formula

$$P = \frac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

Using the model,

(a) calculate the number of mammals at the start of the study,

(1)

(b) calculate the exact value of t when $P = 315$

Give your answer in the form $a \ln k$, where a and k are integers to be determined.

(4)

(c) (i) Find $\frac{dP}{dt}$

(ii) Hence find the value of $\frac{dP}{dt}$ when $t = 8$, giving your answer to 2 decimal places.

(4)

Question Number	Scheme	Marks
9 (a)	$P=450$	B1 **(1)**
(b)	$315=\dfrac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t}-1}\Rightarrow 45e^{\frac{1}{4}t}=315$	M1 A1
	$\Rightarrow e^{\frac{1}{4}t}=7\Rightarrow t=4\ln 7$	M1A1 **(4)**
(c)(i)	$\dfrac{dP}{dt}=\dfrac{\left(3e^{\frac{1}{4}t}-1\right)\times 225e^{\frac{1}{4}t}-900e^{\frac{1}{4}t}\times\frac{3}{4}e^{\frac{1}{4}t}}{\left(3e^{\frac{1}{4}t}-1\right)^2}=\left(\dfrac{-225e^{\frac{1}{4}t}}{\left(3e^{\frac{1}{4}t}-1\right)^2}\right)$	M1A1
(ii)	$\left.\dfrac{dP}{dt}\right\vert_{t=8}=\dfrac{\left(3e^2-1\right)\times 225e^2-900e^2\times\frac{3}{4}e^2}{\left(3e^2-1\right)^2}=\dfrac{-225e^2}{\left(3e^2-1\right)^2}=-3.71$	M1A1
		(4) **[9 marks]**

(a)

B1 $(P=)450$

(b)

M1 Substitutes $P=315$, cross multiplies to reach a form $Ae^{\frac{1}{4}t}=B$ oe

A1 $45e^{\frac{1}{4}t}=315$ or equivalent such as $e^{\frac{1}{4}t}=7$. Note $e^{-\frac{1}{4}t}=\frac{1}{7}$ is correct

M1 $e^{\frac{1}{4}t}=D\ (D>0)\Rightarrow t=\ldots$ using ln's. Allow equivalent working from $e^{-\frac{1}{4}t}=E,\ E>0$

Equivalent work may be seen. $\ln 45+\frac{1}{4}t=\ln 315\Rightarrow t=\ldots$

A1 $t=4\ln 7$ or equivalent such as $t=2\ln 49,\quad \ln 2401$ but not $t=4\ln\left(\frac{315}{45}\right)$ (Scheme requires $a\ln k$)

(c)(i)

M1 Attempts to apply the quotient rule on $P=\dfrac{900\mathrm{e}^{\frac{1}{4}t}}{3\mathrm{e}^{\frac{1}{4}t}-1}$ with $u=900\mathrm{e}^{\frac{1}{4}t}, v=3\mathrm{e}^{\frac{1}{4}t}-1$ to reach an expression of the required form (see below). Condone slips on the coefficients.

Score for $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)\times A\mathrm{e}^{\frac{1}{4}t}-900\mathrm{e}^{\frac{1}{4}t}\times B\mathrm{e}^{\frac{1}{4}t}}{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^2}$, $A, B>0$

Award this mark if the candidate incorrectly multiples out before writing down their $\dfrac{\mathrm{d}P}{\mathrm{d}t}$

$$u=900\mathrm{e}^{\frac{1}{4}t}, u'=225\mathrm{e}^{\frac{1}{4}t}, v=3\mathrm{e}^{\frac{1}{4}t}-1, v'=\frac{3}{4}\mathrm{e}^{\frac{1}{4}t}\Rightarrow\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)\times 225\mathrm{e}^{\frac{1}{4}t}-675\mathrm{e}^{\frac{1}{4}t^2}}{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^2}$$

Alternatively applies the product rule on $900\mathrm{e}^{\frac{1}{4}t}\times\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^{-1}$ or chain rule on $900\left(3-\mathrm{e}^{-\frac{1}{4}t}\right)^{-1}$

For the product rule look for $P\mathrm{e}^{\frac{1}{4}t}\times\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^{-1}\pm Q\mathrm{e}^{\frac{1}{4}t}\times\mathrm{e}^{\frac{1}{4}t}\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^{-2}$ oe

A1 $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)\times 225\mathrm{e}^{\frac{1}{4}t}-900\mathrm{e}^{\frac{1}{4}t}\times\frac{3}{4}\mathrm{e}^{\frac{1}{4}t}}{\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^2}$ which may be left unsimplified.

$\dfrac{\mathrm{d}P}{\mathrm{d}t}=225\mathrm{e}^{\frac{1}{4}t}\times\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^{-1}-675\mathrm{e}^{\frac{1}{4}t}\times\mathrm{e}^{\frac{1}{4}t}\left(3\mathrm{e}^{\frac{1}{4}t}-1\right)^{-2}$ which may be left unsimplified.

(c)(ii)

M1 Substitutes $t = 8$ into their $\dfrac{\mathrm{d}P}{\mathrm{d}t}$ and calculates a value for $\dfrac{\mathrm{d}P}{\mathrm{d}t}$

A1 -3.71 only. Note that this is **not** awrt
If the candidate subsequently writes 3.71 this is A0

Leave blank

10.

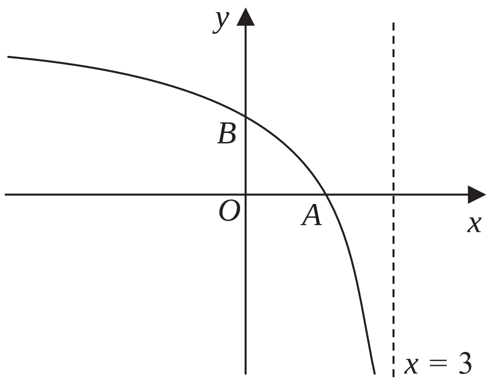


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = g(x)$, where

$$g(x) = \frac{3x - 4}{x - 3}, \quad x \in \mathbb{R}, \quad x < 3$$

The graph cuts the x-axis at the point A and the y-axis at the point B, as shown in Figure 2.

(a) State the range of g.

(1)

(b) State the coordinates of

(i) point A

(ii) point B

(2)

(c) Find $gg(x)$ in its simplest form.

(3)

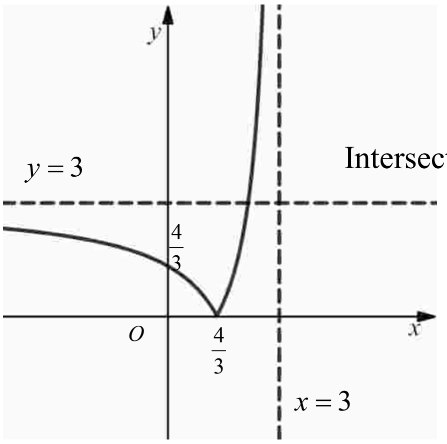
(d) Sketch the graph with equation $y = |g(x)|$

On your sketch, show the coordinates of each point at which the graph meets or cuts the axes and state the equation of each asymptote.

(3)

(e) Find the exact solution of the equation $|g(x)| = 8$

(3)

Question Number	Scheme	Marks
10 (a)	$y < 3$	B1
		(1)
(b)(i)	$\left(\frac{4}{3}, 0\right)$	B1
(ii)	$\left(0, \frac{4}{3}\right)$	B1
		(2)
(c)	$\text{gg}(x) = \dfrac{3\times\frac{3x-4}{x-3}-4}{\frac{3x-4}{x-3}-3} = \dfrac{3(3x-4)-4(x-3)}{3x-4-3(x-3)} = \dfrac{5x}{5} = x$	M1dM1A1
		(3)
(d)	 Shape	B1
	Intersects y - axis at $\left(0, \frac{4}{3}\right)$ meets x- axis at $\left(\frac{4}{3}, 0\right)$	B1ft
	Asymptotes at $x = 3$ and $y = 3$	B1
		(3)
(e)	$\dfrac{3x-4}{x-3} = -8 \Rightarrow 3x-4 = -8x+24 \Rightarrow x = \dfrac{28}{11}$	M1dM1A1
		(3) **[12 marks]**

(a)

B1 Accept $y < 3$, $\text{g}(x) < 3$, $\text{g} < 3$, $-\infty < y < 3$ $(-\infty, 3)$

(b)(i)

B1 $\left(\frac{4}{3}, 0\right)$ Allow candidate to state $x = \frac{4}{3}, y = 0$ or state $x = \frac{4}{3}, \text{g}(x) = 0$

(b)(ii)

B1 $\left(0, \frac{4}{3}\right)$ Allow candidate to state $x = 0, y = \frac{4}{3}$ or state $x = 0, \text{g}(x) = \frac{4}{3}$

SC: For candidates who in(i) write just x or $A = \frac{4}{3}$ and in (ii) write just y or $B = \frac{4}{3}$ score B1 B0 SC

This SC should also be used for candidates who embed the values. So for example in (i) show $0 = \frac{3x-4}{x-3} \Rightarrow x = \frac{4}{3}$ and in (ii) show $y = \frac{3\times 0-4}{0-3} \Rightarrow y = \frac{4}{3}$ as there is no explicit statement in (i) $x = 0$ and (ii) $y = 0$

(c)

M1 Attempts to substitute g into g. $\text{gg}(x)=\dfrac{3\times\dfrac{3x-4}{x-3}-4}{\dfrac{3x-4}{x-3}-3}$ is sufficient

An alternative is using $g(x)=3+\dfrac{5}{x-3}\Rightarrow \text{gg}(x)=3+\dfrac{5}{3+\dfrac{5}{x-3}-3}$

dM1 Multiplies by all terms on the numerator and all terms on the denominator by $(x-3)$ to form a fraction of the form $\dfrac{ax+b}{cx+d}$ Condone poor bracketing

A1 $\text{gg}(x)=x$

(d)

B1 Correct shape with cusp (not a minimum) at *A*. The curve must appear to have the same asymptote as the original curve (at $x=3$ and $y=3$) It should have the correct curvature on the rhs and not appear to bend back on itself

B1ft Intersects y - axis at $\left(0,\dfrac{4}{3}\right)$ and meets x- axis at $\left(\dfrac{4}{3},0\right)$.

Follow through on coordinates from (b). Allow this to be marked *A*, *B* as long as the coordinates of *A* and *B* were correct.

B1 Gives the equation of both asymptotes as $x=3$ and $y=3$

(e)

M1 For writing down a correct equation leading to a solution of $|g(x)|=8$

Allow $\dfrac{3x-4}{x-3}=-8$ oe so allow $-\dfrac{3x-4}{x-3}=8$, $\dfrac{-3x+4}{x-3}=8$, $\dfrac{3x-4}{3-x}=8$ and $\left(\dfrac{3x-4}{x-3}\right)^2=64$

dM1 Solves an allowable equation (see above) by cross multiplying, collecting terms to reach $x=..$

Do not allow a candidate to score this mark from $-\dfrac{3x-4}{x-3}=8\Rightarrow\dfrac{-3x+4}{-x+3}=8$ This scores M1 M0

A1 $x=\dfrac{28}{11}$ **oee only**

If both values are found $\dfrac{3x-4}{x-3}=\pm 8\Rightarrow x=4,\dfrac{28}{11}$ then this mark is scored only when "4" is deleted or $\dfrac{28}{11}$ is chosen as **the** answer

....

Solution from squaring in (c)

M1: $\left(\dfrac{3x-4}{x-3}\right)^2=64$

dM1: $\Rightarrow Ax^2+Bx+c=0$ and solves by usual methods

FYI the correct quadratic is $55x^2-360x+560=0\Rightarrow(11x-28)(x-4)=0$

A1: Selects $x=\dfrac{28}{11}$

Leave blank

11. Relative to a fixed origin O, the line l_1 is given by the equation

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$$

where λ is a scalar parameter.

The line l_2 passes through the origin and is parallel to l_1

(a) Find a vector equation for l_2

(2)

The point A and the point B both lie on l_1 with parameters $\lambda = 0$ and $\lambda = 3$ respectively.

Write down

(b) (i) the coordinates of A,

(ii) the coordinates of B.

(2)

(c) Find the size of the acute angle between OA and l_1

Give your answer in degrees to one decimal place.

(3)

The point D lies on l_2 such that $OABD$ is a parallelogram.

(d) Find the area of $OABD$, giving your answer to the nearest whole number.

(3)

Question Number	Scheme	Marks
11 (a)	$l_2: \ \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$	M1A1
		(2)
(b)(i)	$A = (2,3,-1)$	B1
(ii)	$B = (-1,15,8)$	B1
		(2)
(c)	Uses a correct pair of gradients $a = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $b = k\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ or vice versa	M1
	$a.b = \lvert a \rvert \lvert b \rvert \cos\theta \ \Rightarrow -2+12-3 = \sqrt{14}\sqrt{26}\cos\theta \Rightarrow \theta =$ awrt $68.5°$	dM1, A1
		(3)
(d)	$AB = \sqrt{3^2+12^2+9^2}$ OR $AB = 3\times '\sqrt{26}'$	M1
	Area = $'OA \times AB \times \sin(c)' = \sqrt{14} \times 3\sqrt{26} \times \sin 68.5° =$ awrt $53\left(\text{units}^2\right)$	M1A1
		(3)
		[10 marks]

(a)

M1 Scored for the **rhs** of the equation with gradient $k \times \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ **and** containing the point $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

A1 Correct equation with lhs $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ oe such as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mu \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$.

Allow for gradient any multiple of $\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ Allow with any scalar parameter inc λ.

Condone another constant appearing so $\mathbf{r} = \begin{pmatrix} 1k \\ -4k \\ -3k \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ is acceptable

It must be an equation so $\mathbf{r} =$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ must be on the lhs

(b)(i)

B1 $A = (2,3,-1)$ Accept in vector notation

(b)(ii)

B1 $B = (-1,15,8)$ Accept in vector notation

(c)

M1 Attempts to use a correct pair of gradient vectors. Eg uses their $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and their $\boldsymbol{b} = k\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ or vice versa, Condone one sign slip only

dM1 Correct method for finding acute angle from $a.b = |a||b|\cos\theta$

Expect to see an attempt proceeding to $\cos\theta = ...$ condoning one slip on $a.b$ and an attempt at squaring and adding for $|a|$ & $|b|$ If there is no method shown for $|a|$ or $|b|$ then expect at least one to be correct. It is dependent upon the previous M

A1 $\theta =$ awrt $68.5°$ Note that 1.2 is the radian answer and scores A0.

Allow awrt $68.5°$ coming from $180° - 111.5°$

(d)

M1 Uses a correct method of finding distance AB Accept $\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}$ or $3 \times \begin{vmatrix} -1 \\ 4 \\ 3 \end{vmatrix}$

M1 Uses a correct method of finding the area of $OABD$

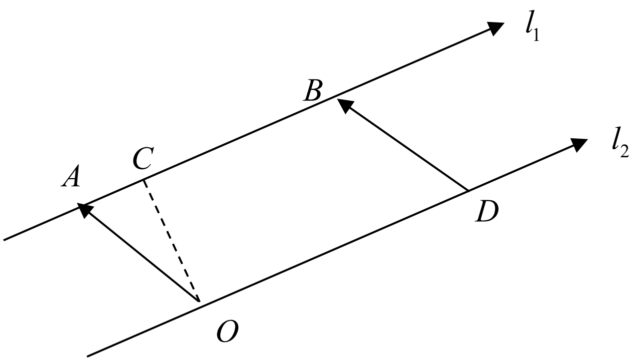
Accept $'OA \times AB \times \sin(c)'$ or two triangles

Note that $'OA \times AB \times \sin(180-c)' = \sqrt{14} \times 3\sqrt{26} \times \sin 111.5° =$ awrt $53\left(\text{units}^2\right)$ is also correct

A1 awrt 53 Accept the exact answer $9\sqrt{35}$ and then isw

....

There are various other ways of finding are $OABD$



Eg by finding the perpendicular distance between AB and OD

M1 Uses a correct method of finding distance AB

M1 It must be a full method. The values are given for a check. (It is a method mark!)

Sets up a point C on l_1 with coordinates $(2-\lambda, 3+4\lambda, -1+3\lambda)$

Uses the fact that OC and AB are perpendicular

$$\begin{pmatrix} 2-\lambda \\ 3+4\lambda \\ -1+3\lambda \end{pmatrix} . \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = 0 \Rightarrow \lambda = -\frac{7}{26}$$

Finds point $C = \left(\frac{59}{26}, \frac{25}{13}, -\frac{47}{26}\right)$ and hence distance $OC = \sqrt{\frac{315}{26}}$

And then multiplies AB by OC

A1 AWRT 53

Note: It is possible to find the area by finding the perpendicular distance between AO and BD

Please scan through the whole response and mark M1 dM1 A1where the first M is a full method to find the perpendicular distance.

Similarly, it is possible to attempt $AO \times OB \sin AOB$ Use the same scoring as above. M1 dM1 A1

Leave blank

12.

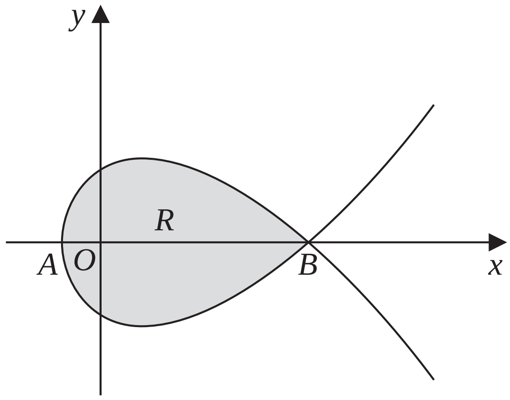


Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = 7t^2 - 5, \qquad y = t(9 - t^2), \qquad t \in \mathbb{R}$$

(a) Find an equation of the tangent to C at the point where $t = 1$

Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.
(5)

The curve C cuts the x-axis at the points A and B, as shown in Figure 3

(b) (i) Find the x coordinate of the point A.

(ii) Find the x coordinate of the point B.
(3)

The region R, shown shaded in Figure 3, is enclosed by the loop of the curve C.

(c) Use integration to find the area of R.
(5)

Question Number	Scheme	Marks
Qu 12 **(a)**	$t=1 \Rightarrow x=2,\ y=8$	B1
	$\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{9-3t^2}{14t}$	M1A1
	Equation of tangent is $y-8=\dfrac{6}{14}(x-2)$	M1
	$3x-7y+50=0$	A1
		(5)
(b)(i)	At A, $x=-5$;	B1
(ii)	$y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$	
	$t=0, 3, -3$	M1
	At $t=3$, $x=7(3)^2-5=58$ or $\left(\text{At } t=-3,\ x=7(-3)^2-5=58\right)$	
	At B, $x=58$	A1
		(3)
(c)	$\displaystyle\int y\,dx=\int y\frac{dx}{dt}\,dt=\int t(9-t^2)14t\,dt$	M1
	$=\displaystyle\int(126t^2-14t^4)\,dt$	A1
	$=\dfrac{126t^3}{3}-\dfrac{14t^5}{5}\ (+C)\quad \left(=42t^3-2.8t^5\ (+C)\right)$	M1
	Either $2\times\left[\dfrac{126t^3}{3}-\dfrac{14t^5}{5}\right]_0^3=2\times\left(42\times3^3-2.8\times3^5\right)=907.2$	ddM1 A1
	Or $\left[\dfrac{126t^3}{3}-\dfrac{14t^5}{5}\right]_{-3}^3=\left(42\times3^3-2.8\times3^5\right)-\left(42\times(-3)^3-2.8\times(-3)^5\right)=907.2$	
		(5) **[13 marks]**

(a)

B1 At $t=1 \Rightarrow x=2,\ y=8$. Score if $(2,8)$ is used in the tangent equation

M1 Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$. Condone for this mark slips in attempts at multiplying out $t(9-t^2)$ before differentiating or slips in the product rule (eg bracketing errors) but do not allow $\frac{dy}{dt}=1\times -2t$

A1 $\frac{dy}{dx}=\frac{9-3t^2}{14t}$ or exact equivalent.

M1 A valid attempt at a tangent to C at $t = 1$ Allow $y-"8"="\frac{6}{14}"(x-"2")$

A1 Allow $k(3x-7y+50=0)$ where k is an integer

(b)

B1 $A=(-5,0)$ Allow $x=-5$ and $A=-5$

M1 For attempting to solve $t(9-t^2)=0$ to produce a non zero value for t and substitute the value into $x=7t^2-5$

A1 $B=(58,0)$ Allow $x=58$ and $B=58$

(c)

M1 Attempts area = $\int y\frac{dx}{dt}dt$. Do not be concerned with limits

A1 Area $=\int(126t^2-14t^4)dt$. Do not be concerned with limits but it must be multiplied out
Alternatively (rare) accept $\int 14t^2(9-t^2)\,dt$ by parts with u' being one of $14t^2$ or $(9-t^2)$ and v being the other

M1 Integrates to a form $At^3+Bt^5\ (+c)$. Condone slips on the coefficients only

ddM1 A full method to find the area of R using their limits. Accept either $2\times[....]_0^3$ or $[....]_{-3}^3$
Dependent upon both M's

A1 907.2 (units^2) or equivalent 4536/5

Attempts made from a Cartesian equation:

(a) Allow slips, but the M1 should be awarded for an attempt at the product and chain rules on functions of the type $y=\sqrt{\frac{x+5}{7}}\left(9-\frac{x+5}{7}\right)$ or $y=\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}\times\frac{(58-x)}{7}$

or the chain rules on a function of this type $\frac{9}{\sqrt{7}}(x+5)^{\frac{1}{2}}-\frac{1}{7\sqrt{7}}(x+5)^{\frac{3}{2}}$

A1 Scored for $\frac{dy}{dx}=\frac{9}{2\sqrt{7}}(x+5)^{-\frac{1}{2}}-\frac{3}{14\sqrt{7}}(x+5)^{\frac{1}{2}}$ or $\frac{dy}{dx}=\frac{(x+5)^{-\frac{1}{2}}}{2\sqrt{7}}\times\frac{(58-x)}{7}-\frac{(x+5)^{\frac{1}{2}}}{7\sqrt{7}}$ oe

(c)

Alt I (c) **Cartesian**	$\int y\,dx=\int\sqrt{\frac{x+5}{7}}\left(9-\frac{x+5}{7}\right)dx,=\int\frac{9}{\sqrt{7}}(x+5)^{\frac{1}{2}}-\frac{1}{7\sqrt{7}}(x+5)^{\frac{3}{2}}\,dx$	M1, A1
	$\frac{6}{\sqrt{7}}(x+5)^{\frac{3}{2}}-\frac{2}{35\sqrt{7}}(x+5)^{\frac{5}{2}}$	M1
	$2\times\left[\frac{6}{\sqrt{7}}(x+5)^{\frac{3}{2}}-\frac{2}{35\sqrt{7}}(x+5)^{\frac{5}{2}}\right]_{-5}^{58}=907.2$	ddM1 A1
Alt II (c) **Cartesian**	Via parts $=\int\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}\times\frac{(58-x)}{7}dx$ with $u=\frac{(58-x)}{7}$ $\frac{dv}{dx}=\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}$	M1, A1
	Integrates by parts to the form $=\frac{2(58-x)(x+5)^{\frac{3}{2}}}{21\sqrt{7}}+\frac{4(x+5)^{\frac{5}{2}}}{105\sqrt{7}}$	M1
	$2\times\left[=\frac{2(58-x)(x+5)^{\frac{3}{2}}}{21\sqrt{7}}+\frac{4(x+5)^{\frac{5}{2}}}{105\sqrt{7}}\right]_{-5}^{58}=907.2$	ddM1 A1

M1 A full attempt to get y in terms of x AND forms $\int y\,dx$

A1 A correct expression for the area and in a form that can be integrated. If by parts is used the correct selection must be made for u and v'

M1 Raises powers correctly to a form $P(x+5)^{\frac{3}{2}}+Q(x+5)^{\frac{5}{2}}$

Or by parts $C(58-x)(x+5)^{\frac{3}{2}}+D(x+5)^{\frac{5}{2}}$

ddM1 A full method to find the area of R using their limits. Dependent upon both M's

Leave blank

13. The volume of a spherical balloon of radius r m is V m^3, where $V = \frac{4}{3}\pi r^3$

(a) Find $\frac{dV}{dr}$ **(1)**

Given that the volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{20}{V(0.05t+1)^3}, \qquad t \geqslant 0$$

(b) find an expression in terms of r and t for $\frac{dr}{dt}$ **(3)**

Given that $V = 1$ when $t = 0$

(c) solve the differential equation

$$\frac{dV}{dt} = \frac{20}{V(0.05t+1)^3}$$

giving your answer in the form $V^2 = f(t)$. **(6)**

(d) Hence find the radius of the balloon at time $t = 20$, giving your answer to 3 significant figures. **(3)**

Question Number	Scheme	Marks
13 (a)	$\frac{dV}{dr} = 4\pi r^2$	B1
		(1)
(b)	Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{20}{V(0.05t+1)^3} = 4\pi r^2 \times \frac{dr}{dt}$	M1
	$\Rightarrow \frac{dr}{dt} = \frac{15}{4\pi^2 r^5 (0.05t+1)^3}$	dM1, A1
		(3)
(c)	$\frac{dV}{dt} = \frac{20}{V(0.05t+1)^3} \Rightarrow \int V\,dV = \int \frac{20dt}{(0.05t+1)^3}$	B1
	$\frac{V^2}{2} = \frac{20(0.05t+1)^{-2}}{-0.1} + c$	M1M1A1
	Sub $V = 1$ when $t = 0 \Rightarrow 0.5 = -200 + c \Rightarrow c = 200.5$	M1
	$\Rightarrow V^2 = 401 - \frac{400}{(0.05t+1)^2}$	A1
		(6)
(d)	Sub $t = 20$ into $(V^2) = 401 - \frac{400}{(1+1)^2} (= 301)$	M1
	Sub $V = \sqrt{301}$ into $V = \frac{4}{3}\pi r^3 \Rightarrow r = \text{awrt } 1.61\text{(m)}$	dM1,A1
		(3) **[13 marks]**

(a)

B1 $\frac{dV}{dr} = 4\pi r^2$ Do not accept $\frac{dV}{dr} = \frac{4}{3}\pi \times 3r^2$

(b)

M1 Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent correctly. (It is not enough to just state this)

Follow through on their $\frac{dV}{dr}$

dM1 Makes $\frac{dr}{dt}$ the subject and attempts to replace V with $\frac{4}{3}\pi r^3$

A1 $\frac{dr}{dt} = \frac{15}{4\pi^2 r^5 (0.05t+1)^3}$ Allow $\frac{dr}{dt} = \frac{A}{B\pi^2 r^5 (0.05t+1)^3}$ or $\frac{dr}{dt} = \frac{A}{B\pi^2 r^5 \left(\frac{1}{20}t+1\right)^3}$

where A and B are integers and $\frac{A}{B}$ cancels to $\frac{15}{4}$

(c)

B1 Separates variables to achieve $\int V\,\mathrm{d}V = \int \frac{20\mathrm{d}t}{(0.05t+1)^3}$ or equivalent (with or without the integral sign)

M1 Integrates lhs to a form aV^2 with or without a constant

M1 Integrates rhs to a form $b(0.05t+1)^{-2}$ with or without a constant

A1 $\frac{V^2}{2} = \frac{20(0.05t+1)^{-2}}{-0.1} + c$ or equivalent including a constant

M1 Substitutes $V = 1$ and $t = 0$ into an integrated expression of the form $pV^2 = q(0.05t+1)^n + c$, where n is an integer, to find the constant c

A1 $V^2 = 401 - \frac{400}{(0.05t+1)^2}$ or equivalent $V^2 = 401 - \frac{160000}{(t+20)^2}$

……..

Note that there is a solution to part (c) via definite integration that marks similarly

$$\int_1^V V\,\mathrm{d}V = \int_0^t \frac{20\mathrm{d}t}{(0.05t+1)^3} \Rightarrow \left[\frac{V^2}{2}\right]_1^V = \left[\frac{20(0.05t+1)^{-2}}{-0.1}\right]_0^t$$ is B1 M1 M1 A1 as before

with the next M1 scored when $\frac{V^2}{2} - \frac{1}{2} = \frac{20(0.05t+1)^{-2}}{-0.1} + 200$ oe

………

(d)

M1 Substitutes $t = 20$ into their equation for V^2 which includes a numerical constant and finds a value for V or V^2 (This cannot be scored for impossible values ie when $V^2 < 0$)

dM1 Substitutes their V into $V = \frac{4}{3}\pi r^3 \Rightarrow r = ...$ Alt substitutes their V^2 into $V^2 = \frac{16}{9}\pi^2 r^6 \Rightarrow r = ...$

A1 cso $r = \text{awrt } 1.61(\text{m})$ in

Watch for candidates who solve (d) using the differential equation $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15}{4\pi^2 r^5 (0.05t+1)^3}$ and find r in terms of t. This is an acceptable method (although unlikely) but do consider this kind of solution carefully.