Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C34

Advanced

Tuesday 19 June 2018 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. (i) Find

$$\int \frac{2x^2 + 5x + 1}{x^2}\,\mathrm{d}x, \quad x > 0$$

(3)

(ii) Find

$$\int x \cos 2x \,\mathrm{d}x$$

(3)

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Question Number	Scheme	Notes	Marks
1. (i)	$\left\{\int \frac{2x^2+5x+1}{x^2}\,dx = \int 2+\frac{5}{x}+\frac{1}{x^2}\,dx\right\}$		
	$= 2x + 5\ln kx - \frac{1}{x} \quad \{+c\}$ Where $k \neq 0$ (k is usually 1)	At least one of either $\pm\frac{A}{x} \to \pm\alpha\ln kx$ or $\pm\frac{B}{x^2} \to \pm\beta x^{-1}$; A, B, α, β non zero.	M1
		At least 2 out of the 3 terms are correct. e.g. 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
		$2x + 5\ln kx - \frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1
			[3]
	(i) Alternative by parts I:		
	$\left\{\int (2x^2+5x+1)x^{-2}\,dx = -\frac{1}{x}(2x^2+5x+1) + \int \frac{1}{x}(4x+5)\,dx\right\}$		
	$= -2x - 5 - \frac{1}{x} + 4x + 5\ln kx \quad \{+c\}$	At least one of either $\pm\frac{A}{x} \to \pm\alpha\ln kx$ or $\pm\frac{B}{x^2} \to \pm\beta x^{-1}$; A, B, α, β non zero.	M1
		At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
	$= 2x - 5 - \frac{1}{x} + 5\ln kx \quad \{+c\}$ Where $k \neq 0$ (k is usually 1)	$2x - 5 - \frac{1}{x} + 5\ln kx$ with or without $+c$ Or $2x + 5\ln kx - \frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

(i) Alternative by parts II:

$$\left\{\int (2x^2+5x+1)x^{-2}\,dx = x^{-2}\left(\frac{2x^3}{3}+\frac{5x^2}{2}+x\right)+\int 2x^{-3}\left(\frac{2x^3}{3}+\frac{5x^2}{2}+x\right)dx\right\}$$

$=\frac{2x}{3}+\frac{5}{2}+\frac{1}{x}+\frac{4x}{3}+5\ln kx-\frac{2}{x}\ \{+c\}$	At least one of either $\pm\frac{A}{x}\to\pm\alpha\ln kx$ or $\pm\frac{B}{x^2}\to\pm\beta x^{-1}$; A, B, α, β non zero.	M1
	At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
$=2x+\frac{5}{2}-\frac{1}{x}+5\ln kx\ \{+c\}$ Where $k\neq 0$ (k is usually 1)	$2x+\frac{5}{2}-\frac{1}{x}+5\ln kx$ with or without $+c$ or $2x+5\ln kx-\frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

(i) Alternative:

$$\left\{\int \frac{2x^2+5x+1}{x^2}\,dx = \int 2+\frac{5x+1}{x^2}\,dx = \int 2+(5x+1)x^{-2}\,dx\right\} = 2x-\frac{1}{x}(5x+1)+\int\frac{5}{x}\,dx$$

$=2x-5-\frac{1}{x}+5\ln kx\ \{+c\}$	At least one of either $\pm\frac{A}{x}\to\pm\alpha\ln kx$ or $\pm\frac{B}{x^2}\to\pm\beta x^{-1}$; A, B, α, β non zero.	M1
	At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
	$2x-5-\frac{1}{x}+5\ln kx\ \{+c\}$ with or without $+c$ or $2x+5\ln kx-\frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

(ii)	$\left\{ \mathrm{I} = \int x\cos 2x \,\mathrm{d}x \right\}$, $\left\{ \begin{array}{l} u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x \Rightarrow v = \frac{1}{2}\sin 2x \end{array} \right\}$		
		$\pm\lambda x\sin 2x \pm \mu\int \sin 2x \{\mathrm{d}x\}$ **BUT if the parts formula is quoted incorrectly score M0**	M1
	$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \{\mathrm{d}x\}$	$\frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \{\mathrm{d}x\}$ simplified or un-simplified	A1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x \{+c\}$	$\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ with or without $+c$, $\frac{1}{2}x\sin 2x - \left(-\frac{1}{4}\cos 2x\right)$ is A0	A1
			[3]
			6

	Question 1 Notes	
(i)	**Note**	The $5\ln x$ can appear in different correct forms e.g. $5\ln 5x$ or $2.5\ln x^2$ etc. and allow modulus signs e.g. $5\ln\|kx\|$
	Note	There are no marks for attempts at $\dfrac{\int 2x^2 + 5x + 1 \,\mathrm{d}x}{\int x^2 \,\mathrm{d}x}$
(ii)	**Note**	There are no marks for attempts at $\int x\cos x \,\mathrm{d}x$

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2. A curve C has parametric equations

$$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t} \qquad t \neq 0$$

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $t = 3$, giving your answer as a fraction in its simplest form.
(3)

(b) Show that a cartesian equation of C can be expressed in the form

$$y = \frac{ax + b}{x + 5} \qquad x \neq k$$

where a, b and k are integers to be found.
(4)

Question Number	Scheme	Notes	Marks
2.	$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t}, \quad t \neq 0$		
(a)	$\frac{dx}{dt} = \frac{3}{2}, \quad \frac{dy}{dt} = 6t^{-2}$	**Both** $\frac{dx}{dt} = \frac{3}{2}$ or $\frac{dt}{dx} = \frac{2}{3}$ **and** $\frac{dy}{dt} = 6t^{-2}$ $\frac{dy}{dt}$ can be simplified or un-simplified. **Note:** This mark can be implied.	B1
	So, $\frac{dy}{dx} = \frac{6t^{-2}}{\left(\frac{3}{2}\right)} \left\{= 4t^{-2}\right\}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\left\{\text{When } t = 3,\right\} \frac{dy}{dx} = \frac{4}{9}$	$\frac{4}{9}$	A1 **cao**
			[3]
(b)	• $t = \frac{x+5}{\left(\frac{3}{2}\right)} \Rightarrow y = 4 - \frac{6}{\left(\frac{x+5}{\left(\frac{3}{2}\right)}\right)}$	An attempt to eliminate t.	M1
	• $t = \frac{6}{4-y} \Rightarrow x = \frac{3}{2}\left(\frac{6}{4-y}\right) - 5$ • $\frac{6}{4-y} = \frac{2}{3}(x+5)$	Achieves a correct equation in x and y only.	A1 o.e.
	$\Rightarrow y = 4 - \frac{9}{x+5}$		
	$\Rightarrow y = \frac{4(x+5) - 9}{x+5}$		
	$\Rightarrow y = \frac{4x+11}{x+5}$	$\underline{a = 4}$ and $\underline{b = 11}$ or $\frac{4x+11}{x+5}$	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$, $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
	Alternative 1 for (b):		
	$y = \frac{ax+b}{x+5} \Rightarrow 4 - \frac{6}{t} = \frac{a(1.5t-5)+b}{1.5t-5+5}$		
	$\Rightarrow 4 - \frac{6}{t} = \frac{1.5at - 5a + b}{1.5t} \Rightarrow 6t - 9 = 1.5at - 5a + b$ $\Rightarrow 6t = 1.5at$ or $-9 = -5a + b$	Substitutes for x and y and "compares coefficients" for term in t or constant term	M1
	$a = 4$ **or** b = 11	Correct value for a **or** b	A1
	$a = 4$ **and** b = 11	Correct values for a **and** b	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$, $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
			7

	Alternative 2 for (b):		
	$y=\frac{4t-6}{t}=\frac{3(4t-6)}{2\frac{3t}{2}}=\frac{3(4t-6)}{2(x+5)}=\frac{4\times\frac{3t}{2}-9}{(x+5)}=\frac{4(x+5)-9}{(x+5)}$ M1: Obtains y in terms of x A1: Correct unsimplified expression		M1A1
	$\Rightarrow\ y=\frac{4x+11}{x+5}$	$\underline{a=4}$ and $\underline{b=11}$ or $\frac{4x+11}{x+5}$	A1
	$x\neq -5$ or $k=-5$	Do not isw so if they have $x\neq -5$, $k\neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]

	Question 2 Notes	
2. (a)	**Note**	M1 can also be obtained by substituting $t=3$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	Note	Some candidates may use the Cartesian form in (a) possibly having done (b) first. E.g. $y=\frac{4x+11}{x+5}\Rightarrow\frac{dy}{dx}=\frac{4(x+5)-4x-11}{(x+5)^2}\left(=\frac{9}{(x+5)^2}\right)\quad t=3\Rightarrow x=\frac{9}{2}-5=-\frac{1}{2}$ $\Rightarrow\frac{dy}{dx}=\frac{9}{\left(-\frac{1}{2}+5\right)^2}=\frac{4}{9}$ This would require a complete method to find the Cartesian equation and then B1 for the correct derivative. Then M1 for a complete method attempting the derivative and substituting for x or t and A1 for 4/9 as in the main scheme. The marks for obtaining the Cartesian equation can score in (b) provided their Cartesian equation is seen or used in (b). (i.e. if they do (a) first)

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3. $$f(x) = 2^{x-1} - 4 + 1.5x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \frac{1}{3}(8 - 2^x)$$

(2)

The equation $f(x) = 0$ has a root α, where $\alpha = 1.6$ to one decimal place.

(b) Starting with $x_0 = 1.6$, use the iteration formula

$$x_{n+1} = \frac{1}{3}(8 - 2^{x_n})$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) By choosing a suitable interval, prove that $\alpha = 1.633$ to 3 decimal places.

(2)

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Question Number	Scheme	Notes	Marks
3.	$f(x) = 2^{x-1} - 4 + 1.5x,\ x \in \mathbb{R};\quad x_{n+1} = \frac{1}{3}\left(8 - 2^{x_n}\right),\ x_0 = 1.6$		
(a)	$0 = 2^{x-1} - 4 + 1.5x \Rightarrow 1.5x = 4 - 2^{x-1}$ or $4 - 2^{x-1} = 1.5x$	Sets $f(x) = 0$ and makes $1.5x$ (or kx) the subject of the formula using correct processing so allow sign errors only.	M1
	$\Rightarrow x = \frac{2}{3}\left(4 - 2^{x-1}\right) \Rightarrow x = \frac{1}{3}\left(8 - 2^x\right)$ (*) or $\Rightarrow x = \frac{\left(4 - 2^{x-1}\right)}{1.5} \Rightarrow x = \frac{1}{3}\left(8 - 2^x\right)$ (*)	$x = \frac{1}{3}\left(8 - 2^x\right)$ by **cso** with at least one intermediate step. Do not accept recovery from earlier errors for the A mark. **Note that the "= 0" must be seen at some point for this mark even if only from f(*x*) = 0 at the start.**	A1 *
	Special case: Starts with $1.5x = 4 - 2^{x-1}$ **and completes method with no f(*x*) = 0 is M1A0**		
			[2]
	Alternative working backwards:		
	$x = \frac{1}{3}\left(8 - 2^x\right) \Rightarrow 3x = 8 - 2^x \Rightarrow 2^x - 8 + 3x = 0$ $x - \frac{1}{3}\left(8 - 2^x\right) = 0 \Rightarrow 3x - 8 + 2^x = 0$	Multiplies by 3 and collects terms to one side or collects terms to one side and multiplies by 3	M1
	$2^x - 8 + 3x = 0 \Rightarrow 2^{x-1} - 4 + 1.5x = 0$	Obtains $2^{x-1} - 4 + 1.5x = 0$ by cso.	A1
			[2]
(b)	$x_1 = \frac{1}{3}\left(8 - 2^{1.6}\right)$	For substituting $x_0 = 1.6$ into $\frac{1}{3}\left(8 - 2^{x_0}\right)$. This mark can be implied by x_1 = awrt 1.66	M1
	$x_1 = 1.656,\ x_2 = 1.616$	x_1 = awrt 1.656 **and** x_2 = awrt 1.616	A1
	$x_3 = 1.645$	$x_3 = 1.645$ only **(not awrt)**	A1 **cao**
	Mark their values in the order given i.e. assume their first calculated value is x_1 etc.		
			[3]
(c)	$f(1.6325) = -0.00100095...$ or awrt -1×10^{-3} $f(1.6335) = 0.00157396...$ or awrt 1×10^{-3} or awrt 2×10^{-3}	Chooses a suitable interval for *x*, which is within 1.633 ± 0.0005 and either side of 1.63288… and attempts to evaluate f(*x*) for both values.	M1
	Sign change (negative, positive) (and $f(x)$ is continuous) therefore root $(\alpha = 1.633)$	Both values correct awrt (or truncated) 1 sf, sign change and a conclusion	A1 **cso**
			[2]
			7

		Question 3 Notes
3. (a)	M1	There are other methods for obtaining the printed equation but the M1 scores for setting f(x) = 0 and making kx the subject of the formula using correct processing e.g.
		$0 = 2^{x-1} - 4 + 1.5x \Rightarrow \frac{2^x}{2} - 4 + 1.5x = 0 \Rightarrow 3x = 8 - 2^x$ M1 $\Rightarrow x = \frac{1}{3}\left(8 - 2^x\right)$ (*) A1
		$0 = 2^{x-1} - 4 + 1.5x \Rightarrow 2^x - 8 + 3x = 0 \Rightarrow 3x = 8 - 2^x$ M1 $\Rightarrow x = \frac{1}{3}\left(8 - 2^x\right)$ (*) A1
3. (c)	A1	**Correct solution only.** Candidate needs to state both of their values for $\text{f}(x)$ to awrt (or truncated) 1sf along with **a reason and conclusion.** Reference to change of sign **or** $\text{f}(1.6325) \times \text{f}(1.6335) < 0$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a conclusion, e.g. $a = 1.633$ (3 dp). Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion could be "change of sign, so true"
	Note	In part (c), candidates can construct their proof using a narrower range than [1.6325, 1.6335] which contains the root 1.632888767

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4. (a) Find the binomial expansion of

$$(1 + px)^{-4}, \quad |px| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant p.

(3)

$$\mathrm{f}(x) = \frac{3 + 4x}{(1 + px)^4} \qquad |px| < 1$$

where p is a positive constant.

In the series expansion of $\mathrm{f}(x)$, the coefficient of x^2 is twice the coefficient of x.

(b) Find the value of p.

(5)

(c) Hence find the coefficient of x^3 in the series expansion of $\mathrm{f}(x)$, giving your answer as a simplified fraction.

(2)

Question Number	Scheme	Notes	Marks
4. (a)	$(1+px)^{-4} = 1+(-4)(px)+\frac{(-4)(-5)}{2!}(px)^2+\frac{(-4)(-5)(-6)}{3!}(px)^3+...$	see notes	M1
	$= 1-4px+10p^2x^2-20p^3x^3+...$ or $= 1-4(px)+10(px)^2-20(px)^3+...$	Three of the four terms correct and simplified.	A1
		All four terms correct and simplified and isw once a correct answer is seen. Must be seen in part (a).	A1
			[3]
(b)	$\left\{f(x)=\frac{3+4x}{(1+px)^4}=\right\}(3+4x)(1-4px+10p^2x^2-20p^3x^3+...)=...$ Attempts to expand $(3+4x)\times$ their part (a) expansion. There should be evidence of at least ($3\times$one term from part (a)) + ($4x\times$one term from part (a))		M1
	Note: $f(x)=3+(4-12p)x+(30p^2-16p)x^2+(40p^2-60p^3)x^3+...$		
	$= 3-12px+30p^2x^2-60p^3x^3+4x-16px^2+40p^2x^3$ $\Rightarrow$ "$30p^2-16p$" = 2"$(4-12p)$" **Or** or 2"$(30p^2-16p)$" = "$(4-12p)$"	**Dependent on the previous M mark** Multiplies out to give exactly two terms in x and exactly 2 terms in x^2 and attempts one coefficient = twice the other. This mark can be implied by later working. Allow x's to be present for this mark	dM1
	$30p^2-16p=2(4-12p)$	Correct equation with no x's	A1
	$30p^2+8p-8=0$ $\Rightarrow(10p-4)(3p+2)=0$ or $(5p-2)(6p+4)=0 \Rightarrow p=...$ or $15p^2+4p-4=0\Rightarrow(5p-2)(3p+2)=0\Rightarrow p=...$	**Dependent on the 1st M mark** Correct method for solving a 3TQ leading to at least one value. If working is shown see general guidance for solving 3TQs. If no working is shown then you may need to check to see if their 3TQ solves correctly.	dM1
	$\left\{p=\frac{2}{5},-\frac{2}{3}\Rightarrow \text{As } p>0\text{, then}\right\}\ p=\frac{2}{5}$	$p=\frac{2}{5}$ only.	A1
			[5]
(c)	$40\left(\frac{2}{5}\right)^2-60\left(\frac{2}{5}\right)^3$	Substitutes their $p=\frac{2}{5}$ from part (b) into their coefficient of x^3 (which comes from exactly 2 terms from their expansion)	M1
	Coefficient of x^3 is $\frac{64}{25}$	Allow $\frac{64}{25}$ or $2\frac{14}{25}$. Condone 2.56. Allow $\frac{64}{25}x^3$, $2\frac{14}{25}x^3$, $2.56x^3$ **If 2 answers are offered, score A0**	A1
			[2]
			10

Question 4 Notes

4. (a)	**M1**	Uses the binomial expansion with $n=-4$ and $'x'=px$.
	Note	M1 can be given for either $1+(-4)(px)$ or $\frac{(-4)(-5)}{2!}(px)^2$ or $\frac{(-4)(-5)(-6)}{3!}(px)^3$
(b)	**Note**	Allow recovery in part (b) from missing brackets in part (a). e.g. px^2 now becoming p^2x^2.

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5. (i) The functions f and g are defined by

$$\mathrm{f} : x \rightarrow \mathrm{e}^{2x} - 5, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \rightarrow \ln(3x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

(3)

(b) Find fg(3), giving your answer in its simplest form.

(2)

(ii) (a) Sketch the graph with equation

$$y = |4x - a|$$

where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

Given that

$$|4x - a| = 9a$$

where a is a positive constant,

(b) find the possible values of

$$|x - 6a| + 3|x|$$

giving your answers, in terms of a, in their simplest form.

(5)

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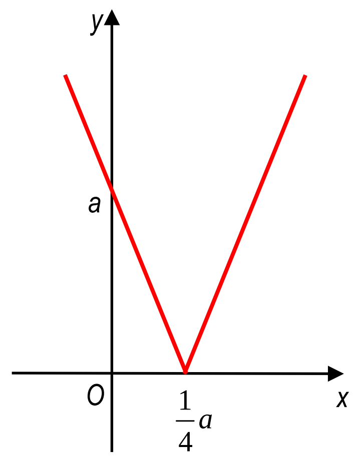
Question Number	Scheme	Notes	Marks
5.	$f: x \to e^{2x} - 5,\ x \in \mathbb{R}\ ;\ \ g: x \to \ln(3x-1),\ x \in \mathbb{R},\ x > \frac{1}{3}$		
(i) (a)	$y = e^{2x} - 5 \Rightarrow x = e^{2y} - 5$ $x + 5 = e^{2y} \Rightarrow \ln(x+5) = 2y$	Attempt to make x (or swapped y) the subject using correct processing so allow sign errors only.	M1
	$(y =)\ \frac{1}{2}\ln(x+5)\ \ \left\{\left(f^{-1}: x \to\right) \frac{1}{2}\ln(x+5)\right\}$	$\frac{1}{2}\ln(x+5)$ or $\frac{1}{2}\ln\lvert x+5\rvert$ or $\ln(x+5)^{\frac{1}{2}}$. Correct expression ignoring how it is referenced **but must be in terms of *x*.** Do not allow $\ln(x+5).\frac{1}{2}$ or e.g. $\ln x + 5$ or $\ln(x+5$ unless the correct answer is seen previously or subsequently.	A1
	Domain: $x > -5$ or $(-5, \infty)$	$x > -5$ or $(-5, \infty)$ Condone domain > -5	B1
			[3]
(b)	$fg(3) = e^{2\ln(3(3)-1)} - 5$ $\left(\text{NB } fg(x) = 9x^2 - 6x - 4\right)$	g goes into f **and** $x = 3$ is substituted into the result **or** finds $g(3)\{= \ln 8\}$ and substitutes into f	M1
	$\left\{= e^{2\ln 8} - 5 = 64 - 5\right\} = 59$	59 cao	A1
			[2]
(ii)(a)		A $\vee$ shape with the vertex on the positive x-axis (with no significant asymmetry about the vertical through the vertex). The left hand branch must extend into the second quadrant. Do not allow a "y" shape unless the part below the x-axis is dotted or "crossed out"	B1
		States $(0, a)$ and $\left(\frac{1}{4}a, 0\right)$ **or** $\frac{1}{4}a$ marked in the correct position on the x-axis **and** a marked in the correct position on the y-axis. Other points marked on the axes can be ignored.	B1
			[2]
(b)	$\{4x - a = 9a \Rightarrow\}\ \ x = \frac{10a}{4}\ \ \left\{\text{or } x = \frac{5a}{2}\right\}$	$x = \frac{10a}{4}$ or $x = \frac{9a+a}{4}$ or $x = \frac{5a}{2}$ (may be implied)	B1
	$-(4x - a) = 9a$ or $4x - a = -9a$	Attempt at the "second" solution. Accept $-(4x - a) = 9a$ or $4x - a = -9a$ or $-4x = 8a$. Do not condone (unless recovered) invisible brackets in this case.	M1
	$x = -2a$	$x = -2a$	A1
	$\left\{x = \frac{5}{2}a \Rightarrow\right\}\ \left\lvert \frac{5}{2}a - 6a\right\rvert + 3\left\lvert\frac{5}{2}a\right\rvert; = 11a$ $\{x = -2a \Rightarrow\}\ \lvert -2a - 6a\rvert + 3\lvert -2a\rvert; = 14a$	Substitutes at least one of their x values from solutions of $\lvert 4x - a\rvert = 9a$ where $x < 6a$ into $\lvert x - 6a\rvert + 3\lvert x\rvert$ and finds at least one value for $\lvert x - 6a\rvert + 3\lvert x\rvert$ **Must apply the modulus.**	M1
		Both $11a$ and $14a$ and no other answers	A1
			[5]
			12

	Question 5 Notes	
(b)	**Note**	The values of x might be found by squaring: $\lvert 4x - a \rvert = 9a \Rightarrow 16x^2 - 8ax + a^2 = 81a^2 \Rightarrow 16x^2 - 8ax - 80a^2 = 0$ $16x^2 - 8ax - 80a^2 = 0 \Rightarrow x = \dfrac{5a}{2}, -2a$ Score as follows: B1 for a correct 3 term quadratic (terms collected after squaring) M1: Solves their 3 term quadratic (usual rules) A1: $x = \dfrac{5a}{2}, -2a$

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6. (a) Express $\sqrt{5}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 significant figures.

(3)

(b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$$

giving your answers to 3 significant figures.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(4)

$$f(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B \qquad \theta \in \mathbb{R}$$

where A and B are constants.

Given that the range of f is

$$-15 \leqslant f(x) \leqslant 33$$

(c) find the value of B and the possible values of A.

(4)

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P 5 0 8 0 3 A 0 1 8 5 2

Question Number	Scheme	Notes	Marks
6.	$\sqrt{5}\cos\theta - 2\sin\theta \equiv R\cos(\theta+\alpha)$		
(a)	$R=3$	$R=3$, **cao** (±3 is B0) ($\sqrt{9}$ is B0)	B1
	$\tan\alpha = \pm\frac{2}{\sqrt{5}}$, $\tan\alpha = \pm\frac{\sqrt{5}}{2} \Rightarrow \alpha = ...$ (Also allow $\cos\alpha = \pm\frac{\sqrt{5}}{3}$ or $\pm\frac{2}{3}$, $\sin\alpha = \pm\frac{2}{3}$ or $\pm\frac{\sqrt{5}}{3} \Rightarrow \alpha = ...$, where "3" is their R.)		M1
	$\alpha = 0.7297276562... \Rightarrow \alpha = 0.7297$ (4 sf)	Anything that rounds to 0.7297 (Degrees is 41.81 and scores A0)	A1
	{**Note**: $\sqrt{5}\cos\theta - 2\sin\theta = 3\cos(\theta + 0.7297)$}		**[3]**
(b)	$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$		
	$3\cos(\theta+0.7297)=0.5$ $\Rightarrow \cos(\theta+0.7297) = \frac{0.5}{3}$	Attempts to use part (a) $"3"\cos(\theta \pm "0.7297") = 0.5$ 9and proceeds to $\cos(\theta \pm "0.7297") = K$, $\lvert K\rvert < 1$ May be implied by $\theta \pm "0.7297" = 1.4033$ or $\theta \pm "0.7297" = \cos^{-1}\left(\frac{0.5}{\text{their } 3}\right)(=1.4033...)$	M1
	$\theta_1 = 0.673648... \Rightarrow \theta_1 = 0.674$ (3 sf)	Anything that rounds to 0.674	A1
	$\theta_2 + "0.7297" = "-1.4033" \Rightarrow \theta_2 = ...$	**dependent on the previous M mark** Correct attempt at a second solution in the range. Usually given for: θ_2 + their 0.7297 = – their 1.4033 $\Rightarrow \theta_2 = ...$	dM1
	$\theta_2 = -2.133048... \Rightarrow \theta_2 = -2.13$ (3 sf)	Anything that rounds to – 2.13	A1
	For solutions in (b) that are otherwise fully correct, if there are extra answers in the range, deduct the final A mark.		
	For candidates who work consistently in degrees in (a) and (b) allow awrt 38.6° and awrt – 122° in part (b) as the A mark will be lost in part (a)		
			[4]
(c)	$f(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B, \theta \in \mathbb{R}; -15 \leqslant f(x) \leqslant 33$ $\Rightarrow -15 \leqslant 3A\cos(\theta + 0.730) + B \leqslant 33$		
	Note that part (c) is now marked as B1M1A1A1		
	$B=9$	Correct value for B	B1
	$3A + B = 33$, $-3A + B = -15$ or $3A + B = -15$, $-3A + B = 33$	Writes down at least one pair of simultaneous equations (or inequalities) of the form $RA + B = 33$, $-RA + B = -15$ or $RA + B = -15$, $-RA + B = 33$ and finds at least one value for A	M1
	$A=8$ **or** $A=-8$	One correct value for A	A1
	$A=8$ **and** $A=-8$	Both values correct	A1
			[4]
			11

(c) **Alt 1**	$B = 9$	Correct value for B	B1
	$(2)(A)(3) = 33 - -15$	$(2)(A)(\text{their } R) = 33 - -15 \Rightarrow A = ...$	M1
	$A = 8$ **or** $A = -8$	One correct value for A	A1
	$A = 8$ **and** $A = -8$	Both values correct	A1
			[4]
(c) **Alt 2**	$B = \frac{33-15}{2} = 9$	Correct value for B	B1
	$3A = 33 - 9 \Rightarrow A = 8$	$(\text{their } R)A = 33 - \text{their } B \Rightarrow A = ...$	M1
	$A = 8$ **or** $A = -8$	One correct value for A	A1
	$A = 8$ **and** $A = -8$	Both values correct	A1
			[4]

	Question 6 Notes	
(c)	**Note**	The M mark may be implied by correct answers so obtaining A = 8 implies M1A1

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7.

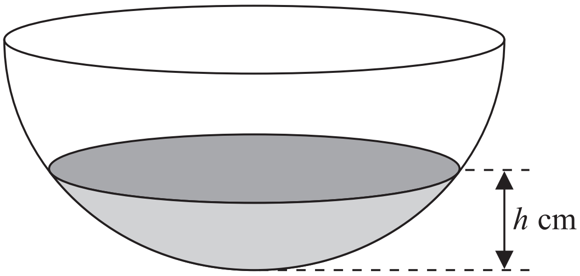

Figure 1

Figure 1 shows a hemispherical bowl.

Water is flowing into the bowl at a constant rate of $180\,\text{cm}^3\,\text{s}^{-1}$.

When the height of the water is h cm, the volume of water V cm^3 is given by

$$V = \frac{1}{3}\pi h^2(90 - h), \qquad 0 \leqslant h \leqslant 30$$

Find the rate of change of the height of the water, in $\text{cm}\,\text{s}^{-1}$, when $h = 15$
Give your answer to 2 significant figures.

(5)

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Question Number	Scheme	Notes	Marks
7.	$V = \frac{1}{3}\pi h^2(90 - h) = 30\pi h^2 - \frac{1}{3}\pi h^3\ ;\ \frac{\mathrm{d}V}{\mathrm{d}t} = 180$		
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 60\pi h - \pi h^2$	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} =\right\} \pm a h \pm b h^2,\ a \neq 0,\ b \neq 0$	M1
		$60\pi h - \pi h^2$ Can be simplified or un-simplified.	A1
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow\right\}\ \left(60\pi h - \pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t} = 180$ $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} \Rightarrow\right\}\ \frac{\mathrm{d}h}{\mathrm{d}t} = 180 \times \frac{1}{60\pi h - \pi h^2}$	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 180$ or $180 \div$ their $\frac{\mathrm{d}V}{\mathrm{d}h}$ This is for a correct **application** of the chain rule and not for just quoting a correct chain rule.	M1
	When $h = 15$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} =\right\} \frac{1}{60\pi(15) - \pi(15)^2} \times 180\ \left\{= \frac{4}{15\pi}\right\}$	**Dependent on the previous M mark.** Substitutes $h = 15$ into an expression which is a result of a quotient (or their rearranged quotient) of their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and 180. May be implied by awrt 0.08 or 0.09.	dM1
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = 0.0848826... \Rightarrow\right\}\ \frac{\mathrm{d}h}{\mathrm{d}t} = \underline{0.085}\ (\mathrm{cm\,s^{-1}})\ (2\text{ sf})$	**Awrt** 0.085 or allow $\frac{4}{15\pi}$ oe (and isw if necessary)	A1 **cao**
			[5]
			5
	Alternative Method for the first M1A1		
	Product rule: $\left\{ u = \frac{1}{3}\pi h^2 \quad v = 90 - h;\quad \frac{\mathrm{d}u}{\mathrm{d}h} = \frac{2}{3}\pi h \quad \frac{\mathrm{d}v}{\mathrm{d}h} = -1 \right\}$		
	$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2}{3}\pi h(90 - h) + \frac{1}{3}\pi h^2(-1)$	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} =\right\} \pm\alpha h(90 - h) \pm \beta h^2(-1),\ \alpha \neq 0,\ \beta \neq 0$ Can be simplified or un-simplified.	M1
		$\frac{2}{3}\pi h(90 - h) + \frac{1}{3}\pi h^2(-1)$ Can be simplified or un-simplified.	A1
	Question 7 Notes		
7.	**Note**	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated for the 1st M1 and/or the 1st A1 but it should be clear that they are differentiating their V.	
	Note	$V = \frac{1}{3}\pi h^2(90 - h) \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2}{3}\pi h(90 - h)$ scores M0A0 even though it satisfies the conditions for the derivative.	

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8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 do not meet.

(4)

The point P is on l_1 where $\lambda = 0$, and the point Q is on l_2 where $\mu = -1$

(b) Find the acute angle between the line segment PQ and l_1, giving your answer in degrees to 2 decimal places.

(5)

(c) Find the shortest distance from the point Q to the line l_1, giving your answer to 3 significant figures.

(2)

Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$; Let $\theta =$ acute angle between PQ and l_1.		
(a)	$\mathbf{i}: \quad 1 + \lambda = 6 + \mu$ **(1)** $\mathbf{j}: -3 + 2\lambda = 4 + \mu$ **(2)** $\mathbf{k}: 2 + 3\lambda = 1 - \mu$ **(3)**		
	(1) and **(2)** yields $\lambda = 2, \mu = -3$ **(1)** and **(3)** yields $\lambda = 1, \mu = -4$ **(2)** and **(3)** yields $\lambda = 1.2, \mu = -4.6$	Attempts to solve a pair of equations to find at least one of either $\lambda = ...$ or $\mu = ...$	M1
		λ and μ are both correct	A1
	Checking **(3)**: $8 \neq 4$ Checking **(2)**: $-1 \neq 0$ Checking **(1)**: $2.2 \neq 1.4$ l_1 and l_2 do not intersect.	Attempts to show a contradiction	M1
		Correct comparison and a conclusion. Accept "do not meet" and accept "are skew". **Requires all previous work to be correct.**	A1
	Allow a calculation that gives "8 = 4 so the lines do not meet"		
			[4]
	Alternative for part (a):		
	(1) and **(2)** yields $\lambda = 2, \mu = -3$ **(1)** and **(3)** yields $\lambda = 1, \mu = -4$ **(2)** and **(3)** yields $\lambda = 1.2, \mu = -4.6$	Attempts to solve a pair of equations to find at least one of either $\lambda = ...$ or $\mu = ...$	M1
		Shows any two of **(1)** and **(2)** yielding $\lambda = 2$ **(1)** and **(3)** yielding $\lambda = 1$ **(2)** and **(3)** yielding $\lambda = 1.2$ **or** shows any two of **(1)** and **(2)** yielding $\mu = -3$ **(1)** and **(3)** yielding $\mu = -4$ **(2)** and **(3)** yielding $\mu = -4.6$	A1
	E.g. So $2 \neq 1$ l_1 and l_2 do not intersect.	Attempts to show a contradiction	M1
		Correct comparison and a conclusion. Accept "do not meet" and accept "are skew". **Requires all previous work to be correct.**	A1
			[4]

(b)	$\overrightarrow{OP} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \ \overrightarrow{OQ} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$		
	$(\overrightarrow{PQ} =) \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$ or $(\overrightarrow{QP} =) \begin{pmatrix} -4 \\ -6 \\ 0 \end{pmatrix}$	Full method of finding $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ where P and Q have been found by using $\lambda = 0$ in l_1 and $\mu = -1$ in l_2	M1
		Correct $\overrightarrow{PQ}$ or $\overrightarrow{QP}$. Also allow for direction, $\mathbf{d}_{PQ} = 2\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$ and allow coordinates e.g. (4, 6, 0)	A1
	$\mathbf{d}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ \mathbf{d}_{PQ} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	M1
	$\cos q = \pm\left(\dfrac{(1)(4)+(2)(6)+(3)(0)}{\sqrt{(1)^2+(2)^2+(3)^2}.\sqrt{(4)^2+(6)^2+(0)^2}}\right)$	**Dependent on the previous M mark.** An attempt to apply the dot product formula between $\pm A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	dM1
	$\cos\theta = \dfrac{16}{\sqrt{14}.\sqrt{52}} \Rightarrow \theta = 53.62985132... = 53.63$ (2 dp)	Anything that rounds to 53.63	A1
			[5]

(c)	$\frac{d}{\sqrt{52}} = \sin q$	Writes down a correct trigonometric equation involving the shortest distance, d. e.g. $\frac{d}{\text{their } PQ} = \sin q$, o.e.	M1
	$\left\{d = \sqrt{52}\sin 53.63... \Rightarrow\right\} d = 5.8064... = 5.81 \text{ (3sf)}$	Anything that rounds to 5.81	A1
			[2]
	Alternative for part (c): (Let M be the point on l_1 closest to Q)		
	$\overrightarrow{OM} = \begin{pmatrix}1\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} \Rightarrow \overrightarrow{QM} = \begin{pmatrix}1\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}5\\3\\2\end{pmatrix}$ $\begin{pmatrix}\lambda-4\\2\lambda-6\\3\lambda\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix} = 0 \Rightarrow \lambda - 4 + 4\lambda - 12 + 9\lambda = 0$ $\begin{pmatrix}\lambda-4\\2\lambda-6\\3\lambda\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix} = \lambda - 4 + 4\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{8}{7}$ $\lambda = \frac{8}{7} \Rightarrow \overrightarrow{QM} = \frac{1}{7}\begin{pmatrix}-20\\-26\\24\end{pmatrix} \Rightarrow \left\lvert\overrightarrow{QM}\right\rvert = \frac{1}{49}\sqrt{20^2 + 26^2 + 24^2}$	Applies a complete and correct method that leads to an expression for the shortest distance	M1
	$= \sqrt{\frac{236}{7}} = 5.81$	Anything that rounds to 5.81	A1
			[2]
			11

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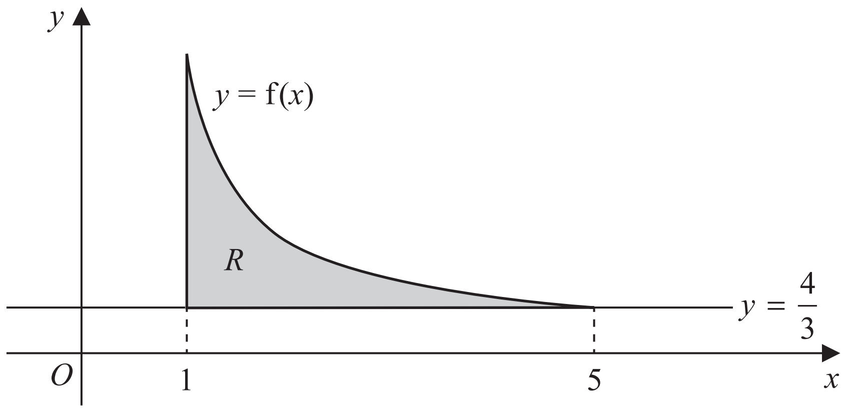

Diagram not drawn to scale

Figure 2

(a) Find

$$\int \frac{1}{(2x-1)^2}\,\mathrm{d}x$$

(2)

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{12}{(2x-1)} \qquad 1 \leqslant x \leqslant 5$$

The finite region R, shown shaded in Figure 2, is bounded by the line with equation $x = 1$, the curve with equation $y = \mathrm{f}(x)$ and the line with equation $y = \frac{4}{3}$.

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid generated, giving your answer in its simplest form.

(6)

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P 5 0 8 0 3 A 0 2 8 5 2

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(x)=\dfrac{12}{(2x-1)},\quad 1\leqslant x\leqslant 5;\ y=\dfrac{4}{3}$		
(a)	$\left\{\int \dfrac{1}{(2x-1)^2}\,\mathrm{d}x\right\} = \dfrac{(2x-1)^{-1}}{(-1)(2)}\ \{+c\}$	$(2x-1)^{-2}\rightarrow \pm\lambda(2x-1)^{-1}$ or $\pm\lambda u^{-1}$ where $u=2x\pm1;\ \lambda\neq 0$	M1
		$\left(\dfrac{(2x-1)^{-1}}{(-1)(2)}\right)$ or $-\dfrac{1}{2(2x-1)}$ oe with or without $+c$. Can be simplified or un-simplified.	A1
			[2]
(b)	$\pi\int\left(\dfrac{12}{2x-1}\right)^2\mathrm{d}x$	For $\pi\int\left(\dfrac{12}{2x-1}\right)^2\mathrm{d}x$ or $\pi\int\dfrac{144}{(2x-1)^2}\mathrm{d}x$ Ignore limits and $\mathrm{d}x$. Can be implied and the π may be recovered later.	B1
	$V_1 = 144\pi\left[\dfrac{-1}{2(2x-1)}\right]_1^5$		
	$=144(\pi)\left(\left(\dfrac{-1}{2(2(5)-1)}\right)-\left(\dfrac{-1}{2(2(1)-1)}\right)\right)$	Applies x-limits of 5 and 1 to an expression of the form $\pm\beta(2x-1)^{-1};\ \beta\neq 0$ and subtracts the correct way round.	M1
		Correct expression for the integrated volume with or without the π. Can be simplified or un-simplified. Can be implied by 64 or 64π.	A1
	$\left\{=-72(\pi)\left(\dfrac{1}{9}-1\right)\ =\ 64(\pi)\right\}$ **Note:** $\pi\int_1^5\left(\dfrac{12}{2x-1}\right)^2\mathrm{d}x$ or $\int_1^5\left(\dfrac{12}{2x-1}\right)^2\mathrm{d}x$ evaluated directly as 64π or 64 **<u>with no incorrect working seen</u>** scores M1A1 (presumably on a calculator)		
	$\{V_{\text{cylinder}}\}=\pi\left(\dfrac{4}{3}\right)^2(4)\ \left\{=\dfrac{64}{9}\pi\right\}$	Attempts to use the formula πr^2h with numerical r and h with at least one of $r=\dfrac{4}{3}$ or $h=4$ correct or attempts $\pi\int_1^5\left(\dfrac{4}{3}\right)^2\mathrm{d}x$ or $\pi\int_0^5\left(\dfrac{4}{3}\right)^2\mathrm{d}x$	M1
		Correct expression for V_{cylinder}: $\pi\left(\dfrac{4}{3}\right)^2(4)$ or $\dfrac{64}{9}\pi$ implies this mark	A1
	$\left\{\mathrm{Vol}(R)=64\pi-\dfrac{64\pi}{9}\right\}\Rightarrow \mathrm{Vol}(R)\ =\ \underline{\dfrac{512}{9}\pi}$	$\dfrac{512}{9}\pi$ or $56\dfrac{8}{9}\pi$	A1
			[6]
			8

Question 9 Notes

9. (b)	**Note**	See extra notes below for how to mark attempts at $\pi\int_1^5\left(\left(\dfrac{12}{2x-1}\right)-\left(\dfrac{4}{3}\right)\right)^2\mathrm{d}x$
	Note	An acceptable approach is $\pi\int_1^5\left(\left(\dfrac{12}{2x-1}\right)^2-\left(\dfrac{4}{3}\right)^2\right)\mathrm{d}x$

Attempts at $\pi\int_1^5\left(\left(\frac{12}{2x-1}\right)-\left(\frac{4}{3}\right)\right)^2 \mathrm{d}x$:

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}-\frac{32}{2x-1}+\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $p\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}-16\ln(2x-1)+\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}-16\ln 9+\frac{80}{9}\right)-\left(-72+\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{640}{9}\pi-48\ln 9\right)$$

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}+\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $p\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}+\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}+\frac{80}{9}\right)-\left(-72+\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{640}{9}\pi\right)$$

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}-\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $p\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}-\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}-\frac{80}{9}\right)-\left(-72-\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{512}{9}\pi\right)$$

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10. The curve C satisfies the equation

$$x\mathrm{e}^{5-2y} - y = 0 \qquad x > 0, \quad y > 0$$

The point P with coordinates $(2\mathrm{e}^{-1}, 2)$ lies on C.

The tangent to C at P cuts the x-axis at the point A and cuts the y-axis at the point B.

Given that O is the origin, find the exact area of triangle OAB, giving your answer in its simplest form.

(7)

P 5 0 8 0 3 A 0 3 2 5 2

Question Number	Scheme	Notes	Marks
10.	$C: xe^{5-2y} - y = 0$ or $\ln x + 5 - 2y - \ln y = 0$; $P(2e^{-1}, 2)$ lies on C.		
	Either • $e^{5-2y} - 2xe^{5-2y}\frac{dy}{dx} - \frac{dy}{dx}(=0)$ • $e^{5-2y} - 2y\frac{dy}{dx} - \frac{dy}{dx}(=0)$ • $\frac{1}{x} - 2\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx}(=0)$ • $\frac{dx}{dy} = e^{2y-5} + 2ye^{2y-5}$ • $e^5 = e^{2y}\frac{dy}{dx} + 2ye^{2y}\frac{dy}{dx}$	Obtains either $\pm Ae^{5-2y} \pm Bxe^{5-2y}\frac{dy}{dx} \pm \frac{dy}{dx}(=0)$ **or** $\pm Ae^{5-2y} \pm By\frac{dy}{dx} \pm \frac{dy}{dx}(=0)$ **or** $\pm\frac{A}{x} \pm K\frac{dy}{dx} \pm \frac{B}{y}\frac{dy}{dx}(=0)$ **or** $\pm\frac{dx}{dy} = \pm Ae^{\pm\alpha\pm 2y} \pm Bye^{\pm\alpha\pm 2y}$ **or** $\pm Ae^{\pm 5} = \pm Be^{\pm 2y}\frac{dy}{dx} \pm Kye^{\pm 2y}\frac{dy}{dx}$ $A, B, K \neq 0$; a, b can be 0	M1
		Correct differentiation. The " = 0" may be implied by later work.	A1
	Ignore any "$\frac{dy}{dx}$ =" in front of their differentiation		
	At P, $e^{5-2(2)} - 2(2e^{-1})e^{5-2(2)}\frac{dy}{dx} - \frac{dy}{dx} = 0$ $\Rightarrow e - 4\frac{dy}{dx} - \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{e}{5}$	Uses $P(2e^{-1}, 2)$ and their gradient equation to find a numerical value for $\frac{dy}{dx}$ or $\frac{dx}{dy}$. Could have extra or fewer $\frac{dy}{dx}$ terms and may have rearranged their expression wrongly before substituting. Accept $\frac{dy}{dx}$ = awrt 0.54 as evidence.	M1
	$\left\{m_T = \frac{e}{5} \Rightarrow\right\}$ • $y - 2 = \frac{e}{5}\left(x - \frac{2}{e}\right)$ **or** $x - \frac{2}{e} = 5e^{-1}(y - 2)$ • $2 = \frac{e}{5}(2e^{-1}) + c \Rightarrow c = \frac{8}{5} \Rightarrow y = \frac{e}{5}x + \frac{8}{5}$	**Dependent on the previous M mark.** A correct attempt at an equation of the tangent at the point $P(2e^{-1}, 2)$ using their numerical $\frac{dy}{dx}$. If using $y = mx + c$ must reach as far as $c = \ldots$	**d**M1
	$y = 0 \Rightarrow -2 = \frac{e}{5}\left(x - \frac{2}{e}\right) \Rightarrow x = -\frac{8}{e}$ $\left\{\Rightarrow A\left(-\frac{8}{e}, 0\right)\right\}$ $x = 0 \Rightarrow y - 2 = \frac{e}{5}\left(-\frac{2}{e}\right) \Rightarrow y = \frac{8}{5}$ $\left\{\Rightarrow B\left(0, \frac{8}{5}\right)\right\}$	Finds at least one correct intercept. For $-\frac{8}{e}$, allow awrt -2.94.	A1
	Area $OAB = \frac{1}{2}\left(\frac{8}{e}\right)\left(\frac{8}{5}\right)$	**Dependent on both previous M marks.** Applies $\frac{1}{2}$(their x_A)(their y_B) where their x_A and y_B are **exact**. Condone a method that gives a negative area.	**dd**M1
	$= \frac{32}{5e}$ or $\frac{32}{5}e^{-1}$	$\frac{32}{5e}$ or $\frac{32}{5}e^{-1}$. Allow $6.4e^{-1}$ but not e.g. $\frac{64}{10e}$	A1
			[7]
			7
	Question 10 Notes		
	Note	Accept the alternative notation for the differentiation e.g. $e^{5-2y}dx - 2xe^{5-2y}dy - dy = 0$	

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11.

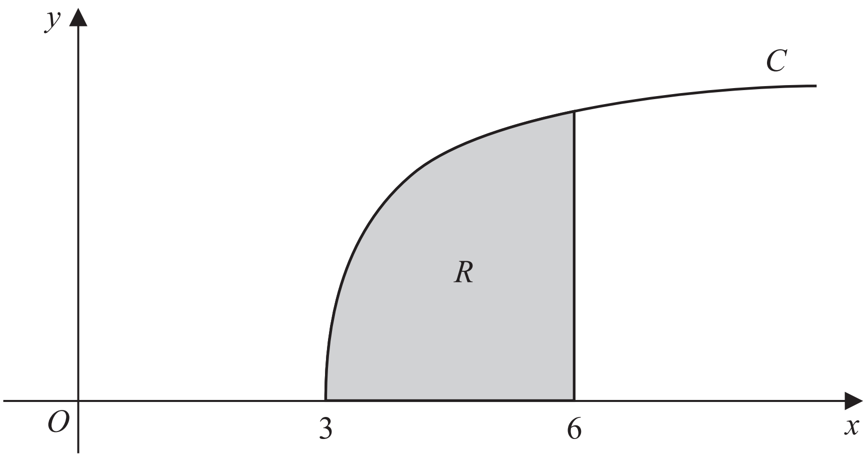

Figure 3

(a) By writing $\sec\theta$ as $\dfrac{1}{\cos\theta}$, show that when $x = 3\sec\theta$,

$$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$$

(2)

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{\sqrt{x^2 - 9}}{x} \qquad x \geqslant 3$$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line with equation $x = 6$

(b) Use the substitution $x = 3\sec\theta$ to find the exact value of the area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

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P 5 0 8 0 3 A 0 3 6 5 2

	Note	The 2nd and 3rd method marks are available for work in decimals but the final method mark requires exact work.
	Note	Accept y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$

Question Number	Scheme	Notes	Marks
11. (a)	$x = 3\sec\theta = \frac{3}{\cos\theta} = 3(\cos\theta)^{-1}$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3(\cos\theta)^{-2}(-\sin\theta)$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \pm k\left((\cos\theta)^{-2}(\sin\theta)\right)$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)(\tan\theta)}} = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3\tan\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$	Convincing proof with no notational or other errors such as missing θ's or missing signs or inconsistent variables. But use of $\cos^{-1}\theta$ as $\frac{1}{\cos\theta}$ is OK. Must see both <u>underlined steps.</u> Allow $3\tan\theta\sec\theta$	A1 *
	If the $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ is included on the lhs it must be correct but condone its omission and apply isw if possible if it appears correctly at some point in their working.		
			[2]
(a) **Alt 1**	$x = 3\sec\theta = \frac{3}{\cos\theta}$		
	$\left\{\begin{array}{ll} u = 3 & v = \cos\theta \\ \frac{\mathrm{d}u}{\mathrm{d}\theta} = 0 & \frac{\mathrm{d}v}{\mathrm{d}\theta} = -\sin\theta \end{array}\right\}$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{0(\cos\theta) - (3)(-\sin\theta)}{(\cos\theta)^2}$	Accept $\frac{0\times(\cos\theta) \pm (3)(\sin\theta)}{(\cos\theta)^2}$ as evidence but if the quotient rule is quoted, it must be correct.	M1
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)(\tan\theta)}} = \underline{\underline{3\sec\theta\tan\theta}}$ *	Convincing proof with no notational or other errors such as missing θ's. Must see both <u>underlined steps.</u> Allow $3\tan\theta\sec\theta$	A1 *
	If the $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ is included on the lhs it must be correct but condone its omission and apply isw if possible if it appears correctly at some point in their working.		
			[2]

(b)	$y=\frac{\sqrt{x^2-9}}{x}, x\geqslant 3;\ x=3\sec\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta}=3\sec\theta\tan\theta$		
	$\int \frac{\sqrt{x^2-9}}{x}\,\mathrm{d}x = \int \frac{\sqrt{((3\sec\theta)^2-9)}}{3\sec\theta}\,3\sec\theta\tan\theta\,\mathrm{d}\theta$	Full substitution of $\frac{\sqrt{x^2-9}}{x}$ in terms of θ and "dx" as their "$\pm k\sec\theta\tan\theta$". This may be implied if they reach $\pm\lambda\int\tan^2\theta\{\mathrm{d}\theta\}$ with no incorrect working seen.	M1
	Note: If $\sqrt{x^2-9}$ is simplified incorrectly to $x-3$ the first mark is still available for a full substitution. (Any subsequent marks are unlikely)		
	$=3\int\tan^2\theta\,\mathrm{d}\theta$	$\pm\lambda\int\tan^2\theta\{\mathrm{d}\theta\}$ (Allow $\pm\lambda\int\tan\theta\tan\theta\{\mathrm{d}\theta\}$)	M1
		$3\int\tan^2\theta\{\mathrm{d}\theta\}$ (Allow $3\int\tan\theta\tan\theta\{\mathrm{d}\theta\}$)	A1
	$=(3)\int(\sec^2\theta-1)\,\mathrm{d}\theta$	**Dependent on the previous M mark** applies $\tan^2\theta=\sec^2\theta-1$	dM1
	$=(3)(\tan\theta-\theta)$	$k\tan^2\theta\to k(\tan\theta-\theta)$	A1
	$\left\{\text{Area}(R)=\int_3^6\frac{\sqrt{(x^2-9)}}{x}\,\mathrm{d}x=\left[3\tan\theta-3\theta\right]_0^{\frac{\pi}{3}}\right\}$		
	$=\left(3\tan\left(\frac{\pi}{3}\right)-3\left(\frac{\pi}{3}\right)\right)-(0)$	Substitutes limits of $\frac{\pi}{3}$ and 0 into an expression that contains a trigonometric and an algebraic function and subtracts the correct way round. [**Note:** Limit of 0 can be implied.] If they return to x, they must substitute the limits 6 and 3 and subtract the correct way round having previously obtained a trigonometric and an algebraic function.	M1
	$=3\sqrt{3}-\pi$	$3\sqrt{3}-\pi$	A1
	$\left[3\tan\theta-3\theta\right]_0^{\frac{\pi}{3}}=3\sqrt{3}-\pi$ can score the final M1A1 but if no substitution is shown and the answer is incorrect, score M0		
			[7]
			9
	Question 11 Notes		
11. (a)	**Note**	$x=\frac{3}{\cos\theta}\Rightarrow x\cos\theta=3\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}\theta}\cos\theta-x\sin\theta=0\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{x\sin\theta}{\cos\theta}=3\sec\theta\tan\theta$ is M1A1. M1 for $\pm A\frac{\mathrm{d}x}{\mathrm{d}\theta}\cos\theta\pm B\,x\sin\theta=0$	
(b)	**Note**	A decimal answer of 2.054559769… (without a correct **exact** answer) is A0.	

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12. (a) Show that

$$\cot x - \tan x \equiv 2\cot 2x, \qquad x \neq 90n°,\ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

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Question Number	Scheme	Notes	Marks
12.	$\cot x - \tan x \equiv 2\cot 2x$		
(a)	$\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$	Attempts to write both $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$ only	M1
	$= \dfrac{\cos^2 x}{\sin x\cos x} - \dfrac{\sin^2 x}{\cos x\sin x}\left(= \dfrac{\cos^2 x - \sin^2 x}{\sin x\cos x}\right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$= \dfrac{\cos 2x}{\frac{1}{2}\sin 2x}\left(= \dfrac{2\cos 2x}{\sin 2x}\right)$	**Dependent on both the previous M marks.** Evidence of correctly applying either $\cos 2x = \cos^2 x - \sin^2 x$ or $\sin 2x = 2\sin x\cos x$	ddM1
	$= 2\cot 2x$ **(*)**	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]
(a) **Alt 1**	$\cot x - \tan x = \dfrac{1}{\tan x} - \tan x$	Writes $\cot x$ in terms of $\tan x$	M1
	$\dfrac{1}{\tan x} - \dfrac{\tan^2 x}{\tan x}\left(= \dfrac{1-\tan^2 x}{\tan x}\right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$\dfrac{2}{\tan 2x}$	**Dependent on both the previous M marks.** Evidence of correctly applying $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$	ddM1
	$= 2\cot 2x$ **(*)**	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1*
			[4]
(a) **Alt 2**	$2\cot 2x = \dfrac{2}{\tan 2x}$	Applies $\cot 2x = \dfrac{1}{\tan 2x}$	M1
	$= \dfrac{2}{\frac{2\tan x}{1-\tan^2 x}}$	**Dependent on the previous M mark** Attempts to apply the double angle formula for $\tan 2x$	dM1
	$= \dfrac{1-\tan^2 x}{\tan x} = \dfrac{1}{\tan x} - \tan x$	**Dependent on both the previous M marks.** Obtains a rational fraction with a single denominator and attempts to split this up into 2 terms	ddM1
	$= \cot x - \tan x$ **(*)**	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]

(b)	$5+\cot(\theta-15°)-\tan(\theta-15°)=0$		
	$\Rightarrow 5+2\cot(...)=0$	Obtains an equation of this form.	M1
	$\cot(...)=-\frac{5}{2} \Rightarrow \tan(...)=-\frac{2}{5}$	Obtains an equation of the form $\tan(...)=\pm\frac{2}{5}$	M1
	$2\theta-30=\tan^{-1}\left(-\frac{2}{5}\right)$	Can be implied by e.g. $2\theta-30=\text{awrt}-21.8$ or $2\theta-30=\text{awrt } 158.2$	A1
	$\theta=\text{awrt } 4.1°$ **or** $\theta=\text{awrt } 94.1°$	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta=\text{awrt } 4.1°$ **and** $\theta=\text{awrt } 94.1°$	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
	Alternative to part (b):		
	$5+\cot(...)-\tan(...)=0 \Rightarrow 5\tan(...)+1-\tan^2(...)$ $\tan^2(...)-5\tan(...)-1=0$ Multiples through by $\tan(...)$ to obtain a 3TQ in $\tan(...)$		M1
	$\tan(...)=\frac{5\pm\sqrt{25+4}}{2}$	Solves their 3TQ and proceeds to tan(…) =	M1
	$(\theta-15°)=\tan^{-1}\left(\frac{5\pm\sqrt{25+4}}{2}\right)$	Can be implied by e.g. $\theta-15=79.099...$ or $\theta-15=-10.900...$	A1
	$\theta=\text{awrt } 4.1°$ **or** $\theta=\text{awrt } 94.1°$	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta=\text{awrt } 4.1°$ **and** $\theta=\text{awrt } 94.1°$	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
			9

Question 12 Notes

(a)	**Note**	Allow candidates to "meet in the middle" e.g. $\text{lhs}=\frac{1}{\tan x}-\tan x=\frac{1-\tan^2 x}{\tan x}$: M1dM1 as in Alt1 $\text{rhs}=2\cot 2x=\frac{2}{\tan 2x}=\frac{2}{\frac{2\tan x}{1-\tan^2 x}}$: ddM1 uses double angle for tan2x on rhs $=\frac{1-\tan^2 x}{\tan x}$ so lhs = rhs A1 Correct proof with conclusion

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13. (a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.
(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x), \qquad t \geqslant 0, \ 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(7)

Given that $k = 0.1$

(c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.
(2)

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Question Number	Scheme	Notes	Marks
13. (a)	$\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)}$ $\Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \ldots$ or $B = \ldots$	Forming a correct identity. For example, $1 \equiv A(2-x) + B(4-x)$ from $\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)}$ and finds at least one of $A = \ldots$ or $B = \ldots$.	M1
	$A = -\frac{1}{2}, B = \frac{1}{2}$ giving $\frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$	$\frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$ or any equivalent form. ***Cannot*** be recovered from part (b) and must be stated as **partial fractions** in (a) **and not just the values of the constants**.	A1
	Correct answer in (a) scores both marks		
			[2]
(b)	$\frac{dx}{dt} = k(4-x)(2-x),\ t \geqslant 0$		
	$\int \frac{1}{(4-x)(2-x)}\, dx = \int k\, dt$	Separates variables correctly. dx and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.	B1 oe
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt\ (+c)$ Or e.g. $\frac{1}{2}\ln(8-2x) - \frac{1}{2}\ln(4-2x) = kt\ (+c)$	$\pm\lambda\ln\alpha(4-x) \pm \mu\ln\beta(2-x)$, $\lambda \neq 0, \mu \neq 0, \alpha \neq 0, \beta \neq 0$	M1
		$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt$ oe Do not condone missing brackets around the $4-x$ and/or the $2-x$ unless they are implied by subsequent work.	A1
	$\{t = 0, x = 0 \Rightarrow\}$ $\frac{1}{2}\ln 4 - \frac{1}{2}\ln 2 = 0 + c$ $\left\{\Rightarrow c = \frac{1}{2}\ln 2\right\}$	Using both $t = 0$ and $x = 0$ in an integrated equation containing a constant of integration.	M1
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt + \frac{1}{2}\ln 2 \Rightarrow \ln\left(\frac{(4-x)}{2(2-x)}\right) = 2kt$		
	$\frac{4-x}{4-2x} = e^{2kt}$	Starting from an equation of the form $\pm\lambda\ln(a-x) \pm \mu\ln(b-x) = \pm kt + c,\ \lambda, \mu, \alpha, \beta \neq 0$, and applies a fully correct method to eliminate their logarithms. (Sign errors only). **Must have a constant of integration that need not be evaluated.**	M1
	$4 - x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4 - 4e^{2kt} = x - 2xe^{2kt}$ $\Rightarrow 4 - 4e^{2kt} = x(1 - 2e^{2kt}) \Rightarrow x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$ (*)	**Dependent on the previous M mark** A complete correct method of rearranging to make x the subject allowing sign errors only. **Must have a constant of integration that need not be evaluated.**	dM1
		Achieves the given answer with no errors.	A1 *
			[7]

(c)	$\left\{\dfrac{4-x}{4-2x} = e^{2kt}\right\} \Rightarrow e^{2kt} = \dfrac{4-1}{4-2}\ \left\{= \dfrac{3}{2}\right\}$	Substitutes $x = 1$ leading to e^{2kt} = value **Note:** $k = 0.1$	M1
	$t = \dfrac{1}{2(0.1)}\ln\left(\dfrac{3}{2}\right) = 2.027325541...\ \{= 2.03\text{ (s) (3 sf)}\}$	Anything that rounds to 2.03 Do not apply isw here and do not accept the exact value.	A1
			[2]
			11
	Question 13 Notes		
(c)	**Note**	May use an earlier form of their equation to find t when x = 1 e.g. $\frac{1}{2}\ln(3) - \frac{1}{2}\ln(1) = 0.1t + \frac{1}{2}\ln 2 \Rightarrow 0.2t = \ln\frac{3}{2}$ M1: For correct processing leading to kt = value $t = \frac{1}{2(0.1)}\ln\left(\frac{3}{2}\right) = 2.027325541...\ \{= 2.03\text{ (s) (3 sf)}\}$ A1: Anything that rounds to 2.03 Do not apply isw here	

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14. Given that

$$y = \frac{(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(a) show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{Ax^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}} \qquad x > 2$$

where A is a constant to be found.

(6)

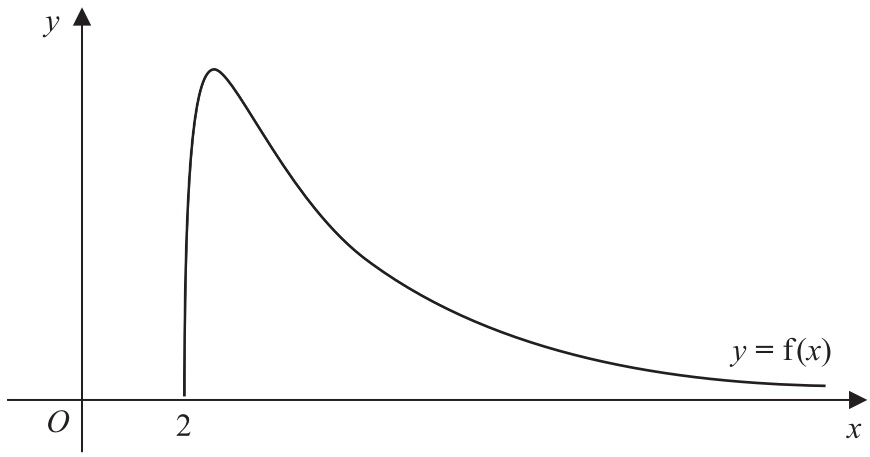

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{24(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(b) Use your answer to part (a) to find the range of f.

(5)

(c) State a reason why f^{-1} does not exist.

(1)

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Question Number	Scheme	Notes	Marks
14.	(a) $y = \dfrac{(x^2-4)^{\frac{1}{2}}}{x^3}$, $x > 2$; (b) $\mathrm{f}(x) = \dfrac{24(x^2-4)^{\frac{1}{2}}}{x^3}$, $x > 2$		
(a)	$u = (x^2-4)^{\frac{1}{2}}$ $\quad v = x^3$	$(x^2-4)^{\frac{1}{2}} \to \pm\lambda x(x^2-4)^{-\frac{1}{2}}$, $\lambda \neq 0$. **Can be implied.**	M1
	$\dfrac{du}{dx} = \dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ $\quad \dfrac{dv}{dx} = 3x^2$	$(x^2-4)^{\frac{1}{2}} \to \dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ un-simplified or simplified. **Can be implied.**	A1
	$\dfrac{dy}{dx} = \dfrac{\frac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}(x^3) - 3x^2(x^2-4)^{\frac{1}{2}}}{(x^3)^2}$	Applies $\dfrac{vu' - uv'}{v^2}$ with $u = (x^2-4)^{\frac{1}{2}}$, $v = x^3$, their u' and their v'	M1
		Correct $\dfrac{dy}{dx}$, un-simplified or simplified.	A1
	$= \dfrac{x^4(x^2-4)^{-\frac{1}{2}} - 3x^2(x^2-4)^{\frac{1}{2}}}{x^6}$		
	Either • $\dfrac{dy}{dx} = \dfrac{(x^2-4)^{-\frac{1}{2}}(x^4 - 3x^2(x^2-4))}{x^6}$ **or** • $\dfrac{dy}{dx} = \dfrac{x^2(x^2-4)^{-\frac{1}{2}} - 3(x^2-4)^{\frac{1}{2}}}{x^4}$	Simplifies $\dfrac{dy}{dx}$ by **either** correctly taking out a factor of $(x^2-4)^{-\frac{1}{2}}$ from their numerator **or** by multiplying numerator and denominator by $(x^2-4)^{\frac{1}{2}}$	M1
	$\dfrac{dy}{dx} = \dfrac{x^2 - 3(x^2-4)}{x^4(x^2-4)^{\frac{1}{2}}} \Rightarrow \dfrac{dy}{dx} = \dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$	Correct algebra leading to $\dfrac{dy}{dx} = \dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$ $\{A = -2\}$	A1
			[6]
	Alternative by product rule:		
	$u = (x^2-4)^{\frac{1}{2}}$ $\quad v = x^{-3}$	$(x^2-4)^{\frac{1}{2}} \to \pm\lambda x(x^2-4)^{-\frac{1}{2}}$, $\lambda \neq 0$. **Can be implied.**	M1
	$\dfrac{du}{dx} = \dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ $\quad \dfrac{dv}{dx} = -3x^{-4}$	$(x^2-4)^{\frac{1}{2}} \to \dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ un-simplified or simplified. **Can be implied.**	A1
	$\dfrac{dy}{dx} = \dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}(x^{-3}) + \left(-3x^{-4}\right)(x^2-4)^{\frac{1}{2}}$	Applies $vu' + uv'$ with $u = (x^2-4)^{\frac{1}{2}}$, $v = x^{-3}$, their u' and their v'	M1
		Correct $\dfrac{dy}{dx}$, un-simplified or simplified.	A1
	$\dfrac{dy}{dx} = \dfrac{1}{x^2(x^2-4)^{\frac{1}{2}}} - \dfrac{3(x^2-4)^{\frac{1}{2}}}{x^4} = \ldots$	Simplifies $\dfrac{dy}{dx}$ by correctly writing as two fractions and attempts a common denominator	M1
	$\dfrac{dy}{dx} = \dfrac{x^2 - 3(x^2-4)}{x^4(x^2-4)^{\frac{1}{2}}} \Rightarrow \dfrac{dy}{dx} = \dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$	Correct algebra leading to $\dfrac{dy}{dx} = \dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$ $\{A = -2\}$	A1
			[6]

(b)	$\left\{ f'(x) = \frac{24(-2x^2+12)}{x^4(x^2-4)^{\frac{1}{2}}} = 0 \Rightarrow \right\}$ $24(-2x^2+12)=0 \Rightarrow x^2=6$	Sets the numerator of their $\frac{dy}{dx}=0$ or the numerator of their $f'(x)=0$ and solves to give $x^2=K$, where $K>0$	M1
	$\Rightarrow x=\sqrt{6}$ or awrt 2.45	$x=\sqrt{6}$ or awrt 2.45 (Allow $x=\pm\sqrt{6}$ or awrt $\pm$ 2.45) (may be implied by their working)	A1
	$f\left(\sqrt{6}\right) = \frac{24(6-4)^{\frac{1}{2}}}{\left(\sqrt{6}\right)^3}; = \frac{24\sqrt{2}}{6\sqrt{6}} = \frac{4}{\sqrt{3}}$ or $\frac{4}{3}\sqrt{3}$	**Dependent on the previous M mark.** Substitutes their found *x* into $f(x)$ or the given expression from part (a). May be implied by awrt 2.3 or may need to check their value.	dM1
		cso leading to $f_{max} = \frac{24\sqrt{2}}{6\sqrt{6}}$ or $\frac{4}{\sqrt{3}}$ or $\frac{4}{3}\sqrt{3}$ **(Must be exact here)**	A1
	Range: $0 < f(x) \leqslant \frac{4}{3}\sqrt{3}$ or $0 < y \leqslant \frac{4}{\sqrt{3}}$ Or e.g. $\left(0, \frac{4}{3}\sqrt{3}\right]$	Correct range of *y* or f(*x*). Also allow ft on their maximum **exact** value if both of the M's have been scored. Allow f or "range" for f(*x*).	A1ft
			[5]
(c)	The function f is many-one	Also accept "the function f is not one-one" or "the inverse is one-many". **This mark should be withheld if there are contradictory statements.**	B1
			[1]
			12

Question 14 Notes

14 (c)	**Note**	Accept • f is many to one (or 2 values in domain of f map to one in the range) • f is not one to one • f^{-1} would be one to many • the inverse would be one to many • it would be one to many • it is not one to one • the graph illustrates a many to one function Do NOT allow • it is many to one • You can't reflect in $y = x$ **Any reference to "it" we must assume refers to the inverse because of the wording in the question**