

MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Differential Equations - F2 (Pearson Edexcel)

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Question Number	Scheme	Notes	Marks
2(a)	$(x^2 + 1) \frac{dy}{dx} + xy - x = 0$		
	$\frac{dy}{dx} + \frac{xy}{(1+x^2)} = \frac{x}{(1+x^2)}$	Correct form.	B1
	$I = e^{\int \frac{x}{1+x^2} dx} = e^{\frac{1}{2} \ln(1+x^2)} = (1+x^2)^{\frac{1}{2}}$	M1: $I = e^{\int \frac{x}{1+x^2} dx} = e^{k \ln(1+x^2)}$ where k is a constant. (Condone missing brackets around the $x^2 + 1$)	M1A1
		A1: Correct integrating factor of $(1+x^2)^{\frac{1}{2}}$	
	$y(1+x^2)^{\frac{1}{2}} = \int \frac{x}{(1+x^2)^{\frac{1}{2}}} dx$	Uses their integration factor to reach the form $yI = \int QI dx$	M1
	$= (1+x^2)^{\frac{1}{2}} (+c)$	Correct integration (+ c not needed)	A1
	$y = 1 + c(1+x^2)^{-\frac{1}{2}}$ oe	Cao with the constant correctly placed. (The “ $y =$ ” must appear at some point)	A1
		(6)	
Way 2	Alternative by separation of variables:		
	$\int \frac{dy}{1-y} = \int \frac{x}{x^2+1} dx$	Separates variables correctly	B1
	$\int \frac{x}{x^2+1} dx = \frac{1}{2} \ln(x^2+1)$	M1: $\int \frac{x}{x^2+1} dx = k \ln(x^2+1)$ where k is a constant. (Condone missing brackets around the $x^2 + 1$)	M1A1
		A1: Correct integration $\frac{1}{2} \ln(x^2+1)$	
	$\int \frac{dy}{1-y} = -\ln(1-y)$	$\int \frac{dy}{1-y} = k \ln(1-y)$ or e.g. $\int \frac{dy}{y-1} = k \ln(y-1)$	M1
	$-\ln(1-y) = \frac{1}{2} \ln(x^2+1) (+c)$	Fully correct integration	A1
	$y = 1 + c(1+x^2)^{-\frac{1}{2}}$ oe	Cao and isw if necessary.	A1
		(6)	
(b)	$2 = 1 + c(1+3^2)^{-\frac{1}{2}} \Rightarrow c = \dots$	Substitutes $x = 3$ and $y = 2$ and attempts to find a value for c .	M1
	$(y =) 1 + \sqrt{10}(1+x^2)^{-\frac{1}{2}}$ oe	Cao. (“ $y =$ ” not needed for this mark) and apply isw if necessary.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks	
6	$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 3y = x^2$			
(a)	$x = e^t \Rightarrow \frac{dx}{dy} = e^t \frac{dt}{dy} \Rightarrow \frac{dy}{dx} = e^{-t} \frac{dy}{dt}$	M1: Attempt first derivative using the chain rule to obtain $\frac{dx}{dy} = e^t \frac{dt}{dy}$	M1A1	
		A1: $\frac{dy}{dx} = e^{-t} \frac{dy}{dt}$ oe		
	$\frac{dy}{dx} = x^{-1} \frac{dy}{dt} \Rightarrow \frac{d^2 y}{dx^2} = -x^{-2} \frac{dy}{dt} + x^{-1} \frac{d^2 y}{dt^2} \cdot \frac{dt}{dx}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1	
		A1: Correct second derivative oe		
	$x^2 \left(\frac{1}{x^2} \frac{d^2 y}{dt^2} - \frac{1}{x^2} \frac{dy}{dt} \right) - 3x \left(\frac{1}{x} \frac{dy}{dt} \right) + 3y = (e^t)^2$	Substitutes their $\frac{d^2 y}{dx^2}$ and $\frac{dy}{dx}$ in terms of t into the differential equation	M1	
	$\frac{d^2 y}{dt^2} - 4 \frac{dy}{dt} + 3y = e^{2t}$	cso	A1	
			(6)	
	Alternative			
	$x = e^t \Rightarrow \frac{dy}{dt} = e^t \frac{dy}{dx} = x \frac{dy}{dx}$	M1: Attempt first derivative using $\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$	M1A1	
		A1: $\frac{dy}{dt} = x \frac{dy}{dx}$ oe		
	$\frac{d^2 y}{dt^2} = \frac{dx}{dt} \frac{dy}{dx} + x \frac{d^2 y}{dx^2} \cdot \frac{dx}{dt} = x \frac{dy}{dx} + x^2 \frac{d^2 y}{dx^2}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1	
		A1: Correct second derivative oe		
	$\frac{d^2 y}{dt^2} - x \frac{dy}{dx} - 3x \frac{dy}{dx} + 3y = e^{2t}$ $= \frac{d^2 y}{dt^2} - \frac{dy}{dt} - 3 \frac{dy}{dt} + 3y = e^{2t}$	Substitutes their $\frac{d^2 y}{dx^2}$ and $x \frac{dy}{dx}$ in terms of t into the differential equation	M1	
	$\frac{d^2 y}{dt^2} - 4 \frac{dy}{dt} + 3y = e^{2t}$	Cso	A1	
		(6)		

(b)	$m^2 - 4m + 3 = 0 \Rightarrow m = 1, 3$	Solves (according to the General Guidance) the correct quadratic (so should be $m = \pm 1, \pm 3$)	M1
	$(y =) Ae^{3t} + Be^t$	Correct CF in terms of t not x . (May be seen later in their GS)	A1
	$y = ke^{2t}, y' = 2ke^{2t}, y'' = 4ke^{2t}$	Correct form for PI and differentiates twice to obtain multiples of e^{2t} each time but do not allow if they are clearly integrating.	M1
	$4ke^{2t} - 8ke^{2t} + 3ke^{2t} = e^{2t} \Rightarrow k = \dots$	Substitutes their y, y', y'' that are of the form αe^{2t} into the differential equation and sets $= e^{2t}$ and proceeds to find their k	M1
	$(y) = -e^{2t}$	Correct PI or $k = -1$	A1
	$y = Ae^{3t} + Be^t - e^{2t}$	Correct ft GS in terms of t (their CF + their PI with non-zero PI). Must be $y = \dots$	B1ft
			(6)
(c)	$(y =) Ax^3 + Bx - x^2$	Allow equivalent expressions in terms of x e.g. $(y =) Ae^{3\ln x} + Be^{\ln x} - e^{2\ln x}$. Note that $y = \dots$ is not needed here.	B1
			(1)
			Total 13

Question Number	Scheme	Notes	Marks
4(a)	$y = 3e^{-x} \cos 3x + Ae^{-x} \sin 3x$ $\Rightarrow \frac{dy}{dx} = -3e^{-x} \cos 3x - 9e^{-x} \sin 3x - Ae^{-x} \sin 3x + 3Ae^{-x} \cos 3x$ $\left(= (-3 + 3A)e^{-x} \cos 3x + (-9 - A)e^{-x} \sin 3x \right)$ <p>Attempts to differentiate the given expression by using the product rule on $3e^{-x} \cos 3x$ to give $\alpha e^{-x} \cos 3x + \beta e^{-x} \sin 3x$ or by using the product rule on $Ae^{-x} \sin 3x$ to give $\alpha Ae^{-x} \cos 3x + \beta Ae^{-x} \sin 3x$</p>		M1
	$\frac{d^2y}{dx^2} = (-24 - 6A)e^{-x} \cos 3x + (18 - 8A)e^{-x} \sin 3x$ <p>(Terms may be uncollected)</p> <p>Uses the product rule again on an expression of the form $e^{-x} \sin 3x$ or $e^{-x} \cos 3x$ to give $\alpha e^{-x} \cos 3x + \beta e^{-x} \sin 3x$. Dependent on the first method mark.</p>		dM1
	$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 10y = (12 - 12A)e^{-x} \cos 3x + (36 + 4A)e^{-x} \sin 3x$ <p>Substitute their results into the differential equation. (May be implied)</p>		M1
	$12 - 12A = 0 \text{ or } 36 + 4A = 40 \Rightarrow A = \dots$	Compares coefficients of $e^{-x} \sin 3x$ or $e^{-x} \cos 3x$ and attempts to find A. Dependent on the previous method mark.	dM1
	$\Rightarrow A = 1$	cao	A1
			(5)
(b)	$m^2 - 2m + 10 = 0 \Rightarrow m = 1 \pm 3i$	M1: Forms and attempts to solve the Auxiliary Equation. See General Principles.	M1 A1
		A1: Correct solution for the AE	
	$(y =) e^x (C \cos 3x + D \sin 3x)$ <p>or</p> $(y =) Ce^{(1+3i)x} + De^{(1-3i)x}$	Correct form for CF using their complex roots from the AE	M1
	$y = e^x (C \cos 3x + D \sin 3x) + 3e^{-x} \cos 3x + e^{-x} \sin 3x$ <p>GS = their CF + their PI (Allow ft on their CF and PI)</p> <p>Must start $y = \dots$ and depends on at least one the M's being scored and must have been using a PI of the form given.</p>		A1ft
			(4)
(c)	$x = 0, y = 3 \Rightarrow 3 = C + 3 (\Rightarrow C = 0)$	Attempts to substitute $x = 0$ and $y = 3$ into their answer to (b)	M1
	$\frac{dy}{dx} = (C + 3D)e^x \cos 3x + (-3C + D)e^x \sin 3x - 10e^{-x} \sin 3x$ <p>Attempt to differentiate their GS with or without their C</p>		M1
	$3 = C + 3D$	Attempt to substitute $x = 0$ and $\frac{dy}{dx} = 3$ into their $\frac{dy}{dx}$	M1
	$y = e^x \sin 3x + 3e^{-x} \cos 3x + e^{-x} \sin 3x$	Correct answer. Must start y = ...	A1cao
			(4)
			Total 13

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6. Find the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = (\cos^2 x) \ln x, \quad 0 < x < \frac{\pi}{2}$$

Give your answer in the form $y = f(x)$.

(8)

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Question Number	Scheme	Notes	Marks
6.	$\cos x \frac{dy}{dx} + y \sin x = (\cos^2 x) \ln x$		
	$\frac{dy}{dx} + y \frac{\sin x}{\cos x} = \cos x \ln x$	Attempt to divide through by $\cos x$. If the intention is not clear, must see at least 2 terms divided by $\cos x$.	M1
	$I = e^{\int \frac{\sin x}{\cos x} dx} = e^{-\ln \cos x}$	M1: $e^{\int \pm \text{their } P(x) (dx)}$. Dependent on the first method mark. A1: $e^{-\ln \cos x}$ or $e^{\ln \sec x}$	dM1A1
	$= \frac{1}{\cos x}$	$\frac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$	A1
	$\frac{y}{\cos x} = \int \ln x dx$ or $\frac{d}{dx} \left(\frac{y}{\cos x} \right) = \ln x$	M1: $y \times \text{their } I = \int Q(x) \times \text{their } I dx$ or $\frac{d}{dx} (y \times \text{their } I) = Q(x) \times \text{their } I$ A1: $\frac{y}{\cos x} = \int \ln x dx$ or $\frac{d}{dx} \left(\frac{y}{\cos x} \right) = \ln x$	M1A1
	$\frac{y}{\cos x} = x \ln x - x + C$	Attempts $\int \ln x dx$ by parts correctly (correct sign needed unless correct formula quoted and used).	M1
	$y = (x \ln x - x + C) \cos x$	Any equivalent with the constant correctly placed and "y = ..." must appear at some stage.	A1
			Total 8
<p>Note: Failure to divide by $\cos x$ at the start would mean that only the 3rd Method mark is available.</p>			

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- 6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$$

(9)

- (b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$
- (5)**

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Question Number	Scheme	Notes	Marks	
	$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$			
6(a)	$m^2 + 3m + 2 = 0 \Rightarrow m = -1, -2$	Correct roots (may be implied by their CF)	B1	
	$y = Ae^{-2x} + Be^{-x}$	M1: CF of the correct form A1: Correct CF	M1A1	
	$y = ax^2 + bx + c$	Correct form for PI	B1	
	$\frac{dy}{dx} = 2ax + b, \frac{d^2y}{dx^2} = 2a \Rightarrow 2a + 3(2ax + b) + 2(ax^2 + bx + c) = 3x^2 + 2x + 1$		M1	
	M1: Differentiates twice and substitutes into the lhs of the given differential equation and puts equal to $3x^2 + 2x + 1$ or substitutes into the lhs of the given differential equation and compares coefficients with $3x^2 + 2x + 1$. For the substitution, at least one of y, y' or y'' must be correctly placed.			
	$a = \frac{3}{2}$		A1	
	$6a + 2b = 2 \Rightarrow b = -\frac{7}{2} \Rightarrow c = \frac{17}{4}$	M1: Solves to obtain one of b or c A1: Correct b and c	M1A1	
	$y = Ae^{-2x} + Be^{-x} + \frac{3}{2}x^2 - \frac{7}{2}x + \frac{17}{4}$	Correct ft (their CF + their PI) but must be $y = \dots$	B1ft	
			(9)	
(b)	$0 = A + B + \frac{17}{4}$	Substitutes $x = 0$ and $y = 0$ into their GS	M1	
	$\frac{dy}{dx} = -2Ae^{-2x} - Be^{-x} + 3x - \frac{7}{2} \Rightarrow 0 = -2A - B - \frac{7}{2}$ Attempts to differentiate and substitutes $x = 0$ and $y' = 0$		M1	
	$0 = A + B + \frac{17}{4}, 0 = -2A - B - \frac{7}{2} \Rightarrow A = \dots, B = \dots$	Solves simultaneously to obtain values for A and B	M1	
	$A = \frac{3}{4}, B = -5$	Correct values	A1	
	$y = \frac{3}{4}e^{-2x} - 5e^{-x} + \frac{3}{2}x^2 - \frac{7}{2}x + \frac{17}{4}$	Correct ft (their CF + their PI) but must be $y = \dots$	B1ft	
			(5)	
			Total 14	