Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34
Advanced

Tuesday 16 January 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. A curve C has equation

$$3^x + xy = x + y^2, \qquad y > 1$$

The point P with coordinates (4, 11) lies on C.

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer in the form $a + b\ln 3$, where a and b are rational numbers.

(6)

Question Number	Scheme	Marks
1	Differentiates wrt x $\quad 3^x \ln 3 + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$	B1, B1, M1, A1
	Substitutes (4, 11) **AND** rearranges to get $\dfrac{dy}{dx} = ..$ $\quad$ Nb $\dfrac{dy}{dx} = \dfrac{3^x \ln 3 + y - 1}{2y - x}$	M1
	$\Rightarrow 81\ln 3 + 4\dfrac{dy}{dx} + 11 = 1 + 22\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{81\ln 3 + 10}{18} = \dfrac{5}{9} + \dfrac{9}{2}\ln 3$	A1
		(6)
		(6 marks)

B1 $\quad 3^x \to 3^x \ln 3$ or $e^{x\ln 3} \to e^{x\ln 3}\ln 3$

B1 $\quad$ Correct product rule to differentiate xy finding $x\dfrac{dy}{dx} + y$

This may appear as $x\,dy + y\,dx$

M1 $\quad$ Differentiates implicitly to get $y^2 \to ky\dfrac{dy}{dx}$

This may appear as $y^2 \to k\,y\,dy$

A1 $\quad$ A correct differential of all terms other than 3^x so $"their\, 3^x \ln 3" + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$

If an extra $\dfrac{dy}{dx}$ is seen, ie $\dfrac{dy}{dx} = "their\, 3^x \ln 3" + x\dfrac{dy}{dx} + y = 1 + 2y\dfrac{dy}{dx}$ you may allow recovery if it is subsequently ignored.

You may see this as $"their\, 3^x \ln 3"dx + x\,dy + y\,dx = 1dx + 2y\,dy$

M1 $\quad$ Substitutes both $x = 4,\ y = 11$ into their expression (seen or implied at least once) and finds a 'numerical' value for $\dfrac{dy}{dx}$ (may rearrange first to give $\dfrac{dy}{dx} =$).

It is dependent upon having two terms in $\dfrac{dy}{dx}$ and proceeding, condoning slips, to $\dfrac{dy}{dx} = ...$

A1 $\quad$ Exact answer only but accept any equivalent e.g. $\dfrac{10}{18} + \dfrac{81}{18}\ln 3,\ \dfrac{5}{9} + 4.5\ln 3$

Remember to isw after sight of the correct answer

Note: If a candidate finds the equation of the tangent without specifically stating $\dfrac{dy}{dx} = ..$ they can score the M mark but not the A. If they give $\dfrac{dy}{dx} = ..$ you can apply isw

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2. $$f(x) = (125 - 5x)^{\frac{2}{3}} \qquad |x| < 25$$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2, giving the coefficient of x and the coefficient of x^2 as simplified fractions.
(4)

(b) Use your expansion to find an approximate value for $120^{\frac{2}{3}}$, stating the value of x which you have used and showing your working. Give your answer to 5 decimal places.
(3)

Question Number	Scheme	Marks
2 (a)	$\mathrm{f}(x)=(125-5x)^{\frac{2}{3}}=125^{\frac{2}{3}}(1-\frac{1}{25}x)^{\frac{2}{3}}$ $\quad 125^{\frac{2}{3}}$ or 25	B1
	$=25\times\left(1+\left(\frac{2}{3}\right)\left(-\frac{1}{25}x\right)+\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2}\left(-\frac{1}{25}x\right)^2+...\right)$	M1A1
	$=25-\frac{2}{3}x-\frac{1}{225}x^2...$	A1 **(4)**
	Alternative: $(125-5x)^{\frac{2}{3}}=125^{\frac{2}{3}}+\left(\frac{2}{3}\right)125^{-\frac{1}{3}}(-5x)+\frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)}{2}125^{-\frac{4}{3}}(-5x)^2$	B1 M1 A1
	$=25-\frac{2}{3}x-\frac{1}{225}x^2...$	A1 **(4)**
(b)	Let $x = 1$ Evaluate $=25-\frac{2}{3}-\frac{1}{225}$ $= 24.32889$	B1 M1 A1 **(3)**
		(7 marks)

(a)
Way 1:

B1 For taking out a factor of $125^{\frac{2}{3}}$ or 25

M1 For the form of the binomial expansion with $n=\frac{2}{3}$ and a term of $\left(\pm\frac{1}{25}x\right)$, $\left(\pm\frac{5}{125}x\right)$ or $\left(\pm\left(\frac{1}{5}\right)^2x\right)$

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign of $\left(-\frac{1}{25}x\right)$. So allow

for either $\left(\frac{2}{3}\right)\left(\pm\frac{1}{25}x\right)$ or $\frac{\left(\frac{2}{3}\right)\times\left(-\frac{1}{3}\right)}{2}\left(\pm\frac{1}{25}x\right)^2$

A1 Any (unsimplified) form of the binomial expansion. Ignore factor preceding the bracket

A1 cao $=25-\frac{2}{3}x-\frac{1}{225}x^2...$ This must be simplified. Ignore extra terms.

(a) Way 2:

B1 For seeing either $125^{\frac{2}{3}}$ or 25 as the first term

M1 It is sufficient to see either term two or term three (unsimplified or simplified).
Allow a slip on the sign of $(-5x)$

So allow for either $\left(\frac{2}{3}\right)125^{-\frac{1}{3}}(\pm 5x)$ or $\frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2}125^{-\frac{4}{3}}(\pm 5x)^2$

The expression $125^{2/3}+\binom{2/3}{1}125^{-\frac{1}{3}}(\pm 5x)+\binom{2/3}{2}125^{-\frac{4}{3}}(\pm 5x)^2+$

does not score the method mark until one of the terms is processed as in the main method

A1 Any (un-simplified) form of the whole binomial expansion.

A1 Must now be simplified cao $=25-\frac{2}{3}x-\frac{1}{225}x^2-...$

(b)

B1: States $x = 1$or is **explicitly seen** to use $x = 1$

M1: See an attempt to substitute a value of x consistently in **their** series expansion, condoning slips.
May be implied by sight of $=25-\frac{2}{3}-\frac{1}{225}$ or the correct answer for their expression.
Allow if they have more terms, but not if they have fewer.

A1: cao 24.32889 * DO NOT ACCEPT AWRT*

Watch: 24.32881 is the calculator answer for $120^{2/3}$

Note: If there is a decimal answer and they don't show their method you will need to use your calculator with $x = 1$ to check their result for the M1
Correct part (a)

Eg 1. (a) $25-\frac{2}{3}x-\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=24.32889$ B0(not stated or seen) M1(implied) A1

Examples 2 to 5: Incorrect part (a)

Eg 2.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $120^{2/3}=24.33778$ B0 M1(implied by calculator check) A0

Eg 3.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $x = 1 \Rightarrow 120^{2/3}=24.33778$ B1 (stated) M1(implied by calculator check) A0

Eg 4.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=25-\frac{2}{3}\times 1+\frac{1}{225}\times 1^2=24.33778$ B1(seen) M1(seen) A0

Eg 5.(a) $25-\frac{2}{3}x+\frac{1}{225}x^2-...$ (b) $\Rightarrow 120^{2/3}=25-\frac{2}{3}+\frac{1}{225}=24.33778$ B0(not stated or seen) M1(implied) A0

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3. $$f(x) = \frac{x^2}{4} + \ln(2x), \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \frac{1}{2}e^{-\frac{1}{4}x^2}$$

(2)

The equation $f(x) = 0$ has a root near 0.5

(b) Starting with $x_1 = 0.5$ use the iterative formula

$$x_{n+1} = \frac{1}{2}e^{-\frac{1}{4}x_n{}^2}$$

to calculate the values of x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval, show that 0.473 is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

Question Number	Scheme	Marks
3. (a)	$f(x)=\frac{x^2}{4}+\ln(2x)=0$ so $\ln(2x)=-\frac{x^2}{4}$ and so $2x=e^{-\frac{x^2}{4}}$ and $x=\frac{1}{2}e^{-\frac{x^2}{4}}$ *	M1 A1* **(2)**
(b)	$x_2=\frac{1}{2}e^{-\frac{(0.5)^2}{4}}$	M1
	$x_2=\text{awrt } 0.4697$,	A1
	$x_3=\text{awrt } 0.4732$ and $x_4=\text{awrt } 0.4728$	A1 **(3)**
(c)	$f(0.4725)=-0.000756...<0$, $f(0.4735)=0.001594..>0$ Sign change (and as $f(x)$ is continuous) therefore root lies in the interval $[0.4725, 0.4735] \Rightarrow root=0.473$ (3 dp)	M1A1 **(2)**
		(7 marks)

(a) **M1:** Puts f(x) = 0, either stated, or implied by sight of $\frac{x^2}{4}+\ln(2x)=0$, then makes the $(2x)$ of $\ln(2x)$ the subject by taking the exponential. Condone slips and the omission of the bracket (very common) but taking the exp of each term, $\frac{x^2}{4}+\ln(2x)=0 \Rightarrow e^{\frac{x^2}{4}}+2x=0$ is M0

Alternatively, award for a correct answer with a missing first step $-\frac{x^2}{4}=\ln(2x) \Rightarrow 2x=e^{-\frac{x^2}{4}} \Rightarrow x=\frac{1}{2}e^{-\frac{x^2}{4}}$

Candidates who work backwards must proceed from $x=\frac{1}{2}e^{-\frac{x^2}{4}}$ to $\frac{x^2}{4}+\ln(2x)=0$ before the M mark is scored. They need to make a comment before the A mark is awarded. Eg. Hence f(x) = 0

A1*: Completely correct work **ignoring** bracketing on $\ln(2x)$ to achieve the printed answer.

(b)
M1: An attempt to substitute $x_1=0.5$ into the iterative formula

This may be implied by sight of $x_2=\frac{1}{2}e^{-\frac{(0.5)^2}{4}}$ $x_2=awrt\ 0.47$

A1: $x_2=awrt\ 0.4697$
A1: $x_3=awrt\ 0.4732$,and $x_4=awrt 0.4728$

Ignore subscripts, mark in the order given.

(c)
M1: Choose suitable interval for x, e.g. $[0.4725,\ 0.4735]$ attempts f(x) at each.
If they use a different function it must be defined or implied by sight of the expression.

For example, candidates could attempt $\pm g(x)$ at each where $g(x)=\left(x-\frac{1}{2}e^{-\frac{x^2}{4}}\right)$.

FYI $g(0.4725)=-0.00036$ $g(0.4735)=+0.00075$

A minority of candidate may choose a tighter range which should include 0.47282 (alpha to 5dp),
This would be acceptable for both marks, provided the conditions for the A mark are met.
Continued iteration is M0

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated) or 3 dp
(ii) sign change stated (or implied by $f(a)\times f(b)<0$) oe and
(iii)some form of conclusion which may be :
$\Rightarrow root = 0.473$ or "so result shown" or qed or tick or equivalent

x	f(x)
0.4725	-0.000756289
0.4726	-0.000521044
0.4727	-0.000285838
0.4728	-5.06723E-05
0.4729	0.000184454
0.473	0.00041954
0.4731	0.000654587
0.4732	0.000889594
0.4733	0.001124561
0.4734	0.001359489
0.4735	0.001594377

x	g(x)
0.4725	-0.000357482
0.4726	-0.000246309
0.4727	-0.000135135
0.4728	-2.39585E-05
0.4729	8.72201E-05
0.473	0.000198401
0.4731	0.000309584
0.4732	0.000420769
0.4733	0.000531956
0.4734	0.000643145
0.4735	0.000754336

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4.

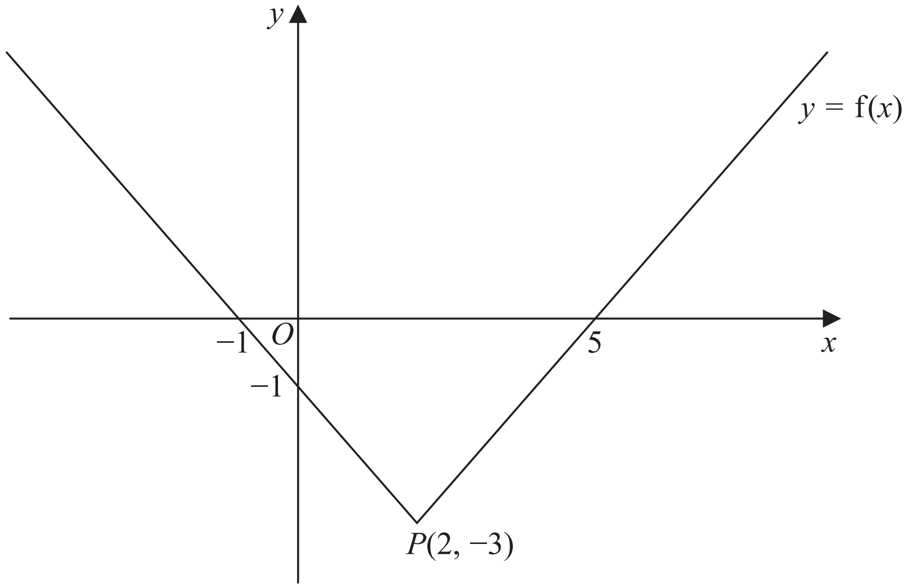


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x)$, $x \in \mathbb{R}$

The graph consists of two half lines that meet at the point P (2, −3), the vertex of the graph.

The graph cuts the y-axis at the point (0, −1) and the x-axis at the points (−1, 0) and (5, 0).

Sketch, on separate diagrams, the graph of

(a) $y = f(|x|)$,

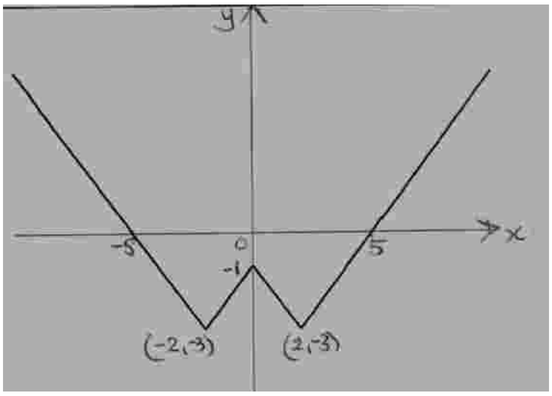
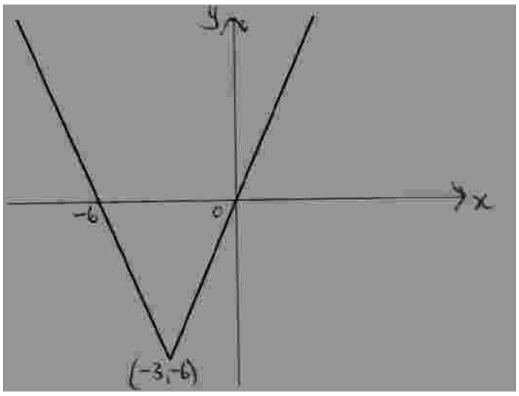
(3)

(b) $y = 2f(x + 5)$.

(3)

In each case, give the coordinates of the points where the graph crosses or meets the coordinate axes.

Also give the coordinates of any vertices corresponding to the point P.

Question Number	Scheme	Marks
4(a)	 **M1:** W shape anywhere, (so allow this mark for $y = \|f(x)\|$). Condone a lack of symmetry **A1:** Intercepts at (5, 0), $(-5,0)$ and $(0, -1)$. Allow 5, -5 and -1 written on the correct axes. Do NOT allow (0,5) for (5,0) etc. **A1:** vertices (corresponding to *P*) at both $(2, -3)$ **and** $(-2, -3)$	M1 A1 A1 **(3)**
(b)	 **M1:** V shape (correct way up) anywhere on the page. Do not however award for the original V. Condone a lack of symmetry or it appearing as a tick. **A1:** intercepts through *O* and $(-6, 0)$ Allow -6 written on the correct axis. Do NOT allow (0, -6) for (-6, 0). Remember the M must have been scored. **A1:** Single vertex at $(-3, -6)$	M1 A1 A1 **(3)** **(6 marks)**

Leave blank

5. (a) Express $\dfrac{9(4+x)}{16-9x^2}$ in partial fractions.

(3)

Given that

$$f(x) = \frac{9(4+x)}{16-9x^2}, \qquad x \in \mathbb{R}, \qquad -\frac{4}{3} < x < \frac{4}{3}$$

(b) express $\int f(x)\,dx$ in the form $\ln(g(x))$, where $g(x)$ is a rational function.

(4)

Question Number	Scheme	Marks
5(a)	$\frac{9(4+x)}{16-9x^2} \equiv \frac{A}{(4-3x)} + \frac{B}{(4+3x)} \Rightarrow A \text{ or } B$	M1
	$A = 6$ or $B = 3$ obtained at any point of the solution	A1
	$\frac{9(4+x)}{16-9x^2} \equiv \frac{6}{(4-3x)} + \frac{3}{(4+3x)}$	A1
		(3)
(b)	$\int \frac{9(4+x)}{16-9x^2}\,dx \equiv \int \frac{A}{(4-3x)} + \frac{B}{(4+3x)}\,dx$	
	$= -\frac{A}{3}\ln(4-3x) + \frac{B}{3}\ln(4+3x) \; (+c)$	M1 A1ft
	$(= -2\ln(4-3x) + \ln(4+3x) \; (+c))$	
	$= \ln\frac{(4+3x)}{(4-3x)^2} + c \;, = \ln\frac{k(4+3x)}{(4-3x)^2}$ or $\ln\left\lvert\frac{k(4+3x)}{(4-3x)^2}\right\rvert$	M1, A1
		(4)
		(7 marks)

This question may be marked as one

(a)

M1: Sets or implies $\frac{9(4+x)}{16-9x^2} \equiv \frac{A}{(4-3x)} + \frac{B}{(4+3x)}$ and proceeds to find at least one unknown

Sets or implies $\frac{(4+x)}{16/9 - x^2} \equiv \frac{A}{\left(4/3 - x\right)} + \frac{B}{\left(4/3 + x\right)}$ and proceeds to find at least one unknown

Sets or implies $\frac{9(4+x)}{16-9x^2} \equiv \frac{A}{(-3x-4)} + \frac{B}{(3x-4)}$ and proceeds to find at least one unknown

Condone $\frac{9(4+x)}{16-9x^2} \equiv \frac{A}{(3x-4)} + \frac{B}{(3x+4)}$ and proceeds to find at least one unknown

A1: Either constant correct or one correct fraction

A1: $\frac{9(4+x)}{16-9x^2} \equiv \frac{6}{(4-3x)} + \frac{3}{(4+3x)}$ in either (a) or within the integral in (b)

Alternative correct forms are;

$\frac{2}{\left(4/3 - x\right)} + \frac{1}{\left(4/3 + x\right)}$, $\quad -\frac{6}{(3x-4)} + \frac{3}{(3x+4)}$,

$\frac{-3}{(-3x-4)} + \frac{-6}{(3x-4)}$ $\quad \frac{1.5}{(1-3/4\,x)} + \frac{0.75}{(1+3/4\,x)}$

Watch out for $\frac{9(4+x)}{16-9x^2} \equiv \frac{6}{(3x-4)} - \frac{3}{(3x+4)}$ where we see 6 and -3 but scores M1 A0 A0

(b)

M1: Uses their partial fractions from part (a) and integrates to obtain $...\ln(4-3x)+...\ln(4+3x)$ or equivalent such as $...\ln\left(\frac{4}{3}-x\right)+...\ln\left(\frac{4}{3}+x\right)$ with or without modulus signs.

If they fail to reach $\frac{9(4+x)}{16-9x^2}\equiv\frac{A}{(4-3x)}+\frac{B}{(4+3x)}$ or an alternative correct form and use say $\frac{9(4+x)}{16-9x^2}\equiv\frac{A}{(x)}+\frac{B}{(16-9x)}$ candidate can (potentially) score the first three marks in part (b) as long as they have two fractions.

A1ft: Correct answer for their *A*, *B* (do not need constant of integration at this stage) – may have modulus signs

M1: For combining their log terms correctly with a constant of integration seen on the same line.

A1: cao. The answer given in the scheme o.e.

Allow $-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{k(4+3x)}{(4-3x)^2}$ without explanation or

$-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{e^c(4+3x)}{(4-3x)^2}$ or

$\frac{-3}{(-3x-4)}+\frac{-6}{(3x-4)}+c\to\ln\left|\frac{k(-3x-4)}{(3x-4)^2}\right|$ **with the modulus sign**

N.B. $\ln\frac{(4+3x)}{(4-3x)^2}+c$ gets M1 A0 as does $-2\ln(4-3x)+\ln(4+3x)+c\to\ln\frac{c(4+3x)}{(4-3x)^2}$

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6.

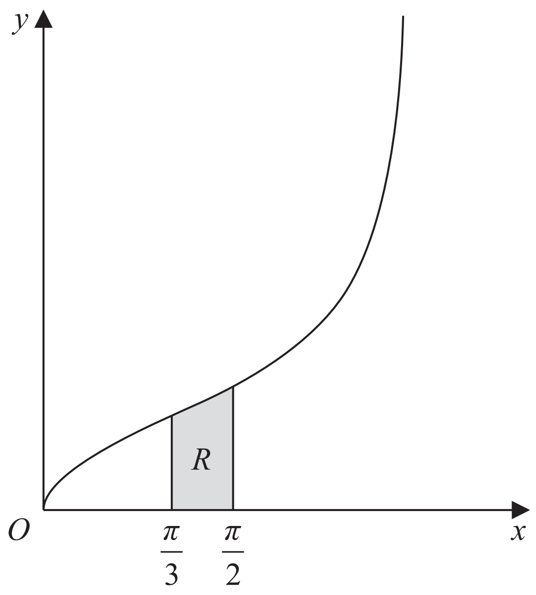


Figure 2

The curve shown in Figure 2 has equation

$$y^2 = 3\tan\left(\frac{x}{2}\right), \qquad 0 < x < \pi, \qquad y > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{\pi}{3}$ the x-axis and the line with equation $x = \frac{\pi}{2}$

The region R is rotated through 360° about the x-axis to generate a solid of revolution.

Show that the exact value of the volume of the solid generated may be written as $A\ln\left(\frac{3}{2}\right)$, where A is a constant to be found.

(5)

Question Number	Scheme	Marks
6	$(V) = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\left(\frac{x}{2}\right)\mathrm{d}x$	B1
	$= (\pi)\left[-6\ln\cos\left(\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $(\pi)\left[6\ln\sec\left(\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	M1A1
	$= (\pi)\left[-6\ln\left(\frac{1}{\sqrt{2}}\right) + 6\ln\left(\frac{\sqrt{3}}{2}\right)\right]$	dM1
	$= (\pi)\left[6\ln\left(\frac{\sqrt{6}}{2}\right)\right] = 3\pi\ln\left(\frac{3}{2}\right)$	A1 **(5)**
		(5 marks)

B1: Need the expression including π and correct limits. The limits and π may be implied by later working. Condone the omission of the $\mathrm{d}x$. You do not need to see V

As a minimum accept $\pi\int 3\tan\frac{x}{2}$ with the limits $\frac{\pi}{2}$ and $\frac{\pi}{3}$ being used later

or $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\frac{x}{2}$ found and subsequently multiplied by π

M1: Achieves k ln cos(x/2) or k lnsec(x/2) where k is constant

A1: cao – do not need π nor limits. It is for $-6\ln\cos\left(\frac{x}{2}\right)$ or $6\ln\sec\left(\frac{x}{2}\right)$ oe

Note that it may be common to see a first line of $(V) = 2\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\tan\left(\frac{x}{2}\right)\mathrm{d}x$.

In this case you would award for $-12\ln\cos\left(\frac{x}{2}\right)$ or $12\ln\sec\left(\frac{x}{2}\right)$

dM1: Dependent on first M1. Substitutes given limits and subtracts (either way around)
A1: cao and depends on having explicitly seen evidence for both M marks

Leave blank

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (13\mathbf{i} + 15\mathbf{j} - 8\mathbf{k}) + \lambda(3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (7\mathbf{i} - 6\mathbf{j} + 14\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection, B. **(6)**

(b) Find the acute angle between the lines l_1 and l_2 **(3)**

The point A has position vector $-5\mathbf{i} -3\mathbf{j} + 16\mathbf{k}$

(c) Show that A lies on l_1 **(1)**

The point C lies on the line l_1 where $\overrightarrow{AB} = \overrightarrow{BC}$

(d) Find the position vector of C. **(3)**

Question Number	Scheme	Marks
7 (a)	$\begin{pmatrix}13\\15\\-8\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-4\end{pmatrix}=\begin{pmatrix}7\\-6\\14\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\2\end{pmatrix}\Rightarrow$ $13+3\lambda=7+2\mu$, $15+3\lambda=-6-3\mu$, $-8-4\lambda=14+2\mu$ **any two** of these	M1
	Full method to find either λ **or** μ	M1
	$(1)-(2)\Rightarrow\mu=-3$	
	Sub $\mu=-3$ into (2) to give $\lambda=-4$ (need both*)	A1
	Check values in 3rd equation $-8-4\times-4=14-6=8$ (True)	B1
	Position vector of intersection is $\begin{pmatrix}13\\15\\-8\end{pmatrix}+-4\begin{pmatrix}3\\3\\-4\end{pmatrix}$ OR $\begin{pmatrix}7\\-6\\14\end{pmatrix}+-3\begin{pmatrix}2\\-3\\2\end{pmatrix}=$	dM1
	$=\begin{pmatrix}1\\3\\8\end{pmatrix}$	A1 **(6)**
(b)	$\cos\theta=\dfrac{\begin{pmatrix}3\\3\\-4\end{pmatrix}.\begin{pmatrix}2\\-3\\2\end{pmatrix}}{\sqrt{3^2+3^2+(-4)^2}\sqrt{2^2+(-3)^2+2^2}}=\dfrac{-11}{17\sqrt{2}}$	M1 A1
	So acute angle is awrt 62.8 degrees or awrt 1.10 radians	A1 **(3)**
(c)	When $\lambda=-6$ this gives $\begin{pmatrix}-5\\-3\\16\end{pmatrix}$ so A lies on l_1	B1 **(1)**
(d)	Vector approach: $\overrightarrow{AB}=6\mathbf{i}+6\mathbf{j}-8\mathbf{k}$ or "Bus stop" approach: At C $\lambda=-2$	M1
	$\overrightarrow{BC}=6\mathbf{i}+6\mathbf{j}-8\mathbf{k}$ and **c** = **b** + $\overrightarrow{BC}$ oe; so $\begin{pmatrix}13\\15\\-8\end{pmatrix}+-2\begin{pmatrix}3\\3\\-4\end{pmatrix}$	M1
	$\overrightarrow{OC}=7\mathbf{i}+9\mathbf{j}$	A1 **(3)**
		(13 marks)

(a)

M1: For writing down **any two equations** that give the coordinates of the point of intersection.
Accept two of $13+3\lambda=7+2\mu$, $15+3\lambda=-6-3\mu$, $-8-4\lambda=14+2\mu$
There must be an attempt to set the coordinates equal but condone one slip in total in the two equations.

M1: A full method to find **either** λ **or** μ.

A1: Both values correct $\mu=-3$ and $\lambda=-4$ (need both). Correct values following correct equations implies M1 A1

NB * It is possible to provide a complete proof by solving two of the three equations to give $\mu = -3$ or $\lambda = -4$ and then to solve the third equation with one of the previous equations to give the same value independently. It is then sufficient for this mark to have just one of $\mu = -3$ or $\lambda = -4$.

B1: The correct values must be substituted into **both** sides of the third equation. There must be some minimal statement (a tick will suffice) that the values are the same. This can also be scored via the substitution of $\mu = -3$ $\lambda = -4$ into **both** of the equations of the lines but there must be the same minimal statement.

For example $8 = 8$ is insufficient evidence but $8 = 8$ ✓ is fine

NB* It is possible to provide a complete proof by solving two of the three equations to give $\mu = -3$ or $\lambda = -4$ and then to solve the third equation with one of the previous equations to give the same value independently. It is then sufficient for this mark to have just one of $\mu = -3$ or $\lambda = -4$ but there must be the same minimal statement that the lines meet.

dM1: Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection.

A1: Correct answer only. Accept as a vector or a coordinate. Accept (1, 3, 8) (A correct answer here implies previous M mark)

Note that it is possible to score 1,1,1,0,1,1

(b)

M1: A clear attempt to use the correct formula for $\cos\theta =$ using the scalar product of the direction vectors. Allow for one slip and proceed to $\cos\theta =$ a fraction or decimal.

If they attempt to use the point of intersection and another point on each line they **must** get a multiple of the direction vectors.

A1: For $\frac{\pm 11}{17\sqrt{2}}$ or equivalent - may be implied by 62.8 or 117.2 or 1.10 radians or 2.04 radians

A1: cao for awrt 62.8 or 1.10 radians

(c)

B1: Shows that $\lambda = -6$ in all three cases **and** draws conclusion – e.g. point lies on line, or result shown, or QED, or tick….

Alternatively substitutes $\lambda = -6$ in $\begin{pmatrix} 13+3\times -6 \\ 15+3\times -6 \\ -8-4\times -6 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \\ 16 \end{pmatrix}$ and gives a (minimal) conclusion Eg □

(d)

M1: A correct attempt at any correct vector in the direction of *AB* or *BA* using $\overrightarrow{OA}$ and their $\overrightarrow{OB}$.

Allow if two components are correct.

For example $\overrightarrow{AB} = 6\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ or $\overrightarrow{BC} = 6\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ or $\overrightarrow{AC} = 12\mathbf{i} + 12\mathbf{j} - 16\mathbf{k}$

If the bus stop approach is used it is for attempting to find $\lambda(=-2)$ at *C*

M1: A fully correct method to find $\overrightarrow{OC}$ Using a vector approach **c** = **b** + their $\overrightarrow{AB}$ or **c** = **a** + $2\times$ their $\overrightarrow{AB}$ or **c** = $-$ **a** + $2\times$**b** Other methods are possible.

A1: $\overrightarrow{OC} = 7\mathbf{i} + 9\mathbf{j}$ or $\overrightarrow{OC} = \begin{pmatrix} 7 \\ 9 \\ 0 \end{pmatrix}$ Do NOT accept just the coordinate $(7, 9, 0)$

The correct vector without working scores 111, the correct coordinates 110

Leave blank

8. Given that

$$y = 8\tan(2x), \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

show that

$$\frac{dx}{dy} = \frac{A}{B + y^2}$$

where A and B are integers to be found.

(4)

question Number	Scheme	Marks
8	Differentiates wrt x $\quad \dfrac{dy}{dx} = 16\sec^2(2x)$ oe	M1
	Inverts to get $\dfrac{dx}{dy} = \dfrac{1}{16\sec^2 2x} = \dfrac{1}{16(1+\tan^2 2x)}$ | $\dfrac{dy}{dx} = 16\left(1+\tan^2(2x)\right) = 16\left(1+\left(\dfrac{y}{8}\right)^2\right)$	dM1
	$\dfrac{dx}{dy} = \dfrac{A}{B+y^2}$	ddM1
	$= \dfrac{4}{64+y^2}$	A1
		(4)
		(4 marks)

M1: Achieves $\dfrac{dy}{dx} = \lambda\sec^2(2x)$ oe or implicitly $1 = \lambda\sec^2(2x)\dfrac{dx}{dy}$

If they change $\tan 2x$ to $\dfrac{\sin 2x}{\cos 2x}$ they can score this mark for $\dfrac{dy}{dx} = \dfrac{\alpha\cos 2x\cos 2x \pm \beta\sin 2x\sin 2x}{(\cos 2x)^2}$

If they change $\tan 2x$ to $\dfrac{2\tan x}{1-\tan^2 x}$ they could never reach the required solution so score M0

dM1: Scored for two of the three processes 1 and 2 (either order) or 2 followed by 3 :

1. The reciprocal must be taken. (The variable cannot change)
2. The identity $1+\tan^2 2x = \sec^2 2x$ must be attempted
3. There must be an attempt to replace $\tan 2x$ by $\dfrac{y}{8}$

ddM1: Scored for attempting all three processes **and** attempting to eliminate the fractions (seen in at least two of the terms in the expression)

A1: cso

Alternative using arctan

M1: Expresses x as $x = \lambda\arctan\left(\frac{y}{8}\right)$ and attempts some differentiation $\Rightarrow \dfrac{dx}{dy} = \dfrac{...}{...+...y^2}$

dM1: As above but achieves $\dfrac{dx}{dy} = \dfrac{C}{(1+\left(\frac{y}{8}\right)^2)}$

ddM1: Eliminates fractions (seen in at least two of the terms in the expression) $\dfrac{dx}{dy} = \dfrac{A}{B+y^2}$

A1: cso $\dfrac{dx}{dy} = \dfrac{4}{64+y^2}$

Leave blank

9. (a) Show that

$$\frac{\cot^2 x}{1 + \cot^2 x} \equiv \cos^2 x$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\frac{\cot^2 x}{1 + \cot^2 x} = 8\cos 2x + 2\cos x$$

Give each solution in degrees to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Question Number	Scheme	Marks
9 (a)	**Way 1 LHS → RHS:** $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{\cos^2 x/\sin^2 x}{\text{cosec}^2 x}, \equiv \frac{\cos^2 x/\sin^2 x}{1/\sin^2 x} \equiv \cos^2 x$	M1 M1, A1*
	Way 2 LHS → RHS:: $\frac{\cot^2 x}{1+\cot^2 x} \times \frac{\sin^2 x}{\sin^2 x} \equiv \frac{\cos^2 x}{\sin^2 x+\cos^2 x}, \equiv \frac{\cos^2 x}{1}, \equiv \cos^2 x$	M1, M1, A1*
	Way 3 LHS → RHS: $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{1/\tan^2 x}{1+1/\tan^2 x} \equiv \frac{1}{1+\tan^2 x} \equiv \frac{1}{\sec^2 x}, \equiv \cos^2 x$	M1, M1, A1*
	Way 4: LHS → RHS:: $\frac{\cot^2 x}{1+\cot^2 x} \equiv \frac{\text{cosec}^2 x-1}{\text{cosec}^2 x} \equiv 1-\sin^2 x \equiv \cos^2 x$	M1 M1 A1*
	Way 5: Considers both sides	
	$\cos^2 x(1+\cot^2 x) = \cos^2 x\left(1+\frac{\cos^2 x}{\sin^2 x}\right) = \frac{\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$	M1 M1
	Hence $\frac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	A1*
		(3)
(b)	$\cos^2 x = 8(2\cos^2 x-1)+2\cos x$	M1
	$15\cos^2 x+2\cos x-8=0$	A1
	So cos$x$ = 2/3 or −4/5	M1
	$\Rightarrow x = 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$	dM1, A1 A1
		(6)
		(9 marks)

(a)
M1: Uses one VALID identity may implied. It is usually one of
$1+\cot^2 x = \text{cosec}^2 x$, $1+\tan^2 x = \sec^2 x$, $\cot^2 x = \cos^2 x/\sin^2 x$, $\cot^2 x = 1/\tan^2 x$ $\text{cosec}^2 x = 1/\sin^2 x$,
$1/\text{cosec}^2 x = \sin^2 x$, $1-\sin^2 x \equiv \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sec^2 x = 1/\cos^2 x$, $1/\sec^2 x = \cos^2 x$
$\cot^2 x \sin^2 x = \cos^2 x$
M1: Uses two VALID identities (not the same one twice) may implied.
A1: See a complete process.

All notation must be correct (~~$\cos x^2$~~ **) including correct use of variables** ~~$\text{cosec}^2 = \frac{1}{\sin^2}$~~

However, condone a lack of variables if it does not form part of their proof (and is an aside)
There will be some combinations of these methods. **A complete method with no errors scores** M1M1A1

Way 5 must contain a conclusion for the A mark.

$$\frac{\text{cosec}^2 x - 1}{\text{cosec}^2 x} = 1 - \sin^2 x = \cos^2 x$$

For example: The above minimum response can be marked as follows

M1: A correct identity $1+\cot^2 x = \text{cosec}^2 x$ used

M1: A second identity $1/\text{cosec}^2 x = \sin^2 x$ implied. Alternatively, could be scored for $1-\sin^2 x \equiv \cos^2 x$ used

A1: Completes proof with no errors and correct notation.

(b)

M1: Attempt to use both part (a) to replace left hand side and the correct double angle formula $\cos 2x = 2\cos^2 x - 1$ on right hand side to form an equation in $\cos x$ only. If, for instance, $\cos 2x = \cos^2 x - \sin^2 x$ is used this mark is not scored until the $\sin^2 x$ has been replaced by $1-\cos^2 x$

Condone a slip or an omission on either of the coefficients 8 and 2.

For example, $\cos^2 x = 2\cos^2 x - 1 + 2\cos x$ or $\cos^2 x = 8(2\cos^2 x - 1) + \cos x$ if fine for M1

A1: Correct three term quadratic with all terms on same side of equation. The = 0 may be implied by subsequent work

M1: Solves quadratic in $\cos x$ by any method – factorising, formula or completion of square or just writing down answers. Correct answers imply this M mark.

It is dependent upon having attempted to replace $\cos 2x$ by $\pm 2\cos^2 x \pm 1$ oe

dM1: For proceeding to find one correct answer for **their** inverse cos. You may have to use a calculator.

It is dependent upon the previous M mark. One correct answer implies this mark

A1: Two correct answers from awrt $48.2°, 143.1°, 216.9°, 311.8°$

or awrt 0.841, 5.442, 2.498 and 3.785 which are the radian solutions.

These cannot fortuitously be awarded from incorrect working.

A1: All four answers in the range, $x = awrt\ 48.2°$, $143.1°$, $216.9°$ and $311.8°$ and no others

Any extra solutions in the range withhold the final A mark.

Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

Due to the complexities in this question we will not be applying the misread rule for students who miscopy the equation in (b)

Leave blank

10. It is given that

$$f(x) = e^{-2x} \qquad x \in \mathbb{R}$$

$$g(x) = \frac{x}{x-3} \qquad x > 3$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of any points where the graph crosses the axes.

(2)

(b) Find the range of g

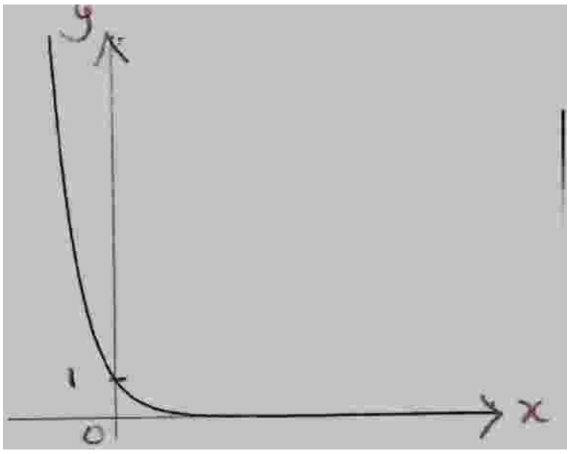
(2)

(c) Find $g^{-1}(x)$, stating the domain of g^{-1}

(4)

(d) Using algebra, find the exact value of x for which $fg(x) = 3$

(4)

Question Number	Scheme	Marks
10(a)	 Correct shape or intercept at 1	M1
	Fully correct	A1
		(2)
(b)	$g(x) > 1$	M1 A1
		(2)
(c)	$y = \dfrac{x}{x-3} \quad \Rightarrow (x-3)y = x \quad \Rightarrow xy - x = 3y$	M1
	$\Rightarrow x = \dfrac{3y}{y-1}$	dM1
	$g^{-1}(x) = \dfrac{3x}{x-1}$, with $x >$ "1"	A1, B1ft
		(4)
(d)	Way 1: $e^{-2\left(\frac{x}{x-3}\right)} = 3$ | Way 2: $g(x) = f^{-1}(3)$	M1
	$-2\left(\dfrac{x}{x-3}\right) = \ln 3$ | $g(x) = -\frac{1}{2}\ln 3$	A1
	$-2x = \ln 3(x-3)$ so $x =$ | $x = g^{-1}\left(-\frac{1}{2}\ln 3\right)$	dM1
	$x = \dfrac{3\ln 3}{2 + \ln 3}$	A1
		(4)
	(since this is outside the range for g(x) there are no solutions.)	(12 marks)

(a)

M1: For a correct shape (any position) or a curve passing through 1 on the +ve y - axis .

For the curve, look for a gradient that is always negative and increasing. Condone slips of the pen.

For the y intercept condone it being marked (1,0) but do not accept e^0

A1: Correct shape with y - intercept at 1 and asymptotic to the x - axis.

As a rule of thumb look for it reaching a point that is half way below the intercept at 1 with some levelling out. Do not condone the y intercept marked as (1,0) for this mark.

(b)

M1: Finds the value 1 but incorrect inequality is possible.

For example $y \geqslant 1, x > 1, \text{f}(x) > 1,\ \text{g} \neq 1,\quad 1 < y < 7$ or even g(x) < 1

A1: Needs g(x) > 1, g > 1, y > 1, $(1,\infty)$ or $(1;\infty)$ but do not accept $\text{f}(x) > 1$

(c)

M1: Setting y = multiplying across and attempting to collect x terms. Award for $\pm xy \pm x = \pm 3y$ but condone numerical slips. Alternatively starting with $x = \dfrac{y}{y-3}$ multiples across and attempts to collect y terms.

If it is attempted by division then expect to see $y = \dfrac{x}{x-3} \Rightarrow y = A + \dfrac{B}{x-3}$ before the A is moved across

dM1: Dependent upon the previous M mark. It is for an attempt at making x or a replaced y the subject of formula. Look for $x = \dfrac{\pm 3y}{\pm y \pm 1}$ but condone numerical slips.

A1: For $\text{g}^{-1}(x) = \dfrac{3x}{x-1}$ or exact equivalent such as $\text{g}^{-1}(x) = -\dfrac{3x}{1-x}$ or $\text{g}^{-1}(x) = 3 + \dfrac{3}{x-1}$ or $\text{g}^{-1}(x) = 3 - \dfrac{3}{1-x}$

Do not allow $\text{y} = \dfrac{3x}{x-1}$ or $\text{f}^{-1}(x) = \dfrac{3x}{x-1}$

B1ft: domain x > 1 or ft their range from part (b) as long as it is in x or set form $(1,\infty)$ Condone $(1;\infty)$

Don't follow through on $y \in \mathbb{R}$ following $x \in \mathbb{R}$

(d)

M1: Way 1 for an attempt at setting fg(x) =3 Condone slips but the order of operations must be correct.
Way 2 for using $\text{g}(x) = \text{f}^{-1}(3)$

A1: Undoes the exponentials to reaches a correct equation in x.

So either $-2\left(\dfrac{x}{x-3}\right) = \ln 3$ or $\dfrac{x}{x-3} = -\tfrac{1}{2}\ln 3$ or $\text{g}(x) = -\tfrac{1}{2}\ln 3$

dM1: A full attempt to make x the subject of the formula from two x terms. It is dependent upon the previous M.
Apply the same rules for change of subject as for M1 dM1 in (c)

Alternatively attempts $\text{g}^{-1}\text{f}^{-1}(3)$ following through on their g^{-1}

A1: $x = \dfrac{3\ln 3}{2 + \ln 3}$ or exact equivalent e.g. ln3 may appear as − ln(1/3) or − 1/2 (ln9)
(Condone lack of the final conclusion)

Leave blank

11.

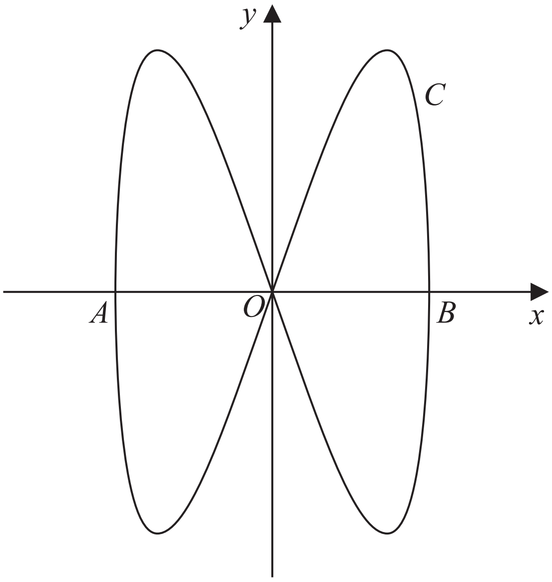


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = 3\cos t, \qquad y = 9\sin 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C meets the x-axis at the origin and at the points A and B, as shown in Figure 3.

(a) Write down the coordinates of A and B.

(2)

(b) Find the values of t at which the curve passes through the origin.

(2)

(c) Find an expression for $\frac{dy}{dx}$ in terms of t, and hence find the gradient of the curve when $t = \frac{\pi}{6}$

(4)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined.

(4)

P 5 1 4 0 3 A 0 3 2 4 8

Question Number	Scheme		Marks
11 (a)	(3,0) and (-3, 0)		B1, B1 **(2)**
(b)	$\frac{\pi}{2}$ and $\frac{3\pi}{2}$		M1 A1 **(2)**
(c)	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{18\cos 2t}{-3\sin t}$		M1 A1
	$=\frac{18\times\frac{1}{2}}{-3\times\frac{1}{2}}=-6$		dM1 A1 **(4)**
(d)	$y^2=81\times 4\sin^2 t\cos^2 t$	$y=9\times 2\sin t\cos t$	M1
	Attempts to replace $\cos^2 t=\frac{x^2}{9}$ *and* $\sin^2 t=1-\frac{x^2}{9}$	Attempts to replace $\cos t=\frac{x}{3}$ *and* $\sin t=\sqrt{1-\frac{x^2}{9}}$	M1
	Correct eqn $y^2=81\times 4\times\left(1-\frac{x^2}{9}\right)\times\frac{x^2}{9}$	Correct eqn $y=9\times 2\times\sqrt{1-\frac{x^2}{9}}\times\frac{x}{3}$	A1
	Obtain $y^2=4x^2(9-x^2)$	Obtain $y^2=4x^2(9-x^2)$	A1 **(4)**
			(12 marks)

(a)

B1: For one correct value of x or for seeing $(0,3)$ or $(0,-3)$

B1: For both **coordinates** correct. You can ignore any reference to A or B or O

(b)

M1: For one correct in degrees or radians. You may well see students who work out both $\cos t=0$ and $\sin 2t=0$ and produce many values of t without selecting the correct ones or even selecting incorrect ones. They can have access to the M mark

A1: For both correct in radians (and no others inside the range)

(c)

M1: Attempts to differentiate both x and y wrt t and uses $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

You may see candidates who attempt to set $y=18\sin t\cos t$ before differentiating.

Condone poor/ incorrect differentiation for the method.

A1: Correct result with no errors seen. $\frac{dy}{dx}=\frac{18\cos 2t}{-3\sin t}$ or exact equivalent, for example $\frac{dy}{dx}=\frac{18\cos^2 t-18\sin^2 t}{-3\sin t}$

(NB if $\frac{dy}{dt}$ and $\frac{dx}{dt}$ have the 'wrong' sign – this is A0)

dM1: Attempts to substitute $t = \frac{\pi}{6}$ into their trig expression for $\frac{dy}{dx} =$

It is dependent upon having scored the previous M1.

A1: cso

Generally

M1: Attempts to use the double angle formula for $\sin 2t$ to reach $y = 18\sin t\cos t$ or equivalent.

You may see this after squaring y so $y^2 = 81\times 4\sin^2 t\cos^2 t$ Condone $y^2 = 162\sin^2 t\cos^2 t$

M1: Uses correct trig identities to form an equation linking y with x

This usually involves using both $x = 3\cos t$ *and* $\sin t = \sqrt{1-\cos^2 t}$

Condone for this mark $x = \cos t$

A1: A correct intermediate equation

A1: cso $y^2 = 4x^2(9-x^2)$

Alt method 1: Uses both sides and given result

M1: Substitutes $x = 3\cos t$ and $y = 9\sin 2t$ into $y^2 = ax^2\left(b - x^2\right)$ and attempts the double angle formula for $\sin 2t$ Eg $81\times 4\sin^2 t\cos^2 t = a9\cos^2 t\left(b - 9\cos^2 t\right)$

M1 : Proceeds so that both sides are of the same form and attempts to find at least one unknown

Eg Replace $\sin^2 t$ by $1-\cos^2 t$ on lhs $\Rightarrow 81\times 4\left(1-\cos^2 t\right)\cos^2 t = a9\cos^2 t\left(b-9\cos^2 t\right)$ multiplies out $\Rightarrow 324\cos^2 t - 324\cos^4 t = 9ab\cos^2 t - 81a\cos^4 t$, and then solves two equations of the form $..a = ..$ **and** $...ab = ...$ to find one unknown.

A1: Solves two equations of the form $..a = ..$ **and** $...ab = ...$ to find both unknowns with one value correct.

A1: Correct equation $y^2 = 4x^2(9-x^2)$ or $a = 4, b = 9$ **and** states hence true

Note that it is possible to find $b = 9$ by substituting $\left(\pm 3, 0\right)$ into $y^2 = ax^2\left(b - x^2\right)$. This scores no marks

Alt method 2: Uses $\sin^2 2t = 1 - \cos^2 2t$ both sides and given result

M1: Attempts to square and use $\sin^2 2t = 1-\cos^2 2t$ Eg $y^2 = k\sin^2 2t = k\left(1-\cos^2 2t\right)$

M1: Attempts to use $\cos 2t = 2\cos^2 t - 1$ and $x = 3\cos t$ to form an equation linking y with x

Condone for this mark $x = \cos t$

A1: In this method it could be $y^2 = 81\left(1 - \left(1 - \frac{2x^2}{9}\right)^2\right)$

A1: cso $y^2 = 4x^2(9-x^2)$

Leave blank

12. (a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 significant figures.
(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leqslant t \leqslant 365$$

where t is the number of days since he began recording.

(b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.
(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(6)

Question Number	Scheme	Marks
12. (a)	$R=\sqrt{4+16}=\sqrt{20}$ *or* $2\sqrt{5}$	B1
	$\tan\alpha=\frac{4}{2}$	M1
	$\Rightarrow\ \alpha=1.11$ (awrt)	A1
		(3)
(b)	Maximum is 12+2*R* or minimum is 12– 2*R*	M1
	maximum = 20.9 (hours) (20h 57m) and minimum = 3.06 (hours) (3 hours 3 m)	A1 A1
		(3)
(c)	$17=12+k\text{"}R\text{"}\sin\left(\frac{2\pi t}{365}\pm\text{"}\alpha\text{"}\right)$	M1
	$\sin\left(\frac{2\pi t}{365}\pm\text{"}\alpha\text{"}\right)=....$	dM1
	For proceeding to one value for *t* from $17=12+2\text{"}R\text{"}\sin\left(\frac{2\pi t}{365}\pm\text{"}\alpha\text{"}\right)$	M1
	t = 99 (days) **or** 212 or 213 (days)	A1
	For finding two values for t	dM1
	t = 99 (days) **and** 212 or 213 (days)	A1
		(6)
		(12 marks)

(a)

B1: $R=\sqrt{20}$ or $2\sqrt{5}$ no working needed. Condone $R=\pm\sqrt{20}$ oe

M1: $\tan\alpha=\pm\frac{4}{2}$ or $\tan\alpha=\pm\frac{2}{4}$ and attempts to find alpha. If *R* is used accept $\sin\alpha=\pm\frac{4}{\text{"}R\text{"}}$ or $\cos\alpha=\pm\frac{2}{\text{"}R\text{"}}$

A1: accept $\alpha=$ awrt 1.11 ; also accept $\sqrt{20}\sin(x-1.11)$. Answers in degrees are A0

(b)

M1: Uses Maximum is 12+2*R* or minimum is 12– 2*R* with their value of *R*

A1: maximum value or minimum value correct allowing exact value(s) $12\pm2\sqrt{20}$ or $12\pm4\sqrt{5}$

A1: maximum **and** minimum value **awrt** 20.9 (20h 57m) 3.06 (3 hours 3 m)
Ignore any units in this part.

Note: It is possible to do this by differentiation. To score M1 you would need to see Differentiation to $\lambda\cos\left(\frac{2\pi t}{365}-\text{'}\alpha\text{'}\right)=0\Rightarrow\frac{2\pi t}{365}-\text{'}\alpha\text{'}=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow t=...$ and then substitute into *H* and find a value.

(c)

M1: For an attempt to interpret the model and writing it in terms of (a), condoning slips

Allow for $17 = 12 + k\,"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$, even $k = 1$ with their value for R and α (Slip on "2")

Allow $17 = 12 + k\,"R"\sin\left(x \pm "\alpha"\right)$ even $k = 1$ with their value for R and α (x instead of $\frac{2\pi t}{365}$)

dM1: For attempting to make $\sin\left(x \pm \text{their } \alpha\right)$ or $\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$ the subject.

M1: For the method of finding at least one value for t, $0 < t < 365$, from a "**correct**" starting point with $2\times$their R.

$17 = 12 + 2\,"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) \rightarrow \sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) = C$ to t = ... by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm "\alpha" = 0.593....$

Condone slips on the $\frac{2\pi}{365}$ for all M marks. Example you may see $\frac{2\pi}{36}$

A1: For one correct value for t, either awrt 99 **or** awrt 212/213.

dM1: For attempting to find a second value for t.

It is dependent upon the previous M mark and it is usually for moving from

$\left(\frac{2\pi t}{365} \pm "\alpha"\right) = \pi - \beta$ (where β was the principal value) to t = ...

by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm "\alpha" = 2.548....$

A1: awrt 99 **and** awrt 212 or 213 only $0 < t < 365$. Remember to ISW

Leave blank

13.

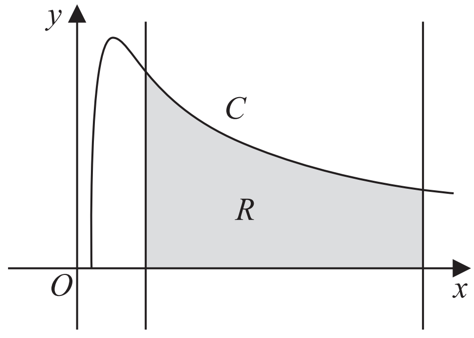


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2x}\ln 2x, \qquad x > \frac{1}{2}$$

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = \mathrm{e}$ and $x = 5\mathrm{e}$.

The table below shows corresponding values of x and y for $y = \frac{1}{2x}\ln 2x$. The values for y are given to 4 significant figures.

x	e	2e	3e	4e	5e
y	0.3114	0.2195	0.1712	0.1416	0.1215

(a) Use the trapezium rule with all the y values in the table to find an approximate value for the area of R, giving your answer to 3 significant figures.
(3)

(b) Using the substitution $u = \ln 2x$, or otherwise, find $\int \frac{1}{2x}\ln 2x \,\mathrm{d}x$
(3)

(c) Use your answer to part (b) to find the true area of R, giving your answer to 3 significant figures.
(2)

(d) Using calculus, find an equation for the tangent to the curve at the point where $x = \frac{\mathrm{e}^2}{2}$, giving your answer in the form $y = mx + c$ where m and c are exact multiples of powers of e.
(5)

Question Number	Scheme	Marks
13 (a)	$\frac{1}{2}\times e\times\{\underline{.........................}\}$	B1 oe
	$\frac{1}{2}\times h\times\underline{\{0.3114+0.1215+2(0.2195+0.1712+0.1416)\}}$	M1
	$=2.04$ (3 sf)	A1
		(3)
(b)	Let $u=\ln 2x$ then $\frac{du}{dx}=\frac{2}{2x}$	B1
	So $\int \frac{1}{2x}\ln 2x\,dx=\int\frac{1}{2}u\,du\ =\frac{1}{4}[\ln(2x)]^2$ oe	M1 A1
		(3)
(c)	$\left[\frac{1}{4}(\ln 2x)^2\right]_e^{5e}\quad=\left[\frac{1}{4}(\ln 10e)^2-\frac{1}{4}(\ln 2e)^2\right]=2.01$	M1 A1
		(2)
(d)	Way 1: $\frac{dy}{dx}=\frac{1}{2x}\times\frac{2}{2x}-\frac{1}{2x^2}\ln 2x$ $=\left(\frac{1}{2x^2}-\frac{1}{2x^2}\ln 2x\right)$ | Way 2: $\frac{dy}{dx}=\frac{2x\times\frac{2}{2x}-(\ln 2x)\times 2}{(2x)^2}$ $=\left(\frac{1-(\ln 2x)}{2x^2}\right)$	M1 A1
	When $x=\frac{e^2}{2}$, $y=\frac{2}{e^2}$	B1
	Uses $\left(\frac{e^2}{2},"\frac{2}{e^2}"\right)$ with their $\left.\frac{dy}{dx}\right\|_{\frac{e^2}{2}}$ to form equation of the tangent	dM1
	$y=-\frac{2}{e^4}x+\frac{3}{e^2}$ cao	A1
		(5)
		(13 marks)

(a)

B1: See $\frac{1}{2}\times e\times$ as part of trapezium rule or $h=e$ stated or used

M1: Correct structure of the terms inside the {.....} of the trapezium rule with their *h*. Expect to see $\frac{1}{2}\times h\times\underline{\{0.3114+0.1215+2(0.2195+0.1712+0.1416)\}}$ condoning slips on the digits of the numbers.

Award this mark if the bracket { } is not present $\frac{1}{2}\times h\times\underline{0.3114+0.1215+2(0.2195+0.1712+0.1416)}$

A1: awrt 2.04 Condone $\frac{599}{800}e$ or awrt $0.749e$

(b) Mark parts (b) and (c) together
Hence

B1: Finds $\frac{du}{dx}=\frac{2}{2x}$ or exact equivalent

M1: Integrates as far as ku^2 or $k[\ln(2x)]^2$ or equivalent

A1: cao $=\frac{1}{4}\left[\ln(2x)\right]^2 \;=\frac{1}{4}\left[\ln 2x\right]^2 \;=\frac{1}{4}\ln^2(2x)$ or $=\frac{1}{4}\ln^2 2x$ FINAL ANSWER
May be awarded in (c) (Does not need constant of integration)
Note $=\frac{1}{4}\ln(2x)^2$ or $=\frac{1}{4}\ln 2x^2$ is incorrect

(b) Otherwise

B1 M1: Integrates as far as $k\left[\ln(2x)\right]^2$
A1: cao $=\frac{1}{4}\left[\ln(2x)\right]^2 \;=\frac{1}{4}\left[\ln 2x\right]^2 \;=\frac{1}{4}\ln^2(2x)$ or $=\frac{1}{4}\ln^2 2x$ FINAL ANSWER
May be awarded in (c) (Does not need constant of integration)
Note $=\frac{1}{4}\ln(2x)^2$ or $=\frac{1}{4}\ln 2x^2$ is incorrect

(c)

M1: Uses correct limits correct way round in an integrated function.
Condone a poor attempt at integrating but the limits cannot be substituted into the original function.
If the integral is left in terms of *u* then the limits must be $\ln 2\text{e}=1.69...$ to $\ln 10\text{e}=3.30...$

A1: Correct answer. Accept awrt 2.01 Allow recovery from incorrect notation $=\frac{1}{4}\ln(2x)^2$,

(d)

M1: Attempts to differentiate $y=\frac{1}{2x}\ln 2x$ using either the product rule or quotient rule to achieve either

$\frac{\text{d}y}{\text{d}x}=\frac{A}{x^2}-\frac{B\ln 2x}{x^2}$ $(A,B>0)$ for the product rule

$\frac{\text{d}y}{\text{d}x}=\frac{\frac{Px}{x}-Q(\ln 2x)}{(2x)^2}$ oe with $(P,Q>0)$ for the quotient rule.

Condone $2x^2$ for $(2x)^2$ on the denominator.

A1: Correct use of quotient or product rule– may not be simplified – accept any correct answer.

B1: Correct simplified *y* coordinate $\frac{2}{\text{e}^2}$ oe such as 2e^{-2} may be awarded within a tangent (or normal equation)

dM1: Uses their $\left(\frac{\text{e}^2}{2},"\frac{2}{\text{e}^2}"\right)$ with their $\left.\frac{\text{d}y}{\text{d}x}\right|_{x=\frac{\text{e}^2}{2}}=\frac{A}{x^2}-\frac{B\ln 2x}{x^2}$ (or equivalent) to form the equation of the tangent. It is dependent upon the M mark for differentiation. Accept $y-"\frac{2}{\text{e}^2}"="-\frac{2}{\text{e}^4}"\left(x-\frac{\text{e}^2}{2}\right)$

If the form $y=mx+c$ is used it is for proceeding as far as $c=..$

A1 cao. It must be simplified. If the simplified *y* coordinate was not written down but this is correct and simplified, you should retrospectively award the B1

Leave blank

14. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$

(a) Find $\frac{dV}{dr}$

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

(b) Using the chain rule, or otherwise, show that

$$\frac{dr}{dt} = \frac{k}{r^n (t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

where k and n are constants to be found.

(2)

Initially, the radius of the balloon is 3 cm.

(c) Using the values of k and n found in part (b), solve the differential equation

$$\frac{dr}{dt} = \frac{k}{r^n (t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

to obtain a formula for r in terms of t.

(6)

(d) Hence find the radius of the balloon when $t = 175$, giving your answer to 3 significant figures.

(1)

(e) Find the rate of increase of the radius of the balloon when $t = 175$. Give your answer to 3 significant figures.

(2)

Question Number	Scheme	Marks
14. (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 **(1)**
(b)	$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \times \frac{1}{4\pi r^2}$	M1
	$= \frac{2250}{r^2(t+81)^{\frac{5}{4}}}$	A1 **(2)**
(c)	$\frac{dr}{dt} = \frac{k}{r^n(t+81)^{\frac{5}{4}}}$, so $\int r^n \, dr = \int \frac{k}{(t+81)^{\frac{5}{4}}} dt$	B1
	$\frac{r^3}{3} = \frac{2250}{-1/4} \times (t+81)^{-\frac{1}{4}} (+c)$	M1 A1ft
	When $t = 0$, $r = 3$ so $c = 9 + 9000 \times (+81)^{-\frac{1}{4}}$	M1
	So $\frac{r^3}{3} = -9000 \times (t+81)^{-\frac{1}{4}} + 3009$	dM1
	$r = \left[9027 - 27000(t+81)^{-\frac{1}{4}}\right]^{\frac{1}{3}}$	A1 **(6)**
(d)	Uses $t = 175$ to give $r = 13.2$	B1 **(1)**
(e)	Uses $t = 175$ to give $\frac{dr}{dt} = 0.0127$ or 0.0126	M1 A1 **(2)**
		(12 marks)

(a)

B1: cao. Condone $= \frac{4}{3} \times 3\pi r^2$ You may isw after a correct answer

(b)

M1: Correct use of chain rule $\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}$ with $\frac{dV}{dt}$ (allow slips) and their $\frac{dV}{dr}$

Allow any correct version $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with $\frac{dV}{dt}$ (allow slips) and their $\frac{dV}{dr}$ leading to $\frac{dr}{dt} =$

A1: cao $\left(\frac{dr}{dt} =\right) \frac{2250}{r^2(t+81)^{\frac{5}{4}}}$ Condone $r \leftrightarrow R, t \leftrightarrow T$

Watch for fudging here. The correct $\frac{dr}{dt}$ can easily appear to be found by incorrect chain rule. This is M0 A0

If the chain rule is written down it must be correct. There needs to be an attempt to use it for the M mark.
For A1 there must not be any incorrect lines.

(c)

B1: Correct separation of variables for their $\frac{\mathrm{d}r}{\mathrm{d}t}$. (Now a B mark) Allow with or without the integral signs but the $\mathrm{d}r$ and $\mathrm{d}t$ must be present and on the correct side as numerators.

M1: Correct method of integration on both sides, following through on *n* only

Look for $\int r^n\,\mathrm{d}r = \int \frac{k}{(t+81)^{\frac{5}{4}}}\,\mathrm{d}t \to ...r^{n+1} = ...(t+81)^{-\frac{1}{4}}$ with or without + *c*

A1ft: Correct integration of both sides ft their *k* and *n* – need not have *c*

M1: The equation must now have a constant. It is for using $t = 0$, $r = 3$ in order to find *c*. Condone poor attempts at integration for this method mark.

dM1: This is dependent upon having achieved $...\frac{r^3}{—} = ...(t+81)^{-\frac{1}{4}} + \text{numerical } c$

It is for proceeding to *r* = using a correct order of operations

Alternatively score this mark for correctly achieving $r^3 = 9027 - 27000(t+81)^{-\frac{1}{4}}$ (SC)

A1: $r = \left[9027 - 27000(t+81)^{-\frac{1}{4}}\right]^{\frac{1}{3}}$

(d)

B1: $r = 13.2$ (accept awrt 13.2)

Note: We can retrospectively award the dM1 in (c), for an answer of 13.2 in (d).

(e)

M1: Substitutes $t = 175$ and their $r = 13.2$ into $\frac{\mathrm{d}r}{\mathrm{d}t} =$

You may need to use a calculator here. It may be implied by 1sf rounded or truncated

A1: $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.0126$ or 0.0127 (awrt)

……..

Special case 1: Where evidence in (d) can be used in (c)

There may be students who do part (c) without ever achieving **the** formula for *r* in terms of *t*

Eg answer to (c) is $r = \sqrt[3]{C - 27000\times(t+81)^{-\frac{1}{4}}}$

Then in part (d) they use $t = 0, r = 3$ and go on to find $r = ..,$ when $t = 175$

If they state $C = 9027$ and go on to correct give $r = 13.2$ they can score all marks in (c)

If they don't find $C = 9027$ but use say

$$\left[\frac{r^3}{3}\right]_3^R = \left[-9000\times(t+81)^{-\frac{1}{4}}\right]_0^{175} \Rightarrow r = 13.2 \text{ just withhold the final A1 in (c)}$$

……..

Special case 2: Candidates who miscopy 9000 as 900 and achieve the following would lose the A mark in (b) and the final A1 in (c). Rules for a misread/miscopy 12-2=10 marks maximum

(b) $= \frac{225}{r^2(t+81)^{\frac{5}{4}}}$ (c) $r = \left[927 - 2700(t+81)^{-\frac{1}{4}}\right]^{\frac{1}{3}}$ (d) $r = 6.32$ (e) $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.00551$