MyStudyBro - Revision Exercise Tool

This Revision Handout includes the Questions and Answers of a total of 5 exercises!

Chapters:

Differential Equations - F2 (Pearson Edexcel)

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2. (a) Find the general solution of the differential equation

$$\left(x^2 + 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + xy - x = 0$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

(b) Find the particular solution for which $y = 2$ when $x = 3$

(2)

Question Number	Scheme	Notes	Marks
2(a)	$\left(x^2+1\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}+xy-x=0$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{xy}{\left(1+x^2\right)}=\dfrac{x}{\left(1+x^2\right)}$	Correct form.	B1
	$I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{\frac{1}{2}\ln\left(1+x^2\right)}=\left(1+x^2\right)^{\frac{1}{2}}$	M1: $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{k\ln\left(1+x^2\right)}$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integrating factor of $\left(1+x^2\right)^{\frac{1}{2}}$	M1A1
	$y\left(1+x^2\right)^{\frac{1}{2}}=\int\dfrac{x}{\left(1+x^2\right)^{\frac{1}{2}}}\mathrm{d}x$	Uses their integration factor to reach the form $yI=\int Q\,\mathrm{I}\,\mathrm{d}x$	M1
	$=\left(1+x^2\right)^{\frac{1}{2}}(+c)$	Correct integration (+ c not needed)	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao with the constant correctly placed. (The "$y=$" must appear at some point)	A1
			(6)
Way 2	**Alternative by separation of variables:**		
	$\int\dfrac{\mathrm{d}y}{1-y}=\int\dfrac{x}{x^2+1}\,\mathrm{d}x$	Separates variables correctly	B1
	$\int\dfrac{x}{x^2+1}\,\mathrm{d}x=\dfrac{1}{2}\ln\left(x^2+1\right)$	M1: $\int\dfrac{x}{x^2+1}\,\mathrm{d}x=k\ln\left(x^2+1\right)$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integration $\dfrac{1}{2}\ln\left(x^2+1\right)$	M1A1
	$\int\dfrac{\mathrm{d}y}{1-y}=-\ln(1-y)$	$\int\dfrac{\mathrm{d}y}{1-y}=k\ln(1-y)$ or e.g. $\int\dfrac{\mathrm{d}y}{y-1}=k\ln(y-1)$	M1
	$-\ln(1-y)=\dfrac{1}{2}\ln\left(x^2+1\right)(+c)$	Fully correct integration	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao and isw if necessary.	A1
			(6)
(b)	$2=1+c\left(1+3^2\right)^{-\frac{1}{2}}\Rightarrow c=...$	Substitutes $x=3$ and $y=2$ and attempts to find a value for c.	M1
	$(y=)1+\sqrt{10}\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao. ("$y=$" not needed for this mark) and apply isw if necessary.	A1
			(2)
			Total 8

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6. (a) Show that the transformation $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^{2t} \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II), expressing y as a function of t.

(6)

(c) Hence find the general solution of the differential equation (I).

(1)

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Question Number	Scheme	Notes	Marks
6	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=x^2$		
(a)	$x=\mathrm{e}^t\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\frac{\mathrm{d}t}{\mathrm{d}y}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$	M1: Attempt first derivative using the chain rule to obtain $\frac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\frac{\mathrm{d}t}{\mathrm{d}y}$	M1A1
		A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$ oe	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=x^{-1}\frac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-x^{-2}\frac{\mathrm{d}y}{\mathrm{d}t}+x^{-1}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}.\frac{\mathrm{d}t}{\mathrm{d}x}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$x^2\left(\frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}\right)-3x\left(\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t}\right)+3y=\left(\mathrm{e}^t\right)^2$	Substitutes their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t into the differential equation	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	cso	A1
			(6)
	Alternative		
	$x=\mathrm{e}^t\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^t\frac{\mathrm{d}y}{\mathrm{d}x}=x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1: Attempt first derivative using $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}\times\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1
		A1: $\frac{\mathrm{d}y}{\mathrm{d}t}=x\frac{\mathrm{d}y}{\mathrm{d}x}$ oe	
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}.\frac{\mathrm{d}x}{\mathrm{d}t}=x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=\mathrm{e}^{2t}$ $=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{\mathrm{d}y}{\mathrm{d}t}-3\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	Substitutes their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $x\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t into the differential equation	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\frac{\mathrm{d}y}{\mathrm{d}t}+3y=\mathrm{e}^{2t}$	Cso	A1
			(6)

(b)	$m^2-4m+3=0 \Rightarrow m=1,\ 3$	Solves (according to the General Guidance) the correct quadratic (so should be *m* = ±1, ±3)	M1
	$(y=)\,Ae^{3t}+Be^{t}$	Correct CF in terms of *t* not *x*. (May be seen later in their GS)	A1
	$y=ke^{2t}, y'=2ke^{2t},\ y''=4ke^{2t}$	Correct form for PI and differentiates twice to obtain multiples of e^{2t} each time but do not allow if they are clearly integrating.	M1
	$4ke^{2t}-8ke^{2t}+3ke^{2t}=e^{2t} \Rightarrow k=...$	Substitutes their $y,\ y',\ y''$ that are of the form αe^{2t} into the differential equation and sets = e^{2t} and proceeds to find their *k*	M1
	$(y)=-e^{2t}$	Correct PI or *k* = -1	A1
	$y=Ae^{3t}+Be^{t}-e^{2t}$	Correct ft GS **in terms of *t*** (their CF + their PI with non-zero PI). **Must be *y* = …**	B1ft
			(6)
(c)	$(y=)\,Ax^3+Bx-x^2$	Allow equivalent expressions in terms of *x* e.g. $(y=)\,Ae^{3\ln x}+Be^{\ln x}-e^{2\ln x}$. **Note that *y* = … is not needed here.**	B1
			(1)
			Total 13

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4. $$y = 3\mathrm{e}^{-x}\cos 3x + A\mathrm{e}^{-x}\sin 3x$$

is a particular integral of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 10y = 40\mathrm{e}^{-x}\sin 3x$$

where A is a constant.

(a) Find the value of A.
(5)

(b) Hence find the general solution of this differential equation.
(4)

(c) Find the particular solution of this differential equation for which both $y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$ at $x = 0$
(4)

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Question Number	Scheme	Notes	Marks
4(a)	$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$ $\Rightarrow \frac{dy}{dx} = -3e^{-x}\cos 3x - 9e^{-x}\sin 3x - Ae^{-x}\sin 3x + 3Ae^{-x}\cos 3x$ $\left(= (-3+3A)e^{-x}\cos 3x + (-9-A)e^{-x}\sin 3x\right)$ Attempts to differentiate the **given expression** by using the product rule on $3e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$ or by using the product rule on $Ae^{-x}\sin 3x$ to give $\alpha Ae^{-x}\cos 3x + \beta Ae^{-x}\sin 3x$		M1
	$\frac{d^2y}{dx^2} = (-24-6A)e^{-x}\cos 3x + (18-8A)e^{-x}\sin 3x$ (Terms may be uncollected) Uses the product rule again on an expression of the form $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$. **Dependent on the first method mark.**		**d**M1
	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = (12-12A)e^{-x}\cos 3x + (36+4A)e^{-x}\sin 3x$ Substitute their results into the differential equation. (May be implied)		M1
	$12-12A=0$ or $36+4A=40 \Rightarrow A = ...$	Compares coefficients of $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ and attempts to find *A*. **Dependent on the previous method mark.**	**d**M1
	$\Rightarrow A = 1$	cao	A1
			(5)
(b) **Marks for (b) can score anywhere in their answer.**	$m^2 - 2m + 10 = 0 \Rightarrow m = 1 \pm 3i$	M1: Forms and attempts to solve the Auxiliary Equation. See General Principles. A1: Correct solution for the AE	M1 A1
	$(y=)e^{x}(C\cos 3x + D\sin 3x)$ or $(y=)Ce^{(1+3i)x} + De^{(1-3i)x}$	Correct form for CF using their complex roots from the AE	M1
	$y = e^{x}(C\cos 3x + D\sin 3x) + 3e^{-x}\cos 3x + e^{-x}\sin 3x$ GS = their CF + their PI (**Allow ft on their CF and PI**) Must start *y* = … and depends on at least one the M's being scored and must have been using a PI of the form given.		A1ft
			(4)
(c)	$x=0, y=3 \Rightarrow 3 = C+3 (\Rightarrow C=0)$	Attempts to substitute *x* = 0 and *y* = 3 into their answer to (b)	M1
	$\frac{dy}{dx} = (C+3D)e^{x}\cos 3x + (-3C+D)e^{x}\sin 3x - 10e^{-x}\sin 3x$ Attempt to differentiate their GS with or without their *C*		M1
	$3 = C + 3D$	Attempt to substitute *x* = 0 and $\frac{dy}{dx} = 3$ into their $\frac{dy}{dx}$	M1
	$y = e^{x}\sin 3x + 3e^{-x}\cos 3x + e^{-x}\sin 3x$	Correct answer. **Must start *y* = …**	A1cao
			(4)
			Total 13

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6. Find the general solution of the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sin x = (\cos^2 x) \ln x, \qquad 0 < x < \frac{\pi}{2}$$

Give your answer in the form $y = \mathrm{f}(x)$.

(8)

Question Number	Scheme	Notes	Marks
6.	$\cos x\frac{dy}{dx}+y\sin x=(\cos^2 x)\ln x$		
	$\frac{dy}{dx}+y\frac{\sin x}{\cos x}=\cos x\ln x$	Attempt to divide through by $\cos x$. If the intention is not clear, must see at least 2 terms divided by $\cos x$.	M1
	$I=e^{\int\frac{\sin x}{\cos x}dx}=e^{-\ln\cos x}$	M1: $e^{\int\pm\text{their P}(x)\,(dx)}$. Dependent on the first method mark. A1: $e^{-\ln\cos x}$ or $e^{\ln\sec x}$	dM1A1
	$=\frac{1}{\cos x}$	$\frac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$	A1
	$\frac{y}{\cos x}=\int\ln x\,dx$ or $\frac{d}{dx}\left(\frac{y}{\cos x}\right)=\ln x$	M1: $y\times$ their $I=\int Q(x)\times$ their I dx or $\frac{d}{dx}(y\times\text{their }I)=Q(x)\times$ their I A1: $\frac{y}{\cos x}=\int\ln x\,dx$ or $\frac{d}{dx}\left(\frac{y}{\cos x}\right)=\ln x$	M1A1
	$\frac{y}{\cos x}=x\ln x-x+C$	Attempts $\int\ln x\,dx$ by parts correctly (correct sign needed unless correct formula quoted and used).	M1
	$y=(x\ln x-x+C)\cos x$	Any equivalent with the constant correctly placed and "$y=\dots$" must appear at some stage.	A1
			Total 8
	Note: Failure to divide by cos x at the start would mean that only the 3rd Method mark is available.		

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$$

(9)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

Question Number	Scheme	Notes	Marks
	$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3x^2+2x+1$		
6(a)	$m^2+3m+2=0 \Rightarrow m=-1,-2$	Correct roots (may be implied by their CF)	B1
	$y=Ae^{-2x}+Be^{-x}$	M1: CF of the correct form A1: Correct CF	M1A1
	$y=ax^2+bx+c$	Correct form for PI	B1
	$\frac{dy}{dx}=2ax+b,\ \frac{d^2y}{dx^2}=2a \Rightarrow 2a+3(2ax+b)+2(ax^2+bx+c)=3x^2+2x+1$		M1
	M1: Differentiates twice and substitutes into the lhs of the given differential equation and puts equal to $3x^2+2x+1$ or substitutes into the lhs of the given differential equation and compares coefficients with $3x^2+2x+1$. For the substitution, at least one of y, y' or y'' must be correctly placed.		
	$a=\frac{3}{2}$		A1
	$6a+2b=2 \Rightarrow b=-\frac{7}{2} \Rightarrow c=\frac{17}{4}$	M1: Solves to obtain one of b or c A1: Correct b and c	M1A1
	$y=Ae^{-2x}+Be^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(9)
(b)	$0=A+B+\frac{17}{4}$	Substitutes $x=0$ and $y=0$ into their GS	M1
	$\frac{dy}{dx}=-2Ae^{-2x}-Be^{-x}+3x-\frac{7}{2} \Rightarrow 0=-2A-B-\frac{7}{2}$ Attempts to differentiate and substitutes $x=0$ and $y'=0$		M1
	$0=A+B+\frac{17}{4},\ 0=-2A-B-\frac{7}{2} \Rightarrow A=..,B=..$	Solves simultaneously to obtain values for A and B	M1
	$A=\frac{3}{4},\quad B=-5$	Correct values	A1
	$y=\frac{3}{4}e^{-2x}-5e^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(5)
			Total 14